



MATHS

BOOKS - OBJECTIVE RD SHARMA MATHS VOL I (HINGLISH)

CONTINUITY AND DIFFERENTIABILITY

Illustration

1. For what value of k the function $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$, is continuous at $x = 2$?

A. 0

B. 4

C. 6

D. none of these

Answer: B



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2. The function $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continuous at $x = 0$ by defining $f(0)$ as

A. 0

B. 1

C. 2

D. -1

Answer: B



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3. If $f(x) = \begin{cases} \frac{1 - \sin x}{(\pi - 2x)^2}, & \text{when } x \neq \frac{\pi}{2} \\ \lambda, & \text{when } x = \frac{\pi}{2} \end{cases}$ the $f(x)$ will be continuous function at $x = \frac{\pi}{2}$, then $\lambda =$

A. $1/8$

B. $1/4$

C. $1/2$

D. none of these

Answer: A

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4. If $f(x) = \frac{\tan\left(\frac{\pi}{4} - x\right)}{\cot 2x}$ for $x \neq \frac{\pi}{4}$, find the value which can be assigned to $f(x)$ at $x = \frac{\pi}{4}$ so that the function $f(x)$ becomes continuous every where in $\left[0, \frac{\pi}{2}\right]$.

A. 1

B. $1/2$

C. 2

D. none of these

Answer: B



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5. If $f(x) = \begin{cases} \frac{\sin(\cos x) - \cos x}{(\pi - 2x)^3}, & \text{when } x \neq \frac{\pi}{2} \\ k, & \text{when } x = \frac{\pi}{2} \end{cases}$ the $f(x)$ will be continuous function at $x = \frac{\pi}{2}$, then $k =$

A. 0

B. $-\frac{1}{6}$

C. $-\frac{1}{24}$

D. $-\frac{1}{48}$

Answer: D



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6. यदि फलन $f(x) = \begin{cases} \frac{(4^x - 1)^3}{\sin(x/4) \log(1 + x^2/3)} & x \neq 0 \\ k & x = 0 \end{cases}$, $x = 0$, पर सतत है तब

$k =$

A. $12(\log 4)^2$

B. $96(\log 2)^3$

C. $(\log 4)^3$

D. none of these

Answer: B



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7. Given a real valued function f such that

$$f(x) = \begin{cases} \frac{\tan^2[x]}{x^2 - [x]^2}, & x < 0 \text{ and } 1, \\ \sqrt{\{x\}\cot\{x\}}, & x < 0 \end{cases}$$

where $[.]$ represents greatest integer function then

A. $A = -3, B = -\sqrt{3}$

B. $A = 3, B = -\frac{\sqrt{3}}{2}$

C. $A = -3, B = -\frac{\sqrt{3}}{2}$

D. $A = -\frac{\sqrt{3}}{2}, B = -3$

Answer: C

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8. greatest integer function $[x]$ is continuous at all points except at .

A. C

B. Z

C. R

D. ϕ

Answer: B

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9. Let $[x]$ be the greatest integer less than or equal to x , Then $f(x) = x \cos(\pi(x + [x]))$ is continuous at

A. $x = -1$

B. $x = 0$

C. $x = 2$

D. $x = 1$

Answer: B



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10. If $f(x) = \begin{cases} x^m \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ and $f(x)$ is continuous at point $x = 0$, then

A. $m \in (0, \infty)$

B. $m \in (-\infty, 0)$

C. $m \in (1, \infty)$

D. $m \in (-\infty, 1)$

Answer: A

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11. माना $f(x) = \frac{1 - \tan x}{4x - \pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right)$, यदि $f(x)$ अंतराल $\left[0, \frac{\pi}{2}\right)$ में सतत है तब $f\left(\frac{\pi}{4}\right) =$

A. 1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -1

Answer: C

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12. The function, $f(x) = \lceil |x| \rceil - \lfloor |x| \rfloor$ where $\lceil \cdot \rceil$ denotes greatest integer function:

- A. continuous everywhere
- B. continuous at integer points only
- C. continuous at non-integer points only
- D. nowhere continuous

Answer: C



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13. Let $f(x) = \begin{cases} \frac{\tan x - \cot x}{x - \frac{\pi}{4}} & x \neq \frac{\pi}{4} \\ a & x = \frac{\pi}{4} \end{cases}$

The value of a so that $f(x)$ is a continuous at $x = \pi/4$ is.

- A. 2
- B. 4
- C. 3

D. 1

Answer: B



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14. $f(x) = \left\{ \begin{array}{ll} \frac{\sqrt{1+px} - \sqrt{1-px}}{x} & -1 \leq x < 0 \\ \frac{2x+1}{x-2} & 0 \leq x \leq 1 \end{array} \right\}$ is continuous in the

interval $[-1, 1]$, then 'p' is equal to: 0

A. -1

B. -1/2

C. 1/2

D. 1

Answer: B



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15. The function $f(x) = \begin{cases} x^2/a & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^2} & \sqrt{2} \leq x < \infty \end{cases}$ and if it is continuous at

$x=1, \sqrt{2}$, then a and b is equal to

- A. -2
- B. -4
- C. -6
- D. -8

Answer: B



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16. If $f(x) = \begin{cases} ax^2 + b, & 0 \leq x < 14 \\ x + 3, & x = 14 \end{cases}$

- A. (2,2)
- B. (3,1)
- C. (4,0)

D. (5,12)

Answer: D



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17. $f: R \rightarrow R$ is defined by $f(x) = \begin{cases} \frac{\cos 3x - \cos x}{x^2}, & x \neq 0 \\ \lambda, & x = 0 \end{cases}$ and f is continuous at $x = 0$; then $\lambda =$

A. -2

B. -4

C. -6

D. -8

Answer: B



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18. If $f(x) = \begin{cases} \frac{1 - \sqrt{2} \sin x}{\pi - 4x}, & x \neq \frac{\pi}{4} \\ a, & x = \frac{\pi}{4} \end{cases}$

is continuous at $x = \frac{\pi}{4}$, then $a =$

A. 4

B. 2

C. 1

D. 1/4

Answer: D



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19. Let $f(x) = \frac{\sin x}{x}$, $x \neq 0$. Then $f(x)$ can be continuous at $x=0$, if

A. $f(0) = 0$

B. $f(0) = 1$

C. $f(0) = 2$

$$D. f(0) = -2$$

Answer: B

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20. Let $a, b \in \mathbb{R}$, ($a \neq 0$). If the function f defined as

$$f(x) = \begin{cases} \frac{2x^2}{a} & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^3} & \sqrt{2} < x < \infty \end{cases} \text{ is a continuous in } [0, \infty). \text{ Then, } (a, b) =$$

A. $(\sqrt{2}, 1 - \sqrt{3})$

B. $(-\sqrt{2}, 1 - \sqrt{3})$

C. $(\sqrt{2}, -1 + \sqrt{3})$

D. $(-\sqrt{2}, 1 + \sqrt{3})$

Answer: A

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21. Let $f(x) = [\cos x + \sin x]$, $0 < x < 2\pi$, where $[x]$ denotes the greatest integer less than or equal to x . The number of points of discontinuity of $f(x)$ is

A. 6

B. 5

C. 4

D. 3

Answer: C



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22. If function $f(x)$ given by

$$f(x) = \begin{cases} (\sin x)^{1/(\pi-2x)} & x \neq \pi/2 \\ \lambda & x = \pi/2 \end{cases} \text{ is continuous at } x = \frac{\pi}{2} \text{ then } \lambda =$$

A. e

B. 1

C. 0

D. none of these

Answer: B



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23. If $f(x) = \{x^2\} - (\{x\})^2$, where $\{x\}$ denotes the fractional part of x , then

A. $f(x)$ is continuous at $x = 2$ but not at $x = -2$

B. $f(x)$ is continuous at $x = -2$ but not at $x = 2$

C. $f(x)$ is continuous at $x = 2$ and $x = -2$

D. $f(x)$ is discontinuous at $x = 2$ and $x = -2$

Answer: A



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24. If $f(x) = [x]\sin\left(\frac{\pi}{[x+1]}\right)$, where $[.]$ denotes the greatest integer function, then the set of point of discontinuity of f in its domain is

- A. \mathbb{Z}
- B. $\mathbb{Z} - \{-1, 0\}$
- C. $\mathbb{R} - [-1, 0)$
- D. none of these

Answer: B



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25. The function $f(x) = (x)$ where (x) denotes the smallest integer $\geq x$ is

- A. everywhere continuous
- B. continuous at $x=n, n \in \mathbb{Z}$
- C. continuous on $\mathbb{R}-\mathbb{Z}$
- D. none of these

Answer: C



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26. Let $f(x) = [x^3 - 3]$, where $[.]$ is the greatest integer function, then the number of points in the interval $(1,2)$ where function is discontinuous is

A. 4

B. 2

C. 6

D. none of these

Answer: C



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27. Let $f(x) = \frac{e^{\tan x} - e^x + \ln(\sec x + \tan x) - x}{\tan x - x}$ be a continuous function at $x=0$. The value $f(0)$ equals

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 2

Answer: C



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28. Let $f(x)$ be given that $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1 - x & \text{if } x \text{ is irrational} \end{cases}$

The number of points at which $f(x)$ is continuous, is

A. ∞

B. 1

C. 0

D. none of these

Answer: C

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29. यदि $f'(a)$ विद्यमान है तब $\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a} =$

A. $f(a) - af'(a)$

B. $f'(a)$

C. $-f'(a)$

D. $f(a) + af'(a)$

Answer: A

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30. If $f(2) = 4$ and $f'(2) = 1$, then $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x - 2}$

A. 2

B. 4

C. -2

D. 1

Answer: A

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31. यदि $f(3) = 6$ तथा $f'(3) = 2$, तब $\lim_{x \rightarrow 3} \frac{xf(3) - 3f(x)}{x - 3} =$

A. 6

B. 4

C. 0

D. none of these

Answer: C

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32. Let $f(x) = [x]$ and $g(x) = |x|$ where $[.]$ denotes the greatest function. Then, $(f \circ g)'(-2)$ is

- A. 0
- B. 1
- C. -1
- D. non-existent

Answer: D



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33. If $f(x)$ is differentiable and strictly increasing function, then the value

of $\lim_{x \rightarrow 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$, is

- A. 1
- B. 0

C. -1

D. 2

Answer: C



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34. If $f(x) = \begin{cases} x - 5f & \text{or } x \leq 1 \\ 4x^2 - 9f & \text{or } 1 < x < 2 \\ 3x + 4f & \text{or } x \geq 2 \end{cases}$ then $f'(2^+) =$

A. 0

B. 2

C. 3

D. 4

Answer: C



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35. If $f: R \rightarrow R$ is defined by $f(x) = \begin{cases} \frac{x-2}{x^2-3x+2} & \text{if } x \in R - (1, 2) \\ 2 & \text{if } x = 1 \\ 1 & \text{if } x = 2 \end{cases}$

then $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} =$

- A. 0
- B. -1
- C. 1
- D. -1/2

Answer: B



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36. If $f(4) = 4$, $f'(4) = 1$, then $\lim_{x \rightarrow 4} \frac{2 - \sqrt{f(x)}}{2 - \sqrt{x}}$ is equal to

- A. -1
- B. 1
- C. 2

D. -2

Answer: B



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37. If $f(x)$ is differentiable function and $f'(0) = a$, then $\lim_{x \rightarrow 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is equal to

A. $3a$

B. $2a$

C. $5a$

D. $4a$

Answer: A



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38. Suppose $f(x)$ is differentiable for all x and

$$\lim_{h \rightarrow 0} \frac{1}{h}(1 + h) = 5 \text{ then } f'(1) \text{ equals}$$

- A. 6
- B. 5
- C. 4
- D. 3

Answer: B



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39. If f is a real-valued differentiable function satisfying

$$|f(x) - f(y)| \leq (x - y)^2, x, y \in \mathbb{R} \text{ and } f(0) = 0, \text{ then } f(1) \text{ equals:}$$

- A. 1
- B. 2
- C. 0

D. -1

Answer: C



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40. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$.

Then, which of the following is true?

- A. $f(x) > 1$ for all $x \in \mathbb{R}$
- B. $f(x)$ is not differentiable at $x=1$
- C. $f(x)$ is everywhere differentiable
- D. $f(x)$ is not differentiable at $x=0$

Answer: C



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41. Let $f(x) = \begin{cases} (x-1)^2 \sin\left(\frac{1}{x-1}\right) - |x| & ; x \neq 1 \\ -1 & ; x = 1 \end{cases}$ then which one of the following is true?

