



MATHS

BOOKS - DHANPAT RAI & CO MATHS

(HINGLISH)

INEQUALITIES

Illustration

1. If $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, then the minimum value of $\cos^3 \theta + \sec^3 \theta$ is

A. 1

B. 2

C. 0

D. none of these.

Answer: B



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2. If $a > 1, b > 1$, then the minimum value of

$\log_b a + \log_a b$ is

A. 0

B. 1

C. 2

D. none of

Answer: C



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3. The minimum value of $9^x + 9^{1-x}$, $x \in R$, is

A. 2

B. 3

C. 4

D. none of these.

Answer: D



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4. If a, b, c are three non-zero numbers of the same sign, then the value of $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$ lies in the interval

A. $[2, \infty)$

B. $[3, \infty)$

C. $[3, \infty)$

D. $[-\infty, 3)$

Answer: B



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5. If a, b are positive real numbers such that $ab=1$, then the least value of the expression $(1 + a)(1 + b)$ is

A. 2

B. 3

C. 4

D. none of these.

Answer: C



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6. $x = \log_5 3 + \log_7 5 + \log_9 7$

A. $a \in [3/2, \infty)$

B. $a \in \left[\frac{1}{2^{1/3}}, \infty \right)$

C. $a \in \left[\frac{3}{2^{1/3}}, \infty \right)$

D. none of these.

Answer: C



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7. If a, b, c are distinct positive real numbers, then

A. $a^2 + b^2 + c^2 > ab + bc + ca$

$$B. a^2 + b^2 + c^2 < ab + bc + ca$$

$$C. a^2 + b^2 + c^2 \geq ab + bc + ca$$

$$D. a^2 + b^2 + c^2 \leq ab + bc + ca$$

Answer: A



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8. If $a_1 > 0$ for all $i = 1, 2, \dots, n$. Then, the least value of

$$(a_1 + a_2 + \dots + a_n) \left(\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \right)$$

, is

A. n^2

B. $2n$

C. n

D. $\frac{1}{n}$

Answer: B



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9. If a, b, c are three distinct positive real numbers such that

$$\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c} > k, \text{ then the}$$

greatest value of k , is

A. 6

B. 3

C. 4

D. 9

Answer: A



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10. If a, b, c are positive integers, then

$$\left(\frac{a^2 + b^2 + c^2}{a + b + c} \right)^{a+b+c} > a^x b^y c^z, \text{ then}$$

A. $x = a, y = b, z = c$

B. $x = b, y = a, z = c$

C. $x = \frac{1}{a}, y = \frac{1}{b}, z = \frac{1}{c}$

D. $x = y = z = 1$

Answer: A



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11. If x, y, z are positive real numbers such that

$x + y + z = a$, then

A. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{a}{9}$

B. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{9}{a}$

C. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{9}{a^2}$

D. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{a^2}{9}$

Answer: B



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12. If $m > 1$ and $n \in \mathbb{N}$, such that

$$1^m + 2^m + 3^m + \dots + n^m > n \left(\frac{n+1}{k} \right)^m$$

Then, $k =$

A. 2

B. n

C. m

D. 1

Answer: A



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13. Find the greatest value of x^2y^3 , where x and y lie in the first quadrant on the line $3x + 4y = 5$.

A. $\frac{3}{4}$

B. $\frac{3}{8}$

C. $\frac{3}{16}$

D. $\frac{1}{16}$

Answer: C



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14. If x is a positive real number, then the greatest value of $(7 - x)(x + 5)^2$, is

A. 256

B. 128

C. 64

D. 16

Answer: A



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15. If $x + y = a$, then the greatest value of x^2y^3 is

A. $\frac{108a^5}{3125}$

B. $\frac{108a^5}{625}$

C. $\frac{54a^5}{3125}$

D. $\frac{108a^4}{3125}$

Answer: A



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16. If x and y are positive real numbers such that

