



MATHS

BOOKS - KVPY PREVIOUS YEAR

KVPY 2021

Part I Mathematics

1. Let [x] be the greatest integer less than or equal to x, for a real number

x. Then the equation $\left[x^2
ight]=x+1$ has

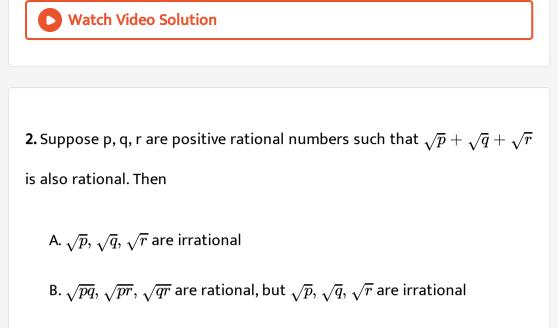
A. two solutions

B. one solution

C. no solution

D. more than two solutions

Answer:



- C. $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are rational
- D. $\sqrt{pq}, \sqrt{pr}, \sqrt{qr}$ are irrational

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3. A,B,E are 3 points of the circumference of a circle of radius 1. If angle AEB = $\frac{\pi}{4}$. Then length of AB is

A. $\sqrt{3}$

B.
$$\frac{4}{3}$$

C. $\frac{3}{\sqrt{2}}$
D. $\sqrt{2}$

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4. Let x and y be two positive real numbers such that x+y=1. Then the

minimum value of
$$\displaystyle rac{1}{x} + \displaystyle rac{1}{y}$$
 is

A. 2

$$\mathsf{B}.\,\frac{5}{2}$$

C. 3

D. 4

Answer:

5. Let ABCD be a qudrilateral such that there exists a point E inside the quadrilateral satisfying AE = BE = CE = DE. Suppose $\angle DAB, \angle ABC, \angle BCD$ is an arithmetic progression. Then the median of the set $\{\angle DAB, \angle ABC, \angle BCD\}$ is

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$

Answer:



6. The number of ordered pairs (x,y) of positive integers satisying $2^x + 3^y = 5^{xy}$ is

A. 1

B. 2

C. 5

D. infinite

Answer:

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7. If the integers from 1 to 2021 are written as a single integer like 123.....91011.....20202021, then the 2021^{st} digit (counted from the left) in the resulting number is

A. 0

B. 1

C. 6

D. 9



8. Let [x] be the greatest integer less than or equal to x, for a real number

x. Then the following sum

$$\left[\frac{2^{2020}+1}{2^{2018}+1}\right] + \left[\frac{3^{2020}+1}{3^{2018}+1}\right] + \left[\frac{4^{2020}+1}{4^{2018}+1}\right] + \left[\frac{5^{2020}+1}{5^{2018}+1}\right] + \left[\frac{6^{2020}+1}{6^{2018}+1}\right]$$

is

A. 80

B. 85

C. 90

D. 95

Answer:

9. Let r be the remainder when 2021^{2020} is divided by 2020^2 . Then r lies between

A. 0 and 5

B. 10 and 15

C. 20 and 100

D. 107 and 120

Answer:

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10. In a triangle ABC, the altitude AD and the median AE divide $\angle A$ into three equal parts. If BC=28, then the nearest integer to AB+ AC is

A. 38

B. 37

C. 36

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11. The number of permutations of the letters a_1 , a_2 , a_3 , a_4 , a_5 in which the first letter a_1 does not occupy the first position (from the left) and the second letter a_2 does not occupy the second position (from the left) is

A. 96

B. 78

C. 60

D. 42

Answer:

12. In a book self if m books have black cover and n books have blue cover and all books are different, then the number of ways black books can be arranged side by side are

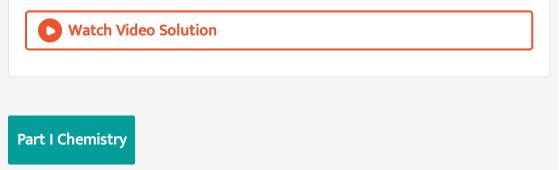
A. m!n!

 $\mathsf{B}.\,m\,!(n+1)\,!$

C.(n+1)!

