



### MATHS

# BOOKS - PEARSON IIT JEE FOUNDATION

# **REMAINDER AND FACTOR THEOREMS**

#### Example

1. Find the remainder when the polynomital

$$p(z)=z^3-3z+2$$
 is divided by z-2.



**3.** If the polynomial  $x^3 + ax^2 - bx - 30$  is exactly divisible by  $x^2 - 2x - 15$ . Find a and b and also the third factor.





**4.** Find the linear polynomial in x which when divided bt (x - 3) leaves 6 as remainder and is exactly divisible by (x + 3).

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5. A quadratic polynomial in x leaves remainders as 4 and 7 respectively when divided by (x+1) and (x-2) . Also it is



quadratic polynomial.



7. Find the remainder when  $x^{999}$  is divided by

$$x^2 - 4x + 3.$$



**9.** If  $f(x + 2) = x^2 + 7x - 13$ , then find the

remainder when f(x) is divided by (x + 2).

$$A. - 25$$

B. - 12

C. - 23

**D**. −11

Answer: A

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10. If (x - 2) and (x - 3) are two factors of  $f(x) = x^3 + ax + b$ , then find the remainder when f(x) is divided by x - 5.

A. 0

B. 15

C. 30

D. 60

#### Answer: D



have a common factor , then find the common

#### factor .

A. x + 2

#### B.x

- C. x + 4
- D. Either (b) or (c)

#### Answer:



12. When a fourth degree polynomial f(x) is divided by (x + 6), the quotient is Q(x) and the remainder is -6. And when f(x) is divided by [Q(x) + 1], the quotient is (x + 6) and the remainder is R(x). Find R(x).

A. 
$$12+x$$
  
B.  $-(x+12)$ 

D. 3

#### Answer:



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A. 0

B. 1

D. 3

#### Answer:

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# Test Your Concepts Very Short Answer Type Questions



polynomial of degree n . If x+1 is one of its

factors, then\_\_\_\_.



**3.** If aandb are distinct integers, prove that a - b is a factor of  $a^n - b^n$ , wherever n is a

positive integer.



**4.** If  $f(x) = x^3 + 2$  is divided by x + 2. Then

the remainder obtained is \_\_\_\_\_.

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5. The condition for which  $ax^2 + bx + a$  is exactly divisible by x - a is \_\_\_\_\_.





**10.** For two odd numbers x and y , if  $x^3 + y^3$  is divisible by  $2^k, k \in \mathsf{N}$  , then x+y is divisible by  $2^k$  . [True/False]



### 11. One of the factors of $2x^{17} + 3x^{15} + 7x^{33}$ is

$$\_.\left(x^{17} \, / \, x^{15} \, / \, x^{23} 
ight)$$

12. If  $\left(x-2
ight)^2$  is the factor of an expression of

the form  $x^3 + bx + c$ , then the other factor is



# 13. What should be added to $3x^3+5x^2-6x+3$ to make it exactly

divisible by x - 1?

14. The remainder when  $2x^6 - 5x^3 - 3$  is divided by  $x^3 + 1$  is \_\_\_\_. Watch Video Solution

15. The remainder when f (x) is divided by f (x) is  $f\left(-\frac{3}{2}\right)$ , then g (x) is necessarily 2x + 3 [True/False]

16. Find the remainder when the polynomial

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x^2 + 13x + 11 is divided by x - 1.
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**18.** The polynomial  $7x^2 - 11x + a$  when divided by x + 1 leaves a remainder of 8 . Then find the value of 'a' .

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19. If x +2 is a factor of f(x) and f(x)  $= x^3 + 4x^2 + kx - 6$ , then find the value of k.



# **21.** Find the value of a if x - a is a factor of the

polynomial

$$x^5 - ax^4 + x^3 - ax^2 + 2x + 3a - 2.$$

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**24.** If the polynomial  $3x^4 - 11x^2 + 6x + k$  is divided by x - 3 , it leaves a remainder 7 . Then

the value of k is \_\_\_\_\_.