$x^2y^3 = 32$ then the least value of $2x + 3y$ is

A. 5

B. 10

C. 20

D. 15

Answer: B



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Section 1 Solved Mcq

1. If p, q, r be three distinct real numbers, then the least possible value of $(p + q)(q + r)(r + p)$, is

A. $> 8pqr$

B. $< 8pqr$

C. $8pqr$

D. none of these

Answer: A



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2. If a, b, c are three positive real numbers then the minimum value of the expression

$$\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c} \text{ is}$$

A. 1

B. 2

C. 3

D. none of these

Answer: D



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3. If $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ lies in the interval

A. $[0,1]$

B. $[-1/2, 1]$

C. $[0, 1/2]$

D. $[1,2]$

Answer: B



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4. If a, b, c are distinct positive real numbers such that $a + \frac{1}{b} = 4, b + \frac{1}{c} = 1, c + \frac{1}{d} = 4$ and $d + \frac{1}{a} = 1$, then

A. $a=c$ and $b=d$

B. $b=d$ and $a \neq c$

C. $ab = 1$ and $cd \neq 1$

D. $cd = 1$ and $ab \neq 1$

Answer: A



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5. If the product of n positive numbers is 1, then their sum is

A. a positive integer

B. divisible by n

C. equal to $n + \frac{1}{n}$

D. greater than or equal to n .

Answer: D



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6. If a, b, c are positive real numbers, then the

least value of $(a + b + c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$, is

A. 9

B. 3

C. $\frac{10}{3}$

D. none of these

Answer: A



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7. If $a_i > 0$ for $i = 1, 2, 3, \dots, n$ and $a_1 a_2 \dots a_n = 1$, then the minimum value of $(1 + a_1)(1 + a_2) \dots (1 + a_n)$, is

A. $2^{n/2}$

B. 2^n

C. 2^{2n}

D. 1

Answer: B



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8. If a, b, c are the sides of a triangle then

$$\frac{a}{b+c-1} + \frac{b}{c+a-b} + \frac{c}{a+b-c} =$$

A. ≤ 3

B. ≤ 3

C. ≤ 2

D. ≤ 2

Answer: B



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9. If a, b, c are positive real number such that λabc is the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$, then $\lambda =$

A. 1

B. 2

C. 3

D. 6

Answer: D



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10. If P_n denotes the product of first n natural numbers, then for all $n \in \mathbb{N}$.

A. $P_n \leq n^n$

B. $P_n + 1 \leq n^n$

C. $P_n \leq \left(\frac{n+1}{2}\right)^n$

D. none of these

Answer: C



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11. If $x_n > 1$ for all $n \in \mathbb{N}$, then the minimum value of the expression

$$\log_{x_2} x_1 + \log_{x_3} x_2 + \dots + \log_{x_n} x_{n-1} + \log_{x_1} x_n$$

is

A. 0

B. 1

C. 2

D. none of these

Answer: D



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12. For all positive values of x and y , the value of

$$\frac{(1 + x + x^2)(1 + y + y^2)}{xy}, \text{ is}$$

A. ≤ 9

B. < 9

C. ≥ 9

D. > 9

Answer: C



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13. If a, b, c are three distinct positive real numbers, then the least value of

$$\frac{(1 + a + a^2)(1 + b + b^2)(1 + c + c^2)}{abc}, \text{ is}$$

A. 3

B. 9

C. 27

D. none of these

Answer: C



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14. If $x + y + z = 1$, then the least value of

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, \text{ is}$$

A. 3

B. 9

C. 27

D. 1

Answer: B



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15. If x, y, z are non-negative real numbers satisfying $x + y + z = 1$, then the minimum value of $\left(\frac{1}{x} + 1\right)\left(\frac{1}{y} + 1\right)\left(\frac{1}{z} + 1\right)$, is

- A. 8
- B. 16
- C. 32
- D. 64

Answer: D



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16. If a , b and c are positive real numbers such that $a \leq b \leq c$, then $\frac{a^2 + b^2 + c^2}{a + b + c}$ lies in the interval

A. $\left(\frac{a^2}{c}, \frac{c^2}{a}\right)$

B. $\left(\frac{a}{c^2}, \frac{c}{a^2}\right)$

C. $\left(\frac{c^2}{a}, \frac{a^2}{c}\right)$

D. $\left(\frac{b^2}{c}, \frac{c^2}{b}\right)$

Answer: A



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17. For any positive real number a and for any

