MATHS

BOOKS - KVPY PREVIOUS YEAR

KVPY

Mathematics

1. Let A denote the matrix $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, where $i^2 = -1$, and let I denote the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Then $I + A + A^2 + \ldots.. + A^{2010}$ is -

A. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

B. $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

C. $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

D. $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Answer: C

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2. Suppose the sides of a triangle form a geometric progression with common ratio r. Then r lies in the interval-

A. $\left(0, \frac{-1+\sqrt{5}}{2}\right)$

B. $\left(\frac{1+\sqrt{5}}{2}, \frac{2+\sqrt{5}}{2}\right)$

C. $\left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

D. $\left(\frac{2+\sqrt{5}}{2}, \infty\right)$

Answer: C

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3. The number of rectangles that can be obtained by joining four of the twelve vertices of a 12-sided regular polygon is 5 K, then the value of K is_____

A. 66

B. 30

C. 24

D. 15

Answer: D

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4. Let I, ω and ω^2 be the cube roots of unity. The least possible degree of a polynomial, with real coefficients having $2\omega^2, 3+4\omega, 3+4\omega^2$ and $5-\omega-\omega^2$ as roots is -

A. 4

B. 5

C. 6

D. 8

Answer: B

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5. A circle touches the parabola $y^2 = 4x$ at $(1, 2)$ and also touches its directrix. The y-coordinates of the point of contact of the circle and the directrix is-

A. $\sqrt{2}$

B. 2

C. $2\sqrt{2}$

D. 4

Answer: C

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6. Let ABC be an equilateral triangle, let KLMN be a rectangle with K, L on BC , M on AC and N on AB. Suppose $AN/NB = 2$ and the area of triangle BKN is 6. The area of the triangle ABC is -

A. 54

B. 108

C. 48

D. not determinable with the above data

Answer: B

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7. Let P be an arbitrary point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b > 0.$ Suppose F_1 and F_2 are the foci are the ellipse. The locus of the centroid of the triangle PF_1F_2 as P moves on the ellipse is- (A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

A. a circle

B. a parabola

C. an ellipse

D. a hyperbola

Answer: C

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8. The number of roots of equation $\cos^7\theta - \sin^4\theta = 1$ that lie in the interval $[0, 2\pi]$ is-

A. 2

B. 3

C. 4

D. 8

Answer: A

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9. Prove that $(1+\tan 1^\circ)(1+\tan 2^\circ)....(1+\tan 45^\circ) = 2^{23}$

A. 2^{21}

B. 2^{22}

C. 2^{23}

D. 2^{25}

Answer: C

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10. Let $f: R \to R$ be a differentiable function such that f (a) = 0 = f (b) and $f'(a)f'(b) > 0$ for some $a < b$. Then the minimum number of roots of f'(x) 0 in the interval (a, b) is

A. 3

B. 2

C. 1

D. 0

Answer: B

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11. The roots of $(x-41)^{49}+(x-49)^{41}+(x-2009)^{2009}=0$ are

A. All necessarily real

B. non-real except one positive real root

C. non-real except three positive real roots

D. non-real except for three real roots of which exactly one is positive

Answer: B

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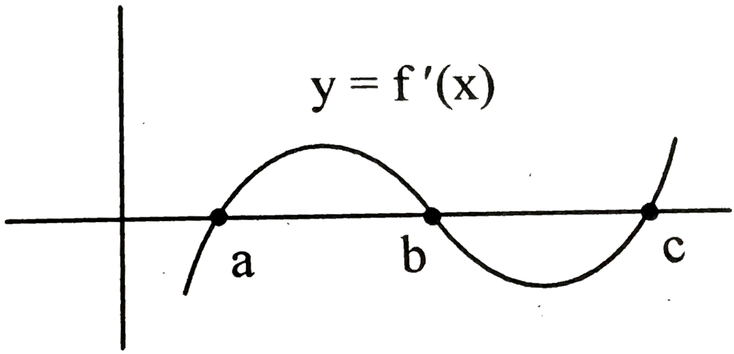
12. The figure shown below is the graph of the derivative of some function $y = f'(x)'$.



Then

A. f has local minima at $x = a, b$ and a local maximum at $x = c$

B. f has local minima at $x = b, c$ and a local maximum at $x = a$

C. f has local minima at $x = c, a$ and a local maximum at $x = b$

D. the given figure is insufficient to conclude any thing about the local minima and local maxima of f

Answer: C

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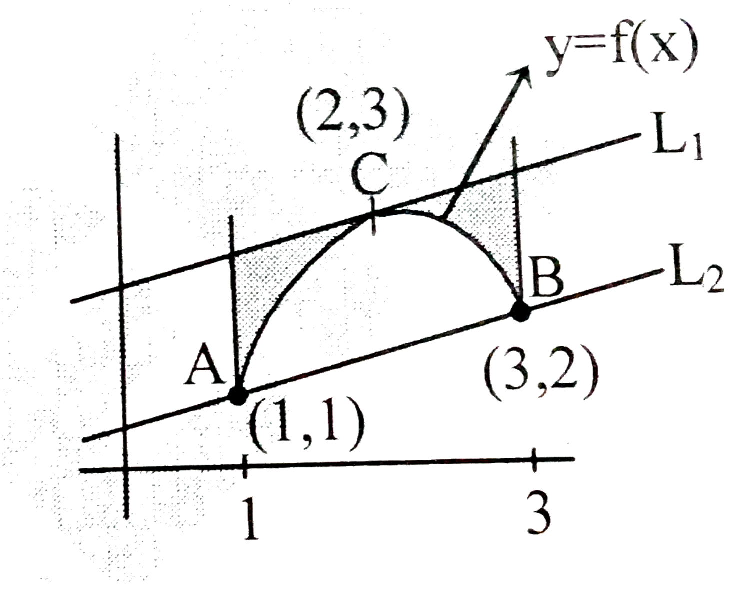
13. The following shows the graph of a continuous function $y = f(x)$on the interval $[1, 3]$. The points A, B, C have coordinates $(1, 1)$, $(3, 2)$, $(2, 3)$ respectively, and the lines L_1 and L_2 are parallel, with L_1 being tangent to the curve at C. If the area under the graph of $y = f(x)$ from $x = 1$ to $x = 3$ is 4 square units, then the area of the shaded region is -



A. 2

B. 3

C. 4

D. 5

Answer: A

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14. Let $I_n = \int_0^1 (\log x)^n dx$, where n is a non-negative integer. Then $I_{2001} + 2011 I_{2010}$ is equal to -

A. $I_{1000} + 999 I_{998}$

B. $I_{890} + 890 I_{889}$

C. $I_{100} + 100 I_{99}$

D. $I_{53} + 54 I_{52}$

Answer: C

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15. Consider the regions $A = \{(x, y) \mid x^2 + y^2 \leq 100\}$ and $= \{(x, y) \mid \sin(x + y) > 0\}$ in the plane. Then the area of the region

$A \cap B$ is -

A. 10π

B. 100

C. 100π

D. 50π

Answer: D

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16. Three vertices are chosen randomly from the seven vertices of a regular 7-sided polygon. The probability that they from the vertices of an isosceles triangle is-

A. $\frac{1}{7}$

B. $\frac{1}{3}$

C. $\frac{3}{7}$

D. $\frac{3}{5}$

Answer: D

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17. Let $\overrightarrow{u} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{v} = -3\hat{j} + 2\hat{k}$ be vectors in R^3 and $\overrightarrow{w}$ be a unit vector in the xy-plane. Then the maximum possible value of $\left|\left(\overrightarrow{u} \times \overrightarrow{v}\right). \overrightarrow{w}\right|$ is-

A. $\sqrt{5}$

B. $\sqrt{12}$

C. $\sqrt{13}$

D. $\sqrt{17}$

Answer: D

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18. How many six-digit numbers are there in which no digit is repeated, even digits appear at even places, odd digits appear at odd places and the number is divisible by 4 ?

A. 3600

B. 2700

C. 2160

D. 1440

Answer: D

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19. The number of natural numbers n in the interval $[1005, 2010]$ for which the polynomial $1 + x + x^2 + x^3 + \ldots..x^{n-1}$ divides the polynomial $1 + x^2 + x^4 + x^6 + \ldots\ldots x^{2010}$ is -

A. 0

B. 100

C. 503

D. 1006

Answer: C

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20. Let $a_0 = 0$ and $a_n = 3a_{n-1} + 1$ for $n \geq 1$. Then the remainder obtained dividing a_{2010} by 11 is -

A. 0

B. 7

C. 3

D. 4

Answer: C

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21. Arrange the expansion of $\left(x^{1/2}+\frac{1}{2x^{1/4}}\right)^{n}$ in decreasing powers of x. Suppose the coefficient of the first three terms form an arithmetic progression. Then the number of terms in the expression having integer powers of x is -

(A) 1

(B) 2

(C) 3

(D) more than 3

A. 1

B. 2

C. 3

D. more than 3

Answer: C

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22. Let r be a real number and $n \in N$ be such that the polynomial $2x^2 + 2x + 1$ divides the polynomial $(x+1)^n - r$. Then (n, r) can be-

(A) $(4000, 4^{1000})$ (B) $\left(4000, \frac{1}{4^{1000}}\right)$ (C) $\left(4^{1000}, \frac{1}{4^{1000}}\right)$ (D) $\left(4000, \frac{1}{4000}\right)$

A. $(4000, 4^{1000})$

B. $\left(4000, \frac{1}{4^{1000}}\right)$

C. $\left(4^{1000}, \frac{1}{4^{1000}}\right)$

D. $\left(4000, \frac{1}{4000}\right)$

Answer: B

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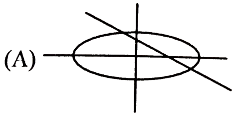
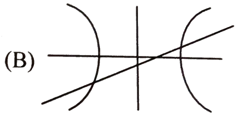
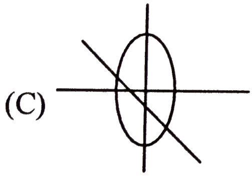
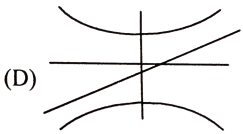
23. Suppose a, b are real numbers such that $ab \neq 0$. Which of the following four figures represents the curve $(y - ax - b)(bx^2 + ay^2 - ab) = 0$?

A. 

B. 

C. 

D. 

Answer: B

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24. Among all cyclic quadrilaterals inscribed in a circle of radius R with one of its angles equal to 120°. Consider the one with maximum possible area. Its area is-

A. $\sqrt{2}R^2$

B. $\sqrt{3}R^2$

C. $2R^2$

D. $2\sqrt{3}R^2$

Answer: B

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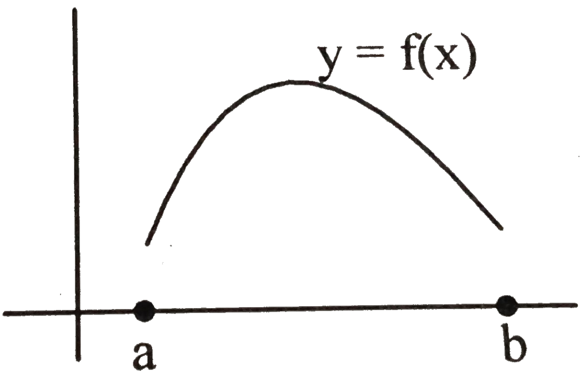
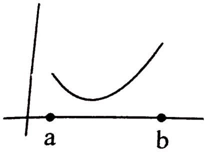
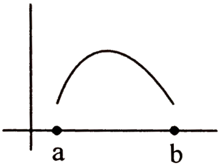
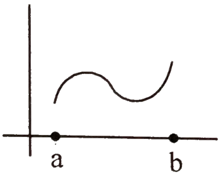
25. The following figure shows the graphs of a differentiable fucntion $y = f(x)$ on the interval $[a, b]$ (not contaning 0).

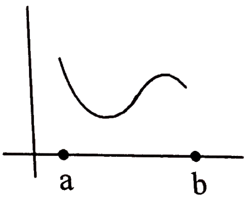


Let $g(x) = f(x)/x$ which of the following is a possible graph of $y = g(x)$?

A. 

B. 

C. 

D. 

Answer: B

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26. Let V_1 be the volume of a given right circular cone with O as the centre of the base and A as its apex. Let V_2 be the maximum volume of the right circular cone inscribed in the given cone whose apex is O and whose base is parallel to the base of the given cone. then the ratio V_2 / V_1 is -

A. $\frac{3}{25}$

B. $\frac{4}{9}$

C. $\frac{4}{27}$

D. $\frac{8}{27}$

Answer: C

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27. Let $f: R \to R$ be a continous function satisfying $f(x) = x + \int_0^x f(t) dt$, for all $x \in R$. Then the numbr of elements in the set $S = \{x \in R: f(x) = 0\}$ is -

A. 1

B. 2

C. 3

D. 4

Answer: A

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28. The value of $\int_{2\pi}^{0} \min\{|x-\pi|, \cos^{-1}(\cos x)\}dx$ is -

A. $\frac{\pi^2}{4}$

B. $\frac{\pi^2}{2}$

C. $\frac{\pi^2}{8}$

D. π^2

Answer: B

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29. Let ABC be a triangle and P be a point inside ABC such that $\overrightarrow{PA} + 2\overrightarrow{PB} + 3\overrightarrow{PC} = \overrightarrow{0}$. The ratio of the area of triangle ABC to that of APC is - (A) 2 (B) $\frac{3}{2}$(C) $\frac{5}{3}$ (D) 3

A. 2

B. $\frac{3}{2}$

C. $\frac{5}{3}$

D. 3

Answer: D

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30. Suppose m, n are positive integers such that $6^m + 2^{m+n}3^m + 2^n = 332$. The value of the expression $m^2 + nm + n^2$ is -

A. 7

B. 13

C. 19

D. 21

Answer: C

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31. Suppose, a, b, c are three distinct real numbers. Let P (x) = $\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-c)(x-a)}{(b-c)(b-a)}+\frac{(x-a)(x-b)}{(c-a)(c-b)}$.

When simplified, P (x) becomes

A. 1

B. x

C. $\frac{x^2+(a+b+c)(ab+bc+ca)}{(a-b)(b-c)(c-a)}$

D. 0

Answer: A

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32. Let a, b, x, y be real numbers such that $a^2 + b^2 = 81, x^2 + y^2 = 121$ and $ax + by = 99$. Then the set of all possible values of ay - bx is -

A. $\left(0, \frac{9}{11}\right]$

B. $\left(0, \frac{9}{11}\right)$

C. $\{0\}$

D. $\left[\frac{9}{11}, \infty\right)$

Answer: C

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33. If $x + \frac{1}{x} = a, x^2 + \frac{1}{x^3} = b,$ then $x^3 + \frac{1}{x^2}$ is-

A. $a^3 + a^2 - 3a - 2 - b$

B. $a^3 - a^2 - 3a + 4 - b$

C. $a^3 - a^2 + 3a - 6 - b$

D. $a^3 + a^2 + 3a - 16 - b$

Answer: A

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34. Let a,b,c,d be real numbers such that |a-b|=2, |b-c|=3, |c-d|=4 Then the sum of all possible values of |a-d|=

A. 9

B. 18

C. 24

D. 30

Answer: B

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35. Below are four equations in x. Assume that $0 < r < 4$. Which of the following has the largest solution for x ?

A. $5\left(1+\frac{r}{\pi}\right)^{x}=9$

B. $5\left(1+\frac{r}{17}\right)^{x}=9$

C. $5(1+2r)^{x}=9$

D. $5\left(1+\frac{1}{r}\right)^{x}=9$

Answer: B

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36. Let ABC be a triangle with $\angle B = 90^{\circ}$. Let AD be the bisector of $\angle A$ with D on BC. Supose AC = 6 cm and the area of the triangle ADC is 10 cm^2. Then the length of BD in cm is equal to

A. $\frac{3}{5}$

B. $\frac{3}{10}$

C. $\frac{5}{3}$

D. $\frac{10}{3}$

Answer: D

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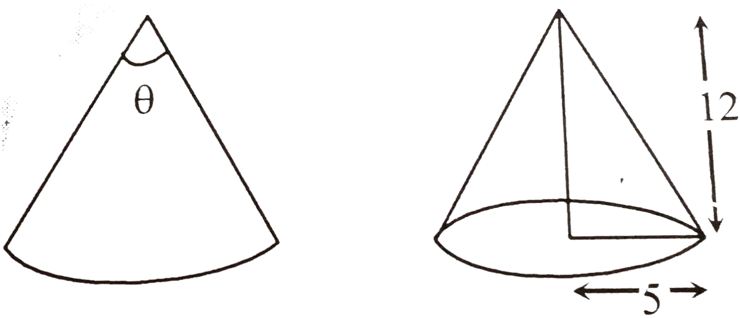
37. A piece of paper in the shape of a sector of a circle (see Fig. 1) is rolled up to form a right- circular cone (see Fig. 2). The value of the angle θ is.



A. $\frac{10\pi}{13}$

B. $\frac{9\pi}{13}$

C. $\frac{5\pi}{13}$

D. $\frac{6\pi}{13}$

Answer: B

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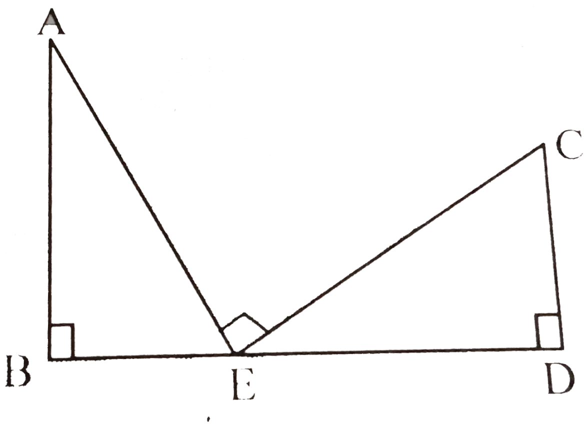
38. In the adjoining figure

$AB = 12cm, CD = 8cm, BD = 20cm, \angle ABD = \angle AEC = \angle EDC = 90^\circ$

. If BE = x, then



A. x has two possible values whose difference is 4

B. x has two possible values whose sum is 28

C. x has only one value and $x \geq 12$

D. x cannot be determined with the given information

Answer: A

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39. Three circles each of radius 1, touch one another externally and they lie between two parallel lines. The minimum possible distance between the lines is

A. $2+\sqrt{3}$

B. $3+\sqrt{3}$

C. 4

D. $2+\frac{1}{\sqrt{3}}$

Answer: A

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40. The number of distinct prime divisors of the number $512^3 - 253^3 - 259^3$ is

A. 4

B. 5

C. 6

D. 7

Answer: C

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41. Consider an incomplete pyramid of balls on a square base having 18 layers; and having 13 balls on each side of the top layer. Then the total number N of balls in that pyramid satisfies

A. $9000 < N < 10000$

B. $8000 < N < 9000$

C. $7000 < N < 8000$

D. $10000 < N < 12000$

Answer: B

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42. A man wants to reach a centain destination. One-sixth of the total distance is muddy while half the distance is tar road. For the remaining distance he takes a boat. His speed of traveling in mud, in water, on tar road is in the ratio $3:4:5$. The ratio ratio of the durations he requires to cross the patch of mud, stream and tar road is

A. $\frac{1}{2}:\frac{4}{3}:\frac{5}{2}$

B. $3:8:15$

C. $10:15:18$

D. $1:2:3$

Answer: C

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43. A frog is presently located at the origin $(0, 0)$ in thexy-plane. It always jumps from a point with integer coordinates to a point with integer coordinates moving adistance of 5 units in each jump. What is the minimum number of jumps required for the frog to go from (0, 0) to (0, 1)

A. 2

B. 3

C. 4

D. 9

Answer: B

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44. A certain 12 - hour digital clock displays the hour and the minute of a day. Due to a defect in the clock whenever the digit 1 is supposed to be displayed it displays 7. What fraction of the day will the clock show the correct time ?