- A. $f(x)$ is differential for all x
- B. f is differentiable for all x except 0
- C. $f(x)$ is differentiable for all x except 0 and 1
- D. $f(x)$ is differentiable for all x except 1

Answer: B



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42. let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \max \{x, x^3\}$. The set of values where $f(x)$ is not differentiable is:

- A. $\{-1, 1\}$
- B. $\{-1, 0\}$

C. $\{0, 1\}$

D. $\{-1, 0, 1\}$

Answer: D

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43. If $f(x) = \{(x, x \leq 1), (x^2 + bx + c, x > 1)$ and $f'(x)$ exists finitely for all $x \in R$, then

A. $b = -1, c \in R$

B. $c = 1, b \in R$

C. $b = 1, c = -1$

D. $b = -1, c = 1$

Answer: D

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44. Let $f(x) = a + b|x| + c|x|^2$, where a, b, c are real constants. The, $f'(0)$ exists if

A. $b=0$

B. $c=0$

C. $a=0$

D. $b=c$

Answer: A



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45. The points where the function $f(x) = [x] + |1 - x|$, $-1 < x < 3$ where $[.]$ denotes the greatest integer function is not differentiable, are

A. $(-1, 0, 1, 2, 3)$

B. $(-1, 0, 2)$

C. $(0, 1, 2, 3)$

D. $(-1, 0, 1, 2)$

Answer: C



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46. The number of points in $(1, 3)$, where $f(x) = a(\lceil x^2 \rceil)$, $a > 1$ is not differential is

A. 0

B. 3

C. 5

D. 7

Answer: D



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47. Let $f(x) = p[x] + qe^{-[x]} + r|x|^2$, where p, q and r are real constants,

If $f(x)$ is differential at $x=0$. Then,

A. $q = 0, r = 0, p \in R$

B. $p = 0, r = 0, q \in R$

C. $p = 0, q = 0, r \in R$

D. none of these

Answer: C



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48. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^n}$, $g'(x)$ is equal to

A. $\frac{1}{1+(g(x))^n}$

B. $1+(g(x))^n$

C. $(g(x))^n - 1$

D. none of these

Answer: B



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49. Let f and g be differentiable functions satisfying $g(a) = b$, $g'(a) = 2$ and $f \circ g = I$ (identity function). then $f'(b)$ is equal to

A. 2

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. none of these

Answer: C



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50. If $f(x) = x + \tan x$ and f is the inverse of g , then $g'(x)$ is equal to

A. $\frac{1}{1 + [g(x) - x]^2}$

B. $\frac{1}{2 + [g(x) - x]^2}$

C. $\frac{1}{2 + [g(x) - x]^2}$

D. none of these

Answer: C



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51. If g is the inverse of a function f and $f'(x) = \frac{1}{1 + x^5}$ then $g(x)$ is equal to (1) $1 + x^5$ (2) $5x^4$ (3) $\frac{1}{1 + \{g(x)\}^5}$ (4) $1 + \{g(x)\}^5$

A. $\frac{1}{1 + (g(x))^5}$

B. $1 + \{g(x)\}^5$

C. $1 + x^5$

D. $5x^4$

Answer: B



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52. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{if } |x| > 2 \\ a + bx^2 & \text{if } |x| \leq 2 \end{cases}$ then $f(x)$ is differentiable at $x=-2$ for

A. $a = \frac{3}{4}, b = \frac{1}{6}$

B. $a = \frac{3}{4}, b = \frac{1}{16}$

C. $a = -\frac{1}{4}, b = \frac{1}{16}$

D. $a = \frac{1}{4}, b = -\frac{1}{16}$

Answer: B



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53. If the function $g(x) = \begin{cases} k\sqrt{x+1}, & 0 \leq x \leq 3 \\ mx+2, & 3 < x \leq 5 \end{cases}$ is differentiable, then the value of $k+m$ is

A. $\frac{10}{3}$

B. 4

C. 2

D. $\frac{16}{5}$

Answer: C



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54. Let a and b be real numbers such that the function

$$g(x) = \begin{cases} -3ax^2 - 2 & x < 1 \\ bx + a^2 & x \geq 1 \end{cases} \text{ is differentiable for all } x \in \mathbb{R}$$

Then the possible value(s) of a is (are)

A. 1, 2

B. 3, 4

C. 5, 6

D. 8, 9

Answer: A



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55. If the function

$f(x) = \begin{cases} -x & x < 1 \\ a + \cos^{-1}(x + b) & 1 \leq x \leq 2 \end{cases}$ is differentiable at $x=1$, then $\frac{a}{b}$ is equal to

A. $\frac{-\pi - 2}{2}$

B. $-1 - \cos^{-1}$

C. $\frac{\pi}{2} + 1$

D. $\frac{\pi}{2} - 1$

Answer: C



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56. Let $g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}$, $0 < x < 2$, m and n are integers, $m \neq 0$, $n > 0$, and let p the left hand derivative of $|x-1|$ at $x=1$. If $\lim_{x \rightarrow 1^+} g(x) = p$, then

A. $n = 1, m = 1$

B. $n = 1, m = -1$

C. $n = 2, m = 2$

D. $n > 2, m = n$

Answer: C



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Section I Solved Mcqs

1. The function $f(x) = [x]^2 - [x^2]$ is discontinuous at (where $[\gamma]$ is the greatest integer less than or equal to γ), is discontinuous at

- A. all integers
- B. all integers except 0 and 1
- C. all integers except 0
- D. all integers except 1

Answer: D

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2. The function $f(x) = [x^2] + [-x]^2$, where $[.]$ denotes the greatest integer function, is

- A. continuous and derivable at $x=2$
- B. neither continuous nor derivable at $x=2$
- C. continuous but not derivable at $x=2$
- D. none of these

Answer: B

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3. Let $f: R \rightarrow R$ be any function. Defining $g: R \rightarrow R$ by $g(x) = |f(x)|$ for $x \in R$. Then g , is

- A. onto if f is onto
- B. one-one if f is one-one
- C. continuous if f is continuous
- D. differentiable if f is differentiable

Answer: C

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4. The left hand derivative of $f(x) = [x]\sin(\pi x)$ at $x = k$, k is an integer, is

- A. $(-1)^k(k-1)\pi$

B. $(-1)^{k-1}(k-1)\pi$

C. $(-1)^k k\pi$

D. $(-1)^{k-1} k\pi$

Answer: A



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5. Which of the following functions is differentiable at $x = 0$?

$\cos(|x|) + |x|$

A. $\cos(|x|) + |x|$

B. $\cos(|x|) - |x|$

C. $\sin(|x|) + |x|$

D. $\sin(|x|) - |x|$

Answer: D



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6. The domain of the derivative of the function:

$$f(x) = \begin{cases} \tan^{-1} x & |x| \leq 1 \\ \frac{1}{2}(|x| - 1) & |x| > 1 \end{cases}$$

A. $R - \{0\}$

B. $R - \{1\}$

C. $4 - \{-1\}$

D. $R - \{-1, 1\}$

Answer: D



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7. The set of all points where the function $f(x) = 3\sqrt{x^2|x|}$ is differentiable, is

A. $[0, \infty)$

B. $(0, \infty)$

C. $(-\infty, \infty)$

D. $(-\infty, 0) \cup (0, \infty)$

Answer: D

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8. Let $f(x) = |x| + |\sin x|$, $x \in (-\pi/2, \pi/2)$. Then, f is

A. nowhere continuous

B. continuous and differentiable everywhere

C. nowhere differentiable

D. differentiable everywhere except at $x=0$

Answer: D

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9. If the function $f(x) = \left[\frac{(x-2)^3}{a} \right] \sin(x-2) + a \cos(x-2)$, $[\cdot]$ denotes the greatest integer function, is continuous in $[4, 6]$, then find the values of a .

A. $a \in [8, 64)$

B. $a \in [0, 8)$

C. $a \in [64, \infty)$

D. none of these

Answer: C

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10. If $\left\{ \frac{\sin\{\cos x\}}{x - \frac{\pi}{2}}, x \neq \frac{\pi}{2} \right\}$ and $1, x = \frac{\pi}{2}$, where $\{\cdot\}$ represents the fractional part function, then $f(x)$ is

A. continuous at $x = \pi/2$

B. $\lim_{x \rightarrow \pi/2} f(x)$ but $f(x)$ is not continuous at $x = \pi/2$

C. $\lim_{x \rightarrow \pi/2} f(x)$ does not exist

D. $\lim_{x \rightarrow \pi/2^-} f(x) = -1$

Answer: B



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11. If $\alpha, \beta(\alpha, \beta)$ are the points of discontinuity of the function $f(f(x))$, where $f(x) = \frac{1}{1-x}$, then the set of values of a for which the points (α, β) and (a, a^2) lie on the same side of the line $x + 2y - 3 = 0$, is

A. $(-3/2, 1)$

B. $[-3/2, 1]$

C. $[1, \infty)$

D. $(-\infty, -3/2]$

Answer: A

12. The function $f(x)$ given by $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is

A. everywhere differentiable such that $f'(x) = -\frac{2}{1+x^2}$

B. such that $f'(x) = \begin{cases} \frac{2}{1+x^2} & -1 < x < 1 \\ \frac{-2}{1+x^2} & |x| > 1 \end{cases}$

C. such that $f'(x) = \begin{cases} \frac{-2}{1+x^2} & -1 < x < 1 \\ \frac{+2}{1+x^2} & |x| > 1 \end{cases}$

D. not differentiable at infinitely many points.

Answer: B

13. Let $f(x)$ be the function given by $f(x) = \arccos\left(\frac{1-x^2}{1+x^2}\right)$. Then

A. $f(x)$ is everywhere differential such that $f'(x) = \frac{2}{1+x^2}$

B. $f'(x) = \begin{cases} \frac{2}{1+x^2} & x > 0 \\ \frac{-2}{1+x^2} & x < 0 \end{cases}$

$$C. f'(x) = \begin{cases} \frac{-2}{1+x^2} & x > 0 \\ \frac{2}{1+x^2} & x < 0 \end{cases}$$

D. $f'(x)$ exists at $x=0$

Answer: B

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14. If $f(x) = \sin^{-1}(2x\sqrt{1-x^2})$, $x \in [-1, 1]$. Then

A. $f'(x) = \frac{2}{\sqrt{1-x^2}}$, for all $x \in (-1, 1)$

B. $f'(x) = \begin{cases} \frac{2}{\sqrt{1-x^2}} & \text{If } |x| < \frac{1}{\sqrt{2}} \\ \frac{-2}{\sqrt{1-x^2}} & \text{If } \frac{1}{\sqrt{2}} < |x| < \frac{1}{2} \end{cases}$

C. $f'(x) = \begin{cases} \frac{-2}{\sqrt{1-x^2}} & \text{If } |x| < \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{1-x^2}} & \text{If } \frac{1}{\sqrt{2}} < |x| < 1 \end{cases}$

D. $f(x)$ exists for all $x \in [-1, 1]$

Answer: B

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15. If $f(x) = \cos^{-1}(2x^2 - 1)$, $x \in [-1, 1]$. Then

A. $f(x)$ is differentiable on $(-1,1)$ such that $f'(x) = \frac{-2}{\sqrt{1-x^2}}$

B. $f(x)$ is differentiable on $(-1, 0) \cup (0, 1)$ such that

$$f'(x) = \frac{-2}{\sqrt{1-x^2}}$$

C. $f(x)$ is differentiable on $(-1, 0) \cup (0, 1)$ such that

$$f'(x) = \begin{cases} \frac{-2}{\sqrt{1-x^2}} & 0 < x < 1 \\ \frac{2}{\sqrt{1-x^2}} & -1 < x < 0 \end{cases}$$

D. $f(x)$ is differentiable on $(-1,1)$ such that

$$f'(x) = \begin{cases} \frac{-2}{\sqrt{1-x^2}} & 0 \leq x < 1 \\ \frac{2}{\sqrt{1-x^2}} & -1 < x \leq 0 \end{cases}$$

Answer: C

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16. If $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, $x \in R$ then $f'(x)$ is given by

A. $f'(x) = \frac{2}{1+x^2}$ for all $x \in R(-1, 1)$

$$B. f'(x) = \frac{2}{1+x^2} \text{ for all } x \in R$$

$$C. F'(x) = \begin{cases} \frac{2}{1+x^2} & \text{if } |x| \leq 1 \\ \frac{-2}{1+x^2} & \text{if } |x| > 1 \end{cases}$$

$$D. f'(x) = \begin{cases} \frac{2}{1+x^2} & \text{if } |x| < 1 \\ \frac{-2}{1+x^2} & \text{if } |x| > 1 \end{cases}$$

Answer: A

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17. If $y = \sin^{-1}(3x - 4x^3)$, then the number of points in $[-1, 1]$, where y is not differentiable is

$$A. f'(x) = -\frac{3}{\sqrt{1-x^2}} \text{ for all } x \in (-1, 1)$$

$$B. f'(x) = \frac{3}{\sqrt{1-x^2}} \text{ for all } x \in [-1, 1]$$

$$C. f'(x) = \begin{cases} \frac{3}{\sqrt{1-x^2}} & \text{if } -\frac{1}{2} < x < \frac{1}{2} \\ \frac{-3}{\sqrt{1-x^2}} & \text{if } \frac{1}{2} < x < 1 \text{ or } -1 < x < -\frac{1}{2} \end{cases}$$

$$D. f'(x) = \begin{cases} \frac{3}{\sqrt{1-x^2}} & \text{if } |x| < \frac{\sqrt{3}}{2} \\ \frac{-3}{\sqrt{1-x^2}} & \text{if } 1 > |x| > \frac{\sqrt{3}}{2} \end{cases}$$

Answer: C



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18. If $f(x) = \cos^{-1}(4x^3 - 3x)$, $x \in [-1, 1]$, then