$n \in \mathbb{N}$, the greatest value of

$$\frac{a^n}{1 + a + a^2 + \dots + a^{2n}}, \text{ is}$$

A. $\frac{1}{2n}$

B. $\frac{1}{2n + 1}$

C. $\frac{1}{2n - 1}$

D. none of these

Answer: B



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18. If $a + b + c = 6$ then the maximum value of

$$\sqrt{4a + 1} + \sqrt{4b + 1} + \sqrt{4c + 1} =$$

A. 9

B. 6

C. 4

D. 12

Answer: A



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19. If a , b and c are distinct positive numbers,

then the expression

$(a + b - c)(b + c - a)(c + a - b) - abc$ is:

A. positive

B. negative

C. non-positive

D. non-negative

Answer: B



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20. $\log_{100} a - \log_a 0.0001, a > 1$

A. 2

B. 3

C. 4

D. none of these

Answer: C



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21. If a, b, c are positive real numbers, then the minimum value of

$$a^{\log b - \log c} + b^{\log c - \log a} + c^{\log a - \log b} \text{ is}$$

- A. 3
- B. 1
- C. 9
- D. 16

Answer: A



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22. If $a, b, c \in \mathbb{R}$, then which one of the following is true:

A. $\max(a, b) < \min(a, b, c)$

B. $\min(a, b) = \frac{1}{2}\{a + b - |a - b|\}$

C. $\min(a, b) < \min(a, b, c)$

D. none of these

Answer: B



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23. If a_1, a_2, \dots, a_n are $n (> 1)$ real numbers,

then

A. $n \sum_{i=1}^n a_i^2 \leq \left(\sum_{i=1}^n a_i \right)^2$

B. $\sum_{i=1}^n a_i^2 \geq \left(\sum_{i=1}^n a_i \right)^2$

C. $\sum_{i=1}^n a_i^2 \leq \left(\sum_{i=1}^n a_i \right)^2$

D. none of these

Answer: D



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24. If $0 < x < \frac{\pi}{2}$, then the minimum value of $(\sin x + \cos x + \cos ec2x)$, is

A. 27

B. $\frac{27}{2}$

C. $\frac{25}{4}$

D. none of these

Answer: D



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25. If $x \in \mathbb{R}$ and $\alpha = \frac{x^2}{1+x^4}$, then α lies in the interval

A. $[0,2]$

B. $[0,1]$

C. $[0, 1/2]$

D. $[1/2, 1]$

Answer: C



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26. If the equation

$$x^4 - 4x^3 + ax^2 + bx + 1 = 0$$

has four positive

roots, then the value of $(a + b)$ is :

A. $a=6, b=-4$

B. $a=-4, b=6$

C. $a=-6, b=4$

D. $a=-6, b=-4$

Answer: A



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27. The least value of $2^{\sin x} + 2^{\cos x}$, is

A. $2^{1/\sqrt{2}}$

B. $2^{1-2^{-1/2}}$

C. $2^{1+2^{-1/2}}$

D. $2^{1-\sqrt{2}}$

Answer: B



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28. If $a_i \in (-\pi, 2\pi)$ for $i = 2, 3, \dots, n$, then

the number of solutions of the inequality

$$2^{1/\sin^2 a_2} \cdot 3^{1/\sin^2 a_3} \cdot 4^{1/\sin^2 a_4} \dots n^{1/\sin^2 a_n} \leq n!, \text{ is}$$

A. infinite

B. 2^{n-1}

C. 3^{n-1}

D. 3^n

Answer: C



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29. If a, b, c are three positive real numbers such that abc^2 has the greatest value $\frac{1}{64}$, then

A. $a = b = \frac{1}{2}, c = \frac{1}{4}$

B. $a = b = \frac{1}{4}, c = \frac{1}{2}$

C. $a = b = c = \frac{1}{3}$

D. none of these

Answer: B



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30. If $a + b + c = 1$ and a, b, c are positive real numbers such that

$$(1 - a)(1 - b)(1 - c) \geq \lambda abc, \text{ then } \lambda =$$

A. 2

B. 4

C. 6

D. 8

Answer: D



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31. If a, b, c are positive real numbers such that a