D. (m + n)!

Answer:



1. The number of ordered pairs of integers(x,y) which satisfy $x^3+y^3=65$

are

A. 0	
B. 2	
C. 4	
D. 6	



2. Consider the following two statements :

I. If n is a composite number, then n divides (n - 1)!.

II. There are infinitely many natural numbers n such that n^3+2n^2+n

divides n!.

Then

A. I and II are true

B. I and II are false

C. I is true and II is false

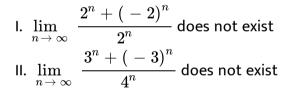
D. I is false and II is true

Answer:



Part I Mathematics

1. Consider the following statements :



Then

A. I is true and II is false

B. I is false and II is true

C. I and II are true

D. neither I nor II is true



2. Consider a regular 10-gon with its vertices on the unit circle. With one vertex fixed, draw straight lines to the other 9 vertices. Call them L_1, L_2, \ldots, L_9 and denote their lengths by l_1, l_2, \ldots, l_9 respectively. Then the product $l_1 l_2 \ldots, l_9$ is

A. 10

B. $10\sqrt{3}$ C. $\frac{50}{\sqrt{3}}$

D. 20

Answer:

3. The value of the integral

$$\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + e^x} dx$$

is
A. $\frac{\pi}{6}$
B. $\frac{\pi}{4}$
C. $\frac{\pi}{2}$
D. $\frac{\pi^2}{2}$

Answer:



4. Let \mathbb{R} be the set of all real numbers and

$$f(x) = \sin^{10}x \left(\cos^8x + \cos^4x + \cos^2x + 1
ight)$$

for $x \in \mathbb{R}.$ Let

 $S=\{\lambda\in\mathbb{R}| ext{ there exists a point }c\in(0,2\pi) ext{ with }f'(c)=\lambda f(c)\}.$

A. $S=\mathbb{R}$

B. $S = \{0\}$

C. $S = [0, 2\pi]$

D. S is a finite set having more than one element

Answer:



5. A person standing on the top of a building of height $60\sqrt{3}$ feel observed the top of a tower to lie at an elevation of 45° . That person descended to the bottom of the building and found that the top of the same tower is now at an angle of elevation of 60° . The height of the tower (in feet) is

A. 30

 $\mathsf{B.}\, 30 \big(\sqrt{3}+3\big)$

 $C.90(\sqrt{3}+1)$

D.
$$150(\sqrt{3}+1)$$



6. Assume that $3.313 \le \pi \le 3.15$. The integer closest to the value of $\sin^{-1}(\sin 1 \cos 4 + \cos 1 \sin 4)$. Where 1 and 4 appearing in sin and cos are given in radians, is

A. −1

B. 1

C. 3

D. 5

Answer:

7. The maximum value of the function $f(x)=e^x+x \ln x$ on the interval $1\leq x\leq 2$ is A. $e^2+\ln 2=1$ B. $e^2+2\ln 2$

C.
$$e^{\pi/2} + rac{\pi}{2} {
m ln} rac{\pi}{2}$$

D. $e^{3/2} + rac{3}{2} {
m ln} rac{3}{2}$

Answer:

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8. Let A be a 2×2 matrix of the form $A = \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}$, where a, b are integers and $-50 \le b \le 50$. The number of such matrices A such that A^{-1} , the inverse of A, exists and A^{-1} contains only integer entries is

A. 101

B. 200

C. 202

 $D.\,101^2$

Answer:

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9. Let $A = (a_{ij})_{1 \le I, j \le 3}$ be a 3×3 invertible matrix where each a_{ij} is a real number. Denote the inverse of the matrix A by A^{-1} . If $\Sigma_{j=1}^3 a_{ij} = 1$ for $1 \le i \le 3$, then

A. sum of the diagonal entries of A is 1

B. sum of each row of A^{-1} is 1

C. sum of each row and each column of A^{-1} is 1

D. sum of the diagonal entries is A^{-1} is 1

Answer:

10. Let x, y be real numbers such that x>2y>0 and

 $2\log(x-2y) = \log x + \log y.$

Then the possible values (s) of $\frac{x}{y}$

A. is 1 only

B. are 1 and 4

C. is 4 only

D. is 8 only

Answer: C

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11. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(b < a)$. Be an ellipse with major axis AB and minor axis CD. Let F_1 and F_2 be its two foci, with A, F_1 , F_2 B in that order on the segment AB. Suppose $\angle F_1CB = 90^\circ$. The eccentricity of the ellipse is