A. $f'(x) = \frac{-3}{\sqrt{1-x^2}}$ for all $x \in [-1, 1]$

B. $f'(x) = \frac{-3}{\sqrt{1-x^2}}$ for all $x \in [-1, 1]$

C. $f'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}} & \text{if } |x| < \frac{1}{2} \\ \frac{3}{\sqrt{1-x^2}} & \text{if } \frac{1}{2} < |x| < \frac{1}{2} \end{cases}$

D. $f'(x) = \begin{cases} \frac{-3}{\sqrt{1-x^2}} & \text{if } |x| < \frac{1}{2} \\ \frac{-3}{\sqrt{1-x^2}} & \text{if } -\frac{1}{2} < x < \frac{1}{2} \end{cases}$

Answer: D



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19. Prove that

$$3 \tan^{-1} x = \begin{cases} \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right) & \text{if } -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right) & \text{if } x > \frac{1}{\sqrt{3}} \\ -\pi + \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right) & \text{if } x < -\frac{1}{\sqrt{3}} \end{cases}$$

A. $f'(3) = \frac{3}{1+x^2}$ for all $x \in R - \left\{ \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\}$

B. $f'(x) = \frac{3}{1+x^2}$ for all $x \in R$

C. $f(x)$ is not differentiable at infinitely many points.

D. none of these

Answer: A

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20. The function, $f(x) = \sin^{-1}(\sin x)$, is

A. continuous but not differentiable at $x = \pi$

B. continuous and differentiable at $x=0$

C. discontinuous at $x = -\pi$

D. none of these

Answer: B

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21. The function, $f(x) = \cos^{-1}(\cos x)$ is

A. discontinuous at infinitely many-points

B. everywhere differentiable such that $f'(x)=1$

C. not differentiable at $x = n\pi, n \in \mathbb{Z}$ and $f'(x) = 1, x \neq n\pi$

D. not differentiable at $x = n\pi, n \in \mathbb{Z}$ and

$$f'(x) = (-1)^n, x \in (n\pi, (n+1)\pi), n \in \mathbb{Z}$$

Answer: D



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22. The function $f(x) = \tan^{-1}(\tan x)$ is

A. everywhere continuous

B. discontinuous at $x = \frac{n\pi}{2}, n \in \mathbb{Z}$

C. not differentiable at x

D. everywhere continuous and differentiable such that $f'(x)=1$ for all

$$x \in \mathbb{R}$$

Answer: C



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23. Number of points where the function $f(x) = \text{Maximum} [\text{sgn}(x), -\sqrt{9-x^2}, x^3]$ is continuous but not differentiable, is

A. 4

B. 2

C. 5

D. 6

Answer: C



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24. The set of points of discontinuity of the function $f(x) = \frac{1}{\log|x|}$, is
- A. $\{0\}$
 - B. $\{-1,1\}$
 - C. $\{-1,0,1\}$
 - D. none of these

Answer: C



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25. Let $f(x) = \frac{\sin(\pi[x - \pi])}{1 + [x^2]}$ where $[\]$ denotes the greatest integer function then $f(x)$ is

- A. continuous at integer points
- B. continuous everywhere

C. differentiable once but $f''(x)$ and $f'''(x)$ do not exist

D. differentiable for all x

Answer: B::D



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26. If $f(x) = \begin{cases} ax^2 - b & a \leq x < 1 \\ 2 & x = 1 \\ x + 1 & 1 \leq x \leq 2 \end{cases}$ then the value of the pair (a,b)

for which $f(x)$ cannot be continuous at $x=1$, is

A. (2,0)

B. (1,-1)

C. (4,2)

D. (1,1)

Answer: D



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27. If $f(x) = \frac{[x]}{|x|}$, $x \neq 0$, where $[.]$ denotes the greatest integer function, then $f'(1)$ is

A. -1

B. 1

C. non-existent

D. none of these

Answer: C



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28. Let $f(x) = [|x|]$ where $[.]$ denotes the greatest integer function, then $f'(-1)$ is

A. 0

B. 1

C. non-existent

D. none of these

Answer: C



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29. If $f(x) = [x][\sin x]$ in $(-1, 1)$ then $f(x)$ is

A. continuous on $(-1,0)$

B. differentiable on $(-1,1)$

C. differentiable at $x=0$

D. none of these

Answer: A



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30. If $f(x - y)$, $f(x)f(y)$ and $f(x + y)$ are in A.P. for all $x, y \in R$ and $f(0)=0$. Then,

- A. $f'(2) = f'(2)$
- B. $f'(-3) = -f'(3)$
- C. $f'(-2) + f'(2) = 0$
- D. none of these

Answer: A



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31. Let $f(x) = \text{Degree of } (u^{x^2} + u^2 + 2u + 3)$. Then, at $x = \sqrt{2}$, $f(x)$ is

- A. continuous but not differentiable
- B. differentiable
- C. discontinuous

D. none of these

Answer: A



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32. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function

$$f(x + 2y) = f(x) + f(2y) + 4xy \text{ for all } x, y \in \mathbb{R}$$

A. $f'(1) = f'(0) + 1$

B. $f'(1) = f'(0) - 1$

C. $f'(0) = f'(1) + 2$

D. $f'(0) = f'(1) - 2$

Answer: D



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33. Let $f: R \rightarrow R$ be a function given by

$$f(x + y) = f(x)f(y) \text{ for all } x, y \in R$$

If $f(x) \neq 0$ for all $x \in R$ and $f'(0)$ exists, then $f'(x)$ equals

- A. $f(x)$ for all $x \in R$
- B. $f(x) f'(0)$ for all $x \in R$
- C. $f(x) + f'(0)$ for all $x \in R$
- D. none of these

Answer: B



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34. Let $f: R \rightarrow R$ be a function given by

$$f(x + y) = f(x)f(y) \text{ for all } x, y \in R$$

If $f(x) \neq 0$, for all $x \in R$ and $f'(0) = \log 2$, then $f(x) =$

- A. x^2

B. 2^x

C. $x(\log 2)$

D. e^{2x}

Answer: B



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35. Let $f: R \rightarrow R$ be a function given by

$$f(x + y) = f(x)f(y) \text{ for all } x, y \in R$$

If $f(x) = 1 + xg(x)$, $\log_e 2$, where $\lim_{x \rightarrow 0} g(x) = 1$. Then, $f'(x) =$

A. $\log_e 2^{f(x)}$

B. $\log_e (f(x))^2$

C. $\log_e 2$

D. none of these

Answer: A



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36. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function given by $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$. If $f'(0) = 2$ then $f(x)$ is equal to`

A. Ae^x

B. Ae^{2x}

C. $2x$

D. none of these

Answer: B



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37. If a differentiable function f defined for $x > 0$ satisfies the relation $f(x^2) = x^3, x > 0$, then what is the value of $f'(4)$?

A. 2

B. 3

C. 4

D. none of these

Answer: B



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38. If $f(x + y) = 2f(x)f(y)$ for all x, y where $f'(0)=3$ and $f(4)=2$, then $f'(4)$ is equal to

A. 6

B. 12

C. 4

D. none of these

Answer: B



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39. Let $f: R \rightarrow R$ be a function given by

$$f(x + y) = f(x)f(y) \text{ for all } x, y \in R$$

If $f(x) = 1 + xg(x) + x^2g(x)\phi(x)$ such that $\lim_{x \rightarrow 0} g(x) = a$ and $\lim_{x \rightarrow 0} \phi(x) = b$

then $f'(x)$ is equal to

A. $(a + b)f(x)$

B. $af(x)$

C. $bf(x)$

D. $abf(x)$

Answer: B



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40. Let $f: R \rightarrow R$ be a function satisfying

$$f(x + y) = f(x) + f(y) \text{ for all } x, y \in R$$

If $f(x) = x^3g(x)$ for all $x, y \in R$, where $g(x)$ is continuous, then $f'(x)$ is equal to

- A. $g(0)$
- B. $g'(x)$
- C. 0
- D. none of these

Answer: C



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41. $2x^2$

- A. $6x - 4$
- B. $x^2 + 3x - 2$
- C. $-x^2 + 3x - 2$
- D. $-x^2 + 9x - 6$

Answer: A



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42. Let $f: R \rightarrow R$ be a function satisfying $f(x + y) = f(x) + \lambda xy + 3x^2y^2$ for all $x, y \in R$ If $f(3)=4$ and $f(5)=52$, then $f'(x)$ is equal to

A. $10x$

B. $-10x$

C. $20x$

D. $128x$

Answer: B



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43. Let f be a differential function satisfying the condition.

$$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)} \text{ for all } x, y (\neq 0) \in R \text{ and } f(y) \neq 0 \text{ If } f'(1)=2', \text{ then}$$

$f'(x)$ is equal to

A. $2f(x)$

B. $\frac{f(x)}{2}$

C. $2x f(x)$

D. $\frac{2f(x)}{x}$

Answer: D



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44. Let $f(x)$ be a real function not identically zero in Z , such that for all

$$x, y \in R \quad f(x + y^{2n+1}) = f(x) = \{f(y)^{2n+1}\}, n \in Z$$

If $f'(0) \geq 0$, then $f'(6)$ is equal to

A. 0

B. 1

C. 2

D. 6

Answer: B



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45. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x) + f(y)}{2}$ for all real x and y . If $f'(0)$ exists and equals -1 and $f(0)=1$, find $f(2)$

A. -1

B. 1

C. 0

D. none of these

Answer: A



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46. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x + y) = f(x) - f(y) + 2xy + 1$ for all $x, y \in \mathbb{R}$. If $f(x)$ is everywhere differentiable and $f'(0) = 1$, then $f'(x) =$

- A. $2x+1$
- B. $2x-1$
- C. $x+1$
- D. $x-1$

Answer: B



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47. If $f(x) = |2 - x| + (2 + x)$, where (x) = the least integer greater than or equal to x , then

A. $\lim_{x \rightarrow 2^-} f(x) = f(2) = 2$

B. $f(x)$ is continuous and differentiable at $x=2$

C. $f(x)$ is neither continuous nor differentiable at $x=2$

D. $f(x)$ is continuous and non-differentiable at $x=2$

Answer: C



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48. If $f(x) = \frac{[x]}{|x|}$, $x \neq 0$ where $[.]$ denotes the greatest integer function, then $f'(1)$ is

A. -1

B. 1

C. non-existent

D. ∞

Answer: C



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49. If $4x + 3|y| = 5y$, then y as a function of x is

A. differentiable at $x=0$

B. continuous at $x=0$

C. $\frac{dy}{dx} = 2$ for all x

D. none of these

Answer: B



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50. Let $f(x) = \log_e |x - 1|$, $x \neq 1$, then the value of $f' \left(\frac{1}{2} \right)$ is

A. -2

B. 2

C. non-existent

D. 1

Answer: A



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51. Let a function $f(x)$ defined on $[3,6]$ be given by

$$f(x) = \begin{cases} \log_e [x] & 3 \leq x < 5 \\ |\log_e x| & 5 \leq x < 6 \end{cases} \text{ then } f(x) \text{ is}$$

A. continuous and differentiable on $[3,6]$

B. continuous on $[3,6]$ but not differentiable at $x=4,5$

C. differentiable on $[3,6]$ but not continuous at $x=4,5$

D. none of these

Answer: D



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52. If $f(x) = \begin{cases} e^x & x < 2 \\ ax + b & x \geq 2 \end{cases}$ is differentiable for all $x \in \mathbb{R}$, then

A. $a = e^2, b = -e^2$

B. $a = -e^2, b = e^2$

C. $a = b = e^2$

D. none of these

Answer: A



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53. If the function $f(x)$ is given by $f(x) = \begin{cases} 2^{1/(x-1)} & x < 1 \\ ax^2 + bx & x \geq 1 \end{cases}$ is

everywhere differentiable, then

A. $a=0, b=1$

B. $a=0, b=0$

C. $a=1, b=0$

D. none of these

Answer: B



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54. Let $f(x) = \sin x$, $g(x) = [x + 1]$ and $h(x) = g \circ f(x)$ where $[.]$ the greatest integer function. Then $h' \left(\frac{\pi}{2} \right)$ is

A. 1

B. -1

C. non-existent

D. none of these

Answer: C



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55. If $f(x) = |x - 2|$ and $g(x) = f(f(x))$, then $g'(x)$ for $x > 2$, is

A. 1

B. 2

C. -1

D. none of these

Answer: A



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56. If $f(x) = \operatorname{sgn}(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ and $g(x) = f(f(x))$,

then at $x = 0$, $g(x)$ is

A. continuous and differentiable

B. continuous but not differentiable

C. differentiable but not continuous

D. neither continuous nor differentiable

Answer: D

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57. Let $f(x) = \cos x$ and $g(x) = [x + 1]$, where $[.]$ denotes the greatest integer function, Then $(gof)'(\pi/2)$ is

A. 0

B. 1

C. -1

D. non-existent

Answer: D

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58. Let $f(x) = \min \{1, \cos x, 1 - \sin x\}$, $-\pi \leq x \leq \pi$, Then, $f(x)$ is

- A. not continuous at $x = \pi/2$
- B. continuous but not differentiable at $x=0$
- C. neither continuous nor differentiable at $x = \pi/2$
- D. none of these