$+ b + c = 18$, find the maximum value of $a^2 b^3 c^4$

A. $2^{19} \times 3^2$

B. $2^{19} \times 3^3$

C. $2^{18} \times 3^3$

D. none of these

Answer: B



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32. If x, y, z are positive real numbers such that $x^3 y^2 z^4 = 7$, then the least value of $2x + 5y + 3z$, is

A. $\left(\frac{525}{128}\right)^{1/9}$

B. $3\left(\frac{525}{128}\right)^{1/9}$

C. $9\left(\frac{525}{128}\right)^{1/9}$

D. none of these

Answer: C



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33. Let a_1, a_2, \dots, a_n be a non-negative real numbers such that $a_1 + a_2 + \dots + a_n = m$

and let $S = \sum_{i < j} a_i a_j$, then

A. $S \leq \frac{m^2}{2}$

B. $S > \frac{m^2}{2}$

C. $S < \frac{m}{2}$

D. $S > \frac{m^2}{2}$

Answer: A



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34. If a, b, c, d are positive real numbers such that

$$a = b + c + d = 2, \text{ then } M = (a + b)(c + d)$$

satisfies the relation $0 \leq M \leq 1$ $1 \leq M \leq 2$

$$2 \leq M \leq 3$$
 $3 \leq M \leq 4$

A. $0 \leq M \leq 1$

B. $1 \leq M \leq 2$

C. $2 \leq M \leq 3$

D. $3 \leq M \leq 4$

Answer: A



35. If a_1, a_2, \dots, a_n are positive real numbers whose product is a fixed number c , then the minimum value of $a_1 + a_2 + \dots + a_{n-1} + 2a_n$ is

$a_{n-1} + 2a_n$ is b. $(n + 1)c^{1/n}$ $2nc^{1/n}$

$(n + 1)(2c)^{1/n}$

A. $n(2c)^{1/n}$

B. $(n + 1)c^{1/n}$

C. $2nc^{1/n}$

D. $(n + 1)(2c)^{1/n}$

Answer: A



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36. If $\alpha \in \left(0, \frac{\pi}{2}\right)$, then $\sqrt{x^2 + x} + \frac{\tan^2 \alpha}{\sqrt{x^2 + x}}$

is always greater than or equal to $2 \tan \alpha$
 $\sec^2 \alpha$

A. $2 \tan \alpha$

B. 1

C. 2

D. $\sec^2 \alpha$

Answer: A



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37. If $a_1, a_2, a_3, \dots, a_n$, are 'n', distinct odd natural numbers, not divisible by any prime number greater than 5, then

$\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \dots + \frac{1}{a_n}$ is less than

(a) $\frac{15}{8}$ (b) $\frac{17}{8}$ (c) $\frac{19}{8}$ (d) $\frac{21}{8}$

A. 1

B. $15/8$

C. $1/2$

D. $3/4$

Answer: B



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38. For any n positive numbers a_1, a_2, \dots, a_n

such that

$\sum_{i=1}^n a_i = \alpha$, the least value of $\sum_{i=1}^n a_i^{-1}$, is

A. $2n - \alpha$

B. $\frac{3n}{\alpha}$

C. $\frac{n(n-1)}{\alpha}$

D. $\frac{n^2}{\alpha}$

Answer: D



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39. If a, b, c denote the sides of a $\triangle ABC$ such that

$$a^2 + b^2 - ab = c^2, \text{ then}$$

A. $\min(a, b) \leq c \leq \max(a, b)$

B. $\min(a, b) < c < \max(a, b)$

C. $c \leq \min(a, b) \leq \max(a, b)$

D. $\min(a, b) \leq \max(a, b) \leq c$

Answer: B



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40.

