MATHS

BOOKS - PEARSON IIT JEE FOUNDATION

MATHEMATICAL INDUCTION AND BIONOMIAL THEOREM

Example

1. Prove that $1+2+3+\ldots..n=\frac{n(n+1)}{2}$

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2. Prove that $1+2+3+\ldots..+n=\frac{n(n+1)}{2}$

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3. Prove that $1 + 2 + 3 + \ldots . + = (2n + 1)) = n^2$

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4. Prove that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots . + n \cdot (n + 1) = \frac{n(n + 1)(n + 2)}{3}$$

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5. Prove that $3^{n+1} > 3(n + 1)$

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6. Prove that 7 is a factor of $2^{3n} - 1$ for all natural numbers n.

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7. Expand $(x+2y)^5$

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8. Find the 3rd term in the expansion of $(3x-5y)^7$

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9. Find the middal term in the expansionof $(2x+3y)^8$

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10. Find the middle the terms in the expansion of $(5x-7y)^7$

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11. Find the term indepnedent of x in $\left(x+\frac{1}{x}\right)^4$

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12. Findt the coefficient of x^2 in $\left(x^2+\frac{1}{x^3}\right)^6$

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13. Find the total number of terms in the expansion of $(2+3x)^{15}+(2-3x)^{15}$

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14. If the expansion $\left(x^2+\frac{1}{x^3}\right)^n$ is to contain and independent term, then what should be the vlaue of m ?

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15. If the coefficient of x^7 in $\left(ax + \frac{1}{x}\right)$ and x^{-7} in $\left(bx - \frac{1}{x}\right)^9$ are equal, find the relation between a and b?

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16. Find the term independent of 'x' in the expanaion of $(1 + x^2)^4 \left(1 + \frac{1}{x^2}\right)^4$

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17. Find the sum of the coefficients of the term expansion $(1 + x + 2x^2)^6$

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18. Find the value of x, if the fourth term in the expansion of $\left(\frac{1}{x^2}+x^2\cdot 2x\right)^6$ is 160

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Test Your Concepts Very Short Answer Type Questions

1. If P(n) is a statement which is true for n=1 and true for (x+1) ,then_____

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2. According to the principle of mathematical induction when, can we say that a statemen X(n)is true for all natural numbers n?

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3. If $p(n) = n(n+1)(n+2)$ then highest common factor of p(n), for different values of mn where n is any ntural number, is _____

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4. Is 2^{3-n} a prime number for all natural numbers n?

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5. The product of (q-1) consecutive integers where $q > 1$ is divisible by _____

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6. An algebraic expressioon with two tems is called a _____

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7. In Pascal, triangle, each row of coefficiets is bounded on both sides by _____

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8. If n is a positive integer, then the number of terms in the expansion of $(x+a)^n$ is

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9. In the expansion of $(x+y)^n$, if the exponent of x in second term is 10, what is the exponent of y in 11th term.

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10. What is the coefficient of a term in a row of Pscal triangle if in the preceding row, the coeficient on the immediate left is 5 on the immediate rigth is 10

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11. In the expansion of various power of $(x+y)^n$ if the expansion contains 49 times, then it is the expansion of _____

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12. In the expression of $(x+y)^{123}$, the sum of the exponents of x and y in 63 rd term is _____

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13. $(n-r)!$= _____

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14. The vlaue of ${}^{n+1}C_r$= _____

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15. If ${}^{n}C_r = 1 \text{ and } n = 6,$ then what may be the value (s) of r be?

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16. In the expansion of $(x+y)^n$, T_{r+1}= _____

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17. ${}^{7}C_{2}$ = _____

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18. ${}^{1230}C_{0}$= _____

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19. The coefficient of x in the expansion of $(2x+3)^{5}$ is _____

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20. The coefficient of y^{7} in the expansion of $(y+z)^{7}$ is _____

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21. $(x+y)^3$=_____

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22. The term which does not contain 'a' in the expansion of $\left(\frac{x}{a}+6x\right)^{12}$ is _____

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23. If ${}^{12}C_r(4)^{12-r}(x)^{12-3r}$ is a contant term in an expansion, then r = _____

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24. Write the first, the middle and the last terms in the expansion of $\left(x^2+1\right)^3$

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25. Constant term in the expansion of $(x+3)^{16}$ is _____