Answer: B



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59. If $[.]$ denotes the greatest integer function, then

$$f(x) = [x] + \left[x + \frac{1}{2} \right]$$

- A. is continuous at $x = \frac{1}{2}$
- B. is discontinuous at $x = \frac{1}{2}$
- C. $\lim_{x \rightarrow \left(\frac{1}{2}\right)} f(x) = 2$

D. $\lim_{x \rightarrow \left(\frac{1}{2}\right)^-} f(x) = 1$

Answer: B



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60. If $f(x) = \operatorname{sgn}(x^5)$, then which of the following is/are false (where sgn denotes signum function)

- A. continuous and differentiable
- B. continuous but not differentiable
- C. differentiable but not continuous
- D. neither continuous nor differentiable

Answer: A



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61. If $g(x) = (x^2 + 2x + 3)f(x)$, $f(0) = 5$ and $\lim_{x \rightarrow 0} \frac{f(x - 5)}{x} = 4$,

then $g'(0)$ is equal to

A. 22

B. 20

C. 18

D. none of these

Answer: A



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62. If $f(x) = \begin{cases} \frac{1}{x} - \frac{2}{e^{2x} - 1} & x \neq 0 \\ 1 & x = 0 \end{cases}$

A. $f(x)$ is differentiable at $x=0$

B. $f(x)$ is not differentiable at $x=0$

C. $f'(0) = \frac{1}{3}$

D. $f(x)$ is continuous but not differentiable at $x=0$

Answer: A



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63. Let $f(x) = (-1)^{[x^3]}$, where $[.]$ denotes the greatest integer function. Then,

A. $f(x)$ is discontinuous at $x = n^{1/3}, n \in \mathbb{Z}$

B. $f(3/2)=1$

C. $f'(0) = 0$ for all $x \in (-1, 1)$

D. none of these

Answer: A



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64. $f(x) = \frac{1}{1-x}$ and $f^n = \text{fofof...of}$, then the points of discontinuity of $f^{(3n)}(x)$ is/are

A. $x=2$

B. $x=0,1$

C. $x=1,2$

D. none of these

Answer: B



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65. Let $f(x) = [n + p \sin x]$, $x \in (0, \pi)$, $n \in \mathbb{Z}$, p a prime number and $[x]$ = the greatest integer less than or equal to x . The number of points at which $f(x)$ is not differentiable is :

A. p

B. $p-1$

C. $2p+1$

D. $2p-1$

Answer: D



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66. Determine the values of x for which the following function fails to be continuous or differentiable:

$f(x) = \begin{cases} 1 - x, & x < 1 \\ (1 - x)(2 - x), & 1 \leq x \leq 2 \\ 3 - x, & x > 2 \end{cases}$ justify your answer.

A. $x=1$

B. $x=2$

C. $x=1,2$

D. none of these

Answer: B



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67. Let $[x]$ denote the greatest integer less than or equal to x and $g(x)$ be

$$\text{given by } g(x) = \begin{cases} [f(x)] & x \in (0, \pi/2) \cup (\pi/2, \pi) \\ 3 & x = \frac{\pi}{2} \end{cases}$$

where, $f(x) = \frac{2(\sin x - \sin^n x) + |\sin x - \sin^n x|}{2(\sin x - \sin^n x) - |\sin x - \sin^n x|}$, $n \in \mathbb{R}^+$ then at $x = \frac{\pi}{2}$, $g(x)$, is

- A. continuous and differentiable when $n > 1$
- B. continuous and differentiable when $0 < n < 1$
- C. continuous but not differentiable when $n > 1$
- D. continuous but not differentiable when $0 < n < 1$

Answer: A



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68. If $f(x) = \begin{cases} \frac{x}{1+|x|} & |x| \geq 1 \\ \frac{x}{1-|x|} & |x| < 1 \end{cases}$ then $f(x)$ is

A. discontinuous and non-differentiable at $x = -1, 1, 0$

B. discontinuous and non-differentiable at $x=-1$, whereas continuous and differentiable at $x=0,1$

C. discontinuous and non-differentiable at $x=-1,1$ whereas continuous and differentiable at $x=0$.

D. none of these

Answer: C



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69. Let $f: [0, 1] \rightarrow [0, 1]$ be a continuous function such that $f(f(x)) = 1 - f(x)$ or $f(x) = 1 - f(f(x))$ for all $x \in [0, 1]$ then:

A. $f(x) = x$ for at least one $x \in (0, 1)$

B. $f(x)$ will be differential in $[0,1]$

C. $f(x)+x=0$ for at least one x such that $0 \leq x \leq 1$

D. none of these

Answer: A



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70. Let $f(x)$ be a continuous function defined for $1 \leq x \leq 3$. If $f(x)$ takes rational values for all x and $f(2) = 10$ then the value of $f(1.5)$ is :

A. 20

B. 5

C. 10

D. none of these

Answer: C



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71. Let $f(x)$ and $g(x)$ be two equal real function such that

$$f(x) = \frac{x}{|x|}g(x), x \neq 0$$

If $g(0)=g'(0)=0$ and $f(x)$ is continuous at $x=0$, then $f'(0)$ is

- A. 0
- B. 1
- C. -1
- D. non-existent

Answer: A



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72. If $f(x)$ is a periodic function with period T , then

- A. f' and f' are also periodic
- B. f' is periodic but f' is not periodic
- C. f' is periodic but f' is not periodic

D. none of these

Answer: A



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73. If $f(x) = \begin{cases} \frac{e^{x[x]} - 1}{x + [x]} & x \neq 0 \\ 1 & x = 0 \end{cases}$ then

A. $\lim_{x \rightarrow 0^+} f(x) = -1$

B. $\lim_{x \rightarrow 0^-} f(x) = \frac{1}{e} - 1$

C. $f(x)$ is continuous at $x=0$

D. $f(x)$ is discontinuous at $x=0$

Answer: D



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74. Let $f(x)$ be defined on $[-2,2[$ such that

$$f(x) = \begin{cases} -1 & -2 \leq x \leq 0 \\ x - 1 & 0 \leq x \leq 2 \end{cases} \text{ and } g(x) = f(|x|) + |f(x)|. \text{ Then } g(x) \text{ is}$$

differentiable in the interval.

A. $[-2,2]$

B. $[-2, 0) \cup (0, 2]$

C. $[-2, 1) \cup (1, 2]$

D. $[-2, 0) \cup (0, 1) \cup (1, 2]$

Answer: D



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75. If $f(x) = \begin{cases} \frac{x^2}{2} & 0 \leq x < 1 \\ 2x^2 - 3x + \frac{3}{2} & 1 \leq x \leq 2 \end{cases}$ then,

A. f, f' and f'' are continuous in $[0,2]$

B. f and f' are continuous in $[0,2]$ whereas f'' is continuous in

$$[0, 1] \cup (1, 2]$$

C. f, f' and f'' are continuous in $[0, 1) \cup (1, 2]$

D. none of these

Answer: A

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76. If $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1)[x] & 2 \leq x < 3 \end{cases}$ where $[.]$ denotes the greatest

integer function, then

- A. both $f'(1)$ and $f'(2)$ do not exist
- B. $f'(1)$ exist but $f'(2)$ does not exist
- C. $f'(2)$ exist but $f'(1)$ does not exist
- D. both $f'(1)$ and $f'(2)$ exist

Answer: A



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77. If $f(x) = \begin{cases} 4 & -3 < x < -1 \\ 5 + x & -1 \leq x < 0 \\ 5 - x & 0 \leq x < 2 \\ x^2 + x - 3 & 2 < x < 3 \end{cases}$ then, $f(|x|)$ is

- A. differentiable but not continuous in $(-3,3)$
- B. continuous but not differentiable in $(-3,3)$
- C. continuous as well as differentiable in $(-3,3)$
- D. neither continuous nor differentiable $(-3,3)$

Answer: B



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78. If $f(x) = \begin{cases} (x - a)^n \cos\left(\frac{1}{x-a}\right) & x \neq a \\ 0 & x = a \end{cases}$

then at $x=a$, $f(x)$ is

A. continuous if $n > 0$ and differentiable if $n > 1$

B. continuous if $n > 1$ and differentiable if $n > 0$

C. continuous and differentiable if $n > 0$

D. none of these

Answer: A

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79. Let $f(x)$ and $g(x)$ be two functions given by

$$f(x) = -1|x - 1|, \quad -1 \leq x \leq 3 \quad \text{and}$$

$$g(x) = 2 - |x + 1|, \quad -2 \leq x \leq 2$$

Then,

A. $f \circ g$ is differentiable at $x=-1$ and $g \circ f$ is differentiable at $x=1$

B. f is differentiable at $x=-1$ and $g \circ f$ is not differentiable at $x=1$

C. $f \circ g$ is differentiable at $x=1$ and $g \circ f$ is differentiable at $x=-1$

D. none of these

Answer: D

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80. The function $y = f(x)$ is defined by $x = 2t - |t|$, $y = t^2 + |t|$, $t \in \mathbb{R}$ in the interval $x \in [-1, 1]$, then

- A. continuous and differentiable in $[-1,1]$
- B. continuous but not differentiable in $[-1,1]$
- C. continuous in $[-1,1]$ and differentiable in $[-1,1]$ only
- D. none of these

Answer: A

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81. Let $f(x)$ be a function defined as

$$f(x) = \begin{cases} \int_0^x (3 + |t - 2|) & \text{if } x > 4 \\ 2x + 8 & \text{if } x \leq 4 \end{cases}$$

Then, $f(x)$ is

- A. continuous at $x=4$
- B. neither continuous nor differentiable at $x=4$
- C. everywhere continuous but not differentiable at $x=4$
- D. everywhere continuous and differentiable

Answer: C



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82. If a function $y=f(x)$ is defined as

$y = \frac{1}{t^2 - t - 6}$ and $t = \frac{1}{x - 2}, t \in R$. Then $f(x)$ is discontinuous at

A. $2, \frac{2}{3}, \frac{7}{3}$

B. $2, \frac{3}{2}, \frac{7}{3}$

C. $2, \frac{2}{3}, \frac{7}{3}$

D. none of these

Answer: B



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83. Let $f(x) = x^3 - x^2 + x + 1$ and $g(x)$ be a function defined by

$$g(x) = \begin{cases} \text{Max}\{f(t) : 0 \leq t \leq x\} & 0 \leq x \leq 1 \\ 3 - x & 1 \leq x \leq 2 \end{cases} \text{ Then, } g(x) \text{ is}$$

- A. continuous and differentiable on $[0,2]$
- B. continuous but not differentiable on $[0,2]$
- C. neither continuous nor differentiable on $[0,2]$
- D. none of these

Answer: B



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84. If $f(x) = \sum_{r=1}^n a_r |x|^r$, where a_i s are real constants, then $f(x)$

is

- A. continuous at $x=0$ for all a_1
- B. differentiable at $x=0$ for all $a_i \in R$
- C. differentiable at $x=0$ for all $a_{2k+1} = 0$
- D. none of these

Answer: A::C

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85. Let $f(x) = \phi(x) + \Psi(x)$, where, $\phi'(x)$ and $\Psi'(a)$ are finite and definite. Then,

- A. $f(x)$ is continuous at $x=a$
- B. $f(x)$ is differentiable on $x=a$
- C. $f'(x)$ is continuous at $x=a$
- D. $f'(x)$ is differentiable at $x=a$

Answer: A::B

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86. A function $f(x)$ is defined in the interval $[1,4)$ as follows

$$f(x) = \begin{cases} \log_e[x] & 1 \leq x < 3 \\ |\log_e x| & 3 \leq x < 4 \end{cases}. \text{ Then, the curve } y=f(x)$$

- A. is broken at two points
- B. is broken at exactly one point
- C. does not have a definite tangent at two points
- D. does not have a definite tangent at more than two points

Answer: A:C

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87. Let $f(x)$ be a function defined by $f(x) = \begin{cases} e^x & x < 2 \\ a + bx & x \geq 2 \end{cases}$ It $f(x)$ is differentiable for all $x \in R$, then

- A. $a+b=0$

B. $a + 2b = e^2$

C. $b = e^2$

D. all of these

Answer: D



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88. Let $f(x) = \min(x^3, x^4)$ for all $x \in \mathbb{R}$. Then,

A. $f(x)$ is continuous for all x

B. $f(x)$ is indifferentiable for all x

C. $f'(x) = 3x^2$ for all $x > 1$

D. $f(x)$ is not differentiable at two points

Answer: A



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89. Let $g(x)$ be a polynomial of degree one and $f(x)$ is defined by $f(x) = \{g(x), x \leq 0 \text{ and } \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}, x > 0\}$ Find $g(x)$ such that $f(x)$ is continuous and $f'(1) = f(-1)$

A. $-\frac{1}{9}(1 + 6 \log_e, 3)x$

B. $\frac{1}{9}(1 + 6 \log_e, 3)$

C. $-\frac{1}{9}(1 - 6 \log_e, 3)x$

D. none of these

Answer: A



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90. Number of points of non-differentiability of $f(x) = \sin \pi(x - [x])$ in $(-\pi/2, [\pi/2])$. Where $[.]$ denotes the greatest integer function is

A. 4

B. 5

C. 3

D. 2

Answer: C



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91. If $f(x) = [\sin^2 x]$ ($[.]$ denotes the greatest integer function), then

A. f is everywhere continuous

B. f is everywhere differentiable

C. f is a constant function

D. none of these

Answer: D



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92. If $f(x) = [x^2] + \sqrt{\{x\}^2}$, where $[]$ and $\{ \}$ denote the greatest integer and fractional part functions respectively, then

- A. $f(x)$ is continuous at all integer points
- B. $f(x)$ is continuous and differentiable at $x=0$
- C. $f(x)$ is continuous for all $x \in \mathbb{Z} - \{1\}$
- D. $f(x)$ is not differentiable on \mathbb{Z}

Answer: C



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93.