For

$0 < x < \frac{\pi}{2}$, $(1 + 4 \cos ex)(1 + 8 \sec x)$, is

A. ≥ 81

B. > 81

C. > 83

D. ≥ 83

Answer: B



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41. If a, b, c are distinct positive integers such that $ab + bc + ca \geq 74$, then the minimum value of $a^3 + b^3 + c^3 - 3abc$, is

A. 42

B. 46

C. 45

D. 48

Answer: C



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42. If $a + b + c = 1$, the greatest value of

$$\frac{ab}{a+b} + \frac{bc}{b+c} + \frac{ca}{c+a}, \text{ is}$$

A. $\frac{1}{3}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: C



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43. If $a, b > 0$, $a + b = 1$, then the least value of

$$\left(1 + \frac{1}{a}\right)\left(1 + \frac{1}{b}\right), \text{ is}$$

A. 3

B. 6

C. 9

D. 12

Answer: C



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44. If a, b, c are positive real numbers such that $a + b + c = 1$, then the greatest value of $\sqrt{(1-a)(1-b)(1-c)}$, is

A. $\frac{1}{27}$

B. $\frac{8}{27}$

C. $\frac{4}{27}$

D. 9

Answer: B



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45. If a , b , c are sides of a triangle, then

$\frac{(a + b + c)^2}{ab + bc + ca}$ always belongs to

A. [1,2]

B. [2,3]

C. [3,4]

D. [4,5]

Answer: C



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46. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circuecirele of the triangle

PQR at the point T . If S is not the centre of the circumeircle, then

A. $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$

B. $\frac{1}{PS} + \frac{1}{ST} = \frac{2}{\sqrt{QS \times SR}}$

C. $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$

D. $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

Answer: D



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47. The minimum value of the sum of real number

$a^{-5}, a^{-4}, 3a^{-3}, 1, a^8,$ and a^{10} with $a > 0$ is

A. 6

B. 7

C. 8

D. 9

Answer: C



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48. Let $a, b, c, d \in R$ such that

$$a^2 + b^2 + c^2 + d^2 = 25$$

A. $ab + bc + cd + da \leq \frac{25}{2}$

B. $ab + bc + cd + da \leq 25$

C. $ab + bc + cd + da \leq 5$

D. $ab + bc + cd + da \leq \frac{5}{2}$

Answer: B



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49. If a , b and c are distinct positive numbers,

then the expression

$(a + b - c)(b + c - a)(c + a - b) - abc$ is:

- A. positive
- B. negative
- C. non-positive
- D. non-negative

Answer: B



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50. If x, y, z are variables and

$$3 \tan x + 4 \tan y + 5 \tan z = 20,$$

then the minimum value of

$$\tan^2 x + \tan^2 y + \tan^2 z, \text{ is}$$

A. 10

B. 15

C. 8

D. 12

Answer: C



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51. If $a > 0, b > 0, c > 0$, then the minimum value of

$$\sqrt{\frac{4a}{b}} + \sqrt[3]{\frac{27b}{c}} + \sqrt[4]{\frac{c}{108a}}, \text{ is}$$

A. 4

B. 5

C. 3

D. 9

Answer: D



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52. If $x + y + z = 1$, then the minimum value of

$$xy(x + y)^2 + yz(y + z)^2 + zx(z + x)^2 \text{ is } ,$$

where $x, y, z \in R^+$

A. $4 xyz$

B. $3 xyz$

C. $2 xyz$

D. $6 xyz$

Answer: A



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53. If $a, b > 0$ then the maximum value of

$$\frac{a^3b}{(a+b)^4}, \text{ is}$$

A. $\frac{81}{512}$

B. $\frac{27}{256}$

C. $\frac{27}{512}$

D. $\frac{81}{256}$

Answer: B



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54. Let x, y, z be positive real numbers such that

$$x + y + z = 12 \quad \text{and} \quad x^3 y^4 z^5 = (0.1)(600)^3.$$

Then $x^3 + y^3 + z^3$ is

A. 270

B. 258

C. 216

D. 342

Answer: C



Section 2 Assertion Reason Type

1. Statement-1 : For any real number

$$x, \frac{2x^2}{1+x^4} \leq 1$$

Statement-2: $A. M. > G. M.$

A. 1

B. 2

C. 3

D. 4

Answer: A



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2. Statement-1 : If angles A, B, C of $\triangle ABC$ are acute,

$$\text{then } \cot A \cot B \cot C \leq \frac{1}{3\sqrt{3}}.$$

Statement-2: If a, b, c are positive real numbers

and $0 < m < 1$, then

$$\frac{a^m + b^m + c^m}{3} < \left(\frac{a + b + c}{3} \right)^m$$

A. 1

B. 2

C. 3

D. 4

Answer: B



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3. Statement-1: If x, y are positive real number satisfying $x + y = 1$, then $x^{1/3} + y^{1/3} > 2^{2/3}$

Statement-2: $\frac{x^n + y^n}{2} < \left(\frac{x + y}{2}\right)^n$, if

$0 < n < 1$ and $x, y > 0$.