25. 
$$(7x - 1)$$
 is a factor of  $7x^3 + 6x^2 - 15x + 2$ . (True/False) Vatch Video Solution

26. If  $ax^2 + bx + c$  is exactly divisible by 2x - 3 , then the relation between a , b and c is .



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**28.** If 
$$(2x-1)$$
 is a factor of  $2x^2+px-2$  ,

then the other factor is \_\_\_\_\_.

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**29.** The expression  $x^m - 1$  is divisible by x + 1

, only if M is \_\_\_\_\_.(even/odd)

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**30.** If x+m is one of the factors of the polynomial  $x^2+mx-m+4$ , then the value of m is  $\ .$ 

1. For what values of m and n is  $2x^4 - 11x^3 + mx + n$  is divisible by  $x^2 - 1?$ 

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**2.** Find a linear polynomial which when divided by (2x + 1) and (3x + 2) leaves remainders 3 and 4, respectively.



**3.** Prove that  $x^m+1$  is a factor of  $x^{mn}-1$  if n

is even.

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**4.** The remainders of a polynomial f(x) in x are 10 and 15 respectively when f (x) is divided by (x -3) and (x-4). Find the remaider when f (x) is divided by (x-3) (x -4).

5. If  $x^{555}$  is divided by  $x^2 - 4x + 3$ , then find

its remainder.



6. If 
$$(x^2-1)$$
 is a factor of  $ax^3-bx^2-ax+d$ , then find the relation between a and c .

7. When  $x^4 - 3x^3 + 4x^2 + p$  is divided by (x-2)

, the remainder is zero Find the value of p.





9. If (x-3) is a factor of  $x^2+q$  (where q $\in Q$ ) , then find the remainder when  $\left(x^2+q
ight)$  is divided by (x-2).







12. The value of a for which x - 7 is a factor of  $x^2 + 11x - 2z$ , is .

13. If a polynomial f (x) is divided by (x-3) and (x-4) it leaves remainders as 7 and 12 respectively, then find the remainder when f(x) is divided by (x-3)(x-4).

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14. Find the ramainder when  $5x^4 - 11x^2 + 6$ 

is divided by  $5x^2 - 6$ .



## 2. Factorize $x^4 - 2x^3 - 9x^2 + 2x + 8$ using

remainder theorem.

**3.** Find the remainder when  $x^{29}$  is divided by  $x^2 - 2x - 3$ .

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**4.** If 
$$x^2 - 2x - 1$$
 is a factor of  $px^3 + qx^2 + 1$ ,

(where p , q are integers) then find the value of p +q.





# 6. If $lx^2 + mx + n$ is exactly divisible by (x-1) and (x+1) and leaves a remainder 1

when divided by x + 2, then find m and n.

**1.** The value of a which the polynomial  $y^3 + ay^2 - 2y + a + 4$  in y has (y + a) as one of its factors is \_\_\_\_\_.

A. 
$$\frac{-3}{4}$$
  
B. 
$$\frac{4}{3}$$
  
C. 
$$\frac{3}{4}$$
  
D. 
$$\frac{-4}{3}$$

#### Answer: D


**2.** If the expression  $2x^3 - 7x^2 + 5x - 3$  leaves a remainder of 5 k - 2 when divided by x +1 , then find the value of k .

A. 3 B. — 3 C. 5

 $\mathsf{D.}-5$ 

Answer: B





**4.** Find the remainder when  $x^6 - 7x^3 + 8$  is divided by  $x^3 - 2$ .

 $\mathsf{A.}-2$ 

B. 2

C. 7

D. 1

#### Answer: A



5. If both the expressions  $x^{1248} - 1$  and  $x^{672} - 1$ , are divisible by  $x^n - 1$ , then the greatest integer value of n is

#### A. 48

B. 96

C. 54

#### D. 112

Answer: B



**6.** When  $x^2 - 7x + 2$  is divided by x - 8 , then

the remainder is \_\_\_\_\_.