A. $\frac{1}{2}$

B. $\frac{5}{8}$

C. $\frac{3}{4}$

D. $\frac{5}{6}$

Answer: A

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45. There are 30 questions in a multiple - choice test. A student gets 1 mark for each unattempted question, 0 mark for each wrong answer and 4 marks for each corrent answer. A student answered x question correctly and scored 60. Then the number of possible value of x is

A. 15

B. 10

C. 6

D. 5

Answer: C

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46. Let f $(x) = ax^2 + bx + c$ where a, b, c are integers. Suppose f (1)=0,

$40 < f(6) < 50, 60 < f(7) < 70, \text{ and } 1000t < f(50) < 1000(t+1)$

for some integer t. Then the value fo t is

A. 2

B. 3

C. 4

D. 5 or more

Answer: C

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47. The expression $\frac{2^2+1}{2^2-1}+\frac{3^2+1}{3^2-1}+\frac{4^2+1}{4^2-1}+\ldots\ldots.+\frac{(2011)^2+1}{(2011)^2-1}$ lies in the interval

A. $\left(2010, 2010\frac{1}{2}\right)$
B. $\left(2011-\frac{1}{2011}, 2011-\frac{1}{2012}\right)$
C. $\left(2011, 2011\frac{1}{2}\right)$
D. $\left(2012, 2012\frac{1}{2}\right)$

Answer: C

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48. The diameter of one of the bases of a truncated cone is 100 mm. If the diameter of this base is increased by 21% such that it still remains a truncated cone with the height and the other base unchanged, the volume also increases by 21%. The radius the other base (in mm) is

A. 65

B. 55

C. 45

D. 35

Answer: B

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49. Two friends A and B are 30 km apart and they start simultaneously on motorcycles to meet each other. The speed of A is 3 times that of B. The distance between them decreases at the rate of 2 km per minute. Ten minutes after they start, A's vehicle breaks down and A stops and waits for B to arrive. After how much time (in minutes) A started riding, does B meet A ?

A. 15

B. 20

C. 25

D. 30

Answer: D

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50. Three taps A, B, C fill up a tank independently in 10 hr, 20 hr, 30 hr, respectively. Initially the tank is empty and exactly one pair of taps is open during each hour and every pair of taps is open at least for one hour. What is the minimum number of hours required to fill the tank?

A. 8

B. 9

C. 10

D. 11

Answer: A

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51. Suppose $\log_a b + \log_b a = c$. The smallest possible integer value of c for all $a, b > 1$ is -

A. 4

B. 3

C. 2

D. 1

Answer: C

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52. Suppose n is a natural number such that $\left|i + 2i^2 + 3i^3 + \ldots\ldots + ni^n\right| = 18\sqrt{2}$ where i is the square root of -1. Then n is

A. 9

B. 18

C. 36

D. 72

Answer: C

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53. Let P be an $m \times m$ matrix such that $P^2 = P$. Then $(1 + P)^n$ equals

A. $I + P$

B. $I + nP$

C. $I + 2^n P$

D. $I + (2^n - 1)P$

Answer: D

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54. Consider the cubic equation $x^3 + ax^2 + bx + c = 0$, where a, b, c are real numbers, which of the following statements is correct?

A. If $a^2 - 2b < 0$, then the equation has one real and two imahinary roots

B. If $a^2 - 2b \geq 0$, then the equation has all real roots

C. If $a^2 - 2b > 0$, then the equation has all real and distinct roots

D. If $4a^3 - 27b^2 > 0$, then the equation has real and distinct roots

Answer: A

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55. All the point (a, y) in the plane satisfying the equation $x^2 + 2x\sin(xy) + 1 = 0$ lie on -

A. a pair of straight lines

B. a family of hyperbolas

C. a parabola

D. an ellipse

Answer: A

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56. Let $A = (4, 0), B = (0, 12)$ be two points in the plane. The locus of a point C such that the area of triangle ABC is 18 sq. units is -

A. $(y + 3x + 12)^2 = 81$

B. $(y + 3x + 81)^2 = 12$

C. $(y + 3x - 12)^2 = 81$

D. $(y + 3x - 81)^2 = 12$

Answer: C

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57. In a recatngle ABCD, the coordinates of A and B are (1, 2) and (3, 6) respectively and some diameter of the circumscribiling circle of ABCD has equation $2x - y + 4 = 0$. Then the area of the rectangle is -

A. 16

B. $2\sqrt{10}$

C. $2\sqrt{5}$

D. 20

Answer: A

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58. In the xy plane three distinct lines l_1, l_2, l_3 are concurrent at $M(\lambda, 0)$. Also the lines l_1, l_2, l_3 are normals to the parabola $y^2 = 6x$ at the points $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ respectively. Then

A. $\lambda < -5$

B. $\lambda > 3$

C. $-5 < \lambda < \ -3$

D. $0 < \lambda < 3$

Answer: B

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59. Let

$f(x) - \cos 5x + A\cos 4x + B\cos 3x + C\cos 2x + D\cos xE$ and $T - f(0)$

then T

A. depends on A, B, C, D, E

B. depends on A, C, E but independent of B and D

C. depends on B, D but independent of A, C, E

D. is independent of A, B, C, D, E

Answer: C

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60. In triangle ABC, we are given that $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$. Then the measure of the angle C is -

A. 30°

B. 150°

C. 60°

D. 75°

Answer: A

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61. Which of the following intervals is possible domain of the function $f(x) = (\log)_{(x)}[x] + (\log)_{[x]}\{x\}$, where [x] is the greatest integer not exceeding $x and \{x\} = x - [x]$? $(0, 1)$ (b) (1,2) (c) (2, 3) (d) (3, 5)

A. (0, 1)

B. (1, 2)

C. (2, 3)

D. (3, 5)

Answer: C

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62. If $f(x) = (2011 + x)^n$, where x is a real variable and n is a positive interger, then value of $f(0) + f'(0) + \frac{f''(0)}{2!} + \ldots + \frac{f^{(n-1)}(0)}{(n-1)!}$ is $- f(0) + f'(0) + \frac{f''(0)}{2!} + \ldots + \frac{f^{(n-1)}(0)}{(n-1)!}$ is -

A. $(2011)^n$

B. $(2012)^n$

C. $(2012)^n - 1$

D. $n(2011)^n$

Answer: C

Watch Video Solution

63. The minimum distance between a point on the curve $y = e^x$ and a point on the curve $y = \log_e x$ is -

A. $\frac{1}{\sqrt{2}}$

B. $\sqrt{2}$

C. $\sqrt{3}$

D. $2\sqrt{2}$

Answer: B

Watch Video Solution

64. Let f : $(2, \infty) \rightarrow N$ be defined by $f(x) =$ the largest prime factor of [x]. Then $\int_2^8 f(x)dx$ is equal to -

A. 17

B. 22

C. 23

D. 25

Answer: B

Watch Video Solution

65. Let [x] denote the largest interger not exceeding x and $\{x\} = x - [x]$. Then $\int_0^{2012} \frac{e^{\cos(\pi\{x\})}}{e^{\cos(\pi\{x\})} + e^{-\cos(\pi\{x\})}} dx$ is equal to -

A. 0

B. 1006

C. 2012

D. 2012π

Answer: B

View Text Solution

66. The value of $\lim_{n \to \infty} \left(\frac{1}{\sqrt{4n^2 - 1}} + \frac{1}{\sqrt{4n^2 - 4}} + \ldots + \frac{1}{\sqrt{4n^2 - n^2}} \right)$ is -

A. $\frac{1}{4}$

B. $\frac{\pi}{12}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: D

Watch Video Solution

67. Two players play the following game : A writes 3, 5, 6 on three different cards, B writes 8, 9, 10 on three different cards. Both draw randomly two cards from their collections. Then A computes the product of two numbers he/she has drawn, and B computes the sum of two numbers he/she has drawn. The player getting the larger number wins. What is the probability that A wins ?

A. $\frac{1}{3}$

B. $\frac{5}{9}$

C. $\frac{4}{9}$

D. $\frac{1}{9}$

Answer: C

Watch Video Solution

68. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors in the xyz space such that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq 0$ If A, B, C are points with position vector $\vec{a}, \vec{b}, \vec{c}$ respectively, then the number of possible position of the centroid of triangle ABC is -

A. 1

B. 2

C. 3

D. 6

Answer: A

View Text Solution

69. The sum of $\left(1^2-1+1\right)(1!)+\left(2^2-2+1\right)(2!)+\ldots+\left(n^2-n+1\right)(n!)$ is -

A. $(n+2)!$

B. $(n-1)((n+1)!)+1$

C. $(n+2)!-1$

D. $n((n+1)!)-1$

Answer: B

View Text Solution

70. Let X be be a ninempty set and let P(X) denote the collection of all subsets of X. Define

$f: X \times P(X) \rightarrow$ by

$$f(x, A) = \begin{cases} 1, & \text{if} \quad x \in A \\ 0, & \text{if} \quad x \notin A \end{cases}$$

Then $f(x, A \cup B)$ equals-

A. $f(x, A) + f(x, B)$

B. $f(x, A) + f(x, B) - 1$

C. $f(x, A) + f(x, B) - f(x, A) f(x, B)$

D. $f(x, A) + |f(x, A) - f(x, B)|$

Answer: C

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71. Let A and B any two $n \times n$ matrices such that the following conditions hold : $AB = BA$ and there exist positive integers k and l such that

$A^k = I$ (the identity matrix) and $B^l = 0$ (the zero matrix). Then-

A. $A + B = I$

B. det (AB) =0

C. det $(A + B) \neq 0$

D. $(A + B)^m = 0$ for some integer m

Answer: B

Watch Video Solution

72. The minimum value of n for which $\dfrac{2^2 + 4^2 + 6^2 + \ldots + (2n)^2}{1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2} < 1.01$

A. is 101

B. is 121

C. is 151

D. does not exist

Answer: C

Watch Video Solution

73. The locus of the point P = (a, b) where a, b are real numbers such that the roots of $x^3 + ax^2 + bx + a = 0$ are in arithmetic progression is -

A. an ellipse

B. a circle

C. a parabola whose vertex in on the y-axis

D. a parabola whose vertex is no the x-axis

Answer: C

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74. The smallest possible positive slope of a line whose y-intercept is 5 and which has a common point with the ellipse $9x^2 + 16y^2 = 144$ is-

A. $\frac{3}{4}$

B. 1

C. $\frac{4}{3}$

D. $\frac{9}{16}$

Answer: B

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75. Let $A = \{\theta \in R \mid \cos^2(\sin\theta) + \sin^2(\cos\theta) = 1\}$ and $B = \{\theta \in R \mid \cos(\sin\theta)\sin(\cos\theta) = 0\}$. Then $A \cap B$

A. is the empty set

B. has exactly one element

C. has more than one but finitely many elements

D. has infinitely many elements

Answer: A

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76. Let $f(x) = x^3 + ax^2 + bx + c$, where a, b, c are real numbers. If $f(x)$ has a local minimum at $x = 1$ and a local maximum at $x = -\frac{1}{3}$ and $f(2) = 0$, then $\int_{-1}^{1} f(x)dx$ equals-

A. $\frac{14}{3}$

B. $\frac{-14}{3}$

C. $\frac{7}{3}$

D. $\frac{-7}{3}$

Answer: B

Watch Video Solution

77. Let $f(x) = x^{12} - x^9 + x^4 - x + 1$. Which of the following is true ?

A. f is one-one

B. f has a real root

C. f' never vabishes

D. f take only positive values

Answer: D

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78. For each positive interger n, define $f_n(x)=$ minimum $\left(\frac{x^n}{n!},\frac{(1-x)^n}{n!}\right)$, for $0\leq x\leq 1$. Let $I_n=\int_0^1 f_n(x)dx, n\geq 1$. Then $I_n=\sum_{n=1}^{\infty} I_n$ is equal to -

A. $2\sqrt{e}-3$

B. $2\sqrt{e}-2$

C. $2\sqrt{e}-1$

D. $2\sqrt{e}$

Answer: A

Watch Video Solution

79. The maximum possible value of $x^2 + y^2 - 4x - 6y, x, y \in \mathbb{R}$ subject to the condition $|x + y| + |x - y| = 4$

A. is 12

B. is 28

C. is 72

D. does not exist

Answer: B

Watch Video Solution

80. The arithemetic mean and the geometric mean of two distinct 2-digit numbers x and y are two integers one of which can be obtained by reserving the digits of the other (in base 10 representation). Then x + y equals

A. 82

B. 116

C. 130

D. 148

Answer: C

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81. Three children, each accompanied by a guardian, seek admission in a school. The principal want to interview all the 6 persons one after the other subject to the condition that no child is interviewed before its guradian. In how many ways can this be done-

A. 60

B. 90

C. 120

D. 180

Answer: B

Watch Video Solution

82. The equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=$ hs

A. No solution

B. Exactly two distinct solutions

C. Exactly four distinct solutions

D. Infinitely may solutions

Answer: D

Watch Video Solution

83. The maxinum value M of $3^x+5^x-9^x+15^x-25^x$, as x varies over reals, satisfies-

A. 3 lt M lt 5

B. 0 lt M lt 2

C. 9 lt M lt 25

D. 5 lt M lt 9

Answer: A::B

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84. Suppose two perpendicular tangents can be drawn from the origin to the circle $x^2 + y^2 - 6x - 2py + 17 = 0$, for some real p. then $|p|$=

A. 0

B. 3

C. 5

D. 17

Answer: C

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85. Let a, b, c, d be numbers in set {1, 2, 3, 4, 5, 6} such that the curves $y = 2x^3 + ax + b$ and $y = 2x^3 + cx + d$ have no point in common. The maximum possible value of $(a - c)^2 + b - d$ is-

A. 0

B. 5

C. 30

D. 36

Answer: B

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86. Consider the conic $ex^2 + \pi y^2 - 2e^2x - 2\pi^2y + e^3 + \pi^3 = \pi e$. Suppose P is any point on the conic and S_1, S_2 are the foci of conic, then the maximum value of $(PS_1 + PS_2)$ is -

A. πe

B. $\sqrt{\pi e}$

C. $2\sqrt{\pi}$

D. $2\sqrt{e}$

Answer: C

Watch Video Solution

87. Let $\mathrm{f}(\mathrm{x}) = \dfrac{\sin(x-a)+\sin(x+a)}{\cos(x-a)-\cos(x+a)}$, then-

A. $f(x+2\pi) = f(x) \text{but f}(x+\alpha) \neq f(x) \text{for any} 0 < \alpha < 2\pi$

B. f is strictly increasing function

C. f is strictly decreasing function

D. f is constant function

Answer: D

Watch Video Solution

88. the value of $\tan 9^{\circ} - \tan 27^{\circ} - \tan 63^{\circ} + \tan 81^{\circ}$ is equal to

A. 0

B. 2

C. 3

D. 4

Answer: D

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89. The mid-point of the domain of the function $f(x) = \sqrt{4 - \sqrt{2x + 5}}$ for real x is -

A. $1/4$

B. $3/2$

C. $2/3$

D. $-2/5$

Answer: B

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90. Let n be a natural and let 'a' be a real number. The number of zeros of $x^{2n+1}-(2n+1)x+a=0$ in the interval $[-1, 1]$ is -

A. 2 if a gt 0

B. 2 if a lt 0

C. At most one for every value of a

D. At least three for every value of a

Answer: C

Watch Video Solution

91. Let $f: R \to R$ be the function $f(x) = (x - a_1)(x - a_2) + (x - a_2)(x - a_3) + (x - a_3)(x - a_1)$ with $a_1, a_2, a_3 \in R$. The fix $f(x) \geq 0$ if and only if -

A. At least two of a_1, a_2, a_3 are equal

B. $a_1 = a_2 = a_3$

C. a_1, a_2, a_3 are all distinct

D. a_1, a_2, a_3, are all positive and distinct

Answer: B

Watch Video Solution

92. The value $\dfrac{\int_0^{\pi/2} (\sin x)^{\sqrt{2}+1} dx}{\int_0^{\pi/2} (\sin x)^{\sqrt{2}-1} dx}$ is -

A. $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$

B. $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$

C. $\dfrac{\sqrt{2}+1}{\sqrt{2}}$

D. $s-\sqrt{2}$

Answer: D

View Text Solution

93. The value of $\int_{-2012}^{2012} \left(\sin\left(x^3\right)+x^5+1\right)\mathrm{dx}$ is -

A. 2012

B. 2013

C. 0

D. 4024

Answer: D

Watch Video Solution

94. Let [x] and {x} be the integer part and fractional part of a real number x respectively. The value of the integral $\int_0^5 [x]\{x\}dx$ is -

A. $5/2$

B. 5

C. 34.5

D. 35.5

Answer: B

Watch Video Solution

95. Let $S_n = \sum_{k=1}^{n} k$ denote the sum of the first n positive integers. The numbers $S_1, S_2, S_3, \ldots\ldots S_{99}$ are written on 99 cards. The probability of drawing a card with an even number written on it is -

A. $1/2$

B. $49/100$

C. $49/99$

D. $48/99$

Answer: C

Watch Video Solution

96. A purse contains 4 copper coins, 3 silver coins and the second purse contains 6 copper coins and 2 silver coins. If a coins is taken out of any purse then what is the probability that it is a copper coin.

A. $41/70$

B. $31/70$

C. $27/70$

D. $1/3$

Answer: A

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97. Let H be the orthocenter of an acute - angled triangle ABC and O be itscircumcenter. Then $\overrightarrow{HA}+\overrightarrow{HB}+\overrightarrow{HC}$

A. is equal to $\overrightarrow{HO}$

B. is equal to $\overrightarrow{3HO}$

C. is equal to $\overrightarrow{2HO}$

D. is not a scalar multiple of $\overrightarrow{HO}$ in general

Answer: C

View Text Solution

98. The number of ordered pairs (m,n), where m, n $\in$ {1,2,3, ...,50}, such that 6^m+9^n is a multiple of 5 is -

A. 1250

B. 2500

C. 625

D. 500

Answer: A

Watch Video Solution

99. Suppose $a_1, a_2, a_3, \ldots\ldots, a_{2012}$ are integers arranged on a circle. Each number is equal to the average of its two adjacent numbers. If the sum of all even indexed numbers is 3018, what is the sum of all numbers ?

A. 0

B. 1509

C. 3018

D. 6036

Answer: D

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100. Let $S = \{1, 2, 3, \ldots\ldots n\}$ and A = $\{a, b) \mid 1 \leq a, b \leq n\} = S \times S$. A subset B of A is said to be a good subset if (x, x) $\in$ B for every $x \in S$. Then number of good subsets of A is -

A. 1

B. 2^n

C. $2^{n(n-1)}$

D. 2^{n^2}

Answer: C

Watch Video Solution

101. Two distinct polynomials f(x) and g(x) defined as defined as follow :

$f(x) = x^2 + ax + 2, g(x) = x^2 + 2x + a$

if the equations f(x) =0 and g(x) =0 have a common root then the sum of roots of the equation f(x) +g(x) =0 is -

A. $-\frac{1}{2}$

B. 0

C. $\frac{1}{2}$

D. 1

Answer: C

Watch Video Solution

102. if n is the smallest natural number such that $n + 2n + 3n + \cdots\cdots\cdots + 99n$ is a perfect squre , then the number of digits in n^2 is -

A. 1

B. 2

C. 3

D. more than 3

Answer: C

Watch Video Solution

103. Let x,y,z be positive reals , which of the which following imples $x = y = z?$

A. $x^3 + y^3 + z^3 = 3xyz$

B. $x^3 + y^3z + yz^2 = 3xyz$

C. $x^3 + y^3z + z^2x = 3xyz$

D. $(x + y + z)^3 = 27xyz$

Answer: B

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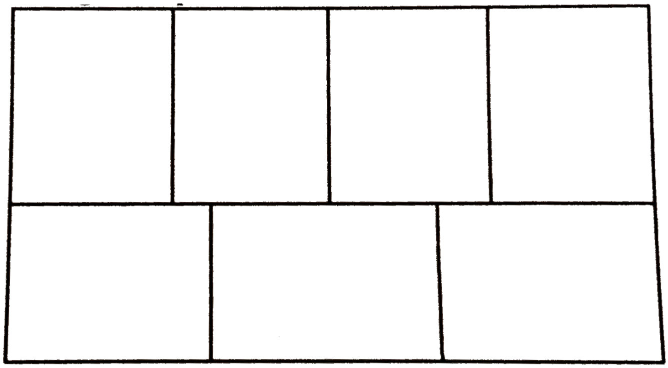
104. in the figure below ,a rectangle of perimeter 763 untis is divided into 7 congruent rectangles :



what is the perimeter of each of the smaller rectangles ?

