MATHS

BOOKS - OBJECTIVE RD SHARMA MATHS VOL I (HINGLISH)

SCALAR AND VECTOR PRODUCTS OF THREE VECTORS

Illustration

1. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors. Then scalar triple product $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ is equal to

A. $\left[\vec{b}\quad\vec{a}\quad\vec{c}\right]$

B. $\left[\vec{a}\,\vec{c}\,\vec{b}\right]$

C. $\left[\vec{c}\,\vec{b}\,\vec{a}\right]$

D. $\left[\vec{b}\,\vec{c}\,\vec{a}\right]$

Answer: D

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2. If $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=1$ then value of $\frac{\vec{a}\,.\,\vec{b}\times\vec{c}}{\vec{c}\times\vec{a}\,.\,\vec{b}}+\frac{\vec{b}\,.\,\vec{c}\times\vec{a}}{\vec{a}\times\vec{b}\,.\,\vec{c}}+\frac{\vec{c}\,.\,\vec{a}\times\vec{b}}{\vec{b}\times\vec{c}\,.\,\vec{a}}$ is

A. 3

B. 1

C. -1

D. None of these

Answer: A

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3. If $\vec{u}, \vec{v}, \vec{w}$ are three vectors such that $\left[\vec{u}\,\vec{v}\,\vec{w}\right] = 1$, then

$3\left[\vec{u}\,\vec{v}\,\vec{w}\right] - \left[\vec{v}\,\vec{w}\,\vec{u}\right] - 2\left[\vec{w}\,\vec{v}\,\vec{u}\right] =$

A. 0

B. 2

C. 3

D. 4

Answer: D

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4. If $\vec{r} = x\left(\vec{a} \times \vec{b}\right) + y\left(\vec{b} \times \vec{c}\right) + z\left(\vec{c} + \vec{a}\right)$

Such that $x + y + z \neq 0$ and $\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right) = x + y + z$, then

$\left[\vec{a}\,\vec{b}\,\vec{c}\right] =$

A. 0

B. 1

C. -1

D. 2

Answer: B

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5. If $\vec{\alpha} = x\left(\vec{a} \times \vec{b}\right) + y\left(\vec{b} \times \vec{c}\right) + z\left(\vec{c} \times \vec{a}\right)$ and $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \frac{1}{8}$, then $x + y + z =$

A. $8\vec{\alpha}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

B. $\vec{\alpha}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

C. $8\left(\vec{a} + \vec{b} + \vec{c}\right)$

D. None of these

Answer: A

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6. If $\overrightarrow{a} = 2\hat{i} + 3\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\overrightarrow{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, then $\left[\overrightarrow{a}\overrightarrow{b}\overrightarrow{c}\right] =$

A. 0.3

B. -0.3

C. 0.15

D. -0.15

Answer: B

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7. Let $\overrightarrow{a} = \hat{i} - \hat{k}, \overrightarrow{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\overrightarrow{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$, then $\left[\overrightarrow{a}\overrightarrow{b}\overrightarrow{c}\right]$ depends on

A. neither x nor y

B. both x and y

C. only x

D. only y

Answer: A

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8. Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$, is

A. π

B. $\pi/2$

C. $\pi/3$

D. $\pi/4$

Answer: A

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9. Let $\overline{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overline{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overline{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overline{PT}, \overline{PQ}$ and $\overline{PS}$ is

A. 5

B. 20

C. 10

D. 30

Answer: A

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10. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}, \lambda\vec{b} + 4\vec{\ }$ and $(2\lambda - 1)\vec{c}$ are non coplanar for

A. no value of λ

B. all except one value of λ

C. all except two values of λ

D. all values of λ

Answer: C

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11. The points with position vectors $\alpha\hat{i} + \hat{j} + \hat{k}, \hat{i} - \hat{j} - \hat{k}, \hat{i} + 2\hat{j} - \hat{k}, \hat{i} + \hat{j} + \beta\hat{k}$ are coplanar if

A. $(1-\alpha)(1+\beta) = 0$

B. $(1-\alpha)(1-\beta) = 0$

C. $(1+\alpha)(1+\beta) = 0$

D. $(1+\alpha)(1-\beta) = 0$

Answer: A

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12. The number of distinct real values of λ for which the vectors $\overrightarrow{a}=\lambda^3\hat{i}+\hat{k}, \overrightarrow{b}=\hat{i}-\lambda^3\hat{j}$ and $\overrightarrow{c}=\hat{i}+(2\lambda-\sin\lambda)\hat{i}-\lambda\hat{k}$ are coplanar is

A. 0

B. 1

C. 1

D. 3

Answer: B

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13. Let $\overrightarrow{a}=\hat{i}+\hat{j}+\hat{k}, \overrightarrow{b}=\hat{i}-\hat{j}+\hat{2}k$ and $\overrightarrow{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vector $\overrightarrow{c}$ lies in the plane of $\overrightarrow{a}$ and $\overrightarrow{b}$ then x equals

A. -4

B. -2

C. 0

D. 1

Answer: B

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14. If $\vec{u}$, $\vec{v}$, $\vec{w}$ are non -coplanar vectors and $p, q,$ are real numbers then the equality

$\left[3\vec{u}\,p\vec{v}\,p\vec{w}\right] - \left[p\vec{v}\,\vec{w}\,q\vec{u}\right] - \left[2\vec{w} - q\vec{v}\,q\vec{u}\right] = 0$ holds for

A. exactly one value of (p, q)

B. exactly two values of (p, q)

C. more than two but not all values of (p, q)

D. all values of (p, q)

Answer: A

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15. The value of $\vec{a}.\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+\vec{b}+\vec{c}\right)$, is

A. $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

B. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

C. 0

D. None of these

Answer: C

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16. The vectors

$\vec{a}=x\hat{i}+(x+1)\hat{j}+(x+2)\hat{k}$,

$\vec{b}=(x+3)\hat{i}+(x+4)\hat{j}+(x+5)\hat{k}$

and $\vec{c}=(x+6)\hat{i}+(x+7)\hat{j}+(x+8)\hat{k}$ are coplanar for

A. all values of x

B. $x < 0$ only

C. $x > 0$ only

D. None of these

Answer: A

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17. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number, then $\left[\lambda\left(\vec{a}+\vec{b}\right) \quad \lambda^2\vec{b} \quad \lambda\vec{c}\right] = \left[\vec{a} \quad \vec{b}+\vec{c} \quad \vec{b}\right]$ for

A. exactly two values of λ

B. exactly two values of λ

C. no value of λ

D. exacty one value of λ

Answer: C

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18. The number of real values of a for which the vectors $\hat{i}+2\hat{j}+\hat{k}, a\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+a\hat{k}$ are coplanar is

A. 1

B. 2

C. 3

D. 0

Answer: A

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19. The number of distinct values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}, \hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is

A. 0

B. 1

C. 2

D. 3

Answer: C

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20. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then

$$\left[2\vec{a} - 3\vec{b} \quad 7\vec{b} - 9\vec{c} \quad 10\vec{c} - 23\vec{a}\right]$$

A. 0

B. $\frac{1}{2}$

C. 24

D. 32

Answer: A

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21. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non -coplanar and l, m, n are distinct scalars such that

$$\left[l\vec{a}+m\vec{b}+n\vec{c} \quad l\vec{b}+m\vec{c}+n\vec{a} \quad l\vec{c}+m\vec{a}+n\vec{b}\right]=0$$ then

A. $lm+mn+nl=0$

B. $l+m+n=0$

C. $l^2+m^2+n^2=0$

D. $l^3+m^3+n^3=0$

Answer: B

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22. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the value of $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right]$ is

A. 0

B. $2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

C. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

D. $-\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

Answer: B

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23. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the value of $\left[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}\right]$, is

A. 0

B. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

C. $-\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

D. $-2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

Answer: A

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24. If $\vec{u}, \vec{v}, \vec{w}$ are three non-coplanar vectors, the $\left(\vec{u} + \vec{v} - \vec{w}\right).\left(\vec{u} - \vec{v}\right) \times \left(\vec{v} - \vec{w}\right)$ equals

A. $\vec{u}.\left(\vec{v} \times \vec{w}\right)$

B. $\vec{u}.\left(\vec{w} \times \vec{v}\right)$

C. $3\vec{u}.\left(\vec{c} \times \vec{w}\right)$

D. 0

Answer: A

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25. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a} - \vec{b} \quad 2\vec{b} - \vec{c} \quad 2\vec{c} - \vec{a}\right] =$

A. 0

B. 1

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: A

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26. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero non coplanar vectors and $\vec{p}, \vec{q}$ and $\vec{r}$ be three vectors given by $\vec{p} = \vec{a} + \vec{b} - 2\vec{c}, \vec{q} = 3\vec{a} - 2\vec{b} + \vec{c}$ and $\vec{r} = \vec{a} - 4vcb + 2\vec{c}$

If the volume of the parallelopiped determined by $\vec{a}, \vec{b}$ and $\vec{c}$ is V_1 and that of the parallelopiped determined by $\vec{a}, \vec{q}$ and $\vec{r}$ is V_2, then $V_2 : V_1 =$

A. $3:1$

B. $7:1$

C. $11:1$

D. $15:1$

Answer: D

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27. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero non-null vectors are $\vec{r}$ is any vector in space then

$\begin{bmatrix} \vec{b} & \vec{c} & \vec{r} \end{bmatrix}\vec{a} + [\vec{c} \quad \vec{a} \quad \vec{r}]\vec{b} + \begin{bmatrix} \vec{a} & \vec{b} & \vec{r} \end{bmatrix}\vec{c}$ is equal to