A. 122

B.4

C. 45

D. 10

#### Answer: D





7. If  $ax^2 + bx + c$  is exactly divisible by 4x + 5

, then

A. 25a - 5b + 16c = 0.

B. 25a + 20b + 16c = 0.

C. 25a - 20b - 16c = 0.

D. 25a - 20b + 16c = 0.

#### Answer: D

8. The expression  $2x^3 + 3x^2 - 5x + p$  when divided by x + 2 leaves a remainder of 3p + 2 . Find p .

A.-2

B. 1

C. 0

D. 2

#### Answer: D

**9.** 3x - 4 is a factor of \_\_\_\_\_.

A.  $18x^4 - 3x^3 - 28x^2 - 3x + 4$ 

$$\mathsf{B}.\, 3x^4 - 10x^3 - 7x^2 + 38x - 24$$

$$\mathsf{C}.\,9x^4 - 6x^3 + 5x^2 - 15$$

 $\mathsf{D}.\,9x^4 + 36x^3 + 17x^2 - 38x - 24$ 

#### **Answer: A**



10. Which of the following is a factor of  $5x^{20} + 7x^{15} + x^9$ ? A.  $x^{20}$  $\mathsf{B.}\,x^{15}$  $\mathsf{C}. x^9$ D.  $x^{24}$ 

#### Answer: C

11. If (x - 2) and (x - 3) are two factors of  $f(x) = x^3 + ax + b$ , then find the remainder when f(x) is divided by x - 5.

A. 1

B. 0

C. 5

D. 4

**Answer: B** 



**12.** The expression  $x^{mn}+1$  is divisible by x +1 , only if

A. n is odd.

B. m is odd.

C. both m and n are even.

D. Cannot say.

Answer: B

13. If both the expressions  $x^{1215} - 1$  and  $x^{945} - 1$ , are divisible by  $x^n - 1$ , then the greatest inteatest integer value of n is \_\_\_\_\_.

A. 135

B. 270

C. 945

D. None of these.

Answer: A

14. If (x-2) is a factor of  $x^2+bx+1$  (where b $\in Q)$  , then find the remainder when  $ig(x^2+bx+1ig)$  is divided by 2x+3.

- A. 7
- B. 8

C. 1

D. 0

#### Answer: A



15. When  $x^3 + 3x^2 + 4x + a$  is divided by (x+2) , the remainder is zero . Find the value of a.

A. 4

B. 6

C. - 8

D. - 12

**Answer:** A

16. If (x + 1) and (x - 1) are the factors of  $ax^3 + bx^2 + cx + d$ , then which of the following is true?

A. 
$$a+b=0$$

- B. b + c = 0
- $\mathsf{C}.b+d=0$
- D. a + c = 0

#### Answer: C





**17.** Find the remainder when  $x^5$  is divided by  $x^2 - 9$ .

A. 81x

- B. 81x + 10
- C.  $3^5x + 34$
- D. 81

#### Answer: A



#### Answer: C







#### **Answer: D**



20. When the polynomial  $p(x) = ax^2 + bx + c$  is divided by (x - 1) and (x + 1), the remainders obtained are 6 and 10 respectively. If the value of p(x) is 5 at x = 0, then the value of 5a - 2b + 5c is \_\_\_\_.

A. 40

#### B.44

D. 42

Answer: B

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**21.** If p - q is a factor of the polynomial  $p^n - q^n$ ,

then n is \_\_\_\_\_.

A. a prime number

B. an odd number

C. an even number

### D. All of these

#### Answer: C



$$A. - 2$$

B. 2

C. 5

D.-5

Answer: A

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### **23.** If $f(x + 1) = 2x^2 + 7x + 5$ , then one of

the factors of f (x) is \_\_\_\_\_.

A. 2x + 3

 $\mathsf{B}.\,2x^2+3$ 

 $\mathsf{C.}\,3x+2$ 

D. 2x + 1

#### Answer: A

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24. If (x - p) and (x - q) are the factors of  $x^2 + px + q$ , then the values of p and q are respectively \_\_\_\_.