A. 38

B. 32

C. 28

D. 19

Answer: C

Watch Video Solution

105. the largest non - negative integer K such that 24^k divides 13! Is -

A. 2

B. 3

C. 4

D. 5

Answer: B

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106. in a triangle ABC , points X and Y are on AB And AC , respectively , such that XY is parallel to BC . Which of the two following equalities always hold? (here [PQR] denotes the area of triangle PQR).

(i) [BCX]=[BCY]

(ii) $[ACX].[ABY] = [AXY].[ABC]$

A. Neither (i) nor (ii)

B. (i) only

C. (ii) only

D. Both (i) and (ii)

Answer: D

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107. Let P be an interior point of a triangle ABC , Let Q and R be the reflections of P in AB and AC , respectively if Q .A , R are collinear then $\angle A$ equals -

A. 30°

B. 60°

C. 90°

D. 120°

Answer: C

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108. Let ABCD be a square of side length 1, and I^{-} a circle passing through B and C, and touching AD. The readius of I^{-} is -

A. $\frac{3}{8}$

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{5}{8}$

Answer: D

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109. Let ABCD be a square fo side length 1. P,Q,R,S be points in the interiors of the sides AD,BC,AB,CD, respectively , such that PQ and RS intersect at right angles .if PQ $= \frac{3\sqrt{3}}{4}$ then RS equals -

A. $\frac{2}{\sqrt{3}}$

B. $\frac{3\sqrt{3}}{4}$

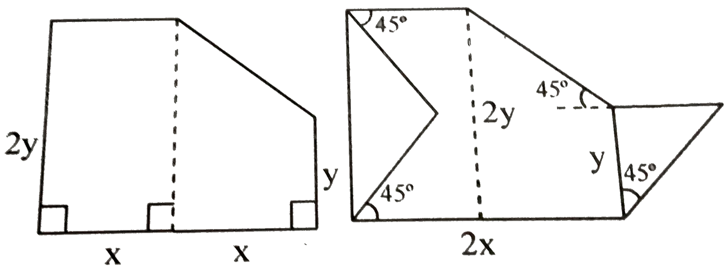
C. $\frac{\sqrt{2}+1}{2}$

D. $4-2\sqrt{2}$

Answer: B

Watch Video Solution

110. in the figure given below , if the areas of the regions are equal then which of the following is true ?



A. x=y

B. x=2y

C. 2x=y

D. x=3y

Answer: B

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111. A man standing on a railway platform noteced that a train took 21 secods to cross the platform (this means the time elpsed from the moment the engine enthers the the platform till the last compartment leaves the platform) which is 88 meters long and that it took 9 seconds to pass him ,ASsuming that the train was moving with unform speed , what is the langth of the train in meters ?

A. 55

B. 60

C. 66

D. 72

Answer: C

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112. the least postitive interger n from which $\sqrt[3]{n+1}-\sqrt[3]{n}<\frac{1}{12}$ is -

A. 6

B. 7

C. 8

D. 9

Answer: C

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113. Let ngt 1 be an interger . Which of the following sets of numbers necessarily contains multiple of 3?

A. $n^{19}-1, n^{19}+1$

B. $n^{19}, n^{38}-1$

C. $n^{38}, n^{38}+1$

D. $n^{38}, n^{19} - 1$

Answer: B

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114. the number of distinct primes dividing $12! + 13! + 14!$ is -

A. 5

B. 6

C. 7

D. 8

Answer: A

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115. How many ways are there to arrange the letters of the word EDUCATION so that all the following three conditions hold ? - the vowels occur in the order (EUAIO) - the consonants occur in the same order (DCTN) - no two consonants are next to each other

A. 15

B. 24

C. 72

D. 120

Answer: A

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116. A triangular corner is cut form a rectangular priece of paper an the resulting pentagon has sides ,5,6,8,9,12 in some order , the ratio of the area of the pentagen to the area of the rectangle is -

A. $\frac{11}{18}$

B. $\frac{13}{18}$

C. $\frac{15}{18}$

D. $\frac{17}{18}$

Answer: D

Watch Video Solution

117. for a real number x, let [x] denote the largest unteger less than or equal to x , and let {x} = x-[x] . The number of solution x to the equation [x] {x} =5 with $0 \leq x \leq 2015$ is -

A. 0

B. 3

C. 2008

D. 2009

Answer: D

Watch Video Solution

118. Let ABCD be a traezium with AD parallel to BC. Assume there is a point M in the interior of the segement BC such that AB=AM and DC=DM . Then the ratio of the area of the tapezium to the area of triabgle AMD is -

A. 2

B. 3

C. 4

D. Not determinable from data

Answer: B

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119. Given area three cylindrical bukets X,Y.Z whose circular bases are of radii 1,2,3 units ,respectively . Intially Water is filled in these bukets upto the same water transferred from Z to X so that they both have the same volume of water Some water is then transferred between X and y so that they both have same volume of water if h_z denote the heights of water at this stage in the bukets y, z, respectively , then the ratio $\frac{h_y}{h_z}$ equals -

A. $\frac{4}{9}$

B. 1

C. $\frac{9}{4}$

D. $\frac{81}{40}$

Answer: D

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120. the average incomes of the people in two villages are P and Q respectively .Assume that $p \neq Q$. A person moves form the first village to the second to the second village . The new average income are P and Q respectively which of the following is not possible ?

A. $P > p$ and $Q > Q$

B. $p > P$ and $Q > Q$

C. $P = P$ and $Q = Q$

D. $P < P$ and $Q < Q$

Answer: C

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121. The number of ordered pairs (x,y) of real numbers that satisfy the simultaneous equations

$x + y^2 = x^2 + y = 12$ is

A. 0

B. 1

C. 2

D. 4

Answer: D

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122. If z a complex number satisfying $\left|z^3 + z^{-3}\right| \leq 2$, then the maximum possible value of $\left|z + z^{-1}\right|$ is -

A. 2

B. $3\sqrt{2}$

C. $2\sqrt{2}$

D. 1

Answer: A

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123. The largest perfect square that divides $2014^3 - 2013^3 + 2012^3 - 2011^3 + \ldots + 2^3 - 1^3$ is

A. 1^2

B. 2^2

C. 1007^2

D. 2014^2

Answer: C

Watch Video Solution

124. Suppose OABC is a reatangle in the xy-plane where O is the origin and A, B lie on the parabola $y = x^2$.Then C must lie on the curve

A. $y = x^2 + 2$

B. $y = 2x^2 + 1$

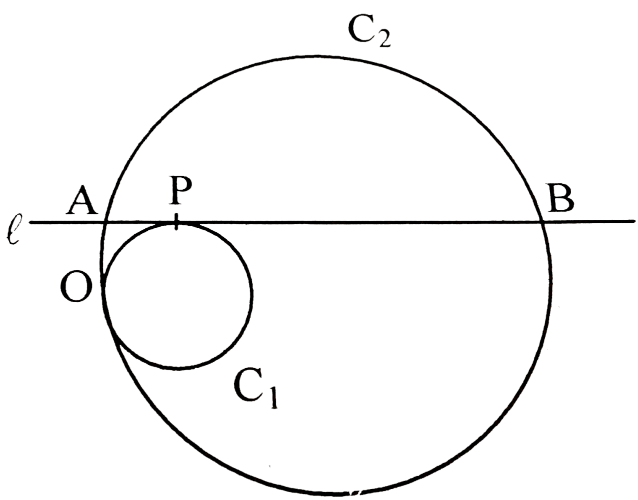
C. $y = -x^2 + 2$

D. $y = -2x^2 + 1$

Answer: A

Watch Video Solution

125. Circles C_1 and C_2 of radii r and R respectively, touch each other as shown in the figure. The lime l, which is parallel to the line joining the centres of C_2 and C_2 is tangent to C_1 at P and intersects C_2 at A,B.If $R^2 = 2r^2$, then $\angle AOB$ equals-



A. $22\frac{1}{2}\circ$

B. 45°

C. 60°

D. $67\frac{1}{2}^\circ$

Answer: B

Watch Video Solution

126. The shortest distance from the origin to a variable point on the sphere $(x-2)^2+(y-3)^2+(z-6)^2=1$ is-

A. 5

B. 6

C. 7

D. 8

Answer: B

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127. The number of real number λ for which the equality

$$\frac{\sin(\lambda\alpha)}{\sin\alpha}-\frac{\cos(\lambda\alpha)}{\cos\alpha}=\lambda-1,$$

holds for all real α which are not integral multiples of $\pi/2$ is-

A. 1

B. 2

C. 3

D. Infinite

Answer: B

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128. Suppose ABCDEF is a hexagon such that AB=BC=CD=1 and DE=EF=FA=2. If the vertices A,B,C,D,E,F, are concylic, the radius of the circle passing throught them is-

A. $\sqrt{\frac{5}{2}}$

B. $\sqrt{\frac{7}{3}}$

C. $\sqrt{\frac{11}{5}}$

D. $\sqrt{2}$

Answer: B

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129. Let P(x) be a polynomial such that p(x)-p'(x)= x^n, where n is a positive integer. Then P(0) equals-

A. n!

B. (n-1)!

C. $\frac{1}{n!}$

D. $\frac{1}{(n-1)!}$

Answer: A

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130. The value of the limit

$\lim_{x \to 0} \left(\frac{x}{\sin x}\right)^{6/x^2}$ is

A. e

B. e^{-1}

C. $e^{-1/6}$

D. e^6

Answer: A

Watch Video Solution

131. Among all sectors of fixed perimeter, choose the one with maximum area. Then the angle at the center of this sector (i.e.,the angle between the boundibg radii) is

A. $\frac{\pi}{3}$

B. $\frac{3}{2}$

C. $\sqrt{3}$

D. 2

Answer: D

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132. Define a function f : R $\rightarrow$ R by

$f(x) = \max\{|x|, |x-1|, \dots |x-n|\}$

where n is a fixed natural number. Then $\int_0^{2n} f(x) dx$ is -

A. n

B. n^2

C. 3n

D. $3n^2$

Answer: D

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133. If P(x) is a cubic polynomial with P(1)=3,P(0)=2 and P(-1)=4, then $\int_{-1}^{1} P(x)dx$ is

A. 2

B. 3

C. 4

D. 5

Answer: D

Watch Video Solution

134. Let $x > 0$ be a fixed reacl number. Then the integral $\int_{0}^{\infty} e^{-1}|x-t|dt$ is equal to -

A. $x+2e^{-x}-1$

B. $x-2e^{-x}+1$

C. $x+2e^{-x}+1$

D. $-x-2e^{-x}+1$

Answer: A

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135. An urn contains marbles of four colours : red, white, blue and green. When four marbles are drawn without replacement, the following events are equally likely :

(1) the selection of four red marbles

the selection of one white and three red marbles

(3) the selection of one white, one blue and two red marbles

(4) the selection of one marble of each colour

The smallest total number of satisfying the given condition is

A. 19

B. 21

C. 46

D. 69

Answer: B

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136. There are boxes labelled $B_1, B_2, \ldots, B_6$. In each trial, two fair dice $D_1D\_(2)$ are thrown. If D_1 shows j and D_2 shows k, then j balls are put into the box the $B(k)$. After n trials, what is the probability that B_1 contains at most one ball ?

A. $\left(\frac{5^{n-1}}{6^{n-1}}\right)+\left(\frac{5^n}{6^n}\right)\left(\frac{1}{6}\right)$

B. $\left(\frac{5^n}{6^n}\right)+\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6}\right)$

C. $\left(\frac{5^n}{6^n}\right)+n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6}\right)$

D. $\left(\frac{5^n}{6^n}\right)+n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6^2}\right)$

Answer: D

Watch Video Solution

137. Let $\vec{a} = 6\vec{i} - 3\vec{j} - 6\vec{k}$ and $\vec{d} = \vec{i} + \vec{j} + \vec{k}$. Suppose that $\vec{a} = \vec{b} + \vec{c}$

A. $5\vec{i} - 4\vec{j} - \vec{k}$

B. $7\vec{i} - 2\vec{j} - 5\vec{k}$

C. $4\vec{i} - 5\vec{j} + \vec{k}$

D. $3\vec{i} + 6\vec{j} - 9\vec{k}$

Answer: B

Watch Video Solution

138. If $\log_{(3x-1)}(x-2) = \log_{(9x^2-6x+1)}(2x^2-10x-2)$, then x equals-

A. $9-\sqrt{15}$

B. $3+\sqrt{15}$

C. $2+\sqrt{5}$

D. $6-\sqrt{5}$

Answer: B

Watch Video Solution

139. Suppose a,b,c are positive integers such that $2^a+4^b+8^c=328$ then $\frac{a+2b+3c}{abc}$ is equal to

A. $\frac{1}{2}$

B. $\frac{5}{8}$

C. $\frac{17}{24}$

D. $\frac{5}{6}$

Answer: C

Watch Video Solution

140. The sides of a right-angled triangle are integers. The length of one of the sides is 12. The largest possible radius of the incircle of such a triangle is-

A. 2

B. 3

C. 4

D. 5

Answer: D

Watch Video Solution

141. $Let x = \left(\sqrt{50}+7\right)^{1/3} - \left(\sqrt{50}-7\right)^{1/3}$.Then-

A. x=2

B. x=3

C. x is a rational number, but not an integer

D. x is an irrational number

Answer: A

Watch Video Solution

142. Let

$\left(1+x+x^2\right)^{2014}=a_0+a_1x+a_2x^2+a_3x^3+\ldots+a_{4028}x^{4028}$, and

let

$A=a_0-a^3+a_6-\ldots+a_{4026}$

$B=a_1-a_4+a_7-\ldots\ldots\ldots-a_{4027}$,

$C=a_2-a_5+a_8-\ldots.+a_{4028}$,

Then-

A. $|A|=|B|>|C|$

B. $|A|=|B|<|C|$

C. $|A| = |C| > |B|$

D. $|A| = |C| < |B|$

Answer: D

Watch Video Solution

143. A mirror in the first quadrant is in the shape of a hyperbola whose equation is xy=1. A light source in the second quadrant emits a beam of light that the hits the mirror at the point (2,1/2). If the reflected ray is parallel to the y-axis , the slope of the incident beam is

A. $\frac{13}{8}$

B. $\frac{1}{4}$

C. $\frac{15}{8}$

D. 2

Answer: C

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144. Let

$$C(\theta) = \sum_{n=0}^{\infty} \frac{\cos(n\theta)}{n!}$$

Which of the following statements is FALSE ?

A. $C(\theta).\,C(\pi) = 1$

B. $C(\theta) + C(\pi) > 2$

C. $C(\theta) > 0 \text{for all} \theta \in R$

D. $C'(\theta) \neq 0 \text{for all} \theta \in R$

Answer: D

Watch Video Solution

145. Let a $>$ 0 be real number. Then the limit

$$\lim_{x \to 2} \frac{a^x + a^{3-x} - (a^2 + a)}{a^{3-x} - a^{x/2}}$$

is-

A. 2 log a

B. $-\frac{4}{3}$ a

C. $\frac{a^2+a}{2}$

D. $\frac{2}{3}(1-a)$

Answer: D

Watch Video Solution

146. Let $f(x)=\alpha x^2-a+\frac{1}{x}$"where"alpha

$is real cons \tan t. The smal \leq st$alpha$f$ or f(x)ge0"for all"xgt0` is-

A. $\frac{2^2}{3^3}$

B. $\frac{2^3}{3^3}$

C. $\frac{2^4}{3^3}$

D. $\frac{2^5}{3^3}$

Answer: D

View Text Solution

147. Let f : R $\rightarrow$ R be a continuous function satisfying

$$f(x) + \int_0^x \mathrm{tf}(t)\mathrm{dt} + x^2 = 0$$

for all x $\in R$. Then-

A. $\lim_{x \to -\infty} f(x) = 2$

B. $\lim_{x \to -\infty} f(x) = -2$

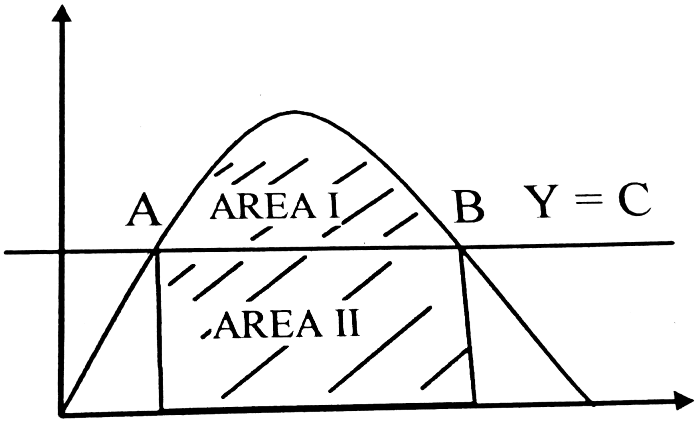
C. f(x) has more than one point in common with x-axis

D. f(x) is an odd functions

Answer: B

Watch Video Solution

148. The figure shows a portion of the graph $y = 2x - 4x^3$. The line y=c is such that the areas of the regions marked I and II are equal. If a, b are the x-coordinates of A,B respectively, then a+b equals-



A. $\frac{2}{\sqrt{7}}$

B. $\frac{3}{\sqrt{7}}$

C. $\frac{4}{\sqrt{7}}$

D. $\frac{5}{\sqrt{7}}$

Answer: A

Watch Video Solution

149. Let $X_n = \{1, 2, 3, \ldots, n\}$ and let a subset A of X_n be chosen so that every pair of elements of A differ by at least 3. (For example, if n=5, A can be $\varnothing$, {2} or {1,5} among others). When n=10, let the probability that 1 $\in$ A be p and let the probability that 2 $\in$ A be q. Then -

A. $p > q$ and $p - q = \frac{1}{6}$

B. $p > q$ and $q - p = \frac{1}{6}$

C. $p > q$ and $p - q = \frac{1}{10}$

D. $p > q$ and $q - p = \frac{1}{10}$

Answer: C

Watch Video Solution

150. The remainder when the determinant

$$\begin{vmatrix} 2014^{2014} & 2015^{2015} & 2016^{2016} \\ 2017^{2017} & 2018^{2018} & 2019^{2019} \\ 2020^{2020} & 2021^{2021} & 2022^{2022} \end{vmatrix}$$

is divided by 5 is-

A. 1

B. 2

C. 3

D. 4

Answer: D

Watch Video Solution

151. A student notices that the roots of the equation $x^2 + bx + a = 0$ are each 1 less than the roots of the equation $x^2 + ax + b = 0$. Then a+b is.

A. Possibly any real number

B. -2

C. -4

D. -5

Answer: C

Watch Video Solution

152. If x, y are real numbers such that $3^{\frac{x}{y}+1} - 3^{\frac{x}{y}-1} = 24$, then the value of (x+y)/(x-y) is

A. 0

B. 1

C. 2

D. 3

Answer: D

Watch Video Solution

153. The number of positive integers n in the set {1, 2, 3, ...100} for which the number $\dfrac{1^2 + 2^2 + 3^2 + \ldots\ldots + n^2}{1 + 2 + 3 + \ldots\ldots + n}$ is an integer is

A. 33

B. 34

C. 50

D. 100

Answer: D

Watch Video Solution

154. The three different face diagonals of a cuboid (rectangular parallelopiped) have lengths 39, 40, 41.The length of the main diagonal of the cuboid which joins a pair of opposite corners :

A. 49

B. $49\sqrt{2}$

C. 60

D. $60\sqrt{2}$

Answer: A

Watch Video Solution

155. The sides of a triangle ABC are positive integers. The smallest side has length 1. Which of the following statement is true ?