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26. The sum of the first n even natural numbers is _____

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27. The sum of the first n odd natural number is

A. $2n-1$

B. $2n+1$

C. n^2

D. n^2-1

Answer: C

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28. The elmennts in the fifth row of Pascal triangle is _____

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29. If ${}^{n}C_{3} = {}^{n}C_{15}$, then ${}^{20}C_{n}$ is _____

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30. The inequality $2^{n} > n$ is true for _____

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Short Answer Type Questions

1. Prove that $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$, for all natural number n.

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2. a-b divides $a^n - b^n, n \in N$

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3. $1^3 + 2^3 + 3^3 + \ldots . + n^3 = \frac{n(n+1)^2}{4}, n \in N$

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4. $2.5 + 3.8 + 4.11 + \ldots . +$ upto n terms $= n\left(n^2 + 4n + 5\right), n \in N$

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5. Prove that :$1^2 + 2^2 + 3^2 + \ + n^2 = \frac{n(n+1)(2n+1)}{6}$

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6.

$a + (a+d) + (a+2d) + ... + [a + (n-1)d] = \frac{n}{2}(2a + (n-1)d)$

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7. Prove the following by the principle of mathematical induction:

$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$

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8. Using the principle of mathmatical induction, prove each of the following for all $n \in N$

$(4^n + 15n - 1)$ is divisible by 9.

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9. Prove the following by the principle of mathematical induction:

$2 + 5 + 8 + 11 + \ + (3n - 1) = \frac{1}{2} n (3n + 1)$

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10. Exapand $\left(3x^2 + \frac{5}{y^2}\right)^6$

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11. Expand $(5x + 3y)^8$

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12. Find the middle term or terms of the expansion of $(x + 5y)^9$

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13. find the middle term of terms or the expanison of $\left(x + \frac{1}{x}\right)^6$

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14. Find the 7th term in the expansion of $\left(5x - \frac{1}{7y}\right)^9$

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Essay Type Questions

1. Evalute: $\left(\sqrt{3}+1\right)^5-\left(\sqrt{3}-1\right)^5$

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2. Find the coefficient of x^{-5} in the expansion of $\left(2x^2-\frac{1}{5x}\right)^8$

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3. Find the term independent of x in $\left(6x^2-\frac{1}{7x^3}\right)^{10}$

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4. Find the coefficient of x^3 in the expansion of $\left(x^2+\frac{1}{3x^3}\right)^4$

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5. Find the term independent of x in $\left(2x^5 + \frac{1}{3x^2}\right)^{21}$

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Level 1

1. $n^2 + n + 1$ is a/an _____ number for all $n \in N$

A. even

B. odd

C. prime

D. none

Answer: B

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2. If the expansion $\left(x^3+\frac{1}{x^2}\right)^n$ contains a term independent of x, then the value of n, can be _____

A. 18

B. 20

C. 24

D. 22

Answer: B

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3. 1+5+9+ . . .+(4n-3)=n(2n-1), for all natural number n.

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4. Prove that 7 is a factor of $2^{3n} - 1$ for all natural numbers n.

A. 3

B. 5

C. 7

D. 2

Answer: B

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5. For what values of n is $14^n + 11^n$ divisiblle by 5 ?

A. When n is an even postive integer

B. For all values of n

C. When n is a prime number

D. When n is odd positive integer

Answer: D

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6. The smallest positive inteter n for which $n! < \frac{(n-1)^n}{2}$ holds , is

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A. 4

B. 3

C. 2

D. 1

Answer: A

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7. The third term form the end in the expansion of $\left(\frac{4x}{3y} - \frac{3y}{2x}\right)^9$ is

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8. In the 8thb term of $(x+y)^n$, the exponetnt of x is 3, then the exponent of x in 5th term is _____

A. 5

B. 7

C. 2

D. 6

Answer: D

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9. The sum of the elements in the sixth row of Pascal triangle is _____

A. 32

B. 63

C. 128

D. 64

Answer: D

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10. In $(n+y)^n-(x-y)^n$, if the number of term is 5, then find n.

A. 6

B. 5

C. 10

D. 9

Answer: D

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11. If the third term the in the exapansion of $\left(x + x^{\log_2^x}\right)^6$, 960, then the value of x is _____

A. 2

B. 3

C. 4

D. 8

Answer: A

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12. Finx the sum of coefficients of all the terms of the exapansin $(ax+y)^n$.