A. 1, -2  
B. 2, -3  
C. 
$$\frac{-1}{3}, \frac{-2}{3}$$
  
D. -2, 1

#### Answer: A

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25. Let 
$$f\left(x-\frac{1}{x}\right)=x^2+\frac{1}{x2}$$
, find the

remainder when f(x) is divided by x - 3.

A. 
$$\frac{82}{9}$$
  
B.  $\frac{8}{3}$   
C. 10

D. 11

#### Answer: D

26. If 
$$(x-2)^2$$
 is a factor of  $f(x) = x^3 + px + q$ , then find the remainder when  $df(x)$  is divided by  $x-1$ .

A. 4

B. - 4

C.-5

D. 5

#### Answer: D

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**27.** A quadratic polynomial in x leaves remainders 4 , 4 and 0 , respectively when

divided by (x-1), (x-2)and (x-3)` . Find the

quadratic polynomial.

$$\mathsf{A.}-2x^2+6x+3$$

$$\mathsf{B.} - 2x^2 + 6x$$

$$\mathsf{C.}-2x^2+6x+5$$

D. 
$$-2x^2+6x-5$$

#### Answer: B

**28.** If  $f(x+3) = x^2 + x - 6$  , then one of the

factors of f(x) is \_\_\_\_\_.

A. x - 3

- B. x 4
- C. x 5
- D. x 6

#### Answer: C

**29.** If  $(x-1)^2$  is a factor of  $f(x) = x^3 + bx + c$  , then find the remainder when f(x) is divided by (x - 2). A. 2  $B_{.} - 3$ C. 4  $D_{-}-4$ Answer: C

30. For what values of m and n , the expression 
$$2x^2 - (m+n)x + 2n$$
 is exactly divisible by  $(x-1)$  and  $(x-2)$ ?

- B. m = 3, n = 4
- C. m = 4, n = 2
- D. m = 2 , n = 4

#### **Answer: C**



**1.** The ratio of the remainders when the expression  $x^2 + bx + c$  is divided by (x - 3) and (x - 2) respec-tively is 4:5. Find b and c, if (x - 1) is a factor of the given expression.

A. 
$$b = \frac{-11}{3}, c = \frac{14}{3}$$
  
B.  $b = \frac{-11}{3}, c = \frac{11}{3}$   
C.  $b = \frac{-14}{3}, c = \frac{11}{3}$ 

D. None of these.

#### Answer: B



2. If the polynomals  $f(x)=x^2+9x+k$  and  $g(x)=x^2+10x+1$  have a common factor, then  $(k-l)^2$  isequal to

A. 9l-10k

B. 10l - 9k

C. Both (a) and (b)

D. None of these.

Answer: A

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**3.** When f(x) is divided by (x - 2), the qoutient is Q (x) and the remainder is zero . And when f(x) is divided by [Q(x) - 1], the qoutient is (x - 2) and the remainder is R (x) . Find the remainder R (x).

A. x + 2

B. - x + 2

$$C. x - 2$$

### D. Cannot be determined

#### Answer: C

4. Find the values of m and n, if 
$$(x-m)$$
 and  $(x-n)$  are the factors of the expression  $x^2 + mx - n$ .

B. 
$$m = 0$$
,  $n = 1$ 

$$\mathsf{C}.\,m=\frac{-1}{2},n=\frac{1}{2}$$

#### Answer: D

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5. Let 
$$f\left(x+rac{1}{x}
ight)=x^2+rac{1}{x^2}$$
, find the

remainder when f(x) is divided by 2x + 1.



#### Answer: A

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### **6.** A polynomial f(x) leaves remainders 10 and

14 respectively when divided by
(x-3) and (x-5) Find the remainder when f(x) is divided by (x-3) and (x-5).