A. The area of ABC is always a rational number

B. The area of ABC is always an irrational number

C. The perimeter of ABC is an even integer

D. The information provided is not sufficient to conclude any of the statement A, B or C above

Answer: B

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156. Consider a square ABCD of side 12 and let M, N be the midpoints of AB, CD respectively. Take a point P on MN and let AP=r, PC=s. Then the area of the triangle whose sides are r, s, 12 is-

A. 72

B. 36

C. $\frac{rs}{2}$

D. $\frac{rs}{7}$

Answer: B

Watch Video Solution

157. A cow is tied to a corner (vertex) of a regular hexagonal fenced area of side a metres by a rope of length 5a/2 metres in a grass field. (The cow cannot graze inside the fenced area.) What is the maximum possible area of the grass field to which the cow has access to graze ?

A. $5\pi a^2$

B. $\frac{5}{2}\pi a^2$

C. $6\pi a^2$

D. $3\pi a^2$

Answer: A

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158. A closed conical vessel is filled with water fully and is placed with its vertex down. The water is flow out at a constant speed. After 21 minutes, it was found that the height of the water column is half of the original height. How much more time in minutes does it require to empty the vessel ?

A. 21

B. 14

C. 7

D. 3

Answer: D

Watch Video Solution

159. I carried 1000 kg of watermelon in summer by train. In the beginning, the water content was 99%. By the time I reached the destination, the water content had dropped to 98%. The reduction in the weight of the watermelon was-

A. 10 kg

B. 50 kg

C. 100 kg

D. 500 kg

Answer: D

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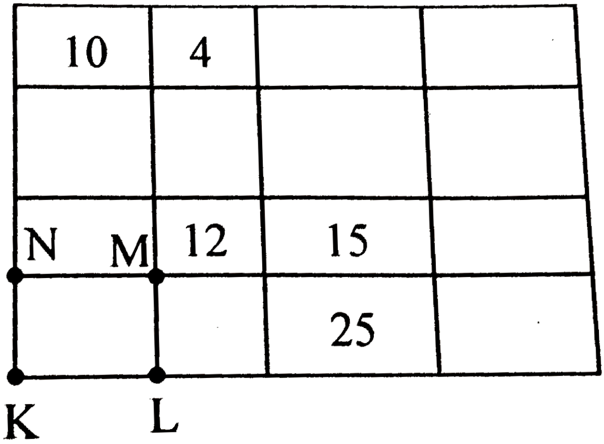
160. A rectangle is divided into 16 sub-rectangles as in the figure, the number in each sub rectangle represents the are of that sub-rectangle.

What is the area of the rectangle KLMS ?



A. 20

B. 30

C. 40

D. 50

Answer: D

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161. In a triangle ABC, D and E are points on AB, AC respectively such that DE is parallel to BC. Suppose BE, CD intersect at O. If the areas of the triangles ADE and ODE are 3 and 1 respectively, find the area of the triangle ABC, with justification

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162. Leela and Madan pooled their music CD's and sold them. They got as many rupees for each CD as the total number of CD's they sold. They share the money as follows Leela first takes 10 rupees, then Madan takes 10 rupees and they continue taking 10 rupees alternately till Madan is left out with less than 10 rupees to take. Find the amount that is left out for Madan at the end, with justification.

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163. (a) Show that for every nutural number n relatively prime to 10, there is another natural number m all of whose digits are 1 's such that n divides m.

(b) Hence or otherwise show that every positive rational number can be expressed in the form $\frac{a}{10^b(10^c - 1)}$ for some natural a, b, c.

View Text Solution

PART-I MATHEMATICS

1. Let $f(x)$ be a quadratic polynomial with $f(2) = -2$. Then the coefficient of x in $f(x)$ is-

A. 1

B. 2

C. 3

D. 4

Answer: C

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2. The square root of $\frac{(0.75)^3}{1-(0.75)} + \left(0.75 + (0.75)^2 + 1\right)$ is-

A. 1

B. 2

C. 3

D. 4

Answer: B

Watch Video Solution

3. The side of a triangle are distinct integers in an arithmetic progression. If the smallest side is 10, the number of such triangles is-

A. 8

B. 9

C. 10

D. Infinitely many

Answer: B

Watch Video Solution

4. If a, b, c, d are positive real numbers such that $\frac{a}{3}=\frac{a+b}{4}=\frac{a+b+c}{5}=\frac{a+b+c+d}{6}, \text{then} \frac{a}{b+2c+3d}$ is-

A. 1//2

B. 1

C. 2

D. Not determinable

Answer: A

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5. For $\frac{2^2 + 4^2 + 6^2 + \ldots\ldots. + (2n)^2}{1^2 + 3^2 + \ldots\ldots + (2n-1)^2}$ to exceed 1.01, the maximum value of n is-

A. 99

B. 100

C. 101

D. 150

Answer: D

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6. In triangle ABC, let AD, BE and CF be the internal angle bisectors with D, E and F on the sides BC, CA and AB respectively. Suppose AD, BE and CF concur at I and B, D, I, F are concyclic, then $\angle IFD$ has measure-

A. 15°

B. 30°

C. 45°

D. Any value $\leq 90^\circ$

Answer: B

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7. A regular octagon is formed by cutting congruent isosceles right-angled triangles from the corners of a square. If the square has side-length 1, the side-length of the octagon is-

A. $\frac{\sqrt{2}-1}{2}$

B. $\sqrt{2}-1$

C. $\frac{\sqrt{5}-1}{4}$

D. $\frac{\sqrt{5}-1}{3}$

Answer: B

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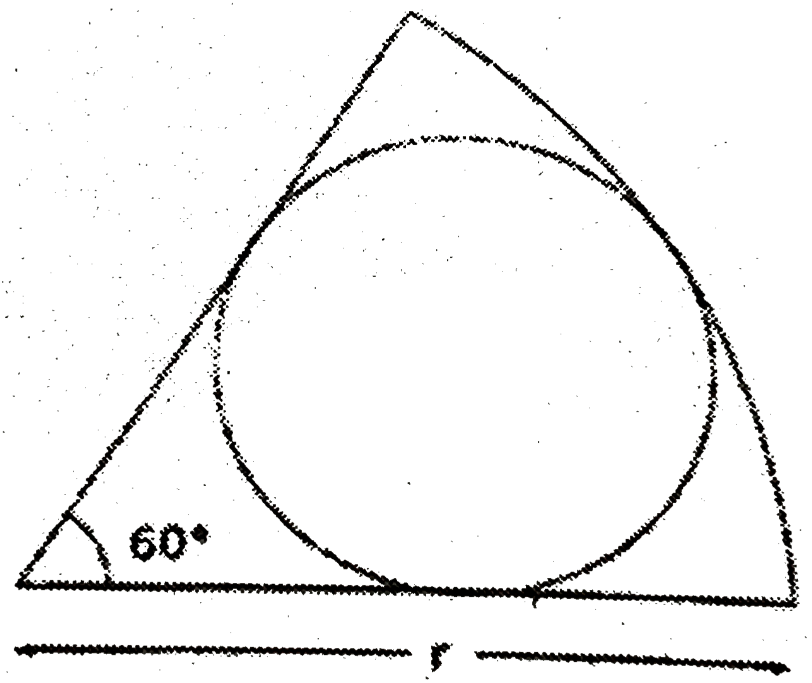
8. A circle is drawn in a sector of a larger circle of radius r, as shown in the adjacent figure. The smaller circle is tangent to the two bounding radii and the are of the sector. The radius of the small circle is-



A. $\frac{r}{2}$

B. $\frac{r}{3}$

C. $\frac{2\sqrt{3}r}{5}$

D. $\frac{r}{\sqrt{2}}$

Answer: B

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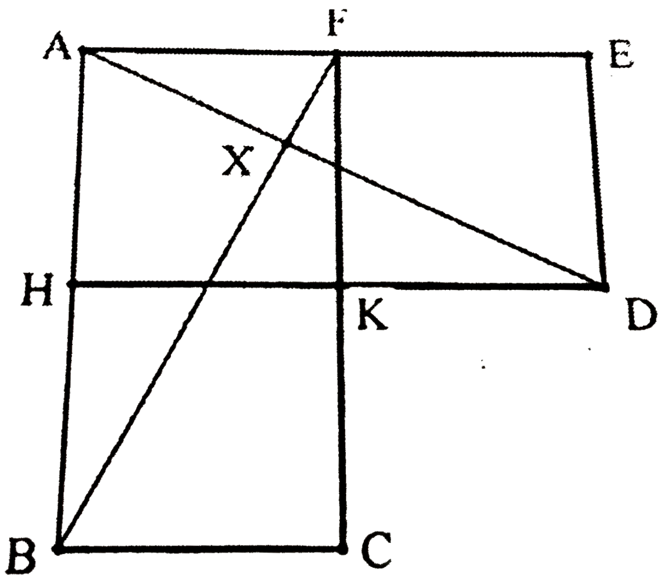
9. In the figure AHKF, FKDE and HBCK are unit squares, AD and BF intersect in X. Then the ratio of the areas fo triangles AXF and ABF is-



A. 1//4

B. 1//5

C. 1//6

D. 1//8

Answer: B

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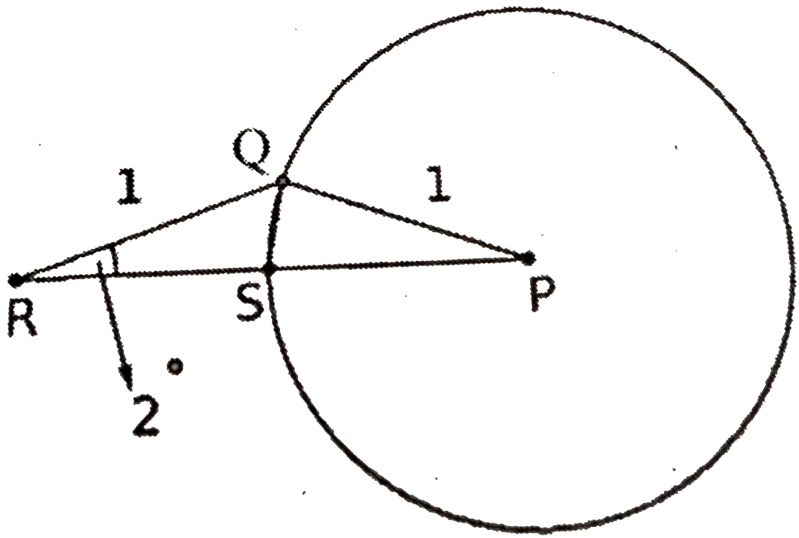
10. Suppose Q is a point on the circle with center P and radius 1, as shown in the figure , R is a point outside the circle such that $QR = 1$ and $\angle QRP = 2^\circ$. Let S be the point where the segment RP intersects the given circle. Then measure of $\angle RQS$ equals-



A. 86°

B. 87°

C. 88°

D. 89°

Answer: D

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11. Observe that, at any instant, the minute and hour hands of a clock make two angles between them whose sum is 360°. At 6:15 the difference between these two angles is-

A. 165°

B. 170°

C. 175°

D. 180°

Answer: A

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12. Two workers A and B are engaged to do a piece of work. Working alone, A takes 8 hours more to complete the work than if both worked together. On the other hand, working alone, B would need $4\frac{1}{2}$ hours more to complete the work than if both worked together. How much time would they take to complete the job working together?

A. 4Hours

B. 5Hours

C. 6 Hours

D. 7 Hours

Answer: C

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13. When a bucket is half full, the weight of the bucket and the water is 10kg. When the bucket is two-thirds full, the total weight is 11kg. What is the total weight, in kg. when the bucket is completely full-

A. 12

B. $12\frac{1}{2}$

C. $12\frac{2}{3}$

D. 13

Answer: D

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14. How many ordered pairs of (m,n) integers satisfy $\frac{m}{12} = \frac{12}{n}$?

A. 30

B. 15

C. 12

D. 10

Answer: A

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15. Let $S = \{1, 2, 3, \ldots, 40\}$ and let A be a subset of S such that notwo elements in A have their sum divisible by 5. What is themaximum number of elements possible in A?

A. 10

B. 13

C. 17

D. 20

Answer: C

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16. Consider the following statements :

I. $\lim_{n \to \infty} \dfrac{2^n + (-2)^n}{2^n}$ does not exist

II. $\lim_{n \to \infty} \dfrac{3^n + (-3)^n}{4^n}$ does not exist

Then

A. I is true and II is false

B. I is false and II is true

C. I and II are true

D. neither I nor II is true

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17. Consider a regular 10-gon with its vertices on the unit circle. With one vertex fixed, draw straight lines to the other 9 vertices. Call them $L_1, L_2, \dots. L_9$ and denote their lengths by $l_1, l_2 \dots. l_9$ respectively. Then the product $l_1 l_2 \dots. l_9$ is

A. 10

B. $10\sqrt{3}$

C. $\frac{50}{\sqrt{3}}$

D. 20

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18. The value of the integral

$$\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+e^x} dx$$

is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{\pi^2}{2}$

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19. Let $\mathbb{R}$ be the set of all real numbers and

$$f(x) = \sin^{10} x \left(\cos^8 x + \cos^4 x + \cos^2 x + 1\right)$$

for $x \in \mathbb{R}$. Let

$S = \{\lambda \in \mathbb{R} |$ there exists a point $c \in (0, 2\pi)$ with $f'(c) = \lambda f(c)\}$.

A. $S = \mathbb{R}$

B. $S = \{0\}$

C. $S = [0, 2\pi]$

D. S is a finite set having more than one element

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20. A person standing on the top of a building of height $60\sqrt{3}$ feel observed the top of a tower to lie at an elevation of 45°. That person descended to the bottom of the building and found that the top of the same tower is now at an angle of elevation of 60°. The height of the tower (in feet) is

A. 30

B. $30(\sqrt{3}+3)$

C. $90(\sqrt{3}+1)$

D. $150(\sqrt{3}+1)$

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21. Assume that $3.313 \leq \pi \leq 3.15$. The integer closest to the value of $\sin^{-1}(\sin 1 \cos 4 + \cos 1 \sin 4)$. Where 1 and 4 appearing in sin and cos are given in radians, is

A. -1

B. 1

C. 3

D. 5

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22. The maximum value of the function $f(x) = e^x + x$ln x on the interval $1 \leq x \leq 2$ is

A. $e^2 + \ln 2 = 1$

B. $e^2 + 2\ln 2$

C. $e^{\pi/2} + \frac{\pi}{2}\ln\frac{\pi}{2}$

D. $e^{3/2} + \frac{3}{2}\ln\frac{3}{2}$

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23. Let A be a 2×2 matrix of the form $A = \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}$, where a, b are integers and $-50 \leq b \leq 50$. The number of such matrices A such that

A^{-1}, the inverse of A, exists and A^{-1} contains only integer entries is

A. 101

B. 200

C. 202

D. 101^2

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24. Let $A = (a_{ij})_{1 \le I, j \le 3}$ be a 3×3 invertible matrix where each a_{ij} is a real number. Denote the inverse of the matrix A by A^{-1}. If $\Sigma_{j=1}^{3} a_{ij} = 1$ for $1 \le i \le 3$, then

A. sum of the diagonal entries of A is 1

B. sum of each row of A^{-1} is 1

C. sum of each row and each column of A^{-1} is 1

D. sum of the diagonal entries is A^{-1} is 1

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25. Let x, y be real numbers such that $x > 2y > 0$ and

$2\log(x-2y) = \log x + \log y.$

Then the possible values (s) of $\frac{x}{y}$

A. is 1 only

B. are 1 and 4

C. is 4 only

D. is 8 only

Answer: C

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26. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (b < a)$. Be an ellipse with major axis AB and minor axis CD. Let F_1 and F_2 be its two foci, with A, F_1, F_2 B in that order on the segment AB. Suppose $\angle F_1CB = 90^\circ$. The eccentricity of the ellipse is

A. $\frac{\sqrt{3}-1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{\sqrt{5}-1}{2}$

D. $\frac{1}{\sqrt{5}}$

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27. Let A denote the set of all real numbers x such that $x^3 - [x]^3 = (x - [x])^3$, where [x] is the greatest integer less than or equal to x. Then

A. A is a discrete set of at least two points

B. A contains an interval, but is not an interval

C. A is an interval, but a proper subset of $(-\infty, \infty)$

D. $A = (-\infty, \infty)$

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28. Define a sequence $\{S_n\}$ of real numbers by

$$S_n = \sum_{k=0}^{n} \frac{1}{\sqrt{n^2 + k}}, \text{ for } n \geq 1.$$

Then $\lim_{n \to \infty} S_n$

A. does not exist

B. exists and lies in the interval (0, 1)

C. exists and lies in the interval [1, 2)

D. exists and lies in the interval $[2, \infty)$

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29. Let

$$f(x) = \begin{cases} \frac{x}{\sin x}, & x \in (0,1) \\ 1, & x = 0 \end{cases}$$

Consider the integral

$$I_n = \sqrt{n}\int_0^{1/n} f(x)e^{-nx}\,dx.$$

Then $\lim_{n \to \infty} I_n$

A. does not exist

B. exists and is 0

C. exists and is 1

D. exists and is $1 - e^{-1}$

Watch Video Solution

30. The value of the integral

$$\int_1^3 \left((x-2)^4 \sin^3(x-2) + (x-2)^{2019} + 1\right)dx$$

is

A. 0

B. 2

C. 4

D. 5

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31. In a 15 sidead polygon a diagnol is chosen at random. Find the probability that it is neither oneof the shortest nor one of the longest

A. $\frac{2}{3}$

B. $\frac{5}{6}$

C. $\frac{8}{9}$

D. $\frac{9}{10}$

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32. Let $M = 2^{30} - 2^{15} + 1$, and M^2 be expressed in base 2. The number of 1's in this base 2 representation of M^2 is

A. 29

B. 30

C. 59

D. 60

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33. Let ABC be a triangle such that AB = 15 and AC = 9. The bisector of $\angle BAC$ meets BC in D. If $\angle ACB = 2\angle ABC$, then BD is

A. 8

B. 9

C. 10

D. 12

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34. The figur in the complex plane given by

$$10z\bar{z} - 3(z^2 + \bar{z}^2) + 4i(z^2 - \bar{z}^2) = 0$$

is

A. a straight line

B. a circle

C. a parabola

D. an ellipse

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PART-2 MATHEMATICS

1. Let a, b, c be positive integers such that $\frac{a\sqrt{2}+b}{b\sqrt{2}+c}$ is a rational number, then which of the following is always an integer?