Let

$f(xy) = f(x)f(y)$ for all $x > 0, y > 0$ and $f(1+x) = 1 + x[1 + g(x)]$, where

- A. $\frac{x^2}{2} + C$
- B. $\frac{x^3}{3} + C$
- C. $\frac{x^2}{3} + C$

D. none of these

Answer: A



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94. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that

$$f\left(\frac{x+y}{3}\right) = \frac{f(x) + f(y)}{3}, f(0) = 0 \text{ and } f'(0) = 3, \text{ then}$$

- A. a quadratic function
- B. continuous but not differentiable
- C. differentiable in \mathbb{R}
- D. bounded in \mathbb{R}

Answer: C



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95. $f(x) = x^2 + 3x^2 - 33x - 33$ for $x > 0$ and g be its inverse such that $kg'(2)=1$, then the value of k is

- A. -36
- B. 42
- C. 12
- D. none of these

Answer: D

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96. $\lim_{h \rightarrow 0} \frac{f(2h + 2 + h^2)}{f(h - h^2 + 1) - f(1)}$ given that $f'(2) = 6$ and $f'(1) = 4$

- A. does not exist
- B. is equal to $-\frac{3}{2}$
- C. is equal to $\frac{3}{2}$

D. is equal to 3

Answer: D

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97. Let $f(x) = (xe)^{\frac{1}{|x|} + \frac{1}{x}}$; $x \neq 0$, $f(0) = 0$, test the continuity & differentiability at $x = 0$

- A. discontinuous everywhere
- B. continuous as well as differential for all x
- C. continuous for all c but not differential at x=0
- D. neither differential nor continuous at x=0

Answer: C

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98. Let $f(x) = \lim_{n \rightarrow \infty} \frac{(2 \sin x)^{2n}}{3^n - (2 \cos x)^{2n}}$, $n \in \mathbb{Z}$. Then

A. at $x = n \pm \frac{\pi}{6}$, $f(x)$ is discontinuous

B. $f\left(\frac{\pi}{3}\right) = 1$

C. $f(0) = 0$

D. all of the above

Answer: D



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99. The function $f(x) = ||x| - 1|$, $x \in \mathbb{R}$, is differentiable at all $x \in \mathbb{R}$ except at the points.

A. 1, 0, -1

B. 1

C. 1, -1

D. -1

Answer: A



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100. If $f(x)$ is continuous and differentiable function such that

$f\left(\frac{1}{n}\right) = 0$ for all $n \in \mathbb{N}$, then

A. $f(x) = 0$ for all $x \in \mathbb{N} \cup (0, 1]$

B. $f(0) = 0, f'(0) = 0$

C. $f'(0) = 0, f''(0) = 0$

D. $f(0)$ and $f'(0)$ may or may not be zero

Answer: B



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101. The second degree polynomial satisfying

$$f(x), f(0) = 0, f(1) = 1, f'(x) > 0 \forall x \in (0, 1)$$

A. $f(x) = \phi$

B. $f(x) = ax + (1 - a)x^2, a \in (0, \infty)$

C. $f(x) = ax + (1 - a)x^2, x \in (0, 2)$

D. non-existent

Answer: C



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102. If $f^x = -f(x)$ and $g(x) = f'(x)$ and

$$F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2 \text{ and given that } F(5) = 5, \text{ then } F(10)$$

is equal to 5 (b) 10 (c) 0 (d) 15

A. 15

B. 10

C. 0

D. 15

Answer: A



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103. If $f(x) = \min(x, x^2, x^3)$, then

A. $f(x)$ is everywhere differentiable

B. $f(x) > 0$ for $x > 1$

C. $f(x)$ is not differentiable at three points but continuous for all

$$x \in \mathbb{R}$$

D. $f(x)$ is not differentiable for two values of x

Answer: C



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104. If $f(x) = \min(1, x^2, x^3)$, then

- A. $f(x)$ is everywhere continuous
- B. $f(x)$ is continuous and differentiable everywhere
- C. $f(x)$ is not differentiable at two points
- D. $f(x)$ is not differentiable at one points

Answer: A::D



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105. Let $f: (-1, 1) \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$ (a) -4 (b) 0 (c) -2 (d)

4

- A. 0
- B. -2
- C. 4

D. -4

Answer: D



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106. If $f(x) = \begin{cases} -x - \frac{\pi}{2} & x \leq -\frac{\pi}{2} \\ -\cos x & -\frac{\pi}{2} < x \leq 0 \\ x - 1 & 0 < x \leq 1 \\ \ln x & x > 1 \end{cases}$ then which one of the

following is not correct?

A. $f(x)$ is continuous at $x = -\frac{\pi}{2}$

B. $f(x)$ is not differentiable at $x=0$

C. $f(x)$ is differentiable at $x = 1, -\frac{3}{2}$

D. $f(x)$ is discontinuous at $x=0$

Answer: D



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107. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$ for all, $x, y \in \mathbb{R}$

If $f(x)$ is differentiable at $x=0$. then, which one of the following is incorrect?

- A. $f(x)$ is continuous for all $x \in \mathbb{R}$
- B. $f'(x)$ is constant for all $x \in \mathbb{R}$
- C. $f(x)$ is differentiable for all $x \in \mathbb{R}$
- D. $f(x)$ is differentiable only in a finite interval containing zero

Answer: D



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108. Let $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{x} \right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in \mathbb{R}$, then f is

- A. differentiable both at $x=0$ and $x=2$
- B. differentiable at $x=0$ but not differentiable at $x=2$

C. not differentiable at $x=0$ but differentiable at $x=2$

D. differentiable neither at $x=0$ nor at $x=2$

Answer: B



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109. Q. For every integer n , let a_n and b_n be real numbers. Let function $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n + 1], \\ -n + \cos \pi x, & \text{for } x \in (2n + 1, 2n) \end{cases}$ for all integers n .

A. $a_n - b_{n+1} = -1$

B. $a_{n-1} - b_{n-1} = 0$

C. $a_n - b_n = 1$

D. $a_{n-1} - b_n = 1$

Answer: B



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110. If f and g are differentiable functions in $[0, 1]$ satisfying

$f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$ (1)

$2f'(c) = g'(c)$ (2) $2f'(c) = 3g'(c)$ (3) $f'(c) = g'(c)$ (4)

$f'(c) = 2g'(c)$

A. $f'(c) = g'(c)$

B. $f'(c) = 2g'(c)$

C. $2f'(c) = g'(c)$

D. $2f'(c) = 3g'(c)$

Answer: B



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111. Let $f_1: \mathbb{R} \rightarrow \mathbb{R}$, $f_2: [0, \infty) \rightarrow \mathbb{R}$, $f_3: \mathbb{R} \rightarrow \mathbb{R}$ and $f_4: \mathbb{R} \rightarrow [0, \infty)$

be defined by

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x > 0 \end{cases} : f_2(x) = x^2, f_3(x) = \begin{cases} \sin x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

and $f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(f_1(x))) - 1 & \text{if } x \geq 0 \end{cases}$ Then, f_4 is

- A. onto but not one-one
- B. neither continuous nor one-one
- C. differentiable but not one-one
- D. continuous and one-one

Answer: A



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112. In Q,NO, 111, f_3 is

- A. onto but not one-one
- B. neither continuous nor one-one
- C. differentiable but not one-one
- D. continuous and one-one

Answer: C



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113. In Q,NO. 111, f_2 or f_1 is

- A. onto but not one-one
- B. neither continuous nor one-one
- C. differentiable but not one-one
- D. continuous and one-one

Answer: B



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114. In Q,NO, 111, f_2 is

- A. onto but not one-one

B. neither continuous nor one-one

C. differentiable but not one-one

D. continuous and one-one

Answer: D



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115. For every pair of continuous functions $f, g: [0, 1] \rightarrow \mathbb{R}$ such that $\max \{f(x) : x \in [0, 1]\} = \max \{g(x) : x \in [0, 1]\}$ then which are the correct statements

A. $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

B. $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

C. $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$

D. $(f(c))^2 + (g(c))^2$ for some $c \in [0, 1]$

Answer: A::D



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116. Let $f: [a, b] \rightarrow [1, \infty)$ be continuous function and let $g: \mathbb{R} \rightarrow \mathbb{R}$ be

$$\text{defined as } g(x) = \begin{cases} 0 & \text{if } x < a \\ \int_a^x dt & \text{if } a \leq x \leq b \\ \int_a^b f(t) & \text{if } x > b \end{cases} \text{ Then,}$$

- A. $g(x)$ is continuous but not differentiable at $x=a$
- B. $g(x)$ is differentiable on \mathbb{R}
- C. $g(x)$ is continuous but not differentiable at $x=b$
- D. $g(x)$ is continuous and differentiable at either $x=a$ or $x=b$ but not both

Answer: A:C



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117. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h: \mathbb{R} \rightarrow \mathbb{R}$ by

$$h(x) = \begin{cases} \max \{f(x), g(x)\}, & \text{if } x \leq 0 \\ \min \{f(x), g(x)\}, & \text{if } x > 0 \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

A. 1

B. 2

C. 3

D. 4

Answer: C



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118. Let $g: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function with

$$g(0) = 0, g'(1) = 0, g'(1) \neq 0. \text{ Let}$$

$$f(x) = \begin{cases} \frac{x}{|x|}g(x), & 0 \neq x \\ 0, & x = 0 \end{cases} \text{ and } h(x) = e^{|x|} \text{ for all } x \in \mathbb{R}.$$

Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then

which of the following is (are) true?

A. f is differentiable at $x = 0$

B. h is differentiable at $x = 0$

C. f is differentiable at $x = 0$

D. h is differentiable at $x = 0$

Answer: A::D



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119. Let $f(x) = \begin{cases} 3 \sin x + a^2 - 10a + 30 & x \in \mathbb{Q} \\ 4 \cos x & x \in \mathbb{Q} \end{cases}$ which one of the following statements is correct?

A. $f(x)$ is continuous for all x when $a=5$

B. $f(x)$ must be continuous for all, x when $a=5$

C. $f(x)$ is continuous for all x ,

$$= 2\pi x - \tan^{-1}\left(\frac{3}{4}\right), n \in \mathbb{Z}, \text{ when } a=5$$

D. $f(x)$ is continuous for all $x = 2\pi x - \tan^{-1}\left(\frac{4}{3}\right), n \in \mathbb{Z}$ when $a=5$

Answer: C



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120. If $(\lim)_{x \rightarrow 0} \frac{\{(a - n)nx - \tan x\} \sin nx}{x^2} = 0$, where n is nonzero real number, the a is 0 (b) $\frac{n+1}{n}$ (c) n (d) $n + \frac{1}{n}$

A. 0

B. $\frac{n}{n+1}$

C. n

D. $n + \frac{1}{n}$

Answer: D



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121. The value of k for which $f(x) = \begin{cases} \frac{x^{32} - 2^{32}x + 4^{16} - 1}{(x-1)^2} & x \neq 1 \\ k & x = 1 \end{cases}$ is

continuous at $x=1$, is

A. $2^{63} - 2^{31}$

B. $2^{65} - 2^{33}$

C. $2^{62} - 2^{31}$

D. $2^{65} - 2^{31}$

Answer: A



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122. The function $f(x) = \begin{cases} \frac{x^2}{a} & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^2} & \sqrt{2} \leq x < \infty \end{cases}$ is a continuous for

$0 \leq x < \infty$. Then which of the following statements is correct?

A. The number of all possible ordered pairs (a,b) is 3

B. The number of all possible ordered pairs (a,b) is 4

C. The product of all possible pairs ,b is -1

D. The product of all possible values of b is 1

Answer: A::C



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123. If $f(x) = \begin{cases} x \left(\left[\frac{1}{x} \right] + \left[\frac{2}{x} \right] + \dots + \left[\frac{n}{x} \right] \right) & x \neq 0 \\ k & x = 0 \end{cases}$ and $n \in \mathbb{N}$.