A. $2\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}\vec{r}$

B. $3\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}\vec{r}$

C. $\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$

D. None of these

Answer: C

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28. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vetors represented by non-current edges of a parallelopiped of volume 4 units, then the value of

$$\left(\vec{a}+\vec{b}\right).\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}+\vec{c}\right).\left(\vec{c}\times\vec{a}\right)+\left(\vec{c}+\vec{a}\right).\left(\vec{a}\times\vec{b}\right)$$

, is

A. 12

B. 4

C. ± 12

D. 0

Answer: C

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29. The three concurrent edges of a parallelopiped represent the vectors $\vec{a}, \vec{b}, \vec{c}$ such that $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=V$. Then the volume of the parallelopiped whose three concurrent edges are the three diagonals of three faces of the given parallelopiped is

A. $2V$

B. $3V$

C. V

D. $6V$

Answer: A

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30. The edges of a parallelopiped are of unit length and a parallel to non-coplanar unit vectors $\widehat{a}, \hat{b}, \hat{c}$ such that $\widehat{a}. \hat{b} = \hat{b}. \hat{c} = \hat{c}. \overrightarrow{a} = 1/2$. Then the volume of the parallelopiped in cubic units is

A. $\dfrac{1}{\sqrt{2}}$

B. $\dfrac{1}{2\sqrt{2}}$

C. $\dfrac{\sqrt{3}}{2}$

D. $\dfrac{1}{\sqrt{3}}$

Answer: A

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31. Let $\vec{a}, \vec{b}$, and $\vec{c}$ be three non coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{x} = p\vec{a} + q\vec{b} + r\vec{c}$ where p,q,r are scalars then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

A. 2

B. 4

C. 6

D. 8

Answer: B

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32. The volume of the tetrahedron whose vertices are the points $\hat{i}, \hat{i}+\hat{j}, \hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+3\hat{j}+\lambda\hat{k}$ is $1/6$ units,

Then the values of λ

A. does not exist

B. is 7

C. is -1

D. is any real value

Answer: D

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33. Let G_1, G_2, G_3 be the centroids of the triangular faces OBC, OCA, OAB of a tetrahedron $OABC$. If V_1 denote the volume of the tetrahedron $OABC$ and V_2 that of the parallelopiped with OG_1, OG_2, OG_3 as three concurrent edges, then

A. $4V_1 = 9V_2$

B. $9V_1 = 4V_2$

C. $3V_1 = 2V_2$

D. $3V_2 = 2V_1$

Answer: A

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34. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the value of $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times \left(\vec{c} \times \vec{a}\right) + \vec{c} \times \left(\vec{a} \times \vec{b}\right)$, is

A. $\vec{0}$

B. $\left[\vec{a} \ \vec{b} \ \vec{c}\right]\vec{a}$

C. $\left[\vec{a} \ \vec{b} \ \vec{c}\right]\vec{b}$

D. $\left[\vec{a} \ \vec{b} \ \vec{c}\right]\vec{c}$

Answer: A

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35. Let $\vec{a}, \vec{b}, \vec{c}$ be any three vectors.Then vectors $\vec{u} = \vec{a} \times \left(\vec{b} \times \vec{c}\right), \vec{v} = \vec{b} \times \left(\vec{c} \times \vec{a}\right)$ and $\vec{w} = \vec{c} \times \left(\vec{a} \times \vec{b}\right)$ are such that they are

A. collinear

B. non-coplanar

C. coplanar

D. none of these

Answer: C

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36. For an vector $\vec{a}$ the value of

$\hat{i} \times \left(\vec{a} \times \vec{i}\right) + \hat{j} \times \left(\vec{a} \times \hat{j}\right) + \hat{k} \times \left(\vec{a} \times \vec{k}\right)$, is

A. $\vec{a}$

B. $2\vec{a}$

C. $3\vec{a}$

D. $\vec{0}$

Answer: B

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37. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: D

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38. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b} \times \left(\vec{c} \times \vec{a}\right)$ and $\left[\vec{,} \vec{b} \quad \vec{c}\right] \neq 0$ then $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is equal to

A. $\vec{0}$

B. $\vec{a} \times \vec{b}$

C. $\vec{b} \times \vec{c}$

D. $\vec{c} \times \vec{a}$

Answer: A

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39. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, then

$\left[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}\right] =$

A. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

B. $2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

C. $3\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

D. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$

Answer: D

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40. If $\left[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}\right] = \lambda\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$, then λ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: B

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41. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar non null vectors such that $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 2$ then $\left\{\left[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}\right]\right\}^2 =$

A. 4

B. 16

C. 8

D. none of these

Answer: B

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42. If $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a}.\vec{b} \neq 0, \vec{b}.\vec{c} \neq 0$ then $\vec{a}$ and $\vec{c}$ are

A. inclined at angle $\frac{\pi}{3}$ between them

B. inclined at angle of $\frac{\pi}{6}$ between them

C. perpendicular

D. parallel

Answer: D

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43. Unit vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ are coplanar. A unit vector $\overrightarrow{d}$ is perpendicular to them.If $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \left(\overrightarrow{c} \times \overrightarrow{d}\right) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{j}k$ and the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is 30°, then $\overrightarrow{c}$ is/are

A. $\frac{1}{3}\left(-2\hat{i} - 2\hat{j}\hat{k}\right)$

B. $\pm\frac{1}{3}\left(-\hat{i} - 2\hat{j} + 2\hat{k}\right)$

C. $\frac{1}{3}\left(2\hat{i} + \hat{j} - \hat{k}\right)$

D. $\pm\frac{1}{3}\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right)$

Answer: D

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44. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a non zero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$ then

A. $\vec{b} = \left(\vec{b} \cdot \vec{z}\right)\left(\vec{z} - \vec{x}\right)$

B. $\vec{a} = \left(\vec{a} \cdot \vec{y}\right)\left(\vec{y} - \vec{z}\right)$

C. $\vec{a} \cdot \vec{b} = -\left(\vec{a} \cdot \vec{y}\right)\left(\vec{b} \cdot \vec{z}\right)$

D. $\vec{a} = \left(\vec{a} \cdot \vec{y}\right)\left(\vec{z} - \vec{y}\right)$

Answer: A::B::C

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45. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ form a reciprocal system of vectors then

$\vec{a}.\vec{a}' + \vec{b}.\vec{b}' + \vec{c}.\vec{c}' =$

A. 0

B. 1

C. 2

D. 3

Answer: D

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46. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ form a reciprocal system of vectors then

$\vec{a}.\vec{a}' + \vec{b}.\vec{b}' + \vec{c}.\vec{c}' =$

A. $\vec{0}$

B. $\vec{a} \times b$

C. $\vec{b} \times \vec{c}$

D. $\vec{c} \times \vec{a}$

Answer: A

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47. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ form a reciprocal system of vectors then $\left[\vec{a}' \quad \vec{b}' \quad \vec{c}'\right] =$

A. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

B. $\dfrac{1}{\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]}$

C. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$

D. $\dfrac{-1}{\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]}$

Answer: B

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48. If $\overrightarrow{a} = -\hat{i} + \hat{j} + \hat{k}, \hat{b} = 2\hat{i} + 0\hat{j} + \hat{k}$, then a vector $\overrightarrow{X}$ satisfying the conditions:

(i) that it is coplanar with $\overrightarrow{a}$ and $\overrightarrow{b}$. (ii) that is perpendicular to $\overrightarrow{b}$

(iii) that $\overrightarrow{a} . \overrightarrow{X} = 7$, is

A. $-3\hat{i} + 5\hat{j} + 6\hat{k}$

B. $\frac{1}{2}\left(-3\hat{i} + 5\hat{j} + 6\hat{k}\right)$

C. $3\hat{i} - 5\hat{j} + 6\hat{k}$

D. $\frac{1}{2}\left(3\hat{i} + 5\hat{j} - 6\hat{k}\right)$

Answer: B

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49. A solution of the vector equation $\overrightarrow{r} \times \overrightarrow{b} = \overrightarrow{a} \times \overrightarrow{b}$, where $\overrightarrow{a}, \overrightarrow{b}$ are two given vectors is

where λ is a parameter.

A. $\vec{r} = \lambda \vec{b}$

B. $\vec{r} = \vec{a} + \lambda \vec{b}$

C. $\vec{r} = \vec{b} + \lambda \vec{a}$

D. $\vec{r} = \lambda \vec{a}$

Answer: B

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50. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then a vector $\vec{r}$ satisfying $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 1$, is

A. $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$

B. $\dfrac{1}{\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]}\left\{\vec{a} \times \vec{b} + \vec{b} \times \overrightarrow{+} \vec{c} \times \vec{a}\right\}$

C. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]\left\{\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \overrightarrow{\times} \vec{a}\right\}$

D. none of these

Answer: B

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Section I Solved Mcqs

1. Which of the following expressions are meaningful?

A. $\vec{u}.\left(\vec{v}\times\vec{w}\right)$

B. $\left(\vec{u}.\vec{v}\right).\vec{w}$

C. $\left(\vec{u}.\vec{v}\right)\vec{w}$

D. $\vec{u}\times\left(\vec{v}.\vec{w}\right)$

Answer: A::C

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2. For three vectors $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not eqal to any of the remaining three?