A. ${}^nC_0 + {}^nC_1^{n-1}x^{n-1}y + {}^bC_2a^{n-2}x^{n-2} + \ldots . + {}^nC_ny^n$

B. ${}^nC_0a^n + {}^nC_1a^{n-1} + {}^nC_2a^{n-2} + \ldots . + {}^nC_n$

C. 2^n

D. None of these

Answer: B

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13. If the sum of the coefficients in the exapansion $\left(4ax - 1 - 3a^2x^2\right)^{10}$ is 0, then the value of a can be _____

A. 2

B. 4

C. 1

D. 7

Answer: C

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14. Find the coefficient of x^4 in the expansion of $\left(2x^2 + \frac{3}{x^3}\right)^7$

A. ${}^7C_2 2^5 3^3$

B. ${}^7C_2 2^5 3^2$

C. ${}^7C_2 3^5 2^2$

D. ${}^7C_2 2^5 3^2$

Answer: B

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15. $n^2 - n + 1$ is an odd number for all _____

A. $n > 1$

B. $n > 2$

C. $n \geq 1$

D. $n \geq 5$

Answer: C

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16. Statement-1: For each natural number n, $(n+1)^7 - n^7 - 1$ is divisible by 7. Statement-2: For each natural number n, $n^7 - n$ is divisible by 7.

A. 10 fol all natural numbers n

B. 10 for odd natural numbers n

C. 10 for even natural numbers n

D. None of these

Answer: C

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17. For $n \in N$, $2^{3n} + 1$ is divisisble by _____

A. 3^{n+11}

B. 3^{n-11}

C. 3^{n+1}

D. 3^{n+111}

Answer: C

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18. $2^n - 1$ gives the set of all odd ntural numbers for all $n \in N$. Comment on he given statement .

A. True for alll values of n

B.

C. True for only odd values of n

D. True for only prime values of n

Answer: B

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19. In the 10th term of $(x + y)^n$, the exponent of x is 3, then the exponent of x in the 7th term is _____

A. 1

B. 7

C. 5

D. 9

Answer: B

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20. If the coefficients of the 6th and 5th terms of expansion $(1+x)^n$ are in the ratio $7:5$, then find the value of n.

A. 11

B. 12

C. 10

D. 9

Answer: A

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21. The third term from the end in the expansion of $(3x - 2y)^{15}$ is

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A. $-{}^{15}C_5 3^{13} 2^2 x^{13} y^2$

B. ${}^{15}C_5 3^{13} 2^2 x^{13} y^2$

C. ${}^{15}C_5 3^2 2^{13} x^2 y^{12}$

D. ${}^{15}C_5 3^2 2^{13} x^2 y^1$

Answer: D

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22. Find the sixth term in the exapnasion of $\left(2x^3 - \frac{3}{7x^2}\right)^{11}$

A. $-{}^{11}C_5 \frac{2^6 3^5}{7^5} x^3$

B. ${}^{11}C_5 \frac{2^6 3^5}{7^5} x^{-3}$

C. $-{}^{11}C_5 \frac{2^6 3^5}{7^5} x^{-3}$

D. $-{}^{11}C_5 \frac{2^3 3^5}{7^5} x^{-3}$

Answer: C

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23. The term independent of x in the exapansion of $\left(x^3 - \frac{1}{x^2}\right)^{10}$ is

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A. ${}^{10}C_6$

B. ${}^{10}C_7$

C. ${}^{10}C_9$

D. $-{}^{10}C_6$

Answer: A

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24. Which term is the constant term in the exapansion of $\left(2x - \frac{1}{3x}\right)^6$

A. 2nd term

B. 3rd term

C. 4th term

D. 5th term

Answer: C

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25. The sum of the coefficients in the expasnio of $(x + y)^7$ is _____

A. 119

B. 64

C. 256

D. 128

Answer: D

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26. The number of terms which are not radicals in the exapansion $\left(\sqrt{7}+4\right)^6+\left(\sqrt{7}-4\right)^6$ after simplification, is _____

A. 6

B. 5

C. 4

D. 3

Answer: C

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27. The coeeficient of x^4 in the exapnsion of (4x^(2)+(3))/(x))^(8)` is