A. $\frac{2a^2+b^2}{2b^2+c^2}$

B. $\frac{a^2+2b^2}{b^2+2c^2}$

C. $\frac{a^2+b^2-c^2}{a+b+c}$

D. $\frac{a^2+b^2+c^2}{a+b-c}$

Answer: D

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2. The number of solutions (x, y, z) to the system of equations

$x+2y+4z=9, 4yz+2xy=13, xyz=3$

Such that at least two of x, y, z are integers is -

A. 3

B. 5

C. 6

D. 4

Answer: B

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3. In a triangle ABC, it is known that AB=AC. Suppose D is the mid-point of AC and BD=BC=2. Then the area of the triangle ABC is-

A. 2

B. $2\sqrt{2}$

C. $\sqrt{7}$

D. $2\sqrt{7}$

Answer: C

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4. A train leaves Pune at 7 :30 am and reaches Mumbai at 11 : 30 am. Another train leaves Mumbai at 9 : 30 am and reaches Pune at 1 : 00 pm. Assuming that the two trains at constant speeds, at what time do the two trains cross each other-

A. $10:20am$

B. $11:30am$

C. $10:26am$

D. Data not sufficinet

Answer: B

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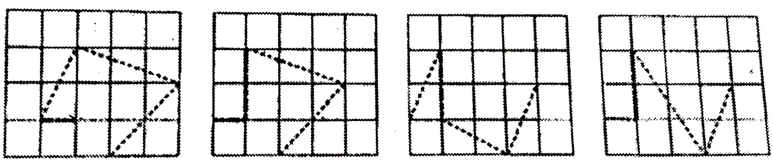
5. In the adjacent figures, which has the shortest path-



A. Fig 1

B. Fig 2

C. Fig 3

D. Fig 4

Answer: B

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PART-2(MATHEMATICS)

1. Suppose a, b, c are real numbers, and each of the equations $x^2 + 2ax + b^2 = 0$ and $x^2 + 2bx + c^2 = 0$ has two distinct real roots. Then the equation $x^2 + 2cx + a^2 = 0$ has - (A) Two distinct positive real roots (B) Two equal roots (C) One positive and one negative root (D) No real roots

A. Two distinct positive real roots

B. Two equal roots

C. One positive and one negative root

D. No real roots

Answer: D

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2. The coefficient of x^{2012} in $\frac{1_x}{(1+x^2)(1-x)}$ is -

A. 2010

B. 2011

C. 2012

D. 2013

Answer: B

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3. Let (x,y) be a variable point on the curve $4x^2+9y^2-8x-36y+15=0.$ Then min $\left(x^2-2x+y^2-4y+5\right)+\max\left(x^2-2x+y^2-4y+5\right)$ is- (A) $\frac{325}{36}$ (B) $\frac{36}{325}$ (C) $\frac{13}{25}$ (D) $\frac{25}{13}$

A. $\frac{325}{36}$

B. $\frac{36}{325}$

C. $\frac{13}{25}$

D. $\frac{25}{13}$

Answer: A

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4. The sum of all $x\in[0,\pi]$ which satisfy the equation sin $x+\frac{1}{2}\cos x=\sin^2\left(x+\frac{\pi}{4}\right)$ is - (A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) π (D) 2π

A. $\frac{\pi}{4}$

B. $\frac{5\pi}{6}$

C. π

D. 2π

Answer: C

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5. A polynomial P(x) with real coefficients has the property that $P^{n}(x) \neq 0$ for all x. Suppose $P(0) = 1$ and P'(0) = -1.

What can you say about P(1)?

A. $P(1) \geq 0$

B. $P(1) \neq 0$

C. $P(1) \leq 0$

D. $-1/2 < P(1) < 1/2$

Answer: C

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6. Define a sequence (a_n) by $a_1 = 5, a_n = a_1 a_2 \ldots a_{n-1} + 4$ for $n > 1$.

Then $\lim_{n \to \infty} \frac{\sqrt{a_n}}{a_{n-1}}$

A. Equals $1/2$

B. equals 1

C. equals 2/5

D. does not exist

Answer: C

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7. The value of the integral $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} \mathrm{dx}$, where $a > 0$, is - (A) π (B) $a\pi$ (C) $\frac{\pi}{2}$ (D) 2π

A. π

B. $a\pi$

C. $\pi/2$

D. 2π

Answer: C

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8. Consider

$L=\sqrt[3]{2012}+\sqrt[3]{2013}+\ldots.+\sqrt[3]{3011}$

$R=\sqrt[3]{2013}+\sqrt[3]{2014}+\ldots..+\sqrt[3]{3012}$

and $I=\int_{2012}^{3012}\sqrt[3]{x}\text{dx}$ Then -

A. $L+R<2I$

B. $L+R>2I$

C. $L+R>2I$

D. $\sqrt{LR}=2I$

Answer: C

Watch Video Solution

9. A man tosses a coin 10 times, scoring 1 point for each head and 2 points for each tail. Let P(K) be the probability of scoring at least K points. The largest value of K such that $P(K) > 1/2$ is -

A. 14

B. 15

C. 16

D. 17

Answer: C

Watch Video Solution

10. Let $f(x)=\dfrac{x+1}{x-1}$ for all $x\neq 1$. Let $f^1(x)=f(x)$, $f^2(x)=f(f(x))$ and generally $f^n(x)=f\left(f^{n-1}(x)\right)$ for n > 1 Let $P=f^1(2)f^2(3)f^3(4)f^4(5)$ Which of the following is a multiple of P- (A) 125 (B) 375 (C) 250 (D) 147

A. 125

B. 375

C. 250

D. 147

Answer: B

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PART -I MATHEMATICS

1. Let C_0 be circle of radius 1. For $n\geq 1$, let C_n be a circle whose area equals the area of a squre inscibed in C_{n-1}. Then $\sum_{i-0}^{\infty}$ Area (C_i)

equals

A. π^2

B. $\frac{\pi - 2}{\pi^2}$

C. $\frac{1}{\pi^2}$

D. $\frac{\pi^2}{\pi - 2}$

Answer: D

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2. For a real number r we denote by [r] the largest integer less than or equal to r. If x,y are real numbers with $x, y \geq 1$ then which of the following statements is always true? A) $[x+y] \leq [x]+[y]$ B) $[xy] \leq [x][y]$ C) $[2^x] \leq 2^x$ D) $\left[\frac{x}{y}\right] \leq \frac{x}{y}$

A. $[x+y] \leq [x]+[y]$

B. $[xy] \leq [x]+[y]$

C. $[2^x] \leq 2^{[x]}$

D. $\left[\frac{x}{y}\right] \leq \frac{[x]}{[y]}$

Answer: D

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3. For each positive integer n, let $A_n = \max\{(C(n, r)0 \leq r \leq n\}$ then the number of elements n in {1,2...20} for $1.9 \leq \frac{A_n}{A_{n-1}} \leq 2$ is

A. 9

B. 10

C. 11

D. 12

Answer: C

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4. Let $b, d > 0$. The locus of all points $P(r, \theta)$ for which the line OP (where O is the origin) cuts the line $r \sin \theta = b$ in Q such that PQ =d is

A. $(r - d)\sin\theta = b$

B. $(r \pm d)\sin\theta = b$

C. $(r - d)\cos\theta = b$

D. $(r \pm d)\cos\theta = b$

Answer: B

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5. Let C be the circle $x^2 + y^2 = 1$ in the xy-plane . For each $t \geq 0$, let L_t be the line passing through (0,1) and (t,0) . Note than L_t interesects C in two points, one of which is (0,1). Let Q_t be the other point. As t varies between 1 and $1 + \sqrt{2}$, the collection of points Q_1 sweeps out an arc on C. The angle subtended by this arc at (0,0) is

A. $\frac{\pi}{8}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{3\pi}{8}$

Answer: B

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6. In an ellipse, its foci and the ends of its major axis are equally spaced. If the length of its semi-minor is $2\sqrt{2}$, them the length of its semi-major axis is

A. 4

B. $2\sqrt{3}$

C. $\sqrt{10}$

D. 3

Answer: D

Watch Video Solution

7. Let ABC be a triangle such that AB=BC. Let F be midpoint of AB and X be a poin on BC such that FX is perpendicular to AB. If BX =3XC then the ratio BC/AC equals

A. $\sqrt{3}$

B. $\sqrt{2}$

C. $\sqrt{\frac{3}{2}}$

D. 1

Answer: C

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8. The number of solutions to the equations $\cos^4 x + \frac{1}{\cos^2 x} = \sin^4 x + \frac{1}{\sin^2 x}$ in the interval $[0, 2\pi]$ is

A. 6

B. 4

C. 2

D. 0

Answer: B

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9. Consider the function f(x)= $\begin{cases} \frac{x+5}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$. Then f(f(x)) is discontinuous

A. at all real numbers

B. at excatly two values of x

C. at ecactly one value of x

D. at exactly three values of x

Answer: B

Watch Video Solution

10. For a real number x let [x] denote the largest number less than or equal to x. For $x \in R$ let f (x)=[x] $\sin \pi x$. Then

A. If is differentiable on R.

B. f is symmetric about the line x=0

C. $\int_{-3}^{3} f(x)dx = 0$

D. For each real α, the equation $f(x) - \alpha = 0$ has infinitely many roots.

Answer: D

Watch Video Solution

11. Let : $[0, \pi] \rightarrow R$ be defined as

$$f(x) = \begin{cases} \sin x & \text{if x is irrational and} x \in [0, \pi] \\ \tan^2 x & \text{if x is rational and} x \in [0, \pi] \end{cases}$$

Then number of points in $[0, \pi]$ at which the fucntion f is continuous is

A. 6

B. 4

C. 2

D. 0

Answer: B

Watch Video Solution

12. Let $f: [0, 1] \to [0, \infty]$ be a continuous function such that $\int_0^1 f(x)dx = 10$. Which of the following statements is NOT necessarily true ?

A. $\int_0^1 e^{-x} f(x)dx \leq 10$

B. $\int_0^1 \frac{f(x)}{(1+x)^2} dx \leq 10$

C. $-10 \le \int_0^1 \sin(100x) fx \le 10$

D. $\int_0^1 f(x)^2 dx \le 100$

Answer: D

View Text Solution

13. A continuous function $f: R \to R$ satisfies the equation $f(x) = x + \int_0^1 f(t)dt$. Which of the following options is true ?

A. $f(x+y) = f(x) + f(y)$

B. $f(x+y) = f(x)f(y)$

C. $f(x+y) = f(x) + f(y) + f(x)f(y)$

D. $f(x+y) = f(xy)$

Answer: C

Watch Video Solution

14. For a real number x let [x] denote the largest integer les than or eqaul to x and {x} =x -[x]. Let n be a positive integer. Then $\int_0^1 \cos(2\pi[x]\{x\})dx$ is equal to

A. 0

B. 1

C. n

D. 2n-1

Answer: B

Watch Video Solution

15. Two persons A and B throw a (fair) die (six faced cube faces numbered from 1 to 6) alternately, starting with. A the first person to get an outcome different from the pervious one by the opponent wins. The probability than B wins is

A. $\frac{5}{6}$

B. $\frac{6}{7}$

C. $\frac{7}{8}$

D. $\frac{8}{9}$

Answer: B

Watch Video Solution

16. Let $n \geq 3$. A list of numbers $x_1, x_2, \ldots., x_n$ has mean μ and standard deviation σ . A new list of numbers $y_1, (y_2, \ldots., y_n$ is made as follows $: y_1 = \frac{x_1 + x_2}{2}, y_2 = \frac{x_1 + x_2}{2}$ and $y_j \text{for} j = 3, 4, \ldots., n$. The mean and the standard deviation of the new list are $\widehat{\mu}$ and $\widehat{\sigma}$. Then whcih of the following is necessarily true?

A. $\mu = \widehat{\mu}$ and $\sigma \leq \widehat{\sigma}$

B. $\mu = \widehat{\mu}$ and $\sigma \geq \widehat{\sigma}$

C. $\sigma - \widehat{\sigma}$

D. $\mu \neq \widehat{\mu}$

Answer: B

View Text Solution

17. What is the angle substended by an edge of regular tetrahedron at its centre?

A. $\cos^{-1}\left(\frac{-1}{2}\right)$

B. $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

C. $\cos^{-1}\left(\frac{-1}{3}\right)$

D. $\cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

Answer: C

View Text Solution

18. Let $S = \{(a, b) : a, b \in Z, 0 \leq a, b \leq 18\}$. The number of elements (x ,y) in Such that 3x+4y+5 is divisible by 19 is

A. 38

B. 19

C. 18

D. 1

Answer: B

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19. For a real number r let [r] denote the largest integer less than or equal to r. Let $a > 1$ be a real number which is not an integer and let k be the smallest positive integer positive integer such that $[a^k] > [a]^k$. Then which of the following statements is always true?

A. $k \leq 2([a] + 1)^2$

B. $k \leq 2([a]+1)^4$

C. $k \leq 2^{[a]+1}$

D. $k \leq \frac{1}{a-[a]}+1$

Answer: B

View Text Solution

20. Let X be a set of 5 elements. The number d of ordered pairs (A,B) of subsets of X such that $A \neq \phi, B \neq \phi, A \cap B = \phi$ satisfies

A. $50 \leq d \leq 100$

B. $101 \leq d \leq 150$

C. $151 \leq d \leq 200$

D. $200 \leq d$

Answer: C

View Text Solution

PART-II MATHEMATICS

1. Let $n \geq 3$ be an integer. For a permutaion $\sigma = (a_1, a_2, \ldots . . , a_n)$ of (1,2,......,n) we let $f_\sigma(x) = a_n X^{n-1} + a_{n-1} X^{x-2} + \ldots . + a_2 x + a_1$. Let S_σ be the sum of the roots of $f_\sigma(x) = 0$ and let S denote the sum over all permutations σ of (1,2,......,n) of the numbers S_σ. Then-

A. $S < 0n!$

B. $-n! < S < 0$

C. $0 < S < n!$

D. $n! \leq S$

Answer: B

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2. If n is a positive integer and $\omega \neq 1$ is a cube of unity, the number of possible values of $\left|e^{\sum_{k=0}^{n} \left(\frac{n}{k}\right) \omega^k}\right|$

A. 2

B. 3

C. 4

D. 6

Answer: C

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3. Suppose a parabola $y = ax^2 + bx + c$ has two x intercepts, one positive and one negative, and its vertex is (2,-2). Then which of the following is true ? (A) ab > 0 (B) bc > 0 (C) ca > 0 (D) a + b + c > 0

A. $ab > 0$

B. $bc > 0$

C. $ca > 0$

D. $a + b + c > 0$

Answer: B

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4. Let $n \geq 3$ and let $C_1, C_2, \ldots., C_n$ be circles witht radii, $r_1, . r_2, \ldots., r_n,$ respectively. Assume that C_1 and C_{i+1} touch external for $2 \leq i \leq n-1$. It is also given that the x-axis and the line $y = 2\sqrt{2}x + 10$ are tangential to each of the ci rcles. Then $r_1, r_2, \ldots., r_n,$ are in-

A. ana arithmetic progeression with common difference $3+\sqrt{3}$

B. a geometric progeression with common ratio $3+\sqrt{3}$

C. an arithmetic progeression with common difference $2+\sqrt{3}$

D. a geometric progeression with common ratio $2+\sqrt{3}$

Answer: D

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5. The number of integers n for which $3x^3 - 25x + n = 0$ has three real roots is-

A. 1

B. 25

C. 55

D. infinite

Answer: C

Watch Video Solution

6. An ellips inscribed in a semi-cicle touches the cicular are at two distinct points and also touches the bounding diameter. Its major axis is parallel to the bounding diameter. When the ellipse has the maximum passible area, its eccentricity is -

A. $\dfrac{1}{\sqrt{2}}$

B. $\dfrac{1}{2}$

C. $\dfrac{1}{\sqrt{3}}$

D. $\sqrt{\dfrac{2}{3}}$

Answer: D

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7. Let $I_n = \int_0^{\pi/2} x^n \cos x dx$, where in is a non-negative integer

Then $\sum_{n=2}^{\infty} \left(\frac{I_n}{n!} + \frac{I_{n-2}}{(n-2)!} \right)$ equals-

A. $e^{\pi/2} - 1 - \dfrac{\pi}{2}$

B. $e^{\pi/2} - 1$

C. $e^{\pi/2} - \dfrac{\pi}{2}$

D. $e^{\pi/2}$

Answer: A

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8. For a real number x let [x] denote the largest intger less than or equal to x. The smalleset positive integer n for which the integer $\int_1^n [x]\left[\sqrt{x}\right]dx$ exceeds 60 is-

A. 8

B. 9

C. 10

D. $\left[60^{2/3}\right]$

Answer: B

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9. Choose a number n uniformly at random from the set {,2,.....,100} . Choose one of the first seven days of the year 2014 at random and consider n consecutive days starting from the chosen day what is the probability that among the chosen n days , the number of Sundays is different from the number of Mondays?

A. $\frac{1}{2}$

B. $\frac{2}{7}$

C. $\frac{12}{49}$

D. $\frac{43}{175}$

Answer: A

View Text Solution

10. Let S={(a,b)|a,b $\in Z, 0 \leq a, b \leq 18\}$. The number of lines in R^2 passing though (0,0) and exactlu one other points in S is-

A. 16

B. 22

C. 28

D. 32

Answer: A

View Text Solution

Part A - Mathematics

1. Let BC be a fixed line segment in the plane . The locus of a point A such that the triangle ABC is isosceles , is (with finitely many possible exceptional points)

A. a line

B. a circle

C. the union of a circle and a line

D. the union of two circles and a line

Answer: D

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2. The number of solution pairs (x , y) of the simultaneous equations $\log_{1/3}(x+y)+\log_3(x-y)=2$ and $2^{y^2}=512^{x+1}$ is

A. 0

B. 1

C. 2

D. 3

Answer: B

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3. The value of the limit $\lim_{x \to -\infty}\left(\sqrt{4x^2-x}+2x\right)$ is

A. $-\infty$

B. $-\dfrac{1}{4}$

C. 0

D. $\frac{1}{4}$

Answer: D

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4. Let R be a relation on the set of all natural numbers given by a R $\Leftrightarrow$ a divides b^2 .

Which of the following properties does R satisfy?

I . Reflexivity

II. Symmetry

III. Transitivity

A. I only

B. III only

C. I and III only

D. I and II only

Answer: A

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5. The fractional part of a real number x is x $-[x]$, where [x] is the greatest integer less than or equal to x . Let F_1 and F_2 be the fractional parts of (44 - $\sqrt{2017})^{2017}$ and $(44+\sqrt{2017})^{2017}$ respectively . Then F_1+F_2 lies between the numbers

A. 0 and 0.45

B. 0.45 and 0.9

C. 0.9 and 1.35

D. 1.35 and 1.8

Answer: C

View Text Solution

6. The number of real solutions of the equation 2 sin 3x + sin 7x - 3 = 0 which lie in the interval $[-2\pi, 2\pi]$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

Watch Video Solution

7. Suppose p,q,r and real number such that $q = p(4-p), r = q(4-q), p = r(4-r)$. The maximum possible value of p+q+r is

A. 0

B. 3

C. 9

D. 27

Answer: C

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8. The parabola $y^2 = 4x + 1$ divides the disc $x^2 + y^2 \leq 1$ into two regions with areas A_1 and A_2. Then $|A_1 - A_2|$ equal

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: B

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9. A shooter can hit a given target with probability $\frac{1}{4}$. She keeps firing a bullet at the target until she hits ig successfully three times and then she stops firing. The probability that she fires exactly six bullets lies in the interval .

A. (0.5272,0.5274)

B. (0.2636,0.2638)

C. (0.1317,0.1319)

D. (0.0658,0.0660)

Answer: D

Watch Video Solution

10. Consider the following events: E_1: Six fair dices are rolled and at least one die shows six. E_2: Twelve fair dice are rolled and at least two dice show six. Let p_1 be the probability of E_1 and p_2 be the probaility of E_2.

Which of the following is true? (A) p1 > p2 (B) p1 = p2 = 0.6651 (C) p1 < p2 (D) p1 = p2 = 0.3349

A. $p_1 > p_2$

B. $p_1 = p_2 = 0.06651$

C. $p_1 < p_2$

D. $p_1 - p_2 = 0.3349$

Answer: A

Watch Video Solution

11. For how many different values of a does the following system have at least two distinct solution ?

ax+y=0

x+(a+10)y=0

A. 0

B. 1

C. 2

D. Infinitely many

Answer: C

Watch Video Solution

12. Let R be the set of real number and $f: R \to R$ be defined by $f(x) = \frac{\{x\}}{1 + [x]^2}$, where [x] is the greatest integer less than or equal to x, and {x}=x-[x]. Which of the following statement are true?

I. The range of f is a closed interval

II. f is continuous on R.

III. f is one - one on R.

A. I only

B. II only

C. III only

D. None of I, II and III

Answer: D

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13. Let $x_n = (2^n + 3^n)^{1/2n}$ for all natural number n.