A. $\vec{u}.\left(\vec{v}\times\vec{w}\right)$

B. $\left(\vec{u}\times\vec{w}\right).\vec{u}$

C. $\vec{v}.\left(\vec{u}\times\vec{w}\right)$

D. $\left(\vec{u}\times\vec{v}\right).\vec{w}$

Answer: C

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3. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $\left|\vec{c}\right|=\sqrt{3}$ then

A. $\alpha=1, \beta=-1$

B. $\alpha=1, \beta=\pm 1$

C. $\alpha=-1, \beta=\pm 1$

D. $\alpha=\pm 1, \beta=1$

Answer: D

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4. The volume of the tetrahedron whose vertices are the points with positon vectors $\hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+7\hat{k}, 5\hat{i}-\hat{j}+\hat{k}$ and $7\hat{i}-4\hat{j}+7\hat{k}$ is 11 cubic units if the value of λ is

A. $-1, 7$

B. $1, 7$

C. -7

D. $-1, -7$

Answer: B

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5. If a vector $\overrightarrow{a}$ is expressed as the sum of two vectors $\overrightarrow{\alpha}$ and $\overrightarrow{\beta}$ along and perpendicular to a given vector $\overrightarrow{b}$ then $\overrightarrow{\beta}$ is equal to

A. $\dfrac{\left(\vec{a} \times \vec{b}\right) \times \vec{b}}{\left|\vec{b}\right|^2}$

B. $\dfrac{\vec{b} \times \left(\vec{a} \times \vec{b}\right)}{\left|\vec{b}\right|^2}$

C. $\dfrac{\vec{b} \times \left(\vec{a} \times \vec{b}\right)}{\left|\vec{b}\right|}$

D. $\left\{\dfrac{\vec{a} \,.\, \vec{b}}{\left(\left|\vec{b}\right|\right)^2}\right\}\vec{b}$

Answer: B

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6. $\vec{a}$ and $\vec{b}$ are two given vectors. On these vectors as adjacent sides a parallelogram is constructed. The vector which is the altitude of the parallelogam and which is perpendicular to $\vec{a}$ is not equal to

A. $\left\{\frac{\left(\vec{a}\cdot\vec{b}\right)}{\left|\vec{a}\right|^2}\right\}\vec{a}-\vec{b}$

B. $\frac{1}{\left|\vec{a}\right|^2}\left\{\left(\vec{a}\cdot\vec{b}\right)\vec{a}-\left(\vec{a}\cdot\vec{a}\right)\vec{b}\right\}$

C. $\frac{\vec{a}\times\left(\vec{a}\times\vec{b}\right)}{\left|\vec{a}\right|^2}$

D. $\frac{\vec{a}\times\left(\vec{b}\times\vec{a}\right)}{\left|\vec{b}\right|^2}$

Answer: D

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7. Let $\hat{a}$ be a unit vector and $\hat{b}$ a non zero vector non parallel to $\vec{a}$. Find the angles of the triangle tow sides of which are represented by the vectors. $\sqrt{3}\left(\widehat{\times}\vec{b}\right)$ and $\vec{b}-\left(\hat{a}.\vec{b}\right)\hat{a}$

A. $\pi/4, \pi/4, \pi/2$

B. $\pi/4, \pi/3, \pi/12$

C. $\pi/6, \pi/3, \pi/2$

D. none of these

Answer: C

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8. The three vectors $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume:

A. $\frac{1}{3}$

B. 4

C. $\frac{3\sqrt{3}}{4}$

D. $\frac{4}{3\sqrt{3}}$

Answer: D

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9. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}. \overrightarrow{=} \left|\vec{c}\right|, \left|\vec{c} - \vec{a}\right| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30°, then $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right| = .$

A. $2/3$

B. $3/2$

C. 2

D. 3

Answer: B

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10. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - \left(\vec{a}.\vec{b}\right)\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $\left|\vec{v}\right|$ is

A. $\left|\vec{u}\right| + \left|\vec{u}.\left(\vec{a} \times \vec{b}\right)\right|$

B. $\left|\overrightarrow{u}\right| + \left|\overrightarrow{u}.\overrightarrow{a}\right|$

C. $\left|\overrightarrow{u}\right| + \left|\overrightarrow{u}.\overrightarrow{b}\right|$

D. $\left|\overrightarrow{u}\right| + \overrightarrow{u}.\left(\overrightarrow{a} + \overrightarrow{b}\right)$

Answer: C

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11. If the vectots $p\hat{i} + \hat{j} + \hat{k}, \hat{i} + q\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + r\hat{k}(p \neq q \neq r \neq 1)$ are coplanar, then the value of $pqr - (p + q + r)$, is

A. 0

B. -1

C. -2

D. 2

Answer: C

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12. If $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \perp \vec{a}$ then $\vec{r}$ is equal to

A. $\dfrac{\vec{a} \times \left(\vec{c} \times \vec{b}\right)}{\vec{a} \, . \, \vec{b}}$

B. $\dfrac{\vec{b} \times \left(\vec{a} \times \vec{c}\right)}{\vec{a} \, . \, \vec{b}}$

C. $\dfrac{\vec{c} \times \left(\vec{a} \times \vec{b}\right)}{\vec{a} \, . \, \vec{b}}$

D. $\dfrac{\vec{c} \times \left(\vec{a} \times \vec{b}\right)}{\vec{b} \, . \, \vec{c}}$

Answer: A

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13. If $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\left(\vec{a} + \vec{b}\right). \vec{c} = \left(\vec{a} - \vec{b}\right) = \vec{c} = 0$ then $\left(\vec{a} \times \vec{b}\right) \times \vec{c}$ is

A. $\vec{0}$

B. $\vec{a}$

C. $\vec{b}$

D. none of these

Answer: A

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14. Let $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$. Then , the value of λ for which the vector $\vec{c} = \lambda\hat{i} + \hat{j} + (2\lambda - 1)\hat{k}$ is parallel to the plane containing $\vec{a}$ and $\vec{b}$. Is

A. 1

B. 0

C. -1

D. 2

Answer: B

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15. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a} . \vec{b} = \vec{a} . \vec{c} = 0$, If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$ then the volume of the parallelopiped whose three coterminous edges are $\vec{a}, \vec{b}, \vec{c}$ is

A. $\frac{\sqrt{3}}{2}$ cubic units

B. $\frac{1}{2}$ cubit unit

C. 1 cubic unit

D. none of these

Answer: A

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16. If $\vec{a}, \vec{b}, \vec{c}$ are three non coplanar, non zero vectors then $\left(\vec{a} . \vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{a} . \vec{b}\right)\left(\vec{c} \times \vec{a}\right) + \left(\vec{a} . \vec{c}\right)\left(\vec{a} \times \vec{b}\right)$ is equal to

A. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]\vec{c}$

B. $\left[\vec{b} \quad \vec{c} \quad \vec{a}\right]\vec{a}$

C. $\left[\vec{c} \quad \vec{a} \quad \vec{b}\right]\vec{b}$

D. none of these

Answer: B

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17. If the acute angle that the vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ makes with the plane of the two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ is $\dfrac{\tan^{-1} 1}{\sqrt{2}}$ then

A. $\alpha(\beta + \gamma) = \beta\gamma$

B. $\beta(\gamma + \alpha) = \gamma\alpha$

C. $\gamma(\alpha + \beta) = \alpha\beta$

D. $\alpha\beta = \beta\gamma + \gamma\alpha = 0$

Answer: A

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18. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are reciprocal vectors, then

$\left(l\vec{a} + m\vec{b} + n\vec{c}\right).\left(l\vec{p} + m\vec{q} + n\vec{r}\right)$ is equal to

A. $l^2 + m^2 + n^2$

B. $lm + mn + nl$

C. 0

D. none of these

Answer: A

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19. If $\vec{a}\,\vec{b}$ are non zero and non collinear vectors, then $\left[\vec{a}\ \vec{b}\ \vec{i}\right]\hat{i} + \left[\vec{a}\ \vec{b}\ \vec{j}\right]\hat{j} + \left[\vec{a}\ \vec{b}\ \vec{k}\right]\hat{k}$ is equal to

A. $\vec{a} + \vec{b}$

B. $\vec{a} \times \vec{b}$

C. $\vec{a} - \vec{b}$

D. $\vec{b} \times \vec{a}$

Answer: B

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20. If $\vec{r}$ is a unit vector such that

$\vec{r} = x\left(\vec{b} \times \vec{c}\right) + y\left(\vec{c} \times \vec{a}\right) + z\left(\vec{a} \times \vec{b}\right)$, then

$\left|\left(\vec{r}.\vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{r}.\vec{b}\right)\left(\vec{c} \times \vec{a}\right) + \left(\vec{r}.\vec{c}\right)\left(\vec{c} \times \vec{b}\right)\right|$ is equal to

A. $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$

B. 1

C. $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$

D. 0

Answer: A

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21. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\left[\vec{a} \ \vec{b} \ \vec{c}\right] = 2$. If $\vec{r} = l\left(\vec{b} \times \vec{c}\right) + m\left(\vec{c} \times \vec{a}\right) + n\left(\vec{a} \times \vec{b}\right)$ be perpendicular to $\vec{a} + \vec{b} + \vec{c}$, then the value of $l + m + n$ is

A. 2

B. 1

C. 0

D. none of these

Answer: C

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22. If $\vec{b}$ is a unit vector, then $\left(\vec{a}\,.\,\vec{b}\right)\vec{b}+\vec{b}\times\left(\vec{a}\times\vec{b}\right)$ is a equal to

A. $\left|\vec{a}\right|^2\vec{b}$

B. $\left(\vec{a}\,.\,\vec{b}\right)\vec{a}$

C. $\vec{a}$

D. $\left(\vec{a}\,.\,\vec{b}\right)\vec{b}$

Answer: C

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23. If $\vec{a},\vec{b},\vec{c}$ are any three non coplanar vectors, then $\left[\vec{a}+\vec{b}+\vec{c}\quad\vec{a}-\vec{c}\quad\vec{a}-\vec{b}\right]$ is equal to

A. 0

B. $\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$

C. $2\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$

D. $= 3\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

Answer: D

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24. If $\vec{a}, \vec{b}, \vec{c}$ are any three non coplanar vectors, then

$\left(\vec{a} + \vec{b} + \vec{c}\right).\left(\vec{b} + \vec{c}\right) \times \left(\vec{c} + \vec{a}\right)$