---

A. ${}^8C_5 12^5$

B. ${}^8C_4 12^4$

C. ${}^8C_3 12^3$

D. ${}^8C_6 12^6$

Answer: B

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28. In the expansion of $(X+Y)^n$, the coefficients of the 17th and the 13th terms equal. Find the number of term in the expansion

A. 26

B. 25

C. 20

D. 24

Answer: B

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29. The number of terms in the expansion of $\left[(2x+3y)^4(4x-6y)^4\right]^9$ is _____

A. 36

B. 37

C. 10

D. 40

Answer: B

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30. If sum of the coefficients of the first two odd terms of the expansion $(x+y)^n$ is 16,then find n

A. 10

B. 8

C. 7

D. 6

Answer: D

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Level 2

1. The number of irrational terms in the expansion of $\left(2^{1/5}+3^{1/10}\right)^{55}$ is

A. 5

B. 6

C. 4

D. 7

Answer: A

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2. The remainder when $9^{49}+7^{49}$ is divide by 64 is _____

A. 24

B. 8

C. 16

D. 38

Answer: C

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3. If $p(n) = (n-2)(n-1)n(n+1)n(n+2)$, then greatest number which divides p(n)for all $n \in N$ is _____

A. 12

B. 24

C. 120

D. None of these

Answer: C

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4. If a, b and n are natural numbers then $a^{2n-1} + b^{2n-1}$ is divisible by

A. $a + b$

B. $(a + b)^2$

C. $a^3 + b^3$

D. $a^2 + b^2$

Answer: A

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5. Find the coefficient of the independent term in the expansion of $\left(x^{1/2} + 7x^{-1/3}\right)^{10}$

A. ${}^{10}C_4 7^4$

B. ${}^{10}C_6 7^6$

C. $^{10}C_6 7^5$

D. $^{10}C_4 7^7$

Answer: B

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6. Find the term which has the exponent of x as 8 in the expansion of

$$\left(x^{5/2} - \frac{3}{x^3\sqrt{x}}\right)^{10}$$

A. T_2

B. T_3

C. T_4

D. Does not exist

Answer: D

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7. The greatest number which divides $25^n - 24n - 1$ for all $n \in N$ is

_____

A. 24

B. 578

C. 27

D. 576

Answer: D

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8. If three consective coefficient in the expansion of $(1 + x)^n$, where n is a natural number are 36,84 and 126 respectively, then n is

_____

A. 8

B. 9

C. 10

D. 36

Answer: B

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9. Find the value of k for which the term independent of x in $\left(x^2+\frac{k}{x}\right)^{12}$ is 7920

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\sqrt{2}$

D. 2

Answer: C

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10. Find the coefficient of x^7 in the expansion of $\left(7x + \frac{2}{x^2}\right)^{13}$

A. $78 \times 8^8 \times 4$

B. $78 \times 7^6 \times 4^2$

C. $78 \times 7^{11} \times 4$

D. $78 \times 7^{11} \times 4^2$

Answer: C

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11. The value of $\left(\sqrt{5} + 2\right)^6 + \left(\sqrt{5} - 2\right)^6$ is

A. a positive integer

B. a negative integer

C. an irration number

D. a rationla number but not n integer

Answer: A

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12. The ratio of the coefficients of $x(4)$ to that of the term independent of x in the expansion of $\left(x^2+\frac{9}{x^2}\right)^{18}$ is _____

A. $1:6$

B. $3:8$

C. $1:10$

D. $1:8$

Answer: C

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13. $\sum_{r=2}^{16} {}^{16}C_r = $ _____

A. $2^{15} - 15$

B. $2^{16} - 16$

C. $2^{16} - 17$

D. $2^{17} - 17$

Answer: C

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14. Number of non-zero terms in the exapansion of $\left(5\sqrt{5}x+\sqrt{7}\right)^6+\left(5\sqrt{5}x-\sqrt{7}\right)^6$ is _____

A. 4

B. 10

C. 12

D. 14

Answer: A

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15. Find the value of $(98)^4$ by using the binomial theorem.