Then

A. $\lim_{n \to \infty} x_n = \infty$

B. $\lim_{n \to \infty} x_n = \sqrt{3}$

C. $\lim_{n \to \infty} x_n = \sqrt{3} + \sqrt{2}$

D. $\lim_{n \to \infty} x_n = \sqrt{5}$

Answer: B

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14. One of the solution of the equation $8\sin^3\theta - 7\sin\theta + \sqrt{3}\cos\theta = 0$

lies in the interval

A. $(0, 10^\circ]$

B. $(10^\circ, 20^\circ]$

C. $(20^\circ, 30^\circ]$

D. $(30^\circ, 40^\circ]$

Answer: B

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15. Let a,b,c,d,e, be real numbers such that $a+b<c+d, b+c<d+e, c+d<e+a, d+e<a+b$. Then

A. The largest is a and the smallest is b

B. The largest is a and the smallest is c

C. The largest is c and the smallest is e

D. The largest is c and the smallest is b

Answer: A

Watch Video Solution

16. If a fair coin is tossed 5 times, the porbability that heads does not occur two or more times in a row is

A. $\frac{12}{2^5}$

B. $\frac{13}{2^5}$

C. $\frac{14}{2^5}$

D. $\frac{15}{2^5}$

Answer: B

Watch Video Solution

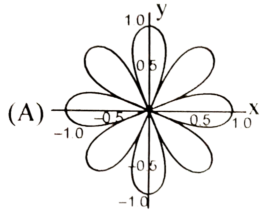
17. Consider the following parametric equation of a curve :

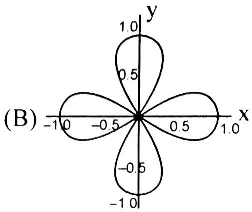
$x(\theta) = |\cos 4\theta| \cos \theta$

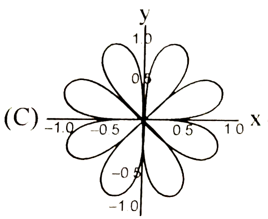
$y(\theta) |\cos 4\theta| \sin \theta$

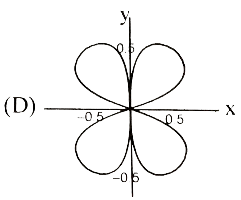
for $0 \leq \theta \leq 2\pi$

Which of the following graphs represents the curve ?

A. 

B. 

C. 

D. 

Answer: A

View Text Solution

18. Let $A = (a_1, a_2)$ and $B = (b_1, b_2)$ be two points in the plane with integer coordinates. Which one of the following is not a possible value of the distance between A an B ?

A. $\sqrt{65}$

B. $\sqrt{74}$

C. $\sqrt{83}$

D. $\sqrt{97}$

Answer: C

Watch Video Solution

19. Let $f(x) = \max\left\{3, x^2, \frac{1}{x^2}\right\}$ for $\frac{1}{2} \leq x \leq 2$. Then the value of integral $\int_{1/2}^{2} f(x)dx$ is

A. $\frac{11}{3}$

B. $\frac{13}{3}$

C. $\frac{14}{3}$

D. $\frac{16}{3}$

Answer: C

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20. Let $a_i = i + \frac{1}{i}$ for i= 1 , 2 .... , 20. Put

$p = \frac{1}{20}(a_1 + a_2 + \dots\dots\dots + a_{20})$ and

$q = \frac{1}{20}\left(\frac{1}{a_1} + \frac{1}{a_2} + \dots\dots\dots + \frac{1}{a_{20}}\right)$. Then

A. $q \in \left(0, \frac{22-p}{21}\right)$

B. $q \in \left(\frac{22-p}{21}, \frac{2(22-p)}{21}\right)$

C. $q \in \left(\frac{2(22-p)}{21}, \frac{22-p}{7}\right)$

D. $q \in \left(\frac{22-p}{7}, \frac{4(22-p)}{21}\right)$

Answer: A

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Part B- Mathematics

1. Let x,y,z be positive integers such that HCF(x,y,z)=1 and $x^2 + y^2 = 2z^2$.

Which of the following statements are true ?

I. 4 divides x or 4 divides y.

II. 3 divides x+y or 3 divides x-y

III. 5 divides $z\left(x^{2-y^2}\right)$

A. I and II only

B. II and III only

C. II only

D. III only

Answer: B

Watch Video Solution

2. How many different (mutually noncongruent) trapeziums can be constructed using four distinct side lengths from the set {1,3,4,5,6} ?

A. 5

B. 11

C. 15

D. 30

Answer: B

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3. A solid hemisphsere is mounted on a solid cylinder, both having equla radii. If the whole solid is to have a fixed surface are and the maximum possible volume, then the ratio of the height of the cylinder to the common radius is

A. $1:1$

B. $1:2$

C. $2:1$

D. $\sqrt{2}:1$

Answer: A

Watch Video Solution

4. Let ABC be an acute scalene triangle, and O and H be its circumcentre and orthocentre respectively. Further let N be the midpoint of OH. The value of the vector sum $\overrightarrow{NA}+\overrightarrow{NB}+\overrightarrow{NC}$ is

A. $\overrightarrow{O}$ (zero vector)

B. $\overrightarrow{HO}$

C. $\frac{1}{2}\overrightarrow{HO}$

D. $\frac{1}{2}\overrightarrow{OH}$

Answer: C

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5. The quotient when $1+x^2+x^4+x^6+.....+x^{34}$ is divided by $1+x+x^2+x^3+.......+x^{17}$ is

A. $x^{17}-x^{15}+x^{13}-x^{11}.....+x$

B. $x^{17}+x^{15}+x^{13}+x^{11}.....+x$

C. $x^{17}+x^{16}+x^{15}+x^{14}.....+1$

D. $x^{17}-x^{16}+x^{15}-x^{14}.....-1$

Answer: D

Watch Video Solution

6. Let R be the region of the disc $x^2+y^2 \le 1$ in the first quadrant. The the area of the largest possible circile contained in R is

A. $\pi(3-2\sqrt{2})$

B. $\pi\left(4-3\sqrt{2}\right)$

C. $\frac{\pi}{6}$

D. $\pi\left(2\sqrt{2}-2\right)$

Answer: A

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7. Let R be the set of real number and $f: R \rightarrow R$ be given by $f(x)=\sqrt{|x|}-\log(1+|x|)$. We now make the following assertions :

I. There exists a real number A such that $f(x) \leq A$ for all x.

II. There exists a real number B such that $f(x) \geq B$ for all x.

A. I is true and II is false

B. I is false and II is true

C. I and II both are true

D. I and II both are false

Answer: B

View Text Solution

8. Define $g(x) = \int_{-3}^{3} f(x-y)f(y)dy,$ for all real x,

where $f(t) = \begin{cases} 1, 0 \leq t \leq 1 \\ 0, \text{elsewhere.} \end{cases}$

Then

A. g(x) is not continuous everywhere

B. g(x) is continuous everywhere but differentiable nowhere

C. g(x) is continuous everywhere and differentiable everywhere except at x=0,1

D. g(x) is continuous everywhere and differentiable everywhere except at x=0,1,2

Answer: D

View Text Solution

9. The integer part of the number

$$\sum_{k=0}^{44} \frac{1}{\cos(k)^{\circ}\cos(k+1)^{\circ}} \text{ is}$$

A. 50

B. 52

C. 57

D. 59

Answer: C

Watch Video Solution

10. The number of continuouss function $f:[0,1] \to R$ that satisfy

A. 0

B. 1

C. 2

D. infinity

Answer: B

View Text Solution

Part 1 Mathematics

1. The number of pairs (a, b) of positive real numbers satisfying $a^4 + b^4 < 1$ and $a^2 + b^2 > 1$ is

A. 0

B. 1

C. 2

D. more than 2

Answer: D

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2. The number of real roots of the polynomial equation $x^4 - x^2 + 2x - 1 = 0$ is

A. 0

B. 2

C. 3

D. 4

Answer: B

Watch Video Solution

3. Suppose the sum of the first m teams of a arithmetic progression is n and the sum of its first n terms is m, where $m \neq n$. Then the sum of the first (m + n) terms of the arithmetic progression is

A. 1- mn

B. mn- 5

C. $-(m = n)$

D. m + n

Answer: C

Watch Video Solution

4. Consider the following two statement:

I. Any pair of consistent linear equations in two variables must have unique solutions.

II. There do not exist two consecutive integers, the sum of whose squares is 365.

Then

A. Both I and II are true

B. both I and II are false

C. I is true and II false

D. I is false and II is true.

Answer: B

Watch Video Solution

5. The number of polynomials p(x) with integer coefficients such that the curve y = p(x) passes through (2, 2) and (4, 5) is

A. 0

B. 1

C. more than 1 but finite

D. infinite

Answer: A

Watch Video Solution

6. find the sum of all three digit natural numbers which are divisible by 7

A. 5497

B. 5498.5

C. 5499.5

D. 5490

Answer: B

Watch Video Solution

7. A solid hemisphere is attached to the top of a cylinder, having the same radius as that of the cylinder. If the height of the cylinder were doubled (keeping both radii fixed), the volume of the entire system would have increased by 50%. By what percentage would the volue have increased if the radii of the hemisphere and the cylinder were doubled (keeping the height fixed)?

A. 3

B. 4

C. 5

D. 6

Answer: C

Watch Video Solution

8. Consider a triangle PQR in which the relation $QR^2 + PR^2 = 5PQ^2$ holds. Let G be the point of intersection of medians PM and QN. Then $\angle QGM$ is always

A. less than 45°

B. obtuse

C. a right angle

D. acute and larger than 45°

Answer: C

Watch Video Solution

9. Let a, b, c be the side-lengths of a triangle, and l, m,n be the lengths of its medians. Put $K = \left(\frac{l+m+n}{a+b+c}\right)$ Then, as a, b, c vary, K can assume every value in the interval

A. $\left(\frac{1}{4}, \frac{4}{5}\right)$

B. $\left(\frac{1}{2}, \frac{4}{5}\right)$

C. $\left(\frac{3}{4}, 1\right)$

D. $\left(\frac{4}{5}, \frac{5}{4}\right)$

Answer: C

Watch Video Solution

10. Let x_0, y_0 be fixed real numbers such that $x_0^2 + y_0^2 > 1$. If x, y are arbitrary real numbers such that $x^2 + y^2 \leq 1$, then the minimum value of $(x - x_0)^2 + (y - y_0)^2$ is

A. $\left(\sqrt{x_0^2 + (y_0^2)} - 1\right)^2$

B. $x_0^2 + y_0^2 - 1$

C. $(|X_0| + |Y_0| - 1)^2$

D. $(|X_0| + |Y_0|)^2 - 1$

Answer: A

Watch Video Solution

11. Let PQR be a triangle which PQ = 3. Form the vertex R, draw the altitude RS to meet PQ at S. Assume that $RS = \sqrt{3}$ and $Ps = QR$. Then PR equals

A. $\sqrt{5}$

B. $\sqrt{6}$

C. $\sqrt{7}$

D. $\sqrt{8}$

Answer: C

Watch Video Solution

12. A 100 mark examination was administered to a class of 50 students. Despite only integer marks being given, the everge score of the class was 47.5. Then , the maximum number of students who could get marks more than the class average is

A. 25

B. 35

C. 45

D. 49

Answer: D

Watch Video Solution

13. Lets be the sum of the digits of the number $15^2 \times 5^{18}$ in base 10. Then

A. s lt 6

B. $6 \in s < 140$

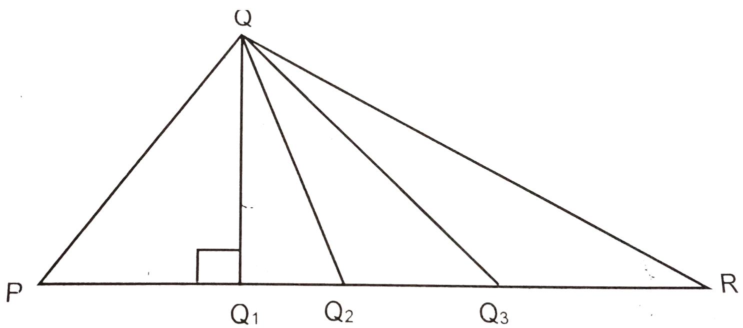
C. $140 \in s < 148$

D. $\sin 148$

Answer: B

Watch Video Solution

14. Let PQR be am acute-angled triangle in which PQ lt QR. From the vertex Q draw altitude $\mathbb{Q}_1$, the angle bisector QQ_2 and the median QQ_3 With Q_1, Q_2, Q_3 lying on PR. Then



A. $PQ_1 < PQ_2 < PQ_3$

B. $PQ_2 < PQ_1 < PQ_3$

C. $PQ_1 < PQ_3 < PQ_2$

D. $PQ_3 < PQ_1 < PQ_2$

Answer: A

Watch Video Solution

15. All the vertices of a rectangle are of the form (a, b) with a, b integers satisfying the equation $(a-8)^2 - (b-7)^2 = 5$. Then the perimeter of the rectangle is

A. 20

B. 22

C. 24

D. 26

Answer: A

Watch Video Solution

16. Let r be a root of the equation $x^2 + 2x + 6 = 0$. The value of $(r+2)(r+3)(r+4)(r+5)$ is equal to-

A. 51

B. -51

C. -126

D. 126

Answer: C

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17. Let R be the set of all real numbers and let f be a function from R to R such that

$$f(X) + \left(X + \frac{1}{2}\right) f(l - X) = 1,$$

for all $\xi n R$. Then $2f(0) + 3f(1)$ is equal to-

A. 2

B. 0

C. -2

D. -4

Answer: C

Watch Video Solution

18. The sum of all positive integers n for which $\frac{1^3 + 2^3 + ... (2n)^3}{1^2 + 2^2 + ... + n^2}$ is also an integer is

A. 8

B. 9

C. 15

D. Infinite

Answer: A

Watch Video Solution

19. Let x and y be two 2-digit numbers such that y is obtained by recersing the digits of x. Suppose they also satisfy $x^2 - y^2 = m^2$ for same positive integer m. The value of x + y + m is-

A. 88

B. 112

C. 144

D. 154

Answer: D

Watch Video Solution

20. Let $p(x) = x^2 - 5x$ and $q(x) - 3x + b$, where a and b are positive integers. Suppose hef $(p(x), q((x)) = x - 1$ and $k(x) = 1cm(p(x), q(x))$. If the coefficient

of the highest degree term of k(x) is 1, the sum of the roots of (x - 1)+ k(x) is-

A. 4

B. 5

C. 6

D. 7

Answer: D

Watch Video Solution

21. In a quadrilateral ABCD, which is not a trapezium, it is known that $\angle DAB = \angle ABC = 60^2$. Moreover $\angle CAB = \angle CBD$. Then

A. $AB = BC + CD$

B. $AB = AD + CD$

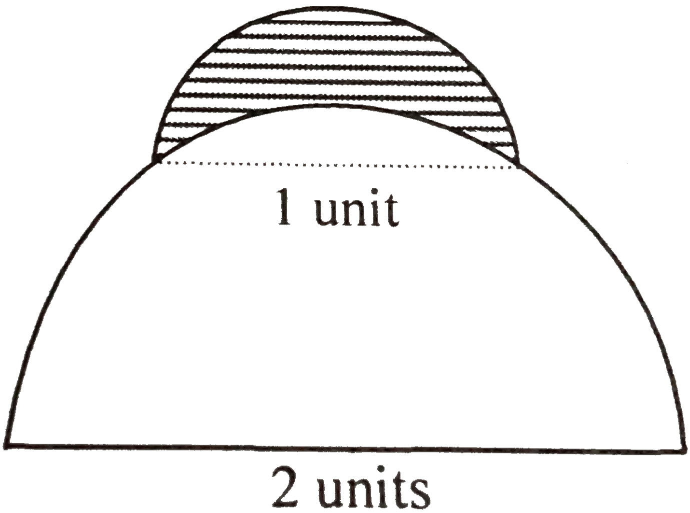
C. $AB = BC + AD$

D. $AB = AC + AD$

Answer: C

Watch Video Solution

22. A semi-circle of diameter 1 unit sits at the top of a semi-circle of diameter2 units. The shaded region inside the smaller semi-cricle but outside the larger semi-circle a lune. The area of the lune is-



A. $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$

B. $\frac{\sqrt{3}}{4} - \pi 24$

C. $\frac{\sqrt{3}}{4} - \frac{\pi}{12}$

D. $\frac{\sqrt{3}}{4} - \frac{\pi}{8}$

Answer: B

Watch Video Solution

23. The angle biosectors BD and CE of a triangle ABC are divied by the incentre 1 in the rartios 3 : 2 and 2 : 1 respecticely. Then the ratio in which I divides the bisector through A is-

A. $3:1$

B. $11:4$

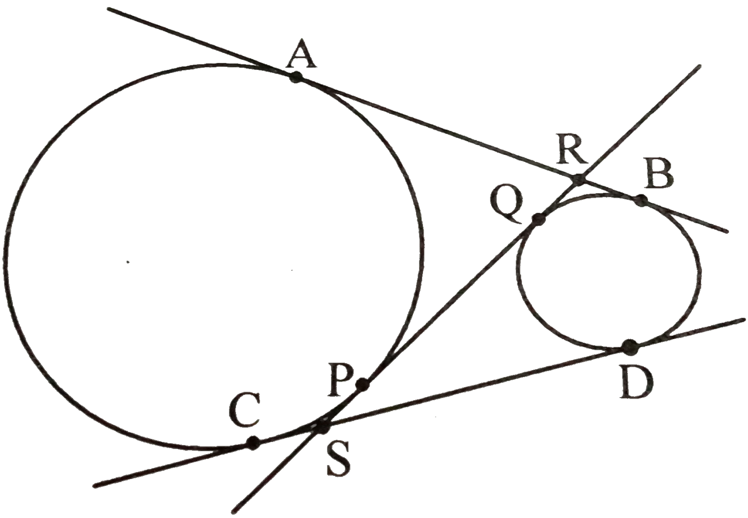
C. $6:5$

D. $7:4$

Answer: B

Watch Video Solution

24. Suppose S_1 and S_2 are two unequal circles, AB and CD are the direct common tangents to these circles. A transverse common tangent PQ cuts AB in R and CD in S. If AB = 10, then RS is -



A. 8

B. 9

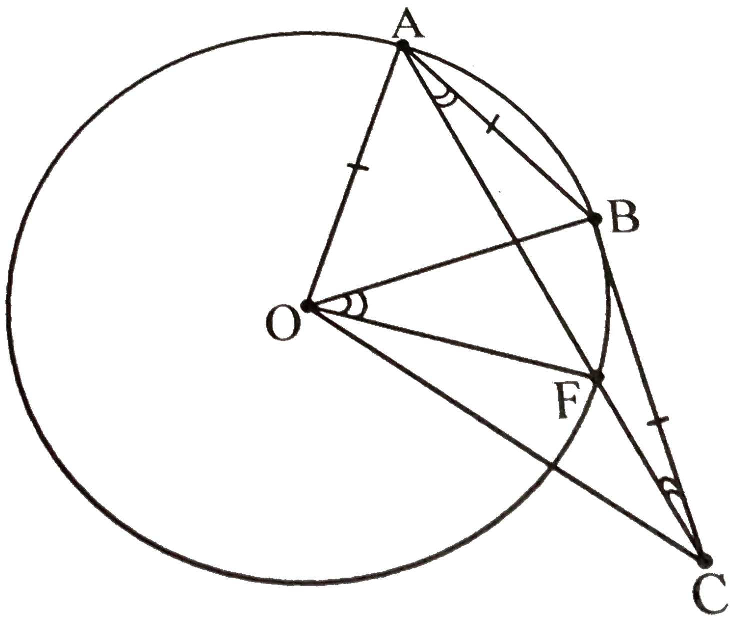
C. 10

D. 11

Answer: C

Watch Video Solution

25. On the circle with centre O, points A, B are such that OA =AB. A point C is located on the tangent at B to the circle such that A and C are on the opposite sides of the line OB and Ab =BC. The line segment AC intersects the circle again at F. Then the ratio $\angle BOF : \angle BOC$ is equal to-



A. $1:2$

B. $2:3$

C. $3:4$

D. $4:5$

Answer: B

Watch Video Solution

26. In a cinema hall, the charge per person is RS. 200. On the first day, only 60% of the seats were filled. The owner decided to reduce the price by 20% abd there was an increasesof 50% in the number of spectators on the next day. The percentage increase in the revenue on the second day was-

A. 50

B. 40

C. 30

D. 20

Answer: D

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27. The population of cattle in a farm increases so that the difference between the populationin year n + 2 and that in year n is proportional to the population in year n + 1. If the populations in years 2010, 2011 and 2013 were 39, 60 and 123, respectively, then the population in 2012 was-

A. 81

B. 84

C. 87

D. 90

Answer: B

Watch Video Solution

28. Find the number of 6-digit numbers of the form ababab (in base 10) each of which is a product of exactly 6distinct primes.