A. 0

B. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

C. $2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

D. $3\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

Answer: B

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25. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three having magnitude 1,1 and 2 respectively such that $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = \vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: C

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26. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} . \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$ then $\vec{b}$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{j} - \hat{k}$

C. $\hat{i}$

D. $2\hat{i}$

Answer: C

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27. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar non-zero vectors, then

$$\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{b} \times \vec{a}\right) + \vec{c} \times \vec{a}\right) \times \left(\vec{\times}$$

is equal to

A. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2 \left(\vec{a} + \vec{b} + \vec{c}\right)$

B. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] \left(\vec{a} + \vec{b} + \vec{c}\right)$

C. $\vec{0}$

D. none of these

Answer: B

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28. If the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are coplanar vectors, then $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right)$ is equal to

A. $\vec{a} + \vec{b} + \vec{c} + \vec{d}$

B. $\vec{0}$

C. $\vec{a} + \vec{b} = \vec{c} + \vec{d}$

D. none of these

Answer: B

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29. $\left(\vec{a} \times \vec{b}\right) . \left(\vec{c} \times \vec{d}\right)$ is not equal to

A. $\vec{a} . \left\{\vec{b} \times \left(\vec{c} \times \vec{d}\right)\right\}$

B. $\left\{\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right\}\vec{d}$

C. $\left(\vec{d} \times \vec{c}\right) . \left(\vec{b} \times \vec{a}\right)$

D. $\left(\vec{a} . \vec{c}\right)\left(\vec{b} . \vec{d}\right) - \left(\vec{a} . \vec{d}\right)\left(\vec{b} . \vec{c}\right)$

Answer: B

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30. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} . \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30°. Then $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right)$ is equal to

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. 2

D. 3

Answer: B

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31. If $\vec{a}, \vec{b}, \vec{c}$ are three non colanar, non =null vectors, and $\vec{r}$ is any vector in space, then

$$\left(\vec{a}\times\vec{b}\right)\times\left(\vec{r}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\times\left(\vec{r}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\times\left(\vec{r}\right.$$

is equal to

A. $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{r}$

B. $3\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{r}$

C. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{r}$

D. none of these

Answer: A

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32. The acute angle betwene any two faces of a regular tetrahedron is

A. $\cos^{-1}\left(\frac{1}{3}\right)$

B. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. $\cos^{-1}\left(\frac{2}{3}\right)$

D. none of these

Answer: A

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33. The acute angle that the vector $2\hat{i} - 2\hat{j} + 2\hat{k}$ makes with the plane determined by the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ is

A. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. $\tan^{-1}(\sqrt{2})$

D. $\cot^{-1}(\sqrt{3})$

Answer: B

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34. If $\vec{a}, \vec{b}, \vec{c}$ are non-null non coplanar vectors, then

$$\left[\vec{a} - 2\vec{b} + \vec{c} \quad \vec{b} - 2\vec{c} + \vec{a} \quad \vec{c} - 2\vec{a} + \vec{b}\right] =$$

A. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

B. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

C. 0

D. $12\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

Answer: C

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35. The three vectors $\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}$ taken two at a time form three planes. The three unit vectors drawn perpendicular to these three planes form a parallelopiped of volume.

A. $\frac{1}{3}$

B. 4

C. $\frac{3\sqrt{3}}{4}$

D. $\frac{4}{3\sqrt{3}}$

Answer: B

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36. Let G_1, G_2, G_3 be the centroids of the triangular faces OBC, OCA, OAB of a tetrahedron $OABC$. If V_1 denote the volume of the tetrahedron $OABC$ and V_2 that of the parallelopiped with OG_1, OG_2, OG_3 as three concurrent edges, then

A. $4V_1 = 9V_2$

B. $9V_1 = 4V_2$

C. $3V_1 = 2V_2$

D. $3V_2 = 2V_1$

Answer: A

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37. Let $\vec{r}, \vec{a}, \vec{b}$ and $\vec{c}$ be four non-zero vectors such that $\vec{r}.\vec{a}=0, \left|\vec{r}\times\vec{b}\right|=\left|\vec{r}\right|\left|\vec{b}\right|, \left|\vec{r}\times\vec{c}\right|=\left|\vec{r}\right|\left|\vec{c}\right|$ then $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=$

A. -1

B. 0

C. 1

D. 2

Answer: B

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38. Let $\vec{V}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{W}=\hat{i}+3\hat{k}$. It $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $\left[\vec{U}\ \vec{V}\ \vec{W}\right]$ is

A. -1

B. $\sqrt{10}+\sqrt{6}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: C

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39. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then the vector $\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)$ is parallel to the vector

A. $\vec{a}+\vec{b}$

B. $\vec{a}-\vec{b}$

C. $2\vec{a}+\vec{b}$

D. $2\vec{a}-\vec{b}$

Answer: B

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40. If $\overrightarrow{\alpha} = 2\hat{i} + 3\hat{j} - \hat{k}, \overrightarrow{\beta} = -\hat{i} + 2\hat{j} - 4\hat{k}, \overrightarrow{\gamma} = \hat{i} + \hat{j} + \hat{k}$, then $\left(\overrightarrow{\alpha} \times \overrightarrow{\beta}\right) . \left(\overrightarrow{\alpha} \times \overrightarrow{\gamma}\right)$ is equal to

A. -74

B. 74

C. 64

D. 60

Answer: A

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41. Let $\overrightarrow{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \overrightarrow{\beta} = b\hat{i} + c\hat{j} + a\hat{k}$ and $\overrightarrow{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplnar vectors with $a \neq b$, and $\overrightarrow{v} = \hat{i} + \hat{j} + \hat{k}$. Then $\overrightarrow{v}$ is perpendicular to

A. $\overrightarrow{\alpha}$

B. $\vec{\beta}$

C. $\vec{\gamma}$

D. all of these

Answer: D

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42. Given $\left|\vec{a}\right| = \left|\vec{b}\right| = 1$ and $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$. If $\vec{c}$ be a vector such that $\vec{c} - \vec{a} - 2\vec{b} = 3\left(\vec{a} \times \vec{b}\right)$, then $\vec{c}.\vec{b}$ is equal to

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. $\frac{3}{2}$

D. $\frac{5}{2}$

Answer: D

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43. If $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w} + (\vec{w} \times \vec{u}) = \vec{v}$ then $\vec{u} \cdot (\vec{v} \times \vec{w})$ will be equal to

A. $1 - \vec{v} \cdot \vec{w}$

B. $1 - \left|\vec{w}\right|^2$

C. $\left|\vec{w}\right|^2 - (\vec{v} \cdot \vec{w})^2$

D. all of these

Answer: D

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44. If $\vec{a}, \vec{b}, \vec{c}$ be three vectors of magnitude $\sqrt{3}, 1, 2$ such that $\vec{a} \times (\vec{a} \times \vec{c}) + 3\vec{b} = \vec{0}$ if θ angle between $\vec{a}$ and $\vec{c}$ then $\cos^2\theta$ is equal to

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. none of these

Answer: A

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45. If the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other then a vector $\vec{v}$ in terms of $\vec{a}$ and $\vec{b}$ satisfying the equations $\vec{v}.\vec{a}=0, \vec{v}.\vec{b}=1$ and $\left[\vec{v}\ \vec{a}\ \vec{b}\right]=1$ is

A. $\frac{\vec{b}}{\left|\vec{b}\right|^2}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|^2}$

B. $\frac{\vec{b}}{\left|\vec{b}\right|}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|^2}$

C. $\frac{\vec{b}}{\left|\vec{b}\right|^2}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$

D. none of these

Answer: A

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46. The value of a so that the volume of the paralelopiped formed by $\hat{i} + a\hat{j} + \hat{k}$, $\hat{j} + a\hat{k}$and $a\hat{i} + \hat{k}$ becomes minimum is

A. $\frac{1}{3}$

B. 3

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

Answer: C

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47. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1,1 and 2 resectively. If $\vec{a} \times \left(\vec{a} \times \vec{c}\right) + \vec{b} = \vec{0}$ then the acute angel between

$\vec{a}$ and $\vec{c}$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. none of these

Answer: B

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48. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\left|\vec{b}\right| = \left|\vec{c}\right|$ then $\left\{\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} + \vec{c}\right)\right\} \times \left(\vec{b} \times \vec{c}\right).\left(\vec{b} + \vec{c}\right) =$

A. 1

B. -1

C. 0

D. none of these

Answer: C

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49. If the magnitude of the moment about the pont $\hat{j}+\hat{k}$ of a force $\hat{i}+\alpha\hat{j}-\hat{k}$ acting through the point $\hat{i}+\hat{j}$ is $\sqrt{8}$, then the value of α is

A. 1

B. 2

C. 3

D. 4

Answer: B

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50. If the volume of the parallelopiped formed by the vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ as three coterminous edges is 27 units, then the volume of the parallelopiped having $\vec{\alpha} = \vec{a} + 2\vec{b} - \vec{c}, \vec{\beta} = \vec{a} - \vec{b}$ and $\vec{\gamma} = \vec{a} - \vec{b} - \vec{c}$ as three coterminous edges, is

A. 27 cubic units

B. 9 cubic units

C. 81 cubic units

D. none of these

Answer: C

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51. If $|\vec{a}| = 5, |\vec{b}| = 3, |\vec{c}| = 4$ and $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$ such that angle between $\vec{b}$ and $\vec{c}$ is $\frac{5\pi}{6}$, then the volume of the parallelopiped having $\vec{a}, \vec{b}$ and $\vec{c}$ as three coterminous edges is

A. 30 cubit units

B. 60 cubic units

C. 20 cubic units

D. none of these

Answer: A

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52. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar vectors, then $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) =$