A. 2x+6

- B. 2x 4
- C. 2x + 4
- $\mathsf{D}.\,2x-6$

#### Answer: C



7. If  $f(x+3) = x^2 - 7x + 2$ , then find the

#### remainder when f(x) is divided by(x + 1).

- A. 8
- $\mathsf{B.}-4$
- C. 20
- D. 46

#### Answer: D

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8. A polynomial f(x) when divided by (x-5) and (x-7) leaves remainders 6 and 16, respectively. Find the remainder when f(x) is divided by (x-5)(x-7).

- A. 5x + 7
- B. 5x 7
- C.5x + 19
- $\mathsf{D.}\,5x-19$

#### Answer: A

9. A polynomial p(x) leaves remainders 75 and 15 , respectively , when divided by (x-1) and (x+2). Then the remainder when f(x) is divided by (x-1)(x+2) is

A. 
$$5(4x + 11)$$

- B. 5(4x 11)
- C. 5(3x + 11)
- D. 5(3x 11)

#### Answer: A



10. The leading coefficient of a polynomial f(x) of degree 3 is 2006. Suppose that f(1) = 5, f(2) = 7 and f(c) = 9. Then find f(x).

A. 2006(x-1)(x-2)(x-3) + 2x + 3

B. 2006(x-1)(x-2)(x-3) + 2x + 1

C. 2006(x-1)(x-2)(x-3) + 2x - 1

D. 2006(x-2)(x-3)(x-1) - (2x-3)

#### Answer: A

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**11.** The ratio of the remainders when the expression  $x^2 + ax + b$  is divided by (x - 2) and (x - 1), respectively is 4:3. Find a and b if (x + 1) is a factor of the expression.

A. 9, -10

B. - 9, 10

C.9,10

D. -9, -10

#### Answer: D

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12. If 
$$x^3 - ax^2 + bx - 6$$
 is exactly divisible by  $x^2 - 5x + 6$ ,then  $\frac{a}{b}$  is \_\_\_\_\_.  
A.  $\frac{6}{11}$ 

$$B. \frac{-6}{11}$$
$$C. \frac{1}{3}$$
$$D. -\frac{1}{3}$$

#### Answer: A

12

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$$f(x) = x^2 + 5x + a ext{ and } g(x) = x^2 + 6x + b$$

lf

have a common factor , then which of the followings is true?

A. 
$$(a - b)^2 + 5(a - b) + b = 0$$
  
B.  $(a + b)^2 + 5(a + b) + a = 0$   
C.  $(a + b)^2 + 6(a + b) + b = 0$   
D.  $(a - b)^2 + 6(a - b) + b = 0$ 

#### Answer: D

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14. If 
$$ax^4 + bx^3 + cx^2 + dx$$
 is exactly divisible  
by  $x^2 - 4$  then  $\displaystyle rac{a}{c}$  is \_\_\_\_\_.



#### Answer: B

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15. If  $x^2 + x + 1$  is a factor of  $x^4 + ax^2 + b$  , then the values of a and b , respectively are

\_\_\_\_\_

A.2,4 B.2,1 C.1,1

D.1,2

#### Answer: C

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16. If (x + 1) and (x - 1) are the factors of

 $x^3 + ax^2 - bx - 2$ , then find the other factor

of the given polynomial.

The following are the steps involved in solving the problem given above . Arrange them in the sequential order.

(A) Put x = -1 in the given polynomial and

obtain the equations in a and b .

(B) Substitute a and b in the given polynomial.

(C) Factorize the polynomial.

(D) Solve the equations in a and b.

A. ADCB

B. ADBC

#### C. ABCD

#### D. ABDC

#### Answer: B

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17. The following are the steps involved in finding the value of a when x - 2 is a factor of  $3x^2 - 7x + a$ . Arrange them in sequential order.