A. 8

B. 10

C. 13

D. 15

Answer: C

Watch Video Solution

29. The houses on one side of a road are numbered using consecutive even numbers .The sum of the numbers of all the houses in that row is 170. If there are at least 6 houses in that row and a is the number of the sixth house then

A. $2 \leq a \leq 6$

B. $8 \leq a \leq 12$

C. $14 \leq a \leq 20$

D. $22 \leq a \leq 30$

Answer: C

Watch Video Solution

30. Suppose $a_2, a_3, a_4, a_5, a_6, a_7$ are integers such that

$$\frac{5}{7} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!} + \frac{a_6}{6!} + \frac{a_7}{7!},$$

where $0 \le a_j < j$ for $j = 2, 3, 4, 5, 6, 7.$ The sum $a_2 + a_3 + a_4 + a_5 + a_6 + a_7$ is-

A. 8

B. 9

C. 10

D. 11

Answer: B

Watch Video Solution

31. Suppose BC is a given line segment in the plane and T is a scalene triangle. The number of points A in the plane such that the triangle with vertices A,B,C (in some order) is similar to triangle T is

A. 4

B. 6

C. 12

D. 24

Answer: C

Watch Video Solution

32. The number of positive integers n in the set $\{2, 3, \ldots., 200\}$ such that $\frac{1}{n}$ has a terminating decimal expansion is

A. 16

B. 18

C. 40

D. 100

Answer: B

Watch Video Solution

33. If a,b,c are real numbers such that a+b+c=0 and $a^2 + b^2 + c^2 = 1$, then

$(3a + 5b - 8c)^2 + (-8a + 3b + 5c)^2 + (5a - 8b + 3c)^2$ is equal to

A. 49

B. 98

C. 147

D. 294

Answer: C

Watch Video Solution

34. Let ABC be a triangle and M be a point on side AC closer to vertex C than A. Let N be a point on side AB such that MN is parallel to BC and let P be a point on side BC such that MP is parallel to AB. If the area of the quadrilateral BNMP is equal to $\frac{5}{18}$th of the area of triangle ABC, then the ratio AM/MC equals.

A. 5

B. 6

C. $\frac{18}{5}$

D. $\frac{15}{2}$

Answer: A

View Text Solution

35. Let $n \geq 4$ be a positive integer and let $l_1, l_2, \ldots., l_n$ be the lengths of the sides of arbitrary n-sided non-degenerate polygon P. Suppose

$\frac{l_1}{l_2} + \frac{l_2}{l_3} + ... \frac{l_{n-1}}{l_n} + \frac{l_n}{l_1} = n.$ Consider the following statements: I. The lengths of the sides of P are equal. II. The angles of P are equal. III. P is a regular polygon if it is cyclic. Then

A. I is true and I implies II

B. II is true

C. III is false

D. I and III are true

Answer: D

Watch Video Solution

36. Consider the following statements. For any integer n,

I. $n^2 + 3$ is never divisible by 17.

II. $n^2 + 4$ is never divisible by 17.

Then

A. both I and II are true

B. both I and II are false

C. I is false and II is true

D. I is true and II is false

Answer: D

Watch Video Solution

37. Let S be the set of all ordered pairs (x,y) of positive intergers, with HCF (x,y)=16 and LCM (x,y) = 48000. The number of elements in S is

A. 4

B. 8

C. 16

D. 32

Answer: B

Watch Video Solution

38. Consider the set A of natural numbers n whose units digit is nonzero, such that if this units digit is erased, then the resulting number divides n. If K is the number of elements in the set A, then

A. K is infinite

B. K is finite but $K > 100$

C. $25 \leq K \leq 100$

D. $K < 25$

Answer: D

View Text Solution

39. There are exactly twelve sundays in the period from january 1 to march 31 in a certain year. Then the day corresponding to february 15 in that year is

A. Tuesday

B. Wednesday

C. Thursday

D. not possible to determine from the given data

Answer: C

Watch Video Solution

40. Consider a three-digit number with the following properties:

I. If its digits in units place and tens place are interchanged, the number increases by 36,

II. If its digits in units place and hundreds place are interchanged, the number decreases by 198. Now suppose that the digits in tens place and hundreds place are interchanged. Then the number.

A. increases by 180

B. decreases by 270

C. increases by 360

D. decreases by 540

Answer: D

Watch Video Solution

41. Consider four triangles having sides (5,12,9), (5,12,11), (5,12,13) and (5,12,15). Among these, the triangle having maximum area has sides

A. (5,12,9)

B. (5,12,11)

C. (5,12,13)

D. (5,12,15)

Answer: C

Watch Video Solution

42. In a classroom, one-fifth of the boys leave the class and the ratio of the remaining boys to girls is 2:3. If further 44 girls leave the class, the ratio of boys to girls is 5:2. How many more boys should leave the class so that the number of boys equals that of girls ?

A. 16

B. 24

C. 30

D. 36

Answer: B

Watch Video Solution

43. Let X,Y,Z be respectively the areas of regular pentagon, regular hexagon and regular heptagon which are inscribed in a circle of radius 1. Then

A. $\frac{X}{5} < \frac{Y}{6} < \frac{Z}{7}$ and $X < Y < Z$

B. $\frac{X}{5} < \frac{Y}{6} < \frac{Z}{7}$ and $X > Y > Z$

C. $\frac{X}{5} > \frac{Y}{6} > \frac{Z}{7}$ and $X > Y > Z$

D. $\frac{X}{5} > \frac{Y}{6} > \frac{Z}{7}$ and $X < Y < Z$

Answer: D

Watch Video Solution

44. The least value of natural number n such that $\binom{n-1}{5} + \binom{n-1}{6} < \binom{n}{r}$, where $\binom{n}{r} = \frac{n!}{(n-r)!r!}, is$

A. 12

B. 13

C. 14

D. 15

Answer: C

Watch Video Solution

45. In a Mathematics test, the average marks of boys is $x\%$ and the average marks of girls is $y\%$ with $x \neq y$. If the average marks of all students is $z\%$ the ration of the number of girls to the total number of students is

A. $\frac{z-x}{y-x}$

B. $\frac{z-y}{y-x}$

C. $\frac{z+y}{y-x}$

D. $\frac{z+x}{y-x}$

Answer: A

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Part 2 Mathematics

1. Let $g: N \to N$ with $g(n)$ being the product of the digits of n.(a) Prove that $g(n) \leq n$ for all $n \in N$.(b) Find all $n \in N$, for which $n^2 - 12n + 36 = g(n)$.

A. 12

B. 13

C. 124

D. 2612

Answer: B

Watch Video Solution

2. Let m (respectively, n) be the number of 5-digit integers obtained by using the digits 1,2,3,4 ,5 with repetitions (respectively, without repetitions) such that the sum of any two adjacent digits is odd. Then $\frac{m}{n}$ is equal to

A. 9

B. 12

C. 15

D. 18

Answer: C

Watch Video Solution

3. The number of solid cones with integer radius and integer height each having its volume numerically equal to its total surface area is

A. 0

B. 1

C. 2

D. infinite vuar

Answer: B

Watch Video Solution

4. Let ABCD be a square. An arc of a circle with A as centre and AB as radius is drawn inside the square joining the points B and D. Points P on AB, S on AD,Q and R on are taken such that PQRS is a square. Further suppose that PQ and RS are parallel to AC. $\text{Then} \frac{\text{area} PQRS}{\text{area} ABCD}$ is

A. $\frac{1}{8}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{2}{5}$

Answer: D

Watch Video Solution

5. Suppose ABCD is a trapezium whose sides and height are integers and AB is parallel to CD. If the area of ABCD is 12 and the sides are distinct, then |AB-CD|

A. 2

B. 4

C. 8

D. connot be determined from the data

Answer: B

Watch Video Solution

6. Let a, b, c be non-zero real numbers such that $a+b+c=0, \quad \text{let} \quad q=a^2+b^2+c^2 \quad \text{and} \quad r=a^4+b^4+c^4$. Then-

A. $q^2 < 2r$ always

B. $q^2 = 2r$ always

C. $q^2 > 2r$ always

D. $q^2 - 2r$ can take both positive and negative values

Answer: B

View Text Solution

7. The value of

$$\sum_{n=0}^{1947} \frac{1}{2^n + \sqrt{2^{1947}}}$$

is equal to

A. $\frac{487}{\sqrt{2^{1945}}}$

B. $\frac{1946}{\sqrt{2^{1947}}}$

C. $\frac{1947}{\sqrt{2^{1947}}}$

D. $\frac{1948}{\sqrt{2^{1947}}}$

Answer: A

Watch Video Solution

8. The number of integers a in the interval [1, 2014] for which the system of equations

$$x + y = a, \frac{x^2}{x-1} + \frac{y^2}{y-1} = 4$$

has finitely many solutions is-

A. 0

B. 1007

C. 2013

D. 2014

Answer: C

Watch Video Solution

9. In a triangle ABC with $\angle A = 90^\circ$, P is a point on BC such that PA : PB = 3 : 4 If $AB = \sqrt{7}$ and $AC = \sqrt{5}$ then BP : PC is-

A. $2:1$

B. $:3$

C. $4:5$

D. $8:7$

Answer: A

Watch Video Solution

10. The number of all 3-digit numbers abc (in base 10)for which

$(a \times b \times c) + (a \times b) + (b \times c) + (c \times a) + a + b + c = 29$ is

A. 6

B. 10

C. 14

D. 18

Answer: C

Watch Video Solution

11. Let ABCD be a trapezium with parallel sides AB and CD such that the circle S with AB as its diameter touches CD. Further, the circle S passes through the midpoints of the diagonals AC and BD of the trapezium. The smallest angle of the trapezium is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{5}$

D. $\frac{\pi}{6}$

Answer: D

View Text Solution

12. Let S be the set of all points $\left(\frac{a}{b}, \frac{c}{d}\right)$ on the circle with radius 1 centred at (0,0) where a and b are relatively prime integers, c and d are relatively prime integers (that is HCF (a, b) = HCF (c,d) = 1), and the integers b and d are even. Then the set S

A. is empty

B. has four elements

C. has eight elements

D. is infinite

Answer: A

Watch Video Solution

13. Suppose we have two circles of radius 2 each in the plane such that the distance between their centres is $2\sqrt{3}$. The area of the region common to both circles lies between

A. 0.5 and 0.6

B. 0.65 and 0.7

C. 0.7 and 0.75

D. 0.8 and 0.9

Answer: C

Watch Video Solution

14. Let C_1, C_2 be two circles touching each other externally at the point A and let AB be the diameter of circle C_1. Draw a secant BA_3 to circle C_2, intersecting circle C_1 at a point $A_1(\neq A)$, and circle C_2 at points A_2 and A_3 . If $BA_1 = 2$, $BA_2 = 3$ and $BA_3 = 4$, then the radii of circles C_1 and C_2 are respectively

A. $\frac{\sqrt{30}}{5}, \frac{3\sqrt{30}}{10}$

B. $\frac{\sqrt{5}}{2}, \frac{7\sqrt{5}}{10}$

C. $\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}$

D. $\frac{\sqrt{10}}{3}, \frac{17\sqrt{10}}{30}$

Answer: A

Watch Video Solution

15. Let a, b, c, d be real numbers between -5 and 5 such that

$$|a| = \sqrt{4 - \sqrt{5 - a}}, |b| = \sqrt{4 + \sqrt{5 - b}},$$

$$|c| = \sqrt{4 - \sqrt{5 + c}}, |d| = \sqrt{4 + \sqrt{5 + d}}, .$$

Then the product abcd is

A. 11

B. -11

C. 121

D. -121

Answer: A

Watch Video Solution

PART-1 MATHEMATICS

1. Suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is real matrix with nonzero entries, ad-bc=0 , and A^2=A. Then a+d equals

A. 1

B. 2

C. 3

D. 4

Answer: A

Watch Video Solution

2. On any given are of positive length on the unit circle |z|=1 in the complex plane,

A. there need not be any root of unity

B. there lies exactly one root of unity

C. there are more than one but finitely many roots of unity

D. there are infinitely many roots of unity

Answer: D

Watch Video Solution

3. For $0 < \theta < \frac{\pi}{2}$, four tangents are drawn at the four points $(\pm 3\cos\theta, \pm 2\sin\theta)$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. If A(θ) denote the area of the quadrilateral formed by these four tangents, the minimum value of $A(\theta)$ is

A. 21

B. 24

C. 27

D. 30

Answer: B

View Text Solution

4. Let S $= \{x \in R : \cos(x) + \cos(\sqrt{2}x) < 2\}$. Then

A. $S = \phi$

B. S is a non-empty finite set

C. S is an infinite proper subset of R\{0}

D. S=R\{0}

Answer: D

Watch Video Solution

5. On a rectangular hyperbola $x^2 - y^2 = a^2, a > 0$, three points A,B,C are taken as follows : A = (-a,0): B and C are placed symmetrically with respect to the x-axis on the branch of the hyperbola not containing A suppose that the triangle ABC is equilateral. If the side-length of the triangle ABC is ka,then k lies in the interval

A. (0,2]

B. (2,4]

C. (4,6]

D. (6,8]

Answer: B

Watch Video Solution

6. The number of real solution x of the equation

$\cos^2(x\sin(2x)) + \frac{1}{1+x^2} = \cos^2 x + \sec^2 x$ is

A. 0

B. 1

C. 2

D. infinite

Answer: B

Watch Video Solution

7. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$, be an ellipse with foci F_1 and F_2. Let AO be its semi-minor axis. Where O is the centre of the ellipse. The lines AF_1 and AF_2, when extended, , cut the ellipse again at point B and C respectively. Suppose that the triangle ABC is equilateral. Then the eccentricity of the ellipse is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer: D

Watch Video Solution

8. Let $a=\cos 1^{\circ}$ and $b=\sin 1^{\circ}$. We say that a real number is algebraic if is a root of a polynomial with integer coefficients. Then

A. a is algebraic but b is not algebraic

B. b is algebraic but a is not algebraic

C. both a and b are algebraic

D. neither a nor b is algebraic

Answer: C

Watch Video Solution

9. A rectangle with its sides parallel to the x-axis and y-axis is inscibed in the region bounded by the curves $y = x^2 - 4$ and $2y = 4 - x^2$. The maximum possible area of such a rectangle is closest to the integer

A. 10

B. 9

C. 8

D. 7

Answer: B

Watch Video Solution

10. Let $f(x) = x|\sin x|, x \in R.$ Then

A. f is differentiable for all x, except at $x = \eta\pi, \eta = 1, 2, 3, \ldots.$

B. f is differentiable for all x, except at $x = \eta\pi, \eta = \pm 1, \pm 2, \pm 3, \ldots$

C. f is differentiable for all x, except at $x = \eta\pi, \eta = 0, 1, 2, 3, \ldots.$

D. f is differentiable for all x, except at $x = \eta\pi, \eta = 0, \pm 1, \pm, 2, \pm 3, \ldots.$

Answer: B

Watch Video Solution

11. Let $f:[-1, 1] \to R$ be a function defined by $f(x) = \left\{x^2 \left|\cos\left(\frac{\pi}{x}\right)\right| \text{for} x \neq 0, \text{ for } x = 0\right.$, The set of points where f is not differentiable is

A. $\{x \in [-1, 1], x \neq 0\}$

B. $\left[x \in 0 - 1, 1\right] : x = 0 \text{ or } x = \frac{2}{2n+1}, n \in Z\Big\}$

C. $\left\{x \in [-1, 1] : x = \frac{2}{2n+1}, n \in Z\right\}$

D. $[-1, 1]$

Answer: C

Watch Video Solution

12. The value of the integral $\int_0^\pi (1 - |\sin 8x|)dx$ is

A. 0

B. $\pi - 1$

C. $\pi - 2$

D. $\pi - 3$

Answer: C

Watch Video Solution

13. Let in x denote the logarithm of x with respect to the base e. Let $S \subset R$ be the set all points where the function In $(x^2 - 1)$ is well-defined. Then the number of function $f: S \to R$ that are differentiable, satisfy

$f'(x) - \quad \text{In}(x^2 - 1)$ for all $x \in S$ and f(2)=0, is

A. 0

B. 1

C. 2

D. infinite

Answer: D

Watch Video Solution

14. Let S be the set of real numbers p such that there is no nonzero continuous function $f: R \to R$ satisfying $\int_0^x f(t)dt = pf(x)$ for all $x \in R$. Then S is

A. the empty set

B. the set of all rational numbers

C. the set of all irrational numbers

D. the whole set R

Answer: D

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15. The porbability of men getting a certain disease is $\frac{1}{2}$ and that of women getting the same disease is $\frac{1}{5}$. The blood test that identifies the disease gives the correct result with probability $\frac{4}{5}$. Suppose a person is chosen at randon from a group of 30 males and 20 females, and the blood test of the person is found to be positive. What is the probability that the chosen person is a man ?

A. $\frac{75}{107}$

B. $\frac{3}{5}$

C. $\frac{15}{19}$

D. $\frac{3}{10}$

Answer: A

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16. The number of function $f:[0,1] \to [0,1]$ satisfying $|f(x)-f(x)| = |x-y|$ for all x, y in $[0,$

A. exactly 1

B. exactly 2

C. more than 2 but finite

D. inifinite

Answer: B

Watch Video Solution

17. Suppose A is a 3×3 matrix consisting of integer entries that are chosen at random from the set $\{-1000, 999, \ldots., 999, 1000\}$. Let P be the probability that either $A^2 = -I$ or A is diagonal, where I is the 3×3 identity matrix. Then

A. $P < \frac{1}{10^{18}}$

B. $P = \frac{1}{10^{18}}$

C. $\frac{5^2}{10^{18}} \leq P \leq \frac{5^3}{10^{18}}$

D. $P > \frac{5^4}{10^{18}}$

Answer: A

Watch Video Solution

18. Let X_k be real number such that $X_k > k^4 + k^2 + 1$ for $1 \le k \le 2018$.

Denot$N = \sum_{k=1}^{2018} k$. Consider the following inequalities:

I.

$$\left(\sum_{k=1}^{2018} kx_k\right)^2 \le N\left(\sum_{k=1}^{2018} kx_k^2\right) \qquad II. \left(\sum_{k=1}^{2018} kx_k\right)^2 \le N\left(\sum_{k=1}^{2018} k^2x_k^2\right)$$

A. both I and II are true

B. I is true and II is false

C. I is false and II is true

D. both I and II are false

Answer: A

Watch Video Solution

19. Let $x^2 = 4ky, k > 0$ be a parabola with vertex A. Let BC be its latus rectum. An ellipse with center on BC touches the parabola at A, and cuts

BC at point D and E such that BD=DE=EC (B,D,E,C in that order). The eccentricity of the ellipse is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{5}}{3}$

D. $\frac{\sqrt{3}}{2}$

Answer: C

Watch Video Solution

20. Let $f:[0'1] \to [-1, 1]$ and $g:[-1, 1] \to [0, 2]$ be two functions such that g is injective and $g^\circ f:[0, 1] \to [0, 2]$ is surjective. Then

A. f must be injective but need not be surjective

B. f must be surjective but need not be injective

C. f must be bijective

D. f must be a constant functions.

Answer: B

Watch Video Solution

PART-2 MATHMATICS

1. Let R be a rectangle , C be a circle, and T be a triangle in the plane. The maximum number of points common to the perimeter of R,C, and T is

A. 3

B. 4

C. 5

D. 6

Answer: D

Watch Video Solution

2. The number of different possible values for the sum x+y+z, where x,y,z are real numbers such that $x^4 + 4y^4 + 16z^4 + 64 = 32xyz$ is (A) 1 (B) 2 (C) 4 (D) 8

A. 1

B. 2

C. 4

D. 8

Answer: C

Watch Video Solution

3. Let Γ be a circle with diameter AB and centre O.Let l be the tangent toΓ at B.For each point M onΓ different from A,consider the tangent t at M and let it interest l at P.Draw a line parallel to AB through P intersecting OM at Q.The locus of Q as M varies over $\Gamma$is

A. an arc of a circle

B. a parabola

C. an arc of an ellipse

D. a branch of a hyperbola

Answer: B

Watch Video Solution

4. The number of solution x of the equation $\sin(x + x^2) - \sin(x^2) = \sin x$ in the interval [2,3] is

A. 0

B. 1

C. 2

D. 3

Answer: C

Watch Video Solution

5. The number of polynomials $p: R \rightarrow R$ satisfying $p(0) = 0, p(x) > x^2$ for all $x \neq 0$, and $p''(0) = \frac{1}{2}$ is

A. 0

B. 1

C. more than 1, but finite

D. infinite

Answer: A

Watch Video Solution

6. Suppose the limit $L = \lim_{n \rightarrow \infty} \sqrt{n} \int_0^1 \frac{1}{(1+x^2)^n} dx$ exist and is larger than $\frac{1}{2}$. Then

A. $\frac{1}{2} < L < 2$

B. $2 < L < 4$

C. $3 < L < 4$

D. $L \geq 4$

Answer: A

View Text Solution

7. Consider the set A_n of point (x,y) such that $0 \leq x \leq n, 0 \leq y \leq n$ where n,x,y are integers. Let S_n be the set of all lines passing through at least two distinct points from A_n. Suppose we choose a line l at random from S_n. Let P_n be the probability that l is tangent to the circle $x^2 + y^2 = n^2\left(1 + \left(1 - \frac{1}{\sqrt{n}}\right)^2\right)$. Then the limit $\lim_{n \to \infty} P_n$ is

A. 0

B. 1

C. $1/\pi$

D. $1/\sqrt{2}$

Answer: A

Watch Video Solution

8. Let $f:[0,1] \rightarrow R$ be an injective continuous function that satisfies the condition $-1 < f(0) < f(1) < 1$

Then the number of functions $g:[-1,1] \rightarrow [0,1]$ such that $(gof)x = x$ for all $x \in [0,1]$ is

A. 0

B. 1

C. more than 1, but finite

D. infinite

Answer: D

View Text Solution

9. The maximum possible area bounded by the parabola $y = x^2 + x + 10$ and a chord of the parabola of length 1 is

A. $\frac{1}{12}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer: B

View Text Solution

10. Suppose z is any root of $11z^8 + 20iz^7 + 10iz - 22 = 0$, where $i = \sqrt{-1}$. Then $s = |z|^2 + |z| + 1$ satisfies

A. $S \leq 3$

B. $3 < S < 7$

C. $7 \leq S < 13$

D. $S \geq 13$

Answer: B

View Text Solution

Matematics

1. Suppose the quadratic polynomial $p(x) = ax^2 + bx + c$ has positive coefficient a, b, c such that $b - a = c - b$. If $p(x) = 0$ has integer roots α and β then what could be the possible value of $\alpha + \beta + \alpha\beta$ if $0 \leq \alpha + \beta + \alpha\beta \leq 8$

A. 3

B. 5

C. 7

D. 14

Answer: C

Watch Video Solution

2. The number of digits in the decimal expansion of $16^5 5^{16}$ is

A. 16

B. 17

C. 18

D. 19

Answer: c

Watch Video Solution

3. Let t be real number such that $t^2 = at + b$ for some positive integers a and b. Then forany choice of positive integers a and b.t^3 is never equal to

A. 4t+3

B. 8t+5

C. 10t+3

D. 6t+5

Answer: b

Watch Video Solution

4. Consider the equation $(1+a+b)^2 = 3(1+a^2+b^2)$. where a, b are real numbers.then

A. There is no solution pair (a,b)

B. there are infinitely many solution pairs (a,b)

C. there are exactly two solution pairs (a,b)

D. there is exactly one solution pair (a,b)

Answer: d

Watch Video Solution

5. Let a_1, a_2, a_{100} be non-zero real numbers such that $a_1 + a_2 + \quad + a_{100} = 0$. Then

A. $\sum_{i=l}^{100} a_i 2^{a_i} >$ and $\sum_{i=l}^{100} a_i 2^{-a_i} < 0$

B. $\sum_{i=l}^{100} a_i 2^{a_i} \geq$ and $\sum_{i=l}^{100} a_i 2^{-a_i} \geq 0$

C. $\sum_{i=l}^{100} a_i 2^{a_i} \leq$ and $\sum_{i=l}^{100} a_i 2^{-a_i} \leq 0$

D. the sign of $\sum_{i=l}^{100} a_i 2^{a_i}$ or $\sum_{i=l}^{100} a_i 2^{-a_i}$ depends on the choice of a_i 's

Answer: a

Watch Video Solution

6. Let ABCD be a trapezium ,in which AB is parallel to CD, AB =11 ,BC=4,CD=6 and DA=3. the distance between AB and CD is

A. 2

B. 2.4

C. 2.8

D. not determinable with the data

Answer: b

Watch Video Solution

7. The points A, B, C, D, E are marked on the circumference of a circle in clockwishdirection such that $\angle ABC = 130^\circ$ and $\angle CDE = 110^\circ$. The measure of $\angle ACE$ degress is

A. 50°

B. 60°

C. 70°

D. 80°

Answer: b

Watch Video Solution

8. Circles of radii $2, 2, 1$ touch each other externally. If a circle of radius r touches all the three circles externally,then r is

A. 1.5

B. 2

C. 2.5

D. 3

Answer: c

Watch Video Solution

9. Let P be a point inside a triangle ABC with $\angle ABC = 90^\circ$. Let P_1 and P_2 be the images of P under reflection in AB and BC respectively. The distance between the circumcenters of triangles ABC and P_1P_2 is

A. $\dfrac{AB}{2}$

B. $\dfrac{AP+BP+CP}{3}$

C. $\frac{AC}{2}$

D. $\frac{AB+BC+AC}{2}$

Answer: c

Watch Video Solution

10. Let a and b be two positive real numbers such that $a+2b\leq 1$. Let A_1 and A_2 be, respectively, the areas of circles with radii ab^3 and b^2. Then the maximum possible value of $\frac{A_1}{A_2}$ is:

A. $\frac{1}{16}$

B. $\frac{1}{64}$

C. $\frac{1}{16\sqrt{2}}$

D. $\frac{1}{32}$

Answer: b

Watch Video Solution

11. There are two candles of same length and same size.both of them burn at uniform rate. The frist one burns in 5 hours and the second one the second one burns in 3 hours. Both the candles are lit together. After many minutes the length of the first candle is 3 times is 3 times that of the other ?

A. 90

B. 120

C. 135

D. 150

Answer: d

Watch Video Solution

12. Consider a cuboid all of whose edges are integers and whose base is square. Suppose the sum of all its edges is numerically equal to the sum of the areas of alll its six faces. Then the sum of all its edges is.

A. 12

B. 18

C. 24

D. 36

Answer: c

Watch Video Solution

13. Let $A_1, A_2 ... A_m$ be non -empty subsets of {1,2,3 ....100}, satisfying the following conditions.

(1) the numbers $|A_1|, |A_2| .... , |A_m|$ are disjoint.

(2) $A_1, A_2, ..., A_m$ are pairwise disjoint.

(Here|A| denotes the number fo elements in the set A.) Then the maximum possible value of m is

A. 13

B. 14

C. 15

D. 16

Answer: a

Watch Video Solution

14. The number of all 2-digit numbers n such that n is equal the sum of the square of digit in its tens place and the cube of the digit in units place is

A. 0

B. 1

C. 2

D. 4

Answer: c

Watch Video Solution

15. Let f be a funcation defined on the set of all positive integers such that $f(x)+f(y)$ for all postive integers x,y,if f (12) =24 and f(8) =15 the value of f(48) is

A. 31

B. 32

C. 33

D. 34

Answer: d

Watch Video Solution

16. Suppose is a positive real number such that $a^5-a^3+a=2$. Then

A. $a^6<2$

B. $2 < a^6 < 3$

C. $3 < a^6 < 4$

D. $4 \leq a^6$

Answer: c

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17. Consider the quadratic equation $nx^2 + 7\sqrt{nx} + n = 0$, where n is a positive intergar. Which of the following statements are necessarliy correct ?

I. For any n, the roots are distinct.

(II) There are infinitely many values of n for which both roots are real.

(III) The product of the roots is necessarlity an integer.

A. III only

B. I and II only

C. II and III only

D. I, II and III

Answer: b

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18. Consider a semicircle of radius 1 unit constructed on the diameter AB, and let O be its centre. Let C be a point on AO such that AC : CO = 2:1. Draw CD perpendicular to AO with D on the semicircle. Draw OE perpendicular to AD with E on AD. Let OE and CD intersects at H. Then DH equals

A. $\frac{1}{\sqrt{5}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{\sqrt{5}-1}{2}$

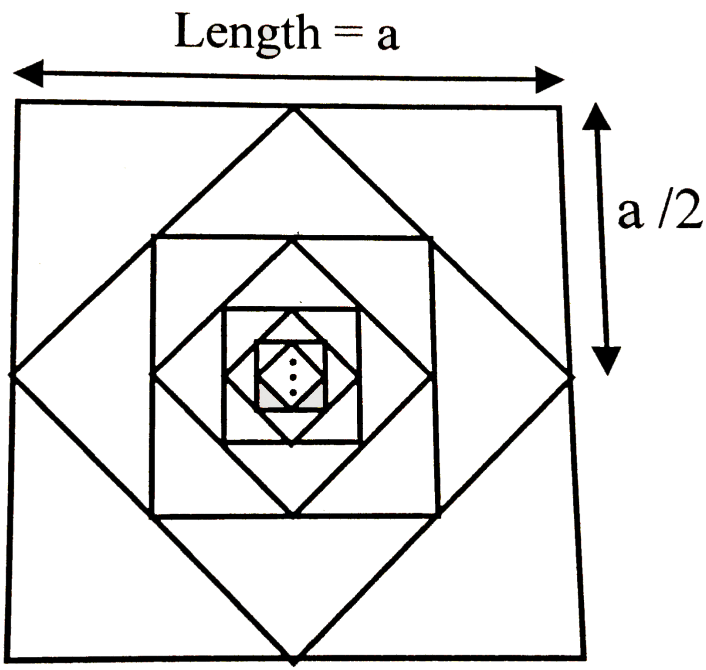
Answer: c

Watch Video Solution

19. Let S_1 be the sum of areas of the squares whose sides are parallel to coordinates axes

Let S_2 be the sum of areas of the slanted squares as shown in the figure.

Then S_1 / S_2 is



A. 2

B. $\sqrt{2}$

C. 1

D. $\frac{1}{\sqrt{2}}$

Answer: a

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20. if a 3-digit number is randomly chosen, what is the probability that either the number itself or some permutation of the number (which is a 3-digit number) is divisible by 4 and 5 ?

A. $\frac{1}{45}$

B. $\frac{29}{180}$

C. $\frac{11}{60}$

D. $\frac{1}{4}$

Answer: b

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exercise

1. The number of ordered pairs of integers(x,y) which satisfy $x^3 + y^3 = 65$ are

A. 0

B. 2

C. 4

D. 6

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2. A,B,E are 3 points of the circumference of a circle of radius 1. If angle AEB = $\frac{\pi}{4}$. Then length of AB is

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3. $[x^2] = x + 1$ how many real roots

A. exactly 2

B. no real roots

C. more than 2

D. none

Watch Video Solution

4. If $x + y = 1$ where x and y are positive numbers, then the minimum value of $\frac{1}{x} + \frac{1}{y}$ is

A. 2

B. 44318

C. 3

D. 4

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5. If all the natural numbers from 1 to 2021 are written as 12345.....20202021, then find the 2021st term

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6.

$$\left[\frac{2^{2020}+1}{2^{2018}+1}\right]+\left[\frac{3^{2020}+1}{3^{2018}+1}\right]+\left[\frac{4^{2020}+1}{4^{2018}+1}\right]+\left[\frac{5^{2020}+1}{5^{2018}+1}\right]+\left[\frac{6^{2020}+1}{6^{2018}+1}\right]$$

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7. Let's say abcde is a 5 digit number which when multiplied by 9 new number formed is edcba then sum a+b+c+d+e

A. 18

B. 27

C. 36

D. 45

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8. I: m is any composite number that divides (m-1)!

II: n is a natural number that $n^3 + 2n^2 + n$ divides n!

A. Both are true

B. Both are false

C. I is true , II is false

D. I is false, II is true

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9. $2^x + 3^y = 5^{xy}$ Number of solutions = ?

Watch Video Solution

10. In a book self if m books have black cover and n books have blue cover and all books are different, then the number of ways black books can be arranged side by side are

A. mn!

B. (m+1)!

C. (n+1)!

D. (n+1)!m!

Watch Video Solution

11. $x > 2y > 0$ and $2\log(x - 2y) = \log xy$ Possible values of $\frac{x}{y}$ is/are

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12. In an equiangular octagon if 6 consecutive sides are 6,8,7,10,9,5 then what is sum of other two sides

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13. If the function f(x) = $2+x^2-e^x$ and g(x) =$f^{-1}(x)$, then the value of $\frac{|g'(1)|}{4}$ equals

Watch Video Solution

14. S= $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{\sqrt{n^2+k^2}}$

A. Does not exist

B. $S\in[0,1)$

C. $S\in[1,2)$

D. $S\in[2,\infty)$

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15. $f(x): R \rightarrow R$

$|f(x)-f(y)|>|x-y|$ forall $x, y \in R$ check one-one/many one & into/onto

Watch Video Solution

16. $x^3-[x]^3=(x-[x])^3$

A. x has discrete values only

B. x contains an interval but is not an interval itself

C. x is a finite interval

D. $x \in (\infty, -\infty)$

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17. S1: $\lim_{n \to \infty} \frac{2^n + (-2)^n}{2^n}$ does not exist

S2: $\lim_{n \to \infty} \frac{3^n + (-3)^n}{4^n}$ does not exist

A. S1 is true s2 is false

B. both are true

C. both are false

D. S1 is false S2 is true

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18. In a 15 sidead polygon a diagnol is chosen at random. Find the probability that it is neither oneof the shortest nor one of the longest

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PART I (Mathematics)

1. Let [x] be the greatest integer less than or equal to x, for a real number x. Then the equation $[x^2] = x + 1$ has

A. two solutions

B. one solution

C. no solution

D. more than two solutions

Watch Video Solution

2. Suppose p, q, r are positive rational numbers such that $\sqrt{p} + \sqrt{q} + \sqrt{r}$ is also rational. Then

A. $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational

B. $\sqrt{pq}, \sqrt{pr}, \sqrt{qr}$ are rational, but $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational

C. $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are rational

D. $\sqrt{pq}, \sqrt{pr}, \sqrt{qr}$ are irrational

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3. A,B,E are 3 points of the circumference of a circle of radius 1. If angle AEB = $\frac{\pi}{4}$. Then length of AB is

A. $\sqrt{3}$

B. $\frac{4}{3}$

C. $\frac{3}{\sqrt{2}}$

D. $\sqrt{2}$

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4. Let x and y be two positive real numbers such that $x + y = 1$. Then the minimum value of $\frac{1}{x} + \frac{1}{y}$ is

A. 2

B. $\frac{5}{2}$

C. 3

D. 4

Watch Video Solution

5. Let ABCD be a qudrilateral such that there exists a point E inside the quadrilateral satisfying $AE = BE = CE = DE$. Suppose $\angle DAB, \angle ABC, \angle BCD$ is an arithmetic progression. Then the median of the set $\{\angle DAB, \angle ABC, \angle BCD\}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

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6. The number of ordered pairs (x,y) of positive integers satisying $2^x + 3^y = 5^{xy}$ is

A. 1

B. 2

C. 5

D. infinite

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7. If the integers from 1 to 2021 are written as a single integer like 123.....91011.....20202021, then the 2021^{st} digit (counted from the left) in the resulting number is

A. 0

B. 1

C. 6

D. 9

Watch Video Solution

8. Let [x] be the greatest integer less than or equal to x, for a real number x. Then the following sum

$$\left[\frac{2^{2020}+1}{2^{2018}+1}\right]+\left[\frac{3^{2020}+1}{3^{2018}+1}\right]+\left[\frac{4^{2020}+1}{4^{2018}+1}\right]+\left[\frac{5^{2020}+1}{5^{2018}+1}\right]+\left[\frac{6^{2020}+1}{6^{2018}+1}\right]$$

is

A. 80

B. 85

C. 90

D. 95

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9. Let r be the remainder when 2021^{2020} is divided by 2020^2. Then r lies between

A. 0 and 5

B. 10 and 15

C. 20 and 100

D. 107 and 120

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10. In a triangle ABC, the altitude AD and the median AE divide $\angle A$ into three equal parts. If BC=28, then the nearest integer to AB+ AC is

A. 38

B. 37

C. 36

D. 33

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11. The number of permutations of the letters a_1, a_2, a_3, a_4, a_5 in which the first letter a_1 does not occupy the first position (from the left) and the second letter a_2 does not occupy the second position (from the left) is

A. 96

B. 78

C. 60

D. 42

Watch Video Solution

Watch Video Solution

12. In a book self if m books have black cover and n books have blue cover and all books are different, then the number of ways black books can be arranged side by side are

A. m!n!

B. $m!(n+1)!$

C. $(n+1)!$

D. $(m+n)!$

Watch Video Solution

PART I (Chemistry)

1. The number of ordered pairs of integers(x,y) which satisfy $x^3+y^3=65$ are

A. 0

B. 2

C. 4

D. 6

Watch Video Solution

2. Consider the following two statements :

I. If n is a composite number, then n divides (n - 1)!.

II. There are infinitely many natural numbers n such that $n^3 + 2n^2 + n$ divides n!.

Then

A. I and II are true

B. I and II are false

C. I is true and II is false

D. I is false and II is true

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PART II MATHEMATICS

1. Let a, b, c be non-zero real roots of the equation $x^3 + ax^2 + bx + c = 0$. Then

A. there are infinitely many such triples a, b, c

B. there is exactly one such triple a, b, c

C. there are exactly two such triples a, b, c

D. there are exactly three such triples a, b, c

Watch Video Solution

2. In a triangle ABC, the angle bisector BD of $\angle B$ intersects AC in D. Suppose $BC = 2, CD = 1$ and $BD = \frac{3}{\sqrt{2}}$. The perimeter of the triangle ABC is

A. $\frac{17}{2}$

B. $\frac{15}{2}$

C. $\frac{17}{4}$

D. $\frac{15}{4}$

Watch Video Solution

3. Len N be set of natural numbers. For $n \in N$ defiine

$$I_n = \int_0^\pi \frac{x \sin^{2n}(x)}{\sin^{2n}(x) + \cos^{2n}(x)} dx.$$

Then for $m, n \in N$

A. $I_m < I_n$ for all $m < n$

B. $I_m > I_n$ for all $m < n$

C. $I_m = I_n$ for all $m \neq n$

D. $I_m < I_n$ for some $m < n$ and $I_m > I_n$ for some $m < n$

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4. Let a = BC, b = CA, c = AB be side lengths of a triangle ABC. And m be the length of the median through A. If a = 8, b-c = 2, m = 6, then the nearest integer to b is

A. 7

B. 8

C. 9

D. 10

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