A. 1

B. $\vec{a}$

C. $\vec{b}$

D. $\vec{0}$

Answer: D

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53. $\left\{\vec{a}.\left(\vec{b}\times\hat{i}\right)\right\}\hat{i}+\left\{\vec{a}.\left(\vec{b}\times\hat{j}\right)\right\}\hat{j}+\left\{\vec{a}.\left(\vec{b}\times\hat{k}\right)\right\}\hat{k}=$

A. $2\left(\vec{a}\times\vec{b}\right)$

B. $3\left(\vec{a}\times\vec{b}\right)$

C. $\vec{a}\times\vec{b}$

D. $-\left(\vec{a}\times\vec{b}\right)$

Answer: C

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54. The unit vector which is orhtogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$, is

A. $\frac{1}{\sqrt{41}}\left(2\hat{i}-6\hat{j}+\hat{k}\right)$

B. $\frac{1}{\sqrt{13}}\left(2\hat{i}-3\hat{j}\right)$

C. $\frac{1}{\sqrt{10}}\left(3\hat{j}-\hat{k}\right)$

D. $\frac{1}{\sqrt{34}}\left(4\hat{i}+3\hat{j}-3\hat{k}\right)$

Answer: C

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55. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that no two are collinear and

$$\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$$

If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$ then $\sin\theta$ equals

A. $\frac{2\sqrt{2}}{3}$

B. $\frac{\sqrt{2}}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer: A

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56. Let $\vec{p}, \vec{q}, \vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation

$$\vec{p} \times \left\{ \left(\vec{x} - \vec{q}\right) \times \vec{p} \right\} + \vec{q} \times \left\{ \left(\vec{x} - \vec{r}\right) \times \vec{q} \right\} + \vec{r} \times \left\{ \left(\vec{x} - \vec{p}\right) \times \vec{r}$$

,

then $\vec{x}$ is given by

A. $\frac{1}{2}\left(\vec{p} + \vec{q} - 2\vec{r}\right)$

B. $\frac{1}{2}\left(\vec{p} + \vec{q} + \vec{r}\right)$

C. $\frac{1}{3}\left(\vec{p} + \vec{q} + \vec{r}\right)$

D. $\frac{1}{3}\left(2\vec{p} + \vec{q} - \vec{r}\right)$

Answer: B

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57. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$

$\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$ then the value of

$\left(2\vec{a}+\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$, is

A. 2

B. 3

C. 4

D. 5

Answer: D

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58. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' make a right angle withe the side AB then the cosine of the angle α is given by

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: B

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59. Let $\overrightarrow{a} = \hat{j} - \hat{k}$ and $\overrightarrow{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\overrightarrow{b}$ satisfying $\overrightarrow{a} \times \overrightarrow{b} + \overrightarrow{c} = \overrightarrow{0}$ and $\overrightarrow{a} . \overrightarrow{b} = 3$ is

A. $\hat{i} - \hat{j} - 2\hat{k}$

B. $\hat{i} + \hat{j} - 2\hat{k}$

C. $-\hat{i} + \hat{j} - 2\hat{k}$

D. $2\hat{i} - \hat{j} + 2\hat{k}$

Answer: C

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60. The vector (s) which is (are) coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$, is/are

A. $\hat{j} - \hat{k}$ and $-\hat{j} + \hat{k}$

B. $-\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$

C. $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$

D. $-\hat{j} + \hat{k}$ and $-\hat{i} + \hat{j}$

Answer: Minimum value at $(\alpha)^{\alpha}$ ^ (x) + alpha^(1-(alpha)^x)` is

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61. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} . \vec{a} = 0$, then the value of $\vec{r} . \vec{b}$ is

A. 4

B. 8

C. 6

D. 9

Answer: D

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62. If $\vec{a} = \frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right), \vec{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right).\left\{\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right\}$ is

A. -5

B. -3

C. 5

D. 3

Answer: A

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63. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = r\hat{i} + \hat{j} + (2r - 1)\hat{k}$ are three vectors such that $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$ then$r$ is equal to,

A. 1

B. 0

C. 2

D. -1

Answer: B

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64. If $\vec{a}, \vec{b}$ are non zero vectors, then $\left(\left(\vec{a} \times \vec{b}\right) \times \vec{a}\right).\left(\left(\vec{b} \times \vec{a}\right) \times \vec{b}\right)$ equals

A. $-\left(\vec{a}.\vec{b}\right)\left|\left(\vec{a} \times \vec{b}\right)\right|$

B. $\left|\vec{a} \times \vec{b}\right|^2 \vec{a}^2$

C. $\left|\vec{a}\times\vec{b}\right|^2\vec{b}^2$

D. $\left(\vec{a}.\vec{b}\right)\left|\vec{a}\times\vec{b}\right|^2$

Answer: A

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Section Ii Assertion Reason Type

1. Statement 1: Let $\vec{r}$ be any vector in space. Then, $\vec{r}=\left(\vec{r}.\hat{i}\right)\hat{i}+\left(\vec{r}.\hat{j}\right)\hat{j}+\left(\vec{r}.\hat{k}\right)\hat{k}$

Statement 2: If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors and $\vec{r}$ is any vector in space then

$$\vec{r}=\left\{\frac{\left[\vec{r}\ \vec{b}\ \vec{c}\right]}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}\right\}\vec{a}+\left\{\frac{\left[\vec{r}\ \vec{c}\ \vec{a}\right]}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}\right\}\vec{b}+\left\{\frac{\left[\vec{r}\ \vec{a}\ \vec{b}\right]}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}\right\}$$

A. 1

B. 2

C. 3

D. 4

Answer: A

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2. Statement 1: If $\vec{a}, \vec{b}$ are non zero and non collinear vectors, then

$$\vec{a} \times \vec{b} = \left[\vec{a} \quad \vec{b} \quad \hat{i}\right]\hat{i} + \left[\vec{a} \quad \vec{b} \quad \hat{j}\right]\hat{j} + \left[\vec{a} \quad \vec{b} \quad \hat{k}\right]\hat{k}$$

Statement 2: For any vector $\vec{r}$

$$\vec{r} = \left(\vec{r}\,.\,\hat{i}\right)\hat{i} + \left(\vec{r}\,.\,\hat{j}\right)\hat{j} + \left(\vec{r}\,.\,\hat{k}\right)\hat{k}$$

A. 1

B. 2

C. 3

D. 4

Answer: A

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3. Statement 1: Let $\vec{a}, \vec{b}, \vec{c}$ be three coterminous edges of a parallelopiped of volume 2 cubic units and $\vec{r}$ is any vector in space then

$$\left|\left(\vec{r}.\vec{a}\right)\left(\vec{b}\times\vec{c}\right)+\left(\vec{r}.\vec{b}\right)\left(\vec{c}\times\vec{a}\right)+\left(\vec{c}.\vec{c}\right)\left(\vec{a}\times\vec{b}\right| = 2|\vec{r}\right|$$

Statement 2: Any vector in space can be written as a linear combination of three non-coplanar vectors.

A. 1

B. 2

C. 3

D. 4

Answer: A

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4. Let $\vec{a}, \vec{b}, \vec{c}$ be any three vectors,

Statement 1: $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right]=2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

Statement 2: $\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right]=\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$

A. 1

B. 2

C. 3

D. 4

Answer: B

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5. Statement 1: Any vector in space can be uniquely written as the linear combination of three non-coplanar vectors.

Stetement 2: If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and $\vec{r}$ is any vector in space then

$$\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]\vec{c}+\left[\vec{b} \quad \vec{c} \quad \vec{r}\right]\vec{a}+\left[\vec{c} \quad \vec{a} \quad \vec{r}\right]\vec{b}=\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]\vec{r}$$

A. 1

B. 2

C. 3

D. 4

Answer: B

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6. Statement 1: Let $\vec{a}, \vec{b}, \vec{c}$ be three coterminous edges of a parallelopiped of volume V. Let V_1 be the volume of the parallelopiped whose three coterminous edges are the diagonals of three adjacent faces of the given parallelopiped. Then $V_1 = 2V$.

Statement 2: For any three vectors, $\vec{p}, \vec{q}, \vec{r}$

$$[\vec{p}+\vec{q} \quad \vec{q}+\vec{r} \quad \vec{r}+\vec{p}] = 2[\vec{p} \quad \vec{q} \quad \vec{r}]$$

A. 1

B. 2

C. 3

D. 4

Answer: A

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7. Statement 1: Let V_1 be the volume of a parallelopiped ABCDEF having $\vec{a}, \vec{b}, \vec{c}$ as three coterminous edges and V_2 be the volume of the parallelopiped $PQRSTU$ having three coterminous edges as vectors whose magnitudes are equal to the areas of three adjacent faces of the parallelopiped $ABCDEF$. Then $V_2 = 2V_1^2$

Statement 2: For any three vectors $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$

$$\left[\vec{\alpha} \times \vec{\beta}, \vec{\beta} \times \vec{\gamma}, \vec{\gamma} \times \vec{\alpha}\right] = \left[\vec{\alpha} \quad \vec{\beta} \quad \vec{\gamma}\right]^2$$

A. 1

B. 2

C. 3

D. 4

Answer: D

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8. Statement 1: If V is the volume of a parallelopiped having three coterminous edges as $\vec{a}, \vec{b}$, and $\vec{c}$, then the volume of the parallelopiped having three coterminous edges as

$\vec{\alpha} = (\vec{a}.\vec{a})\vec{a} + (\vec{a}.\vec{b})\vec{b} + (\vec{a}.\vec{c})\vec{c}$

$\vec{\beta} = (\vec{a}.\vec{b})\vec{a} + (\vec{b}.\vec{b})\vec{b} + (\vec{b}.\vec{c})\vec{c}$

$\vec{\gamma} = (\vec{a}.\vec{c})\vec{a} + (\vec{b}.\vec{c})\vec{b} + (\vec{c}.\vec{c})\vec{c}$ is V^3

Statement 2: For any three vectors $\vec{a}, \vec{b}, \vec{c}$

$$\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}^3$$

A. 1

B. 2

C. 3

D. 4

Answer: C

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9. Statement 1: Unit vectors orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and coplanar with the vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ are $\pm\frac{1}{\sqrt{10}}\left(3\hat{j} - \hat{k}\right)$.