Then the value of k for which $f(x)$ is continuous at $x=0$ is

A. n

B. $n+1$

C. $n(n+1)$

D. $\frac{n(n+1)}{2}$

Answer: D



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124. The value of k for which

$$f(x) = \begin{cases} \left[1 + x \left(e^{-1/x^2}\right) \sin\left(\frac{1}{x^4}\right)\right]^{e^{1/x^2}} & x \neq 0 \\ k & x = 0 \end{cases} \text{ is continuous at}$$

$x=0$, is

A. 1

B. 2

C. 3

D. 4

Answer: A



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125. Let $f(x) = \begin{cases} \sum_{r=0}^{x^2 \left[\frac{1}{|x|} \right]} r & x \neq 0 \\ k & x = 0 \end{cases}$ where $[\cdot]$ denotes the greatest

integer function. The value of k for which is continuous at $x=0$, is

A. 1

B. 2

C. 4

D. $\frac{1}{2}$

Answer: A



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126. Let $f(x) = \begin{cases} |x| - 3 & x < 1 \\ |x - 2| + a & x \geq 1 \end{cases}$ If $h(x) = f(x) + g(x)$ is discontinuous at

$$g(x) = \begin{cases} 2 - |x| & x < 1 \\ \text{Sgn}(x) - b & x \geq 1 \end{cases}$$

exactly one point, then which of the following are correct?

A. $a=3, b=0$

B. $a=-3, b=-1$

C. $a=2, b=1$

D. $a=0, b=3$

Answer: B::C



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127. If $f: R \rightarrow R$ is a continuous function satisfying

$f(0) = 1$ and $f(2x) - f(x) = x$ for all $x \in R$ and $\lim_{x \rightarrow \infty} \left\{ f(x) - f\left(\frac{x}{2^n}\right) \right\}$

$P(x)$, is

- A. a constant function
- B. a linear polynomial in x
- C. a quadratic polynomial in x
- D. a cubic polynomial in x

Answer: B



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128. Let $f: (0, \infty) \rightarrow R$ be a continuous function such that

$F(x) = \int_0^{x^2} tf(t)dt$. If $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2) =$

A. 216

B. 219

C. 222

D. 225

Answer: B



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129. A function $f: R \rightarrow R$ is differentiable and satisfies the equation

$f\left(\frac{1}{n}\right) = 0$ for all integers $n \geq 1$, then

A. $f(x) = 0$ for all $x \in (0, 1]$

B. $f(0) = f'(0)$

C. $f(0) = 0$ but $f'(0)$ need not be equal to 0

D. $|f(x)| \leq 1$ for all $x \in [0, 1]$

Answer: B



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130. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x . Then the value of ab is equal to:

A. 25

B. 9

C. -15

D. -9

Answer: C



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131. If $f(x) = \begin{cases} \alpha + \frac{\sin[x]}{x}, & x > 0 \\ \beta + \left[\frac{\sin x - x}{x^3} \right], & x < 0 \end{cases}$ (whlnotes the greatest integer function) if $f(x)$ is continuous at $x = 0$. then β is equal to

A. $\alpha - 1$

B. $\alpha + 1$

C. $\alpha + 2$

D. $\alpha - 2$

Answer: B



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132. If a function $y=f(x)$ is defined as

$y = \frac{1}{t^2 - t - 6}$ and $t = \frac{1}{x - 2}, t \in R$. Then $f(x)$ is discontinuous at

A. $2, \frac{2}{3}, \frac{7}{3}$

B. $2, \frac{3}{2}, \frac{7}{3}$

C. $2, \frac{3}{2}, \frac{5}{3}$

D. None of these

Answer: B

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133. If $f(x)$ is continuous in $[0,2]$ and $f(0)=f(2)$. Then the equation $f(x)=f(x+1)$ has

- A. no real root in $[0,2]$
- B. at least one real root in $[0,1]$
- C. at least one real root in $[0,2]$
- D. at least one real root in $[1,2]$

Answer: B::C

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134. If $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} [f(x)]$ and $f(x)$ is non-constant continuous function, where $[.]$ denotes the greatest integer function, then

- A. $\lim_{x \rightarrow a} f(x)$ is an integer

B. $\lim_{x \rightarrow a} f(x)$ is not an integer

C. $f(x)$ has a local maximum at $x=a$

D. $f(x)$ has a local minimum at $x=a$

Answer: A::D



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135. Let $f: R \rightarrow R$ differentiable at $x=0$ and satisfies $f(0)=0$ and $f'(0)=1$,

then the value of $\lim_{x \rightarrow 0} \frac{1}{x} \sum_{n=1}^{\infty} (-1)^n f\left(\frac{x}{n}\right)$ is

A. 0

B. $-\ln 2$

C. 1

D. e

Answer: B



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136. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g is not differentiable at $x=0$

B. $g'(0)=\cos(\log 2)$

C. $g'(0)=-\cos(\log 2)$

D. g is differentiable at $x=0$ and $g'(0)=-\sin(\log 2)$

Answer: B



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137. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be differentiable functions such that

$f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ for all $x \in R$, Then, $g'(2)=$

A. $\frac{1}{15}$

B. $\frac{1}{5}$

C. $\frac{1}{3}$

D. 15

Answer: C

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138. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $h: \mathbb{R} \rightarrow \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$.

Then, $h'(1)$ equals.

A. 666

B. 16

C. 66

D. 111

Answer: A

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139. In Example 138, $h(0)$ equals

- A. 6
- B. 16
- C. 2
- D. 15

Answer: B



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140. In Example 138, $h(0)$ equals

- A. 66
- B. 6
- C. 36
- D. 38

Answer: D



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141. Let $a, b \in \mathbb{R}$ and $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$f(x) = a \cos(|x^3 - x|) + b|x| \sin(|x^3 + x|)$. Then f is

- A. differentiable at $x=0$, if $a=0$ and $b=1$
- B. differentiable at $x=1$, if $a=1$ and $b=0$
- C. not differentiable at $x=0$, if $a=1$ and $b=0$
- D. not differentiable at $x=1$, if $a=1$ and $b=1$

Answer: A:B



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142. Let $f: \mathbb{R} \rightarrow (0, \infty)$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be twice differentiable functions such that $f(2)=g(2)=0$, $f''(2) \neq 0$ and $g''(2) \neq 0$, If

$\lim_{x \rightarrow 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

- A. f has a local maximum at $x=2$
- B. f has a local minimum at $x=2$
- C. $f''(2) > f(2)$
- D. $f(x) - f''(x) = 0$ for at least one $x \in R$.

Answer: B::D



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143. Let $f: \left[-\frac{1}{2}, 2\right] \rightarrow R$ and $g: \left[-\frac{1}{2}, 2\right] \rightarrow R$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in R$. Then,

- A. f is discontinuous exactly at three points in $[-1/2, 2]$
- B. f is discontinuous exactly at four points in $[-1/2, 2]$
- C. g is not differentiable exactly at four points in $[-1/2, 2]$

D. g is not differentiable exactly at five points in $[-1/2, 2]$

Answer: B::C



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Section II Assertion Reason Type

1. Statement-1: If $|f(x)| \leq |x|$ for all $x \in \mathcal{R}$ then $|f(x)|$ is continuous at

0. Statement-2: If $f(x)$ is continuous then $|f(x)|$ is also continuous.

A. 1

B. 2

C. 3

D. 4

Answer: A



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2. Let $f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ 1 + [x] + \sin x & 0 \leq x < \pi/2 \\ 3 & x \geq \pi/2 \end{cases}$

Statement-1: F is a continuous on R-[1]

Statement-2: The greatest integer function is discontinuous at every integer point.

A. 1

B. 2

C. 3

D. 4

Answer: B



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3. Statement-1: The function $f(x) = [x] + x^2$ is discontinuous at all integer points.

Statement-2: The function $g(x)=[x]$ has Z as the set of points of its discontinuous from left.

A. 1

B. 2

C. 3

D. 4

Answer: A



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4. Statement-1: If a continuous function on $[0,1]$ satisfy $0 \leq f(x) \leq 1$, then there exist $c \in [0, 1]$ such that $f(c)=c$

Statement-2: $\lim_{x \rightarrow c} f(x) = f(c)$

A. 1

B. 2

C. 3

D. 4

Answer: B

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5. Statement-1: Let $f(x) = [3 + 4 \sin x]$, where $[.]$ denotes the greatest integer function. The number of discontinuities of $f(x)$ in $[\pi, 2\pi]$ is 6

Statement-2: The range of f is $[-1, 0, 1, 2, 3]$

A. 1

B. 2

C. 3

D. 4

Answer: D

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6. The function $f(x) = e^{-|x|}$ is continuous everywhere but not differentiable at $x = 0$ continuous and differentiable everywhere not continuous at $x = 0$ none of these

A. 1

B. 2

C. 3

D. 4

Answer: D



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7. Statement-1: If f and g are differentiable at $x=c$, then $\min(f,g)$ is differentiable at $x=c$.

Statement-2: $\min(f,g)$ is differentiable at $x = c$ if $f(c) \neq g(c)$

A. 1

B. 2

C. 3

D. 4

Answer: D



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8. Statement-1: Let f be a differentiable function satisfying

$$f(x + y) = f(x) + f(y) + 2xy - 1 \text{ for all } x, y \in R \quad \text{and}$$

$$f'(0) = a \text{ where } 0 < a < 1 \text{ then } , f(x) > 0 \text{ for all } x.$$

Statement-2: $f(x)$ is statement-1 is of the form $x^2 + ax + 1$

A. 1

B. 2

C. 3

D. 4

Answer: A



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9. Let f and g be real valued functions defined on interval $(-1,1)$ such that $g''(x)$ is continuous $g(0) = g'(0) \neq 0$ and $f(x) = g(x)\sin x$

Statement-1: $\lim_{x \rightarrow 0} [g(x)\cot x - g(0)\cos ecx] = f''(0)$

Statement-2: $f'(0) = g(0)$

A. 1

B. 2

C. 3

D. 4

Answer: B

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10. Let $f(x) = x|x|$ and $g(x) = s \in x$ Statement 1 : $g \circ f$ is differentiable at $x = 0$ and its derivative is continuous at that point Statement 2: $g \circ f$ is

twice differentiable at $x = 0$ (1) Statement1 is true, Statement2 is true, Statement2 is a correct explanation for statement1 (2) Statement1 is true, Statement2 is true; Statement2 is not a correct explanation for statement1. (3) Statement1 is true, statement2 is false. (4) Statement1 is false, Statement2 is true

A. 1

B. 2

C. 3

D. 4

Answer: C



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11. let $f(x) = 2 + \cos x$ for all real x Statement 1: For each real t , there exists a point c in $[t, t + \pi]$ such that $f'(c) = 0$ Because statement 2: $f(t) = f(t + 2\pi)$ for each real t

A. 1

B. 2

C. 3

D. 4

Answer: B



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12. Define $F(x)$ as the product of two real functions $f_1(x) = x, x \in \mathbb{R}$,

and $f_2(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0, \\ 0 & \text{if } x = 0 \end{cases}$ follows :

$F(x) = \begin{cases} f_1(x) \cdot f_2(x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$ Statement-1 : $F(x)$ is continuous

on \mathbb{R} . Statement-2 : $f_1(x)$ and $f_2(x)$ are continuous on \mathbb{R} .

A. 1

B. 2

C. 3

D. 4

Answer: C



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13. Let $f: [1, 3] \rightarrow \mathbb{R}$ be a function satisfying $\frac{x}{[x]} \leq f(x) \leq \sqrt{6-x}$, for all $x \neq 2$ and $f(2) = 1$, Where \mathbb{R} is the set of all real number and $[x]$ denotes the largest integer less than or equal to x .

Statement-1: $\lim_{x \rightarrow 2} f(x)$ exists.

Statement-2: f is continuous at $x=2$.

A. 1

B. 2

C. 3

D. 4

Answer: C



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Exercise

1. The function $f(x) = \frac{4 - x^2}{4x - x^3}$ is

- A. discontinuous at only one point
- B. discontinuous exactly at two point
- C. discontinuous exactly at three point
- D. None of these

Answer:



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2. Let $f(x) = |x|$ and $g(x) = |x^3|$, then $f(x)$ and $g(x)$ both are continuous at $x = 0$ (b) $f(x)$ and $g(x)$ both are differentiable at $x = 0$ (c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x = 0$ (d) $f(x)$ and $g(x)$ both are not differentiable at $x = 0$

A. $f(x)$ and $g(x)$ both the continuous at $x=0$

B. $f(x)$ and $g(x)$ both the differentiable at $x=0$

C. $f(x)$ is differentiable but $g(x)$ is not differentiable at $x=0$

D. $f(x)$ and $g(x)$ both are not differentiable at $x=0$.

Answer:



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3. The function $f(x) = \sin^{-1}(\cos x)$ is discontinuous at $x = 0$ (b) continuous at $x = 0$ (c) differentiable at $x = 0$ (d) none of these

A. discontinuous at $x=0$

B. continuous at $x=0$

C. differentiable at $x=0$

D. None of these

Answer:

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4. The set of points where the function $f(x) = x|x|$ is differentiable is
(a) $(-\infty, \infty)$ (b) $(-\infty, 0) \cup (0, \infty)$ (c) $(0, \infty)$ (d) $[0, \infty)$

A. $(-\infty, \infty)$

B. $(-\infty, 0) \cup (0, \infty)$

C. $(0, \infty)$

D. $[0, \infty]$

Answer:

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5. On the interval $I = [-2, 2]$, the function

$$f(x) = \begin{cases} (x+1)e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

A. is continuous for all $x \in I - \{0\}$

B. assumes all intermediate values from $f(-2) \rightarrow f(2)$

C. has a maximum value equal to $3/e$.

D. all of the above

Answer:

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6. If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)} & x \neq -2 \\ 2 & x = -2 \end{cases}$, then $f(x)$ is

A. continuous at $x=-2$

B. not continuous at $x=-2$

C. differentiable at $x=-2$

D. continuous but not derivable at $x=-2$

Answer: B

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7. Let $f(x) = (x + |x|)|x|$. Then, for all x f is continuous (b) f is differentiable for some x (c) f' is continuous (d) f is continuous

- A. f and f' are continuous
- B. f is differentiable for some x
- C. f' is not continuous
- D. f'' is continuous

Answer:



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8. The set of all points where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable is

- A. $(-\infty, \infty)$
- B. $(-\infty, 0) \cup (0, \infty)$
- C. $(-1, \infty)$

D. None of these

Answer:



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9. The function $f(x) = e^{|x|}$ is

A. continuous everywhere but not differentiable at $x=0$

B. continuous and differentiable everywhere

C. not continuous at $x=0$

D. None of these

Answer:



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10. The function $f(x) = [\cos x]$ is

A. everywhere continuous and differentiable

B. everywhere continuous but not differentiable at

$$(2n + 1)\pi/2, n \in \mathbb{Z}$$

C. neither continuous nor differentiable at $(2n + 1)\pi/2, n \in \mathbb{Z}$

D. None of these

Answer:



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11. If $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$, then $f(x)$ is continuous on $[-1, 1]$ and differentiable on $(-1, 1)$ continuous on $[-1, 1]$ and differentiable on $(-1, 0) \cup (0, 1)$ (d) none of these

A. continuous of $[-1,1]$ and differentiable on $(-1,1)$

B. continuous on $[-1,1]$ and differentiable aon $(-1, 0) \in (0, 1)$

C. continuous and differentiable on $[-1,1]$

D. None of these

Answer:



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12. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then $f(x)$ is differentiable on

A. $[-1,1]$

B. $R - [-1, 1]$

C. $R - [-1, 1]$

D. None of these

Answer:



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13. If $f(x) = a|\sin x| + be^{|x|} + c|x|^3$ and if $f(x)$ is differentiable at $x = 0$ then

A. $a=b=c=0$

B. $a=0, b=0, c \in R$

C. $b = c = 0, a \in R$

D. $c = 0, a = 0, b \in R$

Answer:



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14. If $f(x) = |x - a|\varphi(x)$, where $\varphi(x)$ is continuous function, then $f'(a^+) = \varphi(a)$ (b) $f'(a^-) = -\varphi(a)$ $f'(a^+) = f'(a^-)$ (d) none of these

A. $f'(a^+) = \varphi(a)$

B. $f'(a^-) = \varphi(a)$

C. $f'(a^+) = f'(a^-)$

D. None of these

Answer:



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15. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \dots + \frac{x^2}{(1+x^2)^n} + \dots$, then at $x = 0$, $f(x)$ has no limit (b) is discontinuous is continuous but not differentiable (d) is differentiable

- A. has no limit
- B. is discontinuous
- C. is continuous but not differentiable
- D. is differentiable

Answer:



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16. If $f(x) = |\log_{10} x|$ then at $x = 1$.

A. $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = -\log_{10} e$

B. $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = \log_{10} e$

C. $f(x)$ is continuous and $f'(1^-) = \log_{10} e$, $f'(1^+) = -\log_{10} e$

D. None of these

Answer:

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17. If $f(x) = |\log_e x|$, then

A. $f'(1^+) = 1$, $f'(1^-) = -1$

B. $f'(1^-) = -1$, $f'(1^+) = 0$

C. $f'(1) = 1$, $f'(1^-) = 0$

D. None of these

Answer:

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18. If $f(x) = |\log_e |x||$, then

A. $f(x)$ is continuous and differentiable for all x in its domain

B. $f(x)$ is continuous for all x in its domain but not differentiable at

$$x = \pm 1$$

C. $f(x)$ is neither continuous nor differentiable at $x = \pm 1$

D. None of these

Answer:



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19. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| > 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$ If $f(x)$ is continuous and differentiable at any point, then

A. $a = \frac{1}{2}, b = -\frac{3}{2}$

B. $a = -\frac{1}{2}, b = \frac{3}{2}$

C. $a=1, b=-1$

D. None of these

Answer:



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20. Let $h(x) = \min \{x, x^2\}$, for every real number of x . Then (A) h is continuous for all x (B) h is differentiable for all x (C) $h'(x) = 1$, for all $x > 1$ (D) h is not differentiable at two values of x

A. h is continuous for all x

B. h is differentiable for all x

C. $h'(x) = 1$ for all $x > 1$

D. h is not differentiable at two values of x

Answer:



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21. If $f(x) = \begin{cases} \frac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at $x=0$, then k

equals

A. $16\sqrt{2} \log 2 \log 3$

B. $16\sqrt{2} \ln 6$

C. $16\sqrt{2} \ln 2 \ln 3$

D. None of these

Answer:



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22. $f(x) = \begin{cases} |x - 4| & \text{if } x \leq 1 \\ \frac{x^3}{2} - x^2 + 3x + \frac{1}{2} & \text{if } x > 1 \end{cases}$, then 1)

$f(x)$ is continuous at $x=1$ and $x=4$ 2) $f(x)$ is differentiable at $x=4$ 3) $f(x)$ is

continuous and differentiable at $x=1$ 4) $f(x)$ is only continuous at $x=1$

A. $f(x)$ is continuous at $x=1$ and $x=4$

B. $f(x)$ is differentiable at $x=4$

C. $f(x)$ is continuous and differentiable at $x=1$

D. $f(x)$ is not continuous at $x=1$

Answer:

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23. Let $f(x) = \begin{cases} \sin 2x & 0 < x \leq x\pi/6 \\ ax + b & \pi/6 < x < 1 \end{cases}$ If $f(x)$ and $f'(x)$ are continuous, then

A. $a = 1, b = \frac{1}{\sqrt{2}} + \frac{\pi}{6}$

B. $a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{2}}$

C. $a = 1, b = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

D. None of these

Answer:



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24. The function f defined by $f(x) = \begin{cases} \frac{\sin x^2}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is

- A. continuous and derivative at $x=0$
- B. neither continuous nor derivative at $x=0$
- C. continuous but not derivable at $x=0$
- D. None of these

Answer:



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25. If $f(x)$ is continuous at $x=0$ and $f(0)=2$, then $\lim_{x \rightarrow 0} \int_0^x f(u) du \rightarrow is$

- A. 0
- B. 2

C. $f(2)$

D. None of these

Answer:



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26. If $f(x)$ defined by

$$f(x) = \begin{cases} \frac{|x^2 - x|}{x^2 - x}, & x \neq 0, 1 \\ 1, & x = 0 \text{ or } 1 \end{cases}$$

(a) x (b) x except at $x = 0$ (c) x except at $x = 1$ (d) x except at $x = 0$ and $x = 1$

A. x

B. x except at $x=0$

C. x except at $x=1$

D. x except at $x=0$ and $x=1$

Answer:



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27. If $f(x) = \begin{cases} \frac{1 - \sin x}{(\pi - 2x)^2} \cdot \frac{\log \sin x}{(\log(1 + \pi^2 - 4\pi x + 4x^2))} & x \neq \frac{\pi}{2} \\ k & x = \frac{\pi}{2} \end{cases}$ is continuous

at $x = \pi/2$, then $k =$

A. $-\frac{1}{16}$

B. $-\frac{1}{32}$

C. $-\frac{1}{64}$

D. $-\frac{1}{28}$

Answer:

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28. The set of points of differentiable of the function

$$f(x) = \begin{cases} \frac{\sqrt{x+1}-1}{\sqrt{x}} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

A. R

B. $[0, \infty)$

C. $(-\infty, 0)$

D. $\mathbb{R} - (0)$

Answer:

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29. The set of points where the function $f(x) = |x - 1|e^x$ is differentiable, is

A. \mathbb{R}

B. $\mathbb{R} - [1]$

C. $\mathbb{R} - [-1]$

D. $\mathbb{R} - (0)$

Answer: B

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30. If $f(x) = (x + 1)^{\cot x}$ be continuous at $x = 0$, the $f(0)$ is equal to 0

(b) $\frac{1}{e}$ (c) e (d) *none of these*

A. 0

B. $1/e$

C. e

D. None of these

Answer:



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31. If $f(x) = \begin{cases} \frac{\sqrt{x+1}-1}{\sqrt{x}} & \text{for } x \neq 0 \\ k & \text{for } x = 0 \end{cases}$ and $f(x)$ is continuous at $x=0$,

then the value of k is

A. $a-b$

B. $a+b$

C. $\log a + \log b$

D. None of these

Answer:



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32. The function $f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & x \neq 0 \\ 0 & x = 0 \end{cases}$

A. is continuous at $x=0$

B. is not continuous at $x=0$

C. is not continuous at $x=0$, but can be made continuous at $x=0$

D. None of these

Answer:



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33. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & x < 4 \\ \frac{x-4}{|x-4|} + b, & x > 4 \end{cases}$ Then $f(x)$ is continuous at $x = 4$ when a. $a = 0, b = 0$ b. $a = 1, b = 1$ c. $a = -1, b = 1$ d. $a = -1, b = -1$

A. $a=0, b=0$

B. $a=1, b=1$

C. $a=-1, b=1$

D. $a=1, b=-1$

Answer:



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34. If the function $f(x) = \begin{cases} (\cos x)^{1/x} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at $x=0$,

then the value of k , is

A. 0

B. 1

C. -1

D. e

Answer:



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35. If the function $f(x) = |x| + |x - 1|$, then

A. $f(x)$ is continuous at $x=0$ as well as at $x=1$

B. $f(x)$ is continuous at $x=0$, but not at $x=1$

C. $f(x)$ is continuous at $x=1$, but not at $x=0$

D. None of these

Answer: A



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36.

Let

$$f(x) = \begin{cases} \frac{x^4 - 5x^2 + 4}{|(x-1)(x-2)|} & , \quad x \neq 1, 2 \\ 112 & , \quad x = 2 \end{cases}$$

. Then, $f(x)$ is continuous on the set R (b) $R - \{1\}$ (c) $R - \{2\}$ (d)

$$R - \{1, 2\}$$

A. R

B. $R - [1]$

C. $R - [2]$

D. $R - [1, 2]$

Answer:



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37. If the function f as defined below is continuous at $x=0$ find the values of

a, b

and

c

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \text{ and } c, & x = 0, \text{ and } \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}} \end{cases}$$

A. $a = -\frac{3}{2}, b = 0, c = \frac{1}{2}$

B. $a = -\frac{3}{2}, b = 1, c = -\frac{1}{2}$

C. $a = -\frac{3}{2}, b \in R - [0], c = \frac{1}{2}$

D. None of these

Answer:

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38. If $f(x) = \begin{cases} mx + 1 & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

A. $m=1, n=0$

B. $m = \frac{n\pi}{2} + 1$

C. $n = \frac{m\pi}{2}$

D. $m = n = \frac{\pi}{2}$

Answer: C

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39. The value of $f(0)$, so that the function

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}}$$
 becomes continuous for all

x , given by $a^{\frac{3}{2}}$ (b) $a^{\frac{1}{2}}$ (c) $-a^{\frac{1}{2}}$ (d) $-a^{\frac{3}{2}}$

A. $a^{3/2}$

B. $a^{1/2}$

C. $-a^{1/2}$

D. $-a^{3/2}$

Answer:



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40. The function $f(x) = \begin{cases} 1 & |x| > 1 \\ \frac{1}{n^2} & \frac{1}{n} < |x| < \frac{1}{n-1}, n = 2, 3 \\ 0 & x = 0 \end{cases}$

A. is discontinuous at finitely many points

B. is continuous everywhere

C. is discontinuous only at $x = \pm \frac{1}{n}, n \in \mathbb{Z} - (0)$ and $x = 0$

D. None of these

Answer:



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41. The value of $f(0)$, so that the function

$$f(x) = \frac{(27 - 2x)^2 - 3}{9 - 3(243 + 5x)^{1/5} - 2} (x \neq 0) \text{ is continuous, is given } \frac{2}{3} \text{ (b) } 6$$

(c) 2 (d) 4

A. $\frac{2}{3}$

B. 6

C. 2

D. 4

Answer:



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42. The value of $f(0)$ so that the function

$$f(x) = \frac{2 - (256 - 7x)^{\frac{1}{8}}}{(5x + 32)^{1/5} - 2}, x \neq 0$$
 is continuous everywhere, is given by

– 1 (b) 1 (c) 26 (d) none of these

A. – 1

B. 1

C. 26

D. None of these

Answer:



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43. The following functions are continuous on $(0, \pi)$ (a) $\tan x$

A. $\tan x$

B. $\int_0^x t \sin \frac{1}{t} dt$

C. $\begin{cases} -1 & 0 < x \leq \frac{3\pi}{4} \\ 2 \sin\left(\frac{2}{9}x\right) & \frac{3\pi}{4} < x < \pi \end{cases}$

D. $\begin{cases} x \sin x & 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x) & \frac{\pi}{2} < x < \pi \end{cases}$