A. 92236846

B. 92239816

C. 9233886

D. 92236806

Answer: B

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16. If the number of terms in the expansion $(2x+y)^n-(2x-y)^n$ is, 8 then the value of n is _____ (where n is odd)

A. 17

B. 19

C. 15

D. 13

Answer: C

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17. If the expansion $\left(2x^5 + \frac{1}{3x^4}\right)^n$ contains a term independent of x, then the value of n can be _____

A. 6

B. 18

C. 3

D. 12

Answer: B

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18. Find the sum of the coefficients in the expansion of $\left(5x^6 - \frac{4}{x^9}\right)^{10}$

A. 5^{10}

B. 1

C. 4^{10}

D. 0

Answer: B

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19. $49^n + 16n - 1$ is an divisible by _____ $(n \in N)$

A. 64

B. 28

C. 48

D. 54

Answer: A

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20. Find the coefficient of x^{11} in the expansion of $\left(1+2x+x^2\right)^6$

A. 1

B. 2

C. 6

D. 12

Answer: D

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Level 3

1. The number of irrational terms in the binomial expansion of $\left(3^{1/5}+7^{1/3}\right)^{100}$ is ___

A. 70

B. 12

C. 75

D. 13

Answer: C

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2. Find the independent term in the expansion of $\left(x^4+\frac{3}{8x^3\sqrt{x}}\right)^{15}$

A. ${}^{15}C_4\left(\frac{3}{8}\right)^{16}$

B. ${}^{15}C_{12}\left(\frac{3}{8}\right)^{8}$

C. ${}^{15}C_8\left(\frac{3}{8}\right)^{8}$

D. ${}^{15}C_7\left(\frac{3}{8}\right)^{8}$

Answer: C

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3. $\sum_{r=1}^{30} r \frac{{}^{30}C_r}{{}^{30}C_{r-1}} =$ ----------

A. 930

B. 465

C. 310

D. 630

Answer: B

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4. For all $n \in N$, $41^n - 40n - 1$ is divisible by _____

A. 41

B. 40

C. 300

D. 500

Answer: B

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5. If m and n are the coeficients of x^{a^2} and x^{b^2} respectively in $(1+x)^{a^2+b^2}$, then relation between m and n is _____

A. $n = 2m$

B. $m + n = 0$

C. $2n = m$

D. $m = n$

Answer: D

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6. For each $n \in N$, $5^{3n} - 1$ is divisible by ____

A. 115

B. 124

C. 5

D. 6

Answer: B

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7. In the exapansion $(6 + 9x)^5$ the coefficient of x^3 is _____

A. $2^2 \times 3^8$

B. $2^4 \times 3^7$

C. $2^3 \times 3^8 \times 5$

D. $2^4 \times 3^7 5$

Answer: C

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8. In the expansion of $(X + y)^n$, the coefficients of the 17th and the 13th terms equal. Find the numbher of term in the expansion

A. 18

B. 22

C. 28

D. 29

Answer: D

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9. Find the coeeficient of x^{-2} in the exapansion of $\left(x^2+\frac{4}{x^5}\right)^6$

A. 240

B. 150

C. 100

D. 180

Answer: A

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10. In the 10th term of $(x+y)^n$, the exponent of x is 3, then the exponent of x in the 7th term is _____

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11. $\sqrt{20}\left\{\left(\sqrt{20}+1\right)^{100}-\left(\sqrt{20}-1\right)^{100}\right\}$ is a/an _____

A. irrational number

B. whole number

C. negtive number

D. rational number

Answer: B

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12. If $x = -{}^{n}C_1 + {}^{n}C_2(2) - {}^{n}C_3(2)^2 + \ldots.$ (where n is odd), then x=_____

A. 1

B. -1

C. 0

D. 12

Answer: B

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13. Find the independent term in the expansion of $\left(5x^2 - \frac{1}{x^4}\right)^6$

A. 8250

B. 8560

C. 9250

D. 9375

Answer: D

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14. The sum of the first three coefficients in the expansion $\left(x+\frac{1}{y}\right)^n$ is 22. Find the value of n.

A. 8

B. 7

C. 6

D. 5

Answer: C

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15. If ${}^{12}C_0 {}^{12}C_1 \ldots . {}^{12}C_{12}$ are the binomial coefficients of the expansion $(1+x)^{12}$, them ${}^{12}C_0 - {}^{12}C_1 + {}^{12}C_{2-}{}^{12}C_3 + \ldots \ldots {}^{12}C_{12} =$

A. 4096

B. 1024

C. 0

D. -1024

Answer: C

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