Statement 2: For any three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ vector $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is orthogonal to $\vec{a}$ and lies in the plane of $\vec{b}$ and $\vec{c}$.

A. 1

B. 2

C. 3

D. 4

Answer: A

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10. Statement If G_1, G_2, G_3 are the centroids of the triangular faces OBC, OCA, OAB of a tetrahedron $OABC$, then the ratio of the volume of the tetrahedron to that of the parallelopiped with OG_1, OG_2, OG_3 as coterminous edges is $9:4$.

Statement 2: For any three vctors, $\vec{a}, \vec{b}, \vec{c}$

$$\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right]=2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$$

A. 1

B. 2

C. 3

D. 4

Answer: A

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11. Statement 1: For any three vectors $\vec{a}, \vec{b}, \vec{c}$

$$\left[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}\right] = 0$$

Statement 2: If $\vec{p}, \vec{q}, \vec{r}$ are linear dependent vectors then they are coplanar.

A. 1

B. 2

C. 3

D. 4

Answer: D

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12. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Statement 1: $\overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq \overrightarrow{0}$

Statement 2: $\overrightarrow{PQ} \times \overrightarrow{RS} = \overrightarrow{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \overrightarrow{0}$

A. 1

B. 2

C. 3

D. 4

Answer: C

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Exercise

1. For non zero vectors $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$

$\left|\left(\overrightarrow{a} \times \overrightarrow{b}\right). \overrightarrow{\quad}\right| = \left|\overrightarrow{a}\right|\left|\overrightarrow{b}\right|\left|\overrightarrow{c}\right|$ holds iff

A. $\overrightarrow{a}. \overrightarrow{b} = \overrightarrow{b}. \overrightarrow{c} = \overrightarrow{a}. \overrightarrow{a} = 0$

B. $\vec{a} . \vec{b} = 0 = \vec{b} . \vec{c}$

C. $\vec{b} . \vec{c} = 0 = \vec{c} . \vec{a}$

D. $\vec{c} . \vec{a} = 0 = \vec{a} . \vec{b}$

Answer: A

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2. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then it is given by

A. $\frac{1}{\sqrt{6}}\left(2\hat{i} - \hat{j} + \hat{j}k\right)$

B. $\frac{1}{\sqrt{2}}\left(\hat{j} + \hat{k}\right)$

C. $\frac{1}{\sqrt{6}}\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

D. $\frac{1}{2}\left(\hat{j} - \hat{k}\right)$

Answer: A

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3. If $\vec{a}$ lies in the plane of vectors $\vec{b}$ and $\vec{c}$, then which of the following is correct?

A. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$

B. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=1$

C. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]=3$

D. $\left[\vec{a}\ \vec{c}\ \vec{a}\right]=1$

Answer: A

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4. The value of $\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right]$, where $\left|\vec{a}\right|=1, \left|\vec{b}\right|=5, \left|\vec{c}\right|=3$, is

A. 0

B. 1

C. 6

D. none of these

Answer: A

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5. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar mutually perpendicular unit vectors, then $\begin{bmatrix}\vec{a} & \vec{b} & \vec{c}\end{bmatrix}$ is

A. ± 1

B. 0

C. -2

D. 2

Answer: A

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6. If $\vec{r}.\vec{a}=\vec{r}.\vec{b}=\vec{r}.\vec{c}=0$ for some non-zero vectro $\vec{r}$, then the value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is

A. 2

B. 3

C. 0

D. none of these

Answer: C

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7. If the vectors $\vec{r}_1=a\hat{i}+\hat{j}+\hat{k}, \vec{r}_2=\hat{i}+b\hat{j}+\hat{k}, \vec{r}_3=\hat{i}+\hat{j}+c\hat{k}(a\neq 1, b\neq 1, c\neq 1$ are coplanar then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is

A. -1

B. 0

C. 1

D. none of these

Answer: C

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8. If $\widehat{a}, \hat{b}, \hat{c}$ are three units vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\widehat{a} \times \left(\hat{b} \times \hat{c}\right) = 1/2\hat{b}$ then the angle between $\widehat{a}$ and $\hat{c}$ is

A. 30°

B. 45°

C. 60°

D. 90°

Answer: C

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9. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the vector $\left(\vec{b} \times \vec{c}\right) \times \vec{a}$ equals

A. $\left(\vec{a}.\vec{b}\right)\vec{c} - \left(\vec{b}.\vec{c}\right)\vec{a}$

B. $\left(\vec{a}.\vec{b}\right)\vec{c} - \left(\vec{a}.\vec{c}\right)\vec{b}$

C. $\left(\vec{b}.\vec{a}\right)\vec{c} - \left(\vec{c}.\vec{a}\right)\vec{b}$

D. none of these

Answer: B

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10. For any these vectors $\vec{a}, \vec{b}, \vec{c}$ the expression $\left(\vec{a} - \vec{b}\right).\left\{\left(\vec{b} - \vec{c}\right) \times \left(\vec{c} - \vec{a}\right)\right\}$ equals

A. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

B. $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

C. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

D. none of these

Answer: D

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11. For any vectors $\overrightarrow{r}$ the value of

$\hat{i} \times \left(\overrightarrow{r} \times \hat{i}\right) + \hat{j} \times \left(\overrightarrow{r} \times \hat{j}\right) + \hat{k} \times \left(\overrightarrow{r} \times \hat{k}\right)$, is

A. $\overrightarrow{0}$

B. $2\overrightarrow{r}$

C. $-2\overrightarrow{r}$

D. none of these

Answer: B

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12. If the vectors $\overrightarrow{a} = \hat{i} + a\hat{j} + a^2\hat{k}, \overrightarrow{b} = \hat{i} + b\hat{j} + b^2\hat{k}, \overrightarrow{c} = \hat{i} + c\hat{j} + c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc is

A. 0

B. 1

C. 2

D. -1

Answer: D

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13. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ be the vectors defined by the relations.

$$\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a} \ \vec{b} \ \vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a} \ \vec{b} \ \vec{c}\right]}, \vec{r} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a} \ \vec{b} \ \vec{c}\right]}$$

Then the value of the expression

$\left(\vec{a} + \vec{b}\right).\vec{p} + \left(\vec{b} + \vec{c}\right).\vec{q} + \left(\vec{c} + \vec{a}\right).\vec{r}$ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: D

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14. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, then $\dfrac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left(\vec{c}\times\vec{a}\right).\vec{b}}+\dfrac{\vec{b}.\left(\vec{a}\times\vec{c}\right)}{\vec{c}.\left(\vec{a}\times\vec{b}\right)}$ is equal to

A. 0

B. 2

C. 1

D. none of these

Answer: A

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15. Let $\overrightarrow{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \overrightarrow{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\overrightarrow{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non zero vectors such that $\overrightarrow{c}$ is a unit vector perpendicular to both $\overrightarrow{a}$ and $\overrightarrow{b}$. If the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

A. 0

B. 1

C. $\frac{1}{4}\left|\overrightarrow{a}\right|^2\left|\overrightarrow{b}\right|^2$

D. $\frac{3}{4}\left|\overrightarrow{a}\right|^2\left|\overrightarrow{b}\right|^2$

Answer: C

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16. If the non zero vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then the solution of the equation $\vec{r} \times \vec{a} = \vec{b}$ is given by

A. $\vec{r} = x\vec{a} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a}\right|^2}$

B. $\vec{r} = x\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

C. $\vec{r} = x\left(\vec{a} \times \vec{b}\right)$

D. $\vec{r} = x\left(\vec{b} \times \vec{a}\right)$

Answer: A

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17. Prove that: [(vecaxxvecb)xx(vecaxxvecc)].vecd=pveca vecb vecc] (veca.vecd)`

A. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\left(\vec{c}\,.\,\vec{d}\right)$

B. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\left(\vec{a}\,.\,\vec{d}\right)$

C. $\left[\left(\vec{a}, \vec{b}, \vec{c}\right)\right]\left[\left(\vec{c} . \vec{d}\right)\right.$

D. none of these

Answer: B

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18. If $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ then

A. $\vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

B. $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{0}$

C. $\vec{c} \times \vec{a} = \vec{a} \times \vec{b}$

D. $\vec{c} \times \vec{b} = \vec{b} \times \vec{a}$

Answer: A

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19. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{p}, \vec{q}, \vec{r}$ are reciprocal system of vectors, then $\vec{a} \times \vec{p} + \vec{b} \times \vec{q} + \vec{c} \times \vec{r}$ equals

A. $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

B. $\left(\vec{p} + \vec{q} + \vec{r}\right)$

C. $\vec{0}$

D. $\vec{a} + \vec{b} + \vec{c}$

Answer: C

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20. $\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)$ equals

A. $\left(\vec{a} . \vec{b}\right)\left(\vec{a} \times \vec{b}\right)$

B. $\left(\vec{a} . \vec{a}\right)\left(\vec{b} \times \vec{a}\right)$

C. $\left(\vec{b} . \vec{b}\right)\left(\vec{a} \times \vec{b}\right)$

D. $\left(\vec{b}\cdot\vec{b}\right)\left(\vec{b}\times\vec{a}\right)$

Answer: B

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21. If $\vec{a}=\hat{i}+\hat{j}-\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}$ is a unit vector perpendiculr to the vector $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then a unit vector $\vec{d}$ perpendicular to both $\vec{a}$ and $\vec{c}$ is