A.
$$rac{\sqrt{3}-1}{2}$$

B.
$$\frac{1}{\sqrt{2}}$$

C. $\frac{\sqrt{5}-1}{2}$
D. $\frac{1}{\sqrt{5}}$

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12. Let A denote the set of all real numbers x such that $x^3 - [x]^3 = (x - [x])^3$, where [x] is the greatest integer less than or equal to x. Then

A. A is a discrete set of at least two points

B. A contains an interval, but is not an interval

C. A is an interval, but a proper subset of $(\,-\infty,\infty)$

D.
$$A=(\,-\infty,\infty)$$

Answer:



13. Define a sequence $\{S_n\}$ of real numbers by

$$S_n=\sum_{k=0}^nrac{1}{\sqrt{n^2+k}}$$
 , for $n\geq 1.$

Then $\lim_{n o \infty} \; S_n$

A. does not exist

B. exists and lies in the interval (0, 1)

C. exists and lies in the interval [1, 2)

D. exists and lies in the interval $[2,\infty)$

Answer:



14. Let

$$f(x)=\left\{egin{array}{c} rac{x}{\sin x}, & x\in(0,1)\ 1, & x=0 \end{array}
ight.$$

Consider the integral

$$I_n=\sqrt{n}{\int_0^{1\,/\,n}f(x)e^{\,-\,nx}dx}.$$

Then $\lim_{n o \infty} \ I_n$

A. does not exist

B. exists and is O

C. exists and is 1

D. exists and is $1 - e^{-1}$

Answer:

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15. The value of the integral

$$\int_{1}^{3} \Big((x-2)^4 \sin^3(x-2) + (x-2)^{2019} + 1 \Big) dx$$
 is

A. 0

B. 2

C. 4

D. 5

Answer:

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16. In a 15 sidead polygon a diagnol is chosen at random. Find the probability that it is neither one of the shortest nor one of the longest

A.
$$\frac{2}{3}$$

B. $\frac{5}{6}$
C. $\frac{8}{9}$
D. $\frac{9}{10}$

Answer:

17. Let $M=2^{30}-2^{15}+1$, and M^2 be expressed in base 2. The number of 1's in this base 2 representation of M^2 is

A. 29

B. 30

C. 59

D. 60

Answer:

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18. Let ABC be a triangle such that AB = 15 and AC = 9. The bisector of

 $\angle BAC$ meets BC in D. If $\angle ACB = 2 \angle ABC$, then BD is

A. 8

B. 9

C. 10

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19. The figur in the complex plane given by

$$10zar{z}-3ig(z^2+ar{z}^2ig)+4iig(z^2-ar{z}^2ig)=0$$

is

A. a straight line

B. a circle

C. a parabola

D. an ellipse

Answer:

1. Let a, b, c be non-zero real roots of the equation $x^3 + ax^2 + bx + c = 0$. Then

A. there are infinitely many such triples a, b, c

B. there is exactly one such triple a, b, c

C. there are exactly two such triples a, b, c

D. there are exactly three such triples a, b, c

Answer:

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2. In a triangle ABC, the angle bisector BD of $\angle B$ intersects AC in D. Suppose BC = 2, CD = 1 and $BD = \frac{3}{\sqrt{2}}$. The perimeter of the triangle ABC is

A.
$$\frac{17}{2}$$

B. $\frac{15}{2}$
C. $\frac{17}{4}$
D. $\frac{15}{4}$

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3. Len N be set of natural numbers. For $n \in N$ define

$$I_n = \int_0^\pi rac{x \sin^{2n}(x)}{\sin^{2n}(x) + \cos^{2n}(x)} dx.$$

Then for $m,n\in N$

- A. $I_m < I_n$ for all m < n
- B. $I_m > I_n$ for all m < n

C.
$$I_m = I_n$$
 for all $m
eq n$

D. $I_m < I_n$ for some m < n and $I_m > I_n$ for some m < n



4. Let a = BC, b = CA, c = AB be side lengths of a triangle ABC. And m be the length of the median through A. If a = 8, b-c = 2, m = 6, then the nearest integer to b is

A. 7 B. 8 C. 9 D. 10

Answer: