MATHS

BOOKS - KVPY PREVIOUS YEAR

KVPY 2021

Part I Mathematics

1. Let [x] be the greatest integer less than or equal to x, for a real number x. Then the equation $\left[x^2\right] = x + 1$ has

A. two solutions

B. one solution

C. no solution

D. more than two solutions

Answer:

Watch Video Solution

2. Suppose p, q, r are positive rational numbers such that $\sqrt{p}+\sqrt{q}+\sqrt{r}$ is also rational. Then

A. $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational

B. $\sqrt{pq}, \sqrt{pr}, \sqrt{qr}$ are rational, but $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational

C. $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are rational

D. $\sqrt{pq}, \sqrt{pr}, \sqrt{qr}$ are irrational

Answer:

Watch Video Solution

3. A,B,E are 3 points of the circumference of a circle of radius 1. If angle AEB = $\frac{\pi}{4}$. Then length of AB is

A. $\sqrt{3}$

B. $\frac{4}{3}$

C. $\frac{3}{\sqrt{2}}$

D. $\sqrt{2}$

Answer:

Watch Video Solution

4. Let x and y be two positive real numbers such that $x + y = 1$. Then the minimum value of $\frac{1}{x} + \frac{1}{y}$ is

A. 2

B. $\frac{5}{2}$

C. 3

D. 4

Answer:

Watch Video Solution

5. Let ABCD be a qudrilateral such that there exists a point E inside the quadrilateral satisfying $AE = BE = CE = DE.$ Suppose $\angle DAB, \angle ABC, \angle BCD$ is an arithmetic progression. Then the median of the set $\{\angle DAB, \angle ABC, \angle BCD\}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer:

Watch Video Solution

6. The number of ordered pairs (x,y) of positive integers satisying $2^x + 3^y = 5^{xy}$ is

A. 1

B. 2

C. 5

D. infinite

Answer:

Watch Video Solution

7. If the integers from 1 to 2021 are written as a single integer like 123.....91011.....20202021, then the 2021^{st} digit (counted from the left) in the resulting number is

A. 0

B. 1

C. 6

D. 9

Answer:

Watch Video Solution

8. Let [x] be the greatest integer less than or equal to x, for a real number x. Then the following sum

$$\left[\frac{2^{2020}+1}{2^{2018}+1}\right]+\left[\frac{3^{2020}+1}{3^{2018}+1}\right]+\left[\frac{4^{2020}+1}{4^{2018}+1}\right]+\left[\frac{5^{2020}+1}{5^{2018}+1}\right]+\left[\frac{6^{2020}+1}{6^{2018}+1}\right]$$

is

A. 80

B. 85

C. 90

D. 95

Answer:

Watch Video Solution

9. Let r be the remainder when 2021^{2020} is divided by 2020^2. Then r lies between

A. 0 and 5

B. 10 and 15

C. 20 and 100

D. 107 and 120

Answer:

Watch Video Solution

10. In a triangle ABC, the altitude AD and the median AE divide $\angle A$ into three equal parts. If BC=28, then the nearest integer to AB+ AC is

A. 38

B. 37

C. 36

D. 33

Answer:

Watch Video Solution

11. The number of permutations of the letters a_1, a_2, a_3, a_4, a_5 in which the first letter a_1 does not occupy the first position (from the left) and the second letter a_2 does not occupy the second position (from the left) is

A. 96

B. 78

C. 60

D. 42

Answer:

Watch Video Solution

12. In a book self if m books have black cover and n books have blue cover and all books are different, then the number of ways black books can be arranged side by side are

A. m!n!

B. $m!(n+1)!$

C. $(n+1)!$

D. $(m+n)!$

Answer:

Watch Video Solution

Part I Chemistry

1. The number of ordered pairs of integers(x,y) which satisfy $x^3+y^3=65$ are

A. 0

B. 2

C. 4

D. 6

Answer:

Watch Video Solution

2. Consider the following two statements :

I. If n is a composite number, then n divides (n - 1)!.

II. There are infinitely many natural numbers n such that $n^3 + 2n^2 + n$ divides n!.

Then

A. I and II are true

B. I and II are false

C. I is true and II is false

D. I is false and II is true

Answer:

Watch Video Solution

Part I Mathematics

1. Consider the following statements :

I. $\lim_{n \to \infty} \frac{2^n + (-2)^n}{2^n}$ does not exist

II. $\lim_{n \to \infty} \frac{3^n + (-3)^n}{4^n}$ does not exist

Then

A. I is true and II is false

B. I is false and II is true

C. I and II are true

D. neither I nor II is true

Answer:

Watch Video Solution

2. Consider a regular 10-gon with its vertices on the unit circle. With one vertex fixed, draw straight lines to the other 9 vertices. Call them $L_1, L_2, \ldots . L_9$ and denote their lengths by $l_1, l_2 \ldots . l_9$ respectively. Then the product $l_1 l_2 \ldots . l_9$ is

A. 10

B. $10\sqrt{3}$

C. $\dfrac{50}{\sqrt{3}}$

D. 20

Answer:

Watch Video Solution

3. The value of the integral

$$\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+e^x} dx$$

is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{\pi^2}{2}$

Answer:

Watch Video Solution

4. Let $\mathbb{R}$ be the set of all real numbers and

$$f(x) = \sin^{10} x \left(\cos^8 x + \cos^4 x + \cos^2 x + 1\right)$$

for $x \in \mathbb{R}$. Let

$S = \{\lambda \in \mathbb{R}|$ there exists a point $c \in (0, 2\pi)$ with $f'(c) = \lambda f(c)\}$.

A. $S = \mathbb{R}$

B. $S = \{0\}$

C. $S = [0, 2\pi]$

D. S is a finite set having more than one element

Answer:

Watch Video Solution

5. A person standing on the top of a building of height $60\sqrt{3}$ feel observed the top of a tower to lie at an elevation of 45°. That person descended to the bottom of the building and found that the top of the same tower is now at an angle of elevation of 60°. The height of the tower (in feet) is

A. 30

B. $30(\sqrt{3} + 3)$

C. $90(\sqrt{3} + 1)$

D. $150(\sqrt{3}+1)$

Answer:

Watch Video Solution

6. Assume that $3.313 \leq \pi \leq 3.15$. The integer closest to the value of $\sin^{-1}(\sin 1 \cos 4 + \cos 1 \sin 4)$. Where 1 and 4 appearing in sin and cos are given in radians, is

A. -1

B. 1

C. 3

D. 5

Answer:

Watch Video Solution

7. The maximum value of the function $f(x) = e^x + x \ln x$ on the interval $1 \leq x \leq 2$ is

A. $e^2 + \ln 2 = 1$

B. $e^2 + 2\ln 2$

C. $e^{\pi/2} + \frac{\pi}{2}\ln\frac{\pi}{2}$

D. $e^{3/2} + \frac{3}{2}\ln\frac{3}{2}$

Answer:

Watch Video Solution

8. Let A be a 2×2 matrix of the form $A = \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}$, where a, b are integers and $-50 \leq b \leq 50$. The number of such matrices A such that A^{-1}, the inverse of A, exists and A^{-1} contains only integer entries is

A. 101

B. 200

C. 202

D. 101^2

Answer:

Watch Video Solution

9. Let $A = (a_{ij})_{1 \le I, j \le 3}$ be a 3×3 invertible matrix where each a_{ij} is a real number. Denote the inverse of the matrix A by A^{-1}. If $\Sigma_{j=1}^{3} a_{ij} = 1$ for $1 \le i \le 3$, then

A. sum of the diagonal entries of A is 1

B. sum of each row of A^{-1} is 1

C. sum of each row and each column of A^{-1} is 1

D. sum of the diagonal entries is A^{-1} is 1

Answer:

Watch Video Solution

10. Let x, y be real numbers such that $x > 2y > 0$ and

$2\log(x - 2y) = \log x + \log y.$

Then the possible values (s) of $\frac{x}{y}$

A. is 1 only

B. are 1 and 4

C. is 4 only

D. is 8 only

Answer: C

Watch Video Solution

11. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(b < a)$. Be an ellipse with major axis AB and minor axis CD. Let $F_1$and F_2 be its two foci, with A, F_1, F_2 B in that order on the segment AB. Suppose $\angle F_1CB = 90^\circ$. The eccentricity of the ellipse is

A. $\frac{\sqrt{3} - 1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{\sqrt{5}-1}{2}$

D. $\frac{1}{\sqrt{5}}$

Answer:

Watch Video Solution

12. Let A denote the set of all real numbers x such that $x^3 - [x]^3 = (x - [x])^3$, where [x] is the greatest integer less than or equal to x. Then

A. A is a discrete set of at least two points

B. A contains an interval, but is not an interval

C. A is an interval, but a proper subset of $(-\infty, \infty)$

D. $A = (-\infty, \infty)$

Answer:

Watch Video Solution

13. Define a sequence $\{S_n\}$ of real numbers by

$$S_n = \sum_{k=0}^{n} \frac{1}{\sqrt{n^2 + k}}, \text{ for } n \geq 1.$$

Then $\lim_{n \to \infty} S_n$

A. does not exist

B. exists and lies in the interval (0, 1)

C. exists and lies in the interval [1, 2)

D. exists and lies in the interval $[2, \infty)$

Answer:

Watch Video Solution

14. Let

$$f(x) = \begin{cases} \frac{x}{\sin x}, & x \in (0,1) \\ 1, & x = 0 \end{cases}$$

Consider the integral

$$I_n = \sqrt{n}\int_0^{1/n} f(x)e^{-nx}\,dx.$$

Then $\lim_{n \to \infty} I_n$

A. does not exist

B. exists and is 0

C. exists and is 1

D. exists and is $1 - e^{-1}$

Answer:

Watch Video Solution

15. The value of the integral

$$\int_1^3 \left((x-2)^4 \sin^3(x-2) + (x-2)^{2019} + 1\right) dx$$

is

A. 0

B. 2

C. 4

D. 5

Answer:

Watch Video Solution

16. In a 15 sidead polygon a diagnol is chosen at random. Find the probability that it is neither oneof the shortest nor one of the longest

A. $\frac{2}{3}$

B. $\frac{5}{6}$

C. $\frac{8}{9}$

D. $\frac{9}{10}$

Answer:

Watch Video Solution

17. Let $M = 2^{30} - 2^{15} + 1$, and M^2 be expressed in base 2. The number of 1's in this base 2 representation of M^2 is

A. 29

B. 30

C. 59

D. 60

Answer:

Watch Video Solution

18. Let ABC be a triangle such that AB = 15 and AC = 9. The bisector of $\angle BAC$ meets BC in D. If $\angle ACB = 2\angle ABC$, then BD is

A. 8

B. 9

C. 10

D. 12

Answer:

Watch Video Solution

19. The figur in the complex plane given by

$$10z\bar{z} - 3(z^2 + \bar{z}^2) + 4i(z^2 - \bar{z}^2) = 0$$

is

A. a straight line

B. a circle

C. a parabola

D. an ellipse

Answer:

Watch Video Solution

1. Let a, b, c be non-zero real roots of the equation $x^3 + ax^2 + bx + c = 0$. Then

A. there are infinitely many such triples a, b, c

B. there is exactly one such triple a, b, c

C. there are exactly two such triples a, b, c

D. there are exactly three such triples a, b, c

Answer:

Watch Video Solution

2. In a triangle ABC, the angle bisector BD of $\angle B$ intersects AC in D. Suppose $BC = 2, CD = 1$ and $BD = \frac{3}{\sqrt{2}}$. The perimeter of the triangle ABC is

A. $\frac{17}{2}$

B. $\frac{15}{2}$

C. $\frac{17}{4}$

D. $\frac{15}{4}$

Answer:

Watch Video Solution

3. Len N be set of natural numbers. For $n \in N$ defiine

$$I_n = \int_0^{\pi} \frac{x \sin^{2n}(x)}{\sin^{2n}(x) + \cos^{2n}(x)} dx.$$

Then for $m, n \in N$

A. $I_m < I_n$ for all $m < n$

B. $I_m > I_n$ for all $m < n$

C. $I_m = I_n$ for all $m \neq n$

D. $I_m < I_n$ for some $m < n$ and $I_m > I_n$ for some $m < n$

Answer:

Watch Video Solution

4. Let a = BC, b = CA, c = AB be side lengths of a triangle ABC. And m be the length of the median through A. If a = 8, b-c = 2, m = 6, then the nearest integer to b is

A. 7

B. 8

C. 9

D. 10

Answer:

Watch Video Solution