A. $\frac{1}{\sqrt{6}}\left(2\hat{i}-\hat{j}+\hat{j}k\right)$

B. $\frac{1}{\sqrt{2}}\left(\hat{j}+\hat{k}\right)$

C. $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

D. $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{k}\right)$

Answer: B

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22. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $3\pi/4$

B. $\pi/4$

C. $\pi/2$

D. π

Answer: A

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23. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lies in a plane then c is

A. the AM of a and b

B. the GM of a and b

C. the HM of a and b

D. equal to zero

Answer: B

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24. If $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$ then

a. $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs and $\left|\vec{a}\right| = \left|\vec{c}\right|, \left|\vec{b}\right| = 1$

b. $\vec{a}, \vec{b}, \vec{c}$ are not orthogonal to each other

c. $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs but $\left|\vec{a}\right| \ne \left|\vec{c}\right|$

d. $\vec{a}, \vec{b}, \vec{c}$ are orthogonal but $\left|\vec{b}\right| = 1$

OR

If $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}$, then

A. $\left|\vec{a}\right| = 1, \vec{b} = \vec{c}$

B. $\left|\vec{c}\right| = 1, \left|\vec{a}\right| = 1$

C. $\left|\vec{b}\right| = 2, \vec{c} = 2\vec{a}$

D. $\left|\vec{b}\right| = 1, \left|\vec{c}\right| = \left|\vec{a}\right|$

Answer: A::D

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25. If $\overrightarrow{p}=\dfrac{\overrightarrow{b}\times\overrightarrow{c}}{\left[\overrightarrow{a}\ \overrightarrow{b}\ \overrightarrow{c}\right]}, \overrightarrow{q}=\dfrac{\overrightarrow{c}\times\overrightarrow{a}}{\left[\overrightarrow{a}\ \overrightarrow{b}\ \overrightarrow{c}\right]}, \overrightarrow{r}=\dfrac{\overrightarrow{a}\times\overrightarrow{b}}{\left[\overrightarrow{a}\ \overrightarrow{b}\ \overrightarrow{b}\right]}$ where $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ are three non-coplanar vectors, then the value of the expression $\left(\overrightarrow{a}+\overrightarrow{b}+\overrightarrow{c}\right).\left(\overrightarrow{p}+\overrightarrow{q}+\overrightarrow{r}\right)$ is

A. 3

B. 2

C. 1

D. 0

Answer: A

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26. If $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}, \vec{r}\times\vec{b}=\vec{a}\times\vec{b}, \vec{a}\neq 0, \vec{b}\neq 0, \vec{a}\neq\lambda\vec{b}, \vec{a}$ is not perpendicular to $\vec{b}$ then $\vec{r}=$

A. $\vec{a}-\vec{b}$

B. $\vec{a}+\vec{b}$

C. $\vec{a}\times\vec{b}+\vec{a}$

D. $\vec{a}\times\vec{b}+\vec{b}$

Answer: B

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27. The vector $\vec{a}$ coplanar with the vectors $\hat{i}$ and $\hat{j}$ perendicular to the vector $\vec{b}=4\hat{i}-3\hat{j}+5\hat{k}$ such that $\left|\vec{a}\right|=\left|\vec{b}\right|$ is

A. $\sqrt{2}\left(3\hat{i}+4\hat{j}\right)$ or $-\sqrt{2}\left(3\hat{i}+4\hat{j}\right)$

B. $\sqrt{2}\left(4\hat{i}+3\hat{j}\right)$ or $-\sqrt{2}\left(4\hat{i}+3\hat{j}\right)$

C. $\sqrt{3}\left(4\hat{i}+5\hat{j}\right)$ ro $-\sqrt{3}\left(4\hat{i}+5\hat{j}\right)$

D. $\sqrt{3}\left(5\hat{i}+4\hat{j}\right)$ or $-\sqrt{3}\left(5\hat{i}+4\hat{j}\right)$

Answer: A

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28. If the vectors $\vec{a}$ and $\vec{b}$ are mutually perpendicular, then $\vec{a}\times\left\{\vec{a}\times\left\{\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right\}\right\}$ is equal to

A. $\left|\vec{a}\right|^2\vec{b}$

B. $\left|\vec{a}\right|^3\vec{b}$

C. $\left|\vec{a}\right|^4\vec{b}$

D. none of these

Answer: C

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29.

$\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)\ \left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\ \left(\vec{c}\times\vec{a}\right)\times\left(\vec{a}\right.\right.$

equal to

A. $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2$

B. $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^3$

C. $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^4$

D. none of these

Answer: C

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30. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k}, \vec{c} = \hat{k} - \hat{i}$. If $\hat{d}$ is a unit vector such that $\vec{a}.\hat{d} = 0 = \left[\vec{b}\ \ \vec{c}\ \ \hat{d}\right]$, then $\hat{d}$ equals

A. $\pm\dfrac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$

B. $\pm\dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

C. $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

D. $\pm\hat{k}$

Answer: A

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31. If the vectors $(\sec^2 A)\hat{i}+\hat{j}+\hat{k}, \hat{i}+(\sec^2 B)\hat{j}+\hat{k}, \hat{i}+\hat{j}+(\sec^2 c)\hat{k}$ are coplanar, then the value of $\cos ec^2 A+\cos ec^2 B+\cos ec^2 C$, is

A. 1

B. 2

C. 3

D. none of these

Answer: B

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32. $\hat{a}$ and $\hat{b}$ are two mutually perpendicular unit vectors. If the vectors $x\hat{a}+x\hat{b}+z(\hat{a}\times\hat{b}), \hat{a}+(\hat{a}\times\hat{b})$ and $z\hat{a}+z\hat{b}+y(\hat{a}\times\hat{b})$ lie in a plane, then z is

A. A.M is x and y

B. G.M. of x and y

C. H.M. of x and y

D. equal to zero

Answer: B

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33. If three concurrent edges of a parallelopiped of volume V represent vectors $\vec{a}, \vec{b}, \vec{c}$ then the volume of the parallelopiped whose three concurrent edges are the three concurrent diagonals of the three faces of the given parallelopiped is

A. V

B. $2V$

C. $3V$

D. none of these

Answer: B

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34. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+\hat{j}, \vec{c}=\hat{i}$ and $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then $\lambda+\mu=$

A. 0

B. 1

C. 2

D. 3

Answer: A

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35. If $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} - 2\hat{k}$, then the volume of the parallelopiped with coterminous edges $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$, $\vec{c} + \vec{a}$ is

A. 2

B. 1

C. -1

D. 0

Answer: D

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36. If $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors, then $\dfrac{\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{b}+\vec{c}\right).\left(5\vec{c}+\vec{a}\right)}{\vec{a}.\left(\vec{b}\times\vec{c}\right)}$ is equal to

A. 10

B. 14

C. 18

D. 12

Answer: D

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37. If $\vec{a}, \vec{b}$ are non-collinear vectors, then

$\left[\vec{a}\ \ \vec{b}\ \ \hat{i}\right]\hat{i}+\left[\vec{a}\ \ \vec{b}\ \ \hat{j}\right]\hat{j}+\left[\vec{a}\ \ \vec{b}\ \ \hat{k}\right]\hat{k}=$

A. $\vec{a}+\vec{b}$

B. $\vec{a}\times\vec{b}$

C. $\vec{a} - \vec{b}$

D. $\vec{b} \times \vec{a}$

Answer: B

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38. If $\left[2\vec{a} + 4\vec{b} \quad \vec{c} \quad \vec{d}\right] = \lambda\left[\vec{a} \quad \vec{c} \quad \vec{d}\right] + \mu\left[\vec{b} \quad \vec{c} \quad \vec{d}\right]$, then $\lambda + \mu =$

A. 6

B. -6

C. 10

D. 8

Answer: A

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39. If the volume of the tetrahedron whose vertices are $(1, -6, 10), (-1, -3, 7), (5, -1, \lambda)$ and $(7, -4, 7)$ is 11 cubit units then $\lambda =$

A. 2,6

B. 3,4

C. 1,7

D. 5,6

Answer: C

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40. $\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right) =$

A. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{c}$

B. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{b}$

C. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{a}$

D. $a \times \left(\vec{b} \times \vec{c}\right)$

Answer: A

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41. When a right handed rectangular Cartesian system $OXYZ$ rotated about z-axis through $\pi/4$ in the counter clock wise sense it is found that a vector $\vec{r}$ has the components $2\sqrt{2}, 3\sqrt{2}$ and 4. The components of $\vec{a}$ in the OXYZ coordinate system ar

A. $5, \ -1, 4$

B. $5, \ -1, 4\sqrt{2}$

C. $-1, \ -5, 4\sqrt{2}$

D. none of these

Answer: D

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42. Prove that vectors

$$\vec{u} = (al + a_1 l_1)\hat{i} + (am + a_1 m_1)\hat{j} + (an + a_1 n_1)\hat{k}$$

$$\vec{v} = (bl + b_1 l_1)\hat{i} + (bm + b_1 m_1)\hat{j} + (bn + b_1 n_1)\hat{k}$$

$$\vec{w} = (wl + c_1 l_1)\hat{i} + (cm + c_1 m_1)\hat{j} + (cn + c_1 n_1)\hat{k}$$

A. form an equilteral triangle

B. are coplanar

C. are collinear

D. are mutually perpendicular

Answer: B

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43. If $\vec{a}\, x\left(\vec{a} \times \vec{b}\right) = \vec{b} \times \left(\vec{b} \times \vec{c}\right)$ and $\vec{a}\,.\,\vec{b} \neq 0,$ and $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] =$

A. 0

B. 1

C. 2

D. 3

Answer: A

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44. $\left[\vec{a} \quad \vec{b} \quad a \times \vec{b}\right] + \left(\vec{a}\,.\,\vec{b}\right)^2 =$

A. $\left|\vec{a}\right|^2\left|\vec{b}\right|^2$

B. $\left|\vec{a} + \vec{b}\right|^2$

C. $\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2$

D. none of these

Answer: A

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45. Let $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$ be the unit vectors such that $\vec{\alpha}$ and $\vec{\beta}$ are mutually perpendicular and $\vec{\gamma}$ is equally inclined to $\vec{\alpha}$ and $\vec{\beta}$ at an angle θ. If $\vec{\gamma} = x\vec{\alpha} + y\vec{\beta} + z\left(\vec{\alpha} \times \vec{\beta}\right)$, then which one of the folllowing is incorrect?