Answer:



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44. If $f(x) = x \sin\left(\frac{1}{x}\right)$, $x \neq 0$ then the value of the function at $x=0$, so that the function is continuous at $x=0$ is

A. 1

B. -1

C. 0

D. intermediate

Answer:



45. Let $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in \mathbb{Z} \\ x^2, & x \in \mathbb{R} - \mathbb{Z} \end{cases}$ then (where $[.]$ denotes greatest integer function)

- A. $\lim_{x \rightarrow 1} f(x)$ exists, but $g(x)$ is not continuous at $x=1$
- B. $\lim_{x \rightarrow 1} g(x)$ does not exist and $f(x)$ is not continuous at $x = 1$
- C. $g \circ f$ is continuous for all x
- D. $f \circ g$ is continuous for all x

Answer:

46. Let $f(x) = \lim_{n \rightarrow \infty} (\sin x)^{2n}$. Then, which one of the following is incorrect?

- A. continuous at $x = \pi/2$

B. discontinuous at $x = \pi/2$

C. discontinuous at $x = -\pi/2$

D. discontinuous at infinite number of points

Answer:



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47. Let $f(x)$ be a function differentiable at $x=c$. Then $\lim_{x \rightarrow c} f(x)$ equals

A. $f'(c)$

B. $f''(c)$

C. $\frac{1}{f(c)}$

D. None of these

Answer:



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48. If $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ exists finitely, then

A. $\lim_{x \rightarrow c} f(x) = f(c)$

B. $\lim_{x \rightarrow c} f'(x) = f'(c)$

C. $\lim_{x \rightarrow c} f(x)$ does not exist

D. $\lim_{x \rightarrow c} f(x)$ may or may not exist

Answer:



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49. If $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1+x^2)} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then

A. $f(x)$ is not continuous at $x=0$

B. $f(x)$ is continuous and differentiable at $x=0$

C. $f(x)$ is continuous at $x=0$ but not differentiable at $x=0$

D. None of these

Answer:



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50. The function $f(x) = |x| + |x - 1|$, is

- A. continuous at $x=1$, but not differentiable
- B. both continuous and differentiable at $x=1$
- C. not continuous at $x=1$
- D. None of these

Answer: A



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51. For the function $f(x) = \begin{cases} |x - 3| & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & x < 1 \end{cases}$ which one of the following is incorrect

A. continuous at $x=1$,

B. derivable at $x=1$

C. continuous at $x=3$

D. derivable at $x=-3$

Answer:



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52. Let $f(x) = \begin{cases} x^n \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ Then $f(x)$ is continuous but not differentiable at $x=0$. If

A. $n \in (0, 1)$

B. $n \in [1, \infty)$

C. $n \in (-\infty, 0)$

D. $n = 0$

Answer:



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53. If $x + 4|y| = 6y$, then y as a function of x is

A. continuous at $x=0$

B. derivable at $x=0$

C. $\frac{dy}{dx} = \frac{1}{2}$ for all x

D. none of these

Answer: A



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54. If $f(x) = x^3 \operatorname{sgn}(x)$, then

A. f is derivable at $x=0$

B. f is continuous but not derivable at $x=0$

C. LHD at $x=0$ is 1

D. RHD at $x=0$ is 1

Answer: A



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55. The function $f(x) = \frac{\tan|\pi[x - \pi]|}{1 + [x]^2}$, where $[x]$ denotes the greatest integer less than or equal to x , is

- A. discontinuous at some x
- B. continuous at all, x but $f'(x)$ does not exist for some x
- C. $f'(x)$ exists for all x , but $f''(x)$ does not exist
- D. $f'(x)$ exists for all x

Answer:



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56. If $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, then

- A. f and f' are continuous at $x=0$
- B. f is derivable at $x=0$ and f' is continuous at $x=0$
- C. f is derivable at $x=0$ and f' is not continuous at $x=0$
- D. f is derivable at $x=0$

Answer:



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57. The following functions are differentiable on $(-1,2)$

A. $\int_x^{2x} (\log t)^2 dt$

B. $\int_x^{2x} \frac{\sin t}{t} dt$

C. $\int_x^{2x} \frac{1-t+t^2}{1+t+t^2} dt$

D. None of these

Answer: C

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58. $f(x) = \sqrt{x + 2\sqrt{2x - 4}} + \sqrt{x - 2\sqrt{2x - 4}}$ then

A. $(-\infty, \infty)$

B. $(2, \infty) - [4]$

C. $[2, \infty)$

D. None of these

Answer:

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59. The derivative of $f(x) = |x|^{3\alpha} \sin x = 0$, is

A. -1

B. 0

C. does not exist

D. None of these

Answer: B

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60. If $f(x) = x(\sqrt{x} + \sqrt{x+1})$, then

A. f is continuous but not differentiable at $x=0$

B. f is differentiable at $x=0$

C. f is differentiable but not continuous at $x=0$

D. f is not differentiable at $x=0$

Answer:

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61. Write the value of the derivative of $f(x) = |x - 1| + |x - 3|$ at $x = 2$

A. -2

B. 0

C. 2

D. does not exist

Answer: B



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62. If $f(x) = [x \sin \pi x]$, then which of the following, is incorrect,

A. $f(x)$ is continuous at $x=0$

B. $f(x)$ is continuous at $(-1,0)$

C. $f(x)$ is differentiable at $x=1$

D. $f(x)$ is differentiable in $(-1,1)$

Answer:



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63. The function $f(x) = 1 + |\sin x|$, is

A. continuous no where

B. continuous everywhere and not differentiable at infinitely many points

C. differentiable no where

D. differentiable at $x=0$

Answer: B



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64. If $f(x) = \begin{cases} 1 & x < 0 \\ 1 + \sin x & 0 \leq x < \frac{\pi}{2} \end{cases}$ then derivative of $f(x)$ at $x=0$

- A. is equal to 1
- B. is equal to 0
- C. is equal to -1
- D. does not exist

Answer:



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65. Let $[x]$ denotes the greatest integer less than or equal to x and $f(x) = [\tan^2 x]$. Then

- A. $f(x)$ does not exist at $x=0$
- B. $f(x)$ is continuous at $x=0$
- C. $f(x)$ is not continuous at $x=0$

D. $f'(0)=1$

Answer:



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66. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies the equation $f(x + y) = f(x)f(y)$, $\forall x, y$ in \mathbb{R} and $f(x) \neq 0$ for any x in \mathbb{R} . Let the function be differentiable at $x = 0$ and $f'(0) = 2$. Show that $f'(x) = 2f(x)$, $\forall x$ in \mathbb{R} . Hence, determine $f(x)$

A. $f(x)$

B. $-f(x)$

C. $2f(x)$

D. None of these

Answer:



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67. Let $f(x)$ be defined on \mathbb{R} such that $f(1) = 2$, $f(2) = 8$ and $f(u + v) = f(u) + kuv - 2v^2$ for all $u, v \in \mathbb{R}$ (k is a fixed constant). Then,

A. $f'(x) = 8x$

B. $f(x) = 8x$

C. $f'(x) = x$

D. None of these

Answer:



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68. Let $f(x)$ be a function satisfying $f(x + y) = f(x) + f(y)$ and $f(x) = xg(x)$ For all $x, y \in \mathbb{R}$, where $g(x)$ is continuous. Then,

A. $f'(x) = g'(x)$

B. $f'(x) = g(x)$

C. $f'(x) = g(0)$

D. None of these

Answer:

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69. If $f(x) = \begin{cases} ax^2 - b & |x| < 1 \\ \frac{1}{|x|} & |x| \geq 1 \end{cases}$ is differentiable at $x=1$, then

A. $a = \frac{1}{2}, b = -\frac{1}{2}$

B. $a = -\frac{1}{2}, b = -\frac{3}{2}$

C. $a = b = \frac{1}{2}$

D. $a = b = -\frac{1}{2}$

Answer: B

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70. If $f(x) = (x - x_0)\phi(x)$ and $\phi(x)$ is continuous at $x=x_0$. Then $f'(x_0)$ is equal to

A. $\phi'(x_0)$

B. $\phi(x_0)$

C. $x_0\phi(x_0)$

D. None of these

Answer:



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71. If $f(x + y) = f(x) \times f(y)$ for all $x, y \in R$ and $f(5) = 2, f'(0) = 3$, then $f'(5) =$

A. 6

B. 3

C. 5

D. None of these

Answer:



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72. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function given by $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$. If $f'(0) = 2$ then $f(x)$ is equal to`

A. 4

B. 1

C. $1/2$

D. 8

Answer:



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73. Let $f(x + y) = f(x)f(y)$ for all $x, y, \in R$, suppose that $f(3) = 3$ and $f'(0) = 2$ then $f'(3)$ is equal to-

A. 22

B. 44

C. 28

D. None of these

Answer:



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74. Let $f(x + y) = f(x) + f(y)$ and $f(x) = x^2g(x) \forall x, y \in R$ where $g(x)$ is continuous then $f'(x)$ is

A. $g'(x)$

B. $g(0)$

C. $g(0)+g'(x)$

D. 0

Answer:



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75. Let $f(x)$ be a function satisfying $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$ and $f(x) = 1 + xg(x)$ where $\lim_{x \rightarrow 0} g(x) = 1$. Then $f'(x)$ is equal to

A. $g'(x)$

B. $g(x)$

C. $f(x)$

D. None of these

Answer:



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76. Let $f(x + y) = f(x)f(y)$ and $f(x) = 1 + (\sin 2x)g(x)$ where $g(x)$ is continuous. Then, $f'(x)$ equals

A. $1+ab$

B. ab

C. a/b

D. None of these

Answer:



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77. Let $f(x + y) = f(x)f(y)$ and $f(x) = 1 + (\sin 2x)g(x)$ where $g(x)$ is continuous. Then, $f'(x)$ equals

A. $f(x)g(0)$

B. $2f(x)g(0)$

C. $2g(0)$

D. None of these

Answer:



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78. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x = c$. Then $g'(f(x))$ equal. $f'(c)$ (b) $\frac{1}{f'(c)}$ (c) $f(c)$ (d) none of these

A. $f'(c)$

B. $\frac{1}{f'(c)}$

C. $f(c)$

D. None of these

Answer:



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79. Let $g(x)$ be the inverse of the function $f(x)$, and $f'(x) = \frac{1}{1+x^3}$

then $g'(x)$ equals

A. $\frac{1}{1+(g(x))^3}$

B. $\frac{1}{1+(f(x))^3}$

C. $1+(g(x))^3$

D. $1+(f(x))^3$

Answer:



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80. Let $f(x) = \begin{cases} x^n \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ Then $f(x)$ is continuous but not

differentiable at $x=0$. If

A. $n \in (0, 1]$

B. $n \in [1, \infty)$

C. $n \in (1, \infty)$

D. $n \in (-\infty, 0)$

Answer:



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81. If for a continuous function f , $f(0) = f(1) = 0$, $f'(1) = 2$ and $y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to a. 1 b. 2 c. 0 d. none of these

A. 1

B. 2

C. 0

D. None of these

Answer:



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82. Let $f(x)$ be a function such that

$f(x + y) = f(x) + f(y)$ and $f(x) = \sin x g(x)$ for all $x, y \in R$. If

$g(x)$ is a continuous function such that $g(0)=k$, then $f'(x)$ is equal to

A. k

B. kx

C. $kg(x)$

D. None of these

Answer:



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83. Let $f: (0, \pi) \rightarrow R$ be defined as

$$f(x) = \begin{cases} \frac{1 - \sin x}{(\pi - 2x)^2} \cdot \frac{\ln \sin x}{(\ln(1 + \pi^2 - 4\pi x + 4x^2))} & x \neq \frac{\pi}{2} \\ k & x = \frac{\pi}{2} \end{cases}$$

If a continuous at

$x = \frac{\pi}{2}$, then the value of $8\sqrt{|k|}$, is

A. 1

B. 2

C. 3

D. 4

Answer:



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84. If $f(x) = \frac{e^{2x} - (1 + 4x)^{1/2}}{\ln(1 - x^2)}$ for $x \neq 0$, then f has

A. an irremovable discontinuity at $x=0$

B. a removable discontinuity at $x=0$ and $f(0)=-4$

C. a removable discontinuity at $x=0$ and $f(0) = -\frac{1}{4}$

D. a removable discontinuity at $x=0$ and $f(0) = 4$

Answer:



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85. Let $f(x) = \begin{cases} \frac{e^{x^2} - \frac{2}{\pi} \sin^{-1} \sqrt{1-x}}{\ln(1+\sqrt{x})} & x \in (0, 1) \\ k & x \leq 0 \end{cases}$ be a continuous at $x=0$,

then the value of k , is

A. $1 + \frac{2}{\pi}$

B. $1 - \frac{2}{\pi}$

C. $\frac{2}{\pi}$

D. $-\frac{2}{\pi}$

Answer: C



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86. Let $f(x) = \begin{cases} x^3 & x < 1 \\ ax^2 + bx + c & : x \geq 1 \end{cases}$. If $f''(1)$ exists, then the value of $(a^2 + b^2 + c^2)$ is

A. 20

B. 21

C. 19

D. 17

Answer:



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