(A)  $12-14+a=0 \Rightarrow a=2$ 

(B) By factor theorem

,

 $f(2)=0\Rightarrow 3(2)^2-7(2)+a=0$  (C ) Let  $f(x)=3x^2-7x+a$ 

#### A. CBA

B. BCA

C. CAB

D. BAC

Answer: A



**18.** If  $px^3 + qx^2 + rx + s$  is exactly divisible by  $x^2-1$  , then which of the following is /are necessarily true ? (A) p = r)B) q = s (C) p =- r (D) q =- s A. Both (A) and (B)

B. Both (C) and (D)

C. Both (A) and (B)

D. Both (B) and (C)

#### Answer: B

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19. Which of the following is a factor of  $x^3 + 3px^2 - 3pqx - q^3$ ? (where p and q are constants)

A. x +p

B. x + q

С. х - р

D. x -q

#### Answer: D

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20. If (x - k) is a common factor of  $x^2 + 3x + a$  and  $x^2 + 4x + b$ , then find the value of k in terms of a and b.

A. a + b

B. a - b

C. 2a + 3b

D. 2 a - 3 b

#### Answer: B

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# Level 3

**1.** Find the remainder when  $x^{33}$  is divided by

$$x^2 - 3x - 4.$$



#### Answer: B

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2. If  $6x^2 = 3x - 1$  is a factor of  $ax^3 + bx - 1$ (where a , b are integers), then find the value of b.

A. 1

B. 3

C.-5

D.-7

#### Answer: C



3. If the polynomials  $f(x) = x^2 + 6x + p$  and  $g(x) = x^2 + 7x + q$  have a common factor , then which of the following is true?

A. 
$$p^2 + q^2 + 2pq + 6p - 7q = 0$$

B. 
$$p^2 + q^2 - 2pq + 7p - 6 = 0$$

C. 
$$p^2+q^2-2pq+6p-7p=0$$

D. 
$$p^2 + q^2 - 2pq + 7p - 6q = 0$$

#### **Answer: B**



**4.** A polynomial of degree 2 in x , when divided by (x + 1), (x + 2) and (x + 3), leaves remainders 1 , 4 and 3 respectively . Find the polynomial.

A. 
$$\frac{1}{2}(x^2 + 9x + 6)$$
  
B.  $\frac{1}{2}(x^2 - 9x + 6)$   
C.  $\frac{-1}{2}(x^2 - 9x + 6)$   
D.  $\frac{-1}{2}(x^2 + 9x + 6)$ 

#### Answer: D



5. When a third degree polynomial f(x) is divided by (x - 3), the quotient is Q(x) and the remainder is zero. Also when f (x) is divided by [Q(x) + x + 1], the quotient is (x - 4) and remainder is R (x). Find the remainder R(x).

A. 
$$Q(x)+3x+4+x^2$$

B. 
$$Q(x)+4x+4-x^2$$

C. 
$$Q(x)+3x+4-x^2$$

#### D. Cannot be determined

#### Answer: C

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6. If the expression  $x^2 + 3x - 3$ , is divided by (x - p) , then it leaves remainder 1 . Find the value of p .

A. 1

B.-3

 $\mathsf{C}.-4$ 

D. either (a) or (c)

#### Answer: D

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7. If 
$$ax^3 - 5x^2 + x + p$$
 is divisible by  $x^2 - 3x + 2$ , then find the values of a and p .

D. 
$$a = 1, p = 2$$

#### Answer: A

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# 8. Which of the following should be added to $9x^3 + 6x^2 + x + 2$ , so that the sum is divisible by (3x + 1)?

B.-3C.-2

 $A_{-}-4$ 

D. - 1

#### Answer: C

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**9.** If the expression  $6x^2 + 13x + k$  is divisible

by 2x + 3. Then which of the following is the

factor of the expression ?

A. 3x + 1

- B. 3x + 4
- C. 3x + 2
- D. 3x + 5

#### Answer: C

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10. Given  $ax^2 + bx + c$  is a quadratic polynomial in x and leaves remainders 6 , 11 and 18 , respectively, when divided by (x + 1), (x + 2) and (x + 3). Find the value of a + b + c.

A. 1

B. 2

C. 3

D. 4

Answer: B