A. $z^2 = 1 - 2x^2$

B. $z^2 = 1 - 2y^2$

C. $z^2 = 1 - x^2 - y^2$

D. $x^2 + y^2 = 1$

Answer: D

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46. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then

$\left[2\vec{a} - 3\vec{b} \quad 7\vec{b} - 9\vec{c} \quad 12\vec{c} - 23\vec{b}\right]$ is equal to

A. 0

B. $1/2$

C. 24

D. 32

Answer: A

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47. If $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 3$, then the volume (in cubic units) of the parallelopiped with $2\vec{a} + \vec{b}, 2\vec{b} + \vec{c}$ and $2\vec{c} + \vec{a}$ as coterminous edges is

A. 15

B. 22

C. 25

D. 27

Answer: D

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48. If V is the volume of the parallelopiped having three coterminous edges as $\vec{a}, \vec{b}$ and $\vec{c}$, then the volume of the parallelopiped having three coterminous edges as

$$\vec{\alpha} = (\vec{a}.\vec{a})\vec{a} + (\vec{a}.\vec{b})\vec{b} + (\vec{a}.\vec{c})\vec{c}$$
$$\vec{\beta} = (\vec{a}.\vec{b})\vec{a} + (\vec{b}.\vec{b})\vec{b} + (\vec{b}.\vec{c})\vec{c}$$
$$\vec{\gamma} = (\vec{a}.\vec{c})\vec{a} + (\vec{b}.\vec{c})\vec{b} + (\vec{c}.\vec{c})\vec{c}$$ is

A. V^3

B. $3V$

C. V^2

D. $2V$

Answer: A

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49. The unit vector $\vec{a}$ and $\vec{b}$ are perpendicular, and the unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c} = \alpha\vec{a} + \beta\vec{b} + \gamma\left(\vec{a} \times \vec{b}\right)$, then which one of the following is incorrect?

A. $\alpha \neq \beta$

B. $\gamma^2 = 1 - 2\alpha^2$

C. $\gamma^2 = -\cos 2\theta$

D. $\beta^2 = \dfrac{1 + \cos 2\theta}{2}$

Answer: A

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50. If the vector $\overrightarrow{AB} = -3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - \lambda\hat{j} + 4\hat{k}$ where $\lambda > 0$ are the sides of ΔABC and the length of the median through A is $\sqrt{18}$, then the length of the side BC, is

A. $2\sqrt{26}$

B. $4\sqrt{13}$

C. $6\sqrt{13}$

D. none of these

Answer: D

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51. Let $\vec{a}$ and $\vec{b}$ be two mutually perpendicular unit vectors and $\vec{c}$ be a unit vector inclned at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c} = x\vec{a} + x\vec{b} + y\left(\vec{a} \times \vec{b}\right)$, where $x, y \varepsilon R$, then the exhaustive range of θ is

A. $\left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$

B. $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. none of these

Answer: B

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52. Let the position vectors of vertices A, B, C of ΔABC be respectively $\vec{a}$, $\vec{b}$ and $\vec{c}$. If $\vec{r}$ is the position vector of the mid point of the line segment joining its orthocentre and centroid then $(\vec{a}-\vec{r})+(\vec{b}-\vec{r})+(\vec{c}-\vec{r})=$

A. 1

B. 2

C. 3

D. none of these

Answer: C

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53. The position vector of a point P is $\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}$ where $x, y, z\varepsilon N$ and $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$. If $\overrightarrow{r}.\overrightarrow{a} = 10$, then the number of possible position of P is

A. 36

B. 72

C. 66

D. none of these

Answer: A

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54. $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors that are mutually perpendicular. A unit vector that if equally inclined to $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{a} \times \overrightarrow{b}$ is equal to

A. $\frac{1}{\sqrt{2}}\left(\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{a} \times \overrightarrow{b}\right)$

B. $\frac{1}{2}\left(\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{a} \times \overrightarrow{b}\right)$

C. $\frac{1}{\sqrt{3}}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

D. $\frac{1}{3}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

Answer: C

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55. If the vectors $2a\hat{i}+b\hat{j}+c\hat{k}, b\hat{i}+c\hat{j}+2a\hat{k}$ and $c\hat{i}+2a\hat{j}+b\hat{k}$ are coplanar vectors, then the straight lines $ax+by+c=0$ will always pass through the point

A. $(1, 2)$

B. $(2, -1)$

C. $(2, 1)$

D. $(1, -2)$

Answer: C

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56. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}$, $\vec{b} = b\hat{i} + c\hat{j} + a\hat{k}$ and $\vec{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ are three coplanar vectors with $a \neq b$ and $\vec{\gamma} = \hat{i} + \hat{j} + \hat{k}$. Then $\vec{\gamma}$ is perpendicular to

A. $\vec{\alpha}$

B. $\vec{\beta}$

C. $\vec{\gamma}$

D. all of these

Answer: D

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57. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors having same magnitude and $\vec{r}$ is a vector satisfying

$$\vec{a} \times \left(\left(\vec{r} - \vec{b}\right) \times \vec{a}\right) + \vec{b} \times \left(\left(\vec{r} - \vec{c}\right) \times \vec{b}\right) + \vec{c} \times \left(\left(\vec{r} - \vec{a}\right)\right.$$

then $\vec{r}$ is equal to

A. $\frac{1}{3}\left(\vec{a}+\vec{b}+\vec{c}\right)$

B. $\frac{1}{2}\left(\vec{a}+\vec{b}+\vec{c}\right)$

C. $\frac{3}{2}\left(\vec{a}+\vec{b}+\vec{c}\right)$

D. $2\left(\vec{a}+\vec{b}+\vec{c}\right)$

Answer: B

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58. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the three non-coplanar vectors and $\vec{d}$ be a non zero vector which is perpendicular to $\vec{a}+\vec{b}+\vec{c}$ and is represented as $\vec{d}=x\left(\vec{a}\times\vec{b}\right)+y\left(\vec{b}\times\vec{c}\right)+z\left(\vec{c}\times\vec{a}\right)$. Then,

A. $x^3+y^3+z^3=3xyz$

B. $xy+yz+zx=0$

C. $x=y=z$

D. $x^2+y^2+z^2=xy+yz+zx$

Answer: A

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59. Let $\vec{r}$ be a unit vector satisfying $\vec{r} \times \vec{a} = \vec{b}$, where $\left|\vec{a}\right| = \sqrt{3}$ and $\left|\vec{b}\right| = \sqrt{2}$. Then $\vec{r}$ -

A. $\frac{2}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

B. $\frac{1}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

C. $\frac{2}{3}\left(\vec{a} - \vec{a} \times \vec{b}\right)$

D. $\frac{1}{3}\left(-\vec{a} + \vec{a} \times \vec{b}\right)$

Answer: B

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60. Let $\vec{a}$ and $\vec{c}$ be unit vectors such that $\left|\vec{b}\right| = 4$ and $\vec{a} \times \vec{b} = 2\left(\vec{a} \times \vec{c}\right)$. The angle between $\vec{a}$ and $\vec{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$. If

$\vec{b} - 2\vec{c} = \lambda\vec{a}$ then $\lambda =$

A. $\frac{1}{3}, \frac{1}{4}$

B. $-\frac{1}{3}, -\frac{1}{4}$

C. $3, -4$

D. $-3, 4$

Answer: C

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61. If $4\vec{a} + 5\vec{b} + 9\vec{c} = \vec{0}$ then $\left(\vec{a} \times \vec{b}\right).\left\{\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\right\}$ is equal to

A. $\vec{0}$

B. $\vec{a}$

C. $\vec{b}$

D. $\vec{c}$

Answer: A

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62. If in a triagle $ABC, \overrightarrow{AB} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|}$ and $\vec{A}C = 2\frac{\vec{u}}{|\vec{u}|}$,where $|\vec{u}| = |\vec{v}|$, then

A. $1 + \cos 2A + \cos 2B + \cos 2C = 0$

B. $1 + \cos 2A + \cos 2B + \cos 2C = 2$

C. both a and b

D. none of these

Answer: A

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63. Let $A\left(2\hat{i}+3\hat{j}+5\hat{k}\right)$, $B\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$ and $C\left(\lambda\hat{i}+5\hat{j}+\mu\hat{k}\right)$ be the vertices of ΔABC and its median through A be equally inclined to the positive directions of the coordinate axds. Then, the value of $2\lambda-\mu$ is

A. 0

B. 1

C. 4

D. 3

Answer: C

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64. A plane is parallel to the vectors $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{k}$ and another plane is parallel to the vectors $\hat{i}+\hat{j}$ and $\hat{i}-\hat{k}$. The acute angle between the line of intersection of the two planes and the vector $\hat{i}-\hat{j}+\hat{k}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

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65. If A,B,C,D are four points in space, then $\left|\overrightarrow{AB}x\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}\right|=k(areof\triangle ABC)wherek=$

(A) 5 (B) 4 (C) 2 (D) none of these

A. 2

B. 1

C. 3

D. 4

Answer: D

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