A. 1

B. 2

C. 3

D. 4

Answer: D



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4. Statement-1: If a, b are positive real numbers such that $a^3 + b^3 = 16$, then $a + b \leq 4$.

Statement-2: If a, b are positive real numbers

and $n > 1$, then

$$\frac{a^n + b^n}{2} \geq \left(\frac{a + b}{2} \right)^n$$

A. 1

B. 2

C. 3

D. 4

Answer: B



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5.

Statement-1:

$$1^3 + 3^3 + 5^3 + 7^3 + \dots + (2n - 1)^3 < n^4, n \in \mathbb{N}$$

Statement-2: If $a_1, a_2, a_3, \dots, a_n$ are n distinct

positive real numbers and $m > 1$, then

$$\frac{a_1^m + a_2^m + \dots + a_n^m}{n} > \left(\frac{a_1 + a_2 + \dots + a_n}{n} \right)^m$$

A. 1

B. 2

C. 3

D. 4

Answer: D



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6. Statement -1: If $n \in N$, then

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \dots + \sqrt{n} < \sqrt{\frac{n^2(1+1)}{2}}$$

Statement-2: If $0 < m < 1$, then

Arithmetic mean of m^{th} powers $<$ m^{th} power
of Arithmetic mean

A. 1

B. 2

C. 3

D. 4

Answer: A



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7. Statement-1: For every natural number $n \geq 2$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}$$

Statement-2: For every natural number $n \geq 2$,

$$\sqrt{n(n+1)} < n+1$$

A. 1

B. 2

C. 3

D. 4

Answer: B



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8. Statement-1: If a, b, c are positive numbers in

AP such that $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = 1$, then the

least value of b is $\sqrt{3}$.

Statement-2: $A. M. \geq G. M.$

A. 1

B. 2

C. 3

D. 4

Answer: A



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9. If a, b, c are three unequal positive numbers, then

Statement-1: The product of their sum and the sum of their reciprocals exceeds 9.

Statement-2: AM of n positive numbers exceeds their HM

A. 1

B. 2

C. 3

D. 4

Answer: A



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1. If $0 < \theta < \pi$, then the minimum value of $\sin^5 \theta + \operatorname{cosec}^5 \theta$, is

A. 0

B. 1

C. 2

D. none of these

Answer: C



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2. If $a > 1, b > 1, c > 1, d > 1$ then the minimum value of

$$\log_b a + \log_a b + \log_d c + \log_c d, \text{ is}$$

A. 1

B. 2

C. 3

D. 4

Answer: D



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3. the minimum value of $4^x + 4^{1-x}$, $x \in R$ is

A. 1

B. 2

C. 4

D. none of these

Answer: C



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4.

If

$$x = \log_{2^2} 2 + \log_{2^3} 2^2 + \log_{2^4} 2^3 + \dots + \log_{2^{n+1}} 2^n,$$

then

A. $x \geq \left(\frac{1}{n+1} \right)^{1/n}$

B. $x \geq n \left(\frac{1}{n+1} \right)^{1/n}$

C. $x \geq \left(\frac{n}{n+1} \right)^{1/n}$

D. none of these

Answer: B



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5. The least value of $5^{\sin x - 1} + 5^{-\sin x - 1}$, is

A. 10

B. $\frac{5}{2}$

C. $\frac{2}{5}$

D. $\frac{1}{5}$

Answer: C



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6. If a_1, a_2, a_3 be any positive real numbers, then which of the following statement is not true.

A. $3a_1, a_2, a_3 \leq a_1^3 + a_2^3 + a_3^3$

B. $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_1} \geq 3$

C. $(a_1 + a_2 + a_3) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right) \geq 9$

D. $(a_1 + a_2 + a_3) \left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right)^3 \leq 27$

Answer: C



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7. For $\theta > \frac{\pi}{3}$, the value of $f(\theta) = \sec^2 \theta + \cos^2 \theta$ always lies in the interval

A. 0,2

B. 0,1

C. 1,2

D. 2, ∞

Answer: D



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8. If a and b are two different positive real numbers then which of the following statement is true?

A. $2\sqrt{ab} > a + b$

B. $2\sqrt{ab} < a + b$

C. $2\sqrt{ab} = a + b$

D. none of these

Answer: B



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9. If $0 < x < \frac{\pi}{2}$ then the minimum value of

$$\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x}, \text{ is}$$

A. $\sqrt{3}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. 1

Answer: D



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10. If a, b, c are positive real numbers such that $a + b + c = 2$ then, which one of the following is true?

A. $(2 - a)(2 - b)(2 - c) \geq 8abc$

B. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 2$

C. $(2 - a)(2 - b)(2 - c) < 8abc$

D. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$

Answer: A



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11. If a, b, c are positive real numbers such that $a + b + c = p$ then, which of the following is true?

A. $(p - a)(p - b)(p - c) \leq \frac{8}{27}p^3$

B. $(p - a)(p - b)(p - c) \geq 8abc$

C. $\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \geq p$

D. none of these

Answer: A



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12. Evaluate: $\int \frac{x^3}{(x-1)(x-2)} dx$

A. a positive integer

B. divisible by n

C. equal to $n + \frac{1}{n}$

D. never less than n^2

Answer: D



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13. If roots of the equation $x^4 - 8x^3 + bx^2 + cx + 16 = 0$ are positive, then

A. $b = 8 = c$

B. $b = -24, c = -32$

C. $b = 24, c = -32$

D. $b = 24, c = 32$

Answer: C



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14. If a, b are distinct positive real numbers, then which one of the following is true?

A. $a^4 + b^4 > a^3b + ab^3$

B. $a^4 + b^4 < a^3b + ab^3$

C. $a^3 + b^3 < a^2b + ab^2$

D. none of these

Answer: A



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15. $2^{\sin^2 x} + 2^{\cos^2 x}$ is

A. ≤ 2

B. ≥ 2

C. ≤ 1

D. ≥ 1

Answer: B



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16. If $xyz = abc$, then the least value of $bcx + cay + abz$, is

A. $3abc$

B. $6abc$

C. abc

D. $4abc$

Answer: A



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17. The number of ordered 4-tuples (x, y, z, w) where $x, y, z, w \in [0, 10]$ which satisfy the inequality

$$2^{\sin^2 x} \times 3^{\cos^2 y} \times 4^{\sin^2 z} \times 5^{\cos^2 w} \geq 120, \text{ is}$$

A. 81

B. 144

C. 0

D. infinite

Answer: B



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18. The range of ab if

$|a| \leq 1$ and $a + b = 1, (a, b \in R)$, is

A. $[0, 1/4]$

B. $[-2, 1/4]$

C. $[1/4, 2]$

D. $[0, 2]$

Answer: B



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19. If $A = x^2 + \frac{1}{x^2}$, $B = x - \frac{1}{x}$ then
minimum value of $\frac{A}{B}$ is

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $\sqrt{2} + 2$

D. none of these

Answer: A



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20. The least perimeter of a cyclic quadrilateral of given area A square units, is

A. \sqrt{A}

B. $2\sqrt{A}$

C. $3\sqrt{A}$

D. $4\sqrt{A}$

Answer: D



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21. A stick of length 20 units is to be divided into n parts so that the product of the length of the parts is greater than unity. The maximum possible value of n , is

A. 18

B. 19

C. 20

D. 21

Answer: B



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22. If x, y, z are positive real numbers such that

$$x^2 + y^2 + z^2 = 27, \text{ then } x^3 + y^3 + z^3 \text{ has}$$

- A. minimum value 81
- B. maximum value 81
- C. minimum value 27
- D. maximum value 27

Answer: A



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23. If x_1, x_2, \dots, x_n are real numbers, then the largest value of the expression

$$\sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \dots + \sin x_n \cos x_1,$$

is

A. n

B. $\frac{n}{2}$

C. $\frac{n}{4}$

D. $\sqrt{n^2 - 1}$

Answer: B



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24. The minimum value of the sum of the lengths of diagonals of a cyclic quadrilateral of area a^2 square units, is

A. $\sqrt{2}a$

B. $2\sqrt{2}a$

C. $2a$

D. none of these

Answer: B



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25. The minimum value of

$$|\sin x + \cos x + \tan x + \sec x + \operatorname{cosec} x + \cot x|$$

, is

A. $2\sqrt{2} - 1$

B. $2\sqrt{2} + 1$

C. $\sqrt{2} - 1$

D. $\sqrt{2} + 1$

Answer: A



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26. The expression

$$(a + b + c)(b + c - a)(c + a - b)(a + b - c)$$

$\leq kb^2c^2$ then k can take the value

A. 4

B. $\frac{1}{4}$

C. 1

D. $\frac{1}{2}$

Answer:



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27. If n is even and $n \geq 4$, $x_1, x_2, \dots, x_n \geq 0$

and $x_1 + x_2 + \dots + x_n = 1$, then

$P = x_1 x_2 x_3 \dots x_n$ cannot exceed

A. $\frac{1}{n+1}$

B. $\frac{1}{n+2}$

C. $\frac{1}{2n}$

D. none of these

Answer: D



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28. The least value of n such that

$$(n - 2)x^2 + 8x + n + 4 > 0, \text{ where } n \in \mathbb{N},$$

A. 2

B. 3

C. 4

D. 5

Answer: D



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29. 1. The number of solution (s) of equation $\sin \sin^{-1}([x]) + \cos^{-1} \cos x = 1$ (where $[x]$ denotes the greatest integer function) is (a) one (b) two (c) three (d) none of these

A. 1

B. 2

C. 3

D. none of these

Answer: B



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30. Number of solutions of

$$\left| \frac{1}{|x| - 1} \right| = x + \sin x, \text{ is}$$

A. 1

B. 2

C. 3

D. none of these

Answer: A



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31. The solution set of equation

$$(x + 2)^2 + [x - 2]^2 = (x - 2)^2 + [x + 2]^2,$$

where $[.]$ represents the greatest integer function, is

A. R

B. N

C. I

D. Q

Answer: C



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32. The number of pairs of positive integers (x,y) where x and y are prime numbers and $x^2 - 2y^2 = 1$, is

A. 1

B. 2

C. 3

D. 4

Answer: A



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33. The number of solution of the equation

$$16(x^2 + 1) + \pi^2 = |\tan x| + 8\pi x \text{ is equal to}$$

A. 2

B. 4

C. 1

D. infinite

Answer: A



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34. The set of values of 'a' for which $ax^2 - (4 - 2a)x - 8 < 0$ for exactly three integral values of x, is

A. $2 \leq a < 4$

B. $1 \leq a < 2$

C. $2 < a < 4$

D. $1 < a < 2$

Answer: A



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35. The number of positive integral solutions (x, y) of the equation $2xy - 4x^2 + 12x - 5y = 11$, is

A. 0

B. 1

C. 2

D. infinitely many

Answer: C



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36. Number of integers, which satisfy the

inequality $\frac{(16)^{\frac{1}{x}}}{(2^{x+3})} > 1$, is equal to

A. 0

B. 3

C. 4

D. infinite

Answer: D



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37. The numbers of integral solutions of the equations

$$y^2(5x^2 + 1) = 25(2x^2 + 13) \quad \text{where } x, y \in I$$

, is

A. 2

B. 4

C. 8

D. infinite

Answer: B



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Section II Assertion Reason Type

1. If $\log_4(x + 2y) + \log_4(x - 2y) = 1$, then

Statement-1: $(|x| - |y|)_{\min} = \sqrt{3}$

Statement-2: $AM \geq GM$

A. 1

B. 2

C. 3

D. 4

Answer:



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