PHYSICS

BOOKS - BHARATI BHAWAN PHYSICS (HINGLISH)

ROTATIONAL DYNAMICS, MOMENT OF INERTIA

Examples

1. A solid body rotates about a stationary axis so that the rotation angle θ varies with time as $\theta = 6t - 2t^3$ radian. Find

(a) the angular acceleration at the moment when the body stops and

(b) the average value of angular velocity and angular acceleration averaged over the time interval between $t = 0$ and the complete stop.

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2. calculate the angular momentum of a projectile of mass m at time t after its projection in a vertical plane with velocity v at angle α with the horizontal.

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3. Two particles P and Q are rotating in the anticlockwise direction about a common center O in circles of radii 2m and 5m with linear speeds $63m/s$. And $21m/s$ respectively.

Calculate the relative angular velocity with reference to the common center O. What is the maximum angular velocity of P relative to Q in the clockwise direction and that in the anticlockwise direction?

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4. Two solid bodies rotate about stationary and mutually perpendicular intersecting axes with constant angualr velocities $w_1 = 2rad/s$

and $w_2 = 4.5 rad/s$. Find the angualr velocity and acceleration of one relative to the other.

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5. Calculate the radius of gyration of a thin rod of mass 1kg and length 100cm about an axis passing through its center of gravity and perpendicular to its length.

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6. A flywheel of mass 500 kg and diameter 1m is set in rotation. In 5 seconds it starts rotating at the rate of 20 resolutions per second. Calcualte the torque applied.

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7. A solid sphere of mass 1 kg and radius 10 cm rolls without slipping with the uniform velocity of 100 cm per second along a straight

line on a horizontal table. Calculate its total energy.

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8. A horizontal disc rotating freely about a vertical axis makes 100 rpm. A small piece of wax of mass 10 g falls vertically on the disc and adheres to it at a distance of 9 cm from the axis if the number of revolution per minute is thereby reduced to 90. Calculate the moment of inertia of disc.

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9. How long does a disc take to roll down an inclined plane of length 10m without slipping? Inclination of the plane with the horizontal = 30° and acceleration due to gravity =$9.8ms^{-2}$

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10. A man stands holding a weight in each hand and with his arms outstretched on a frictionless platform which is rotating at a

speed of 1 revolution per sec. In ths position the total rotational intertia of the man, the weights and the platform is $6kgm^2$. If by drawing in his arms, the man decreases the rotational inertia of the system by $2kgm^2$, calcualte the resulting speed of the platform and the increase in kinetic energy. How do you account for the increase of kinetic energy?

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11. a) show that the energy of a rolling body which is not slipping is equivalent to that of pure rotatioin with the same angular speed about an axis through the point of contact of the rolling body.

b) Show that the angular momentum of a body of mass M which is rolling without slippint about any point is equal to its angular momentum about its center plus the moment of momentum of a particle of the same mass M concentrated at its center and moving with

the speed of the center of the mass of the body about the same point

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12. A ball is thrown down a lawn in such a way that it initially slides with a speed v_0 without rolling. It gradually picks up rotational motion. Prove that it will be without sliding, that is, its motion will be pure rolling when its speed falls to $\frac{5}{7}v_0$.

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13. Show that a cylinder will slip on an inclined plane if the coefficient of static friction between the plane and the cylinder is less than $\frac{1}{3}\tan\theta$ where θ is the angle of the inclination with the horizontal.

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14. A round unifrom body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle

theta with the horizontal. Then its acceleration

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Exercises

1. A sphere of mass 10 kg and diameter 10 cm rolls without slipping with a velocity of 50 cm per second. Calculate its total kinetic energy.

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2. Calculate the moment of inertia of a cylinder of length $1.5m$, radius $0.05m$ and density $8 \times 10^3 kg/m^3$ about the axis of the cylinder.

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3. A uniform rod rotates in a horizontal plane about an axis perpendicular to its length and passing through an extreme end. Calculate the moment of inertia and kinetic energyof the rod if its length is 3m, its mass 6 kg and angular velocity 20 raidans per second.

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4. A sphere starting from rest rolls for 5.3s without slipping along a plane which is 1m in length. The upper end of the plane is 1 cm higher than the lower end. Find the accelerationi due to gravity.

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5. A couple of 10 Nm is applied to a flywheel of mass 10 kg and radius 50cm. What is the resulting acceleration? For how many seconds must the couple be applied and then removed to produce 100 rpm (revolutions per minute) in the wheel?

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6. If the earth suddenly contracted to half of its present radius without losing any mass,

how many hours would there be in a day? (hint: Apply the principle of conservation of angular momentum).

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7. A ring, a disc and a sphere all of the same radius and mass roll down an inclined plane from the same height h. Which of the three reaches the bottom (i) earliest (ii) latest ?

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8. A flywheel of mass 5 kg and radius 10 cm rolls down a plane inclined at 30° to the horizontal. It rolls from rest through 100 cm in 10s. Neglecting friction find a) the kinetic energy of the flywheel at the end of 10s, (b) the momentum of intertia of the flywheel about its axis.

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9. Two point masses m_1 and m_2 are joined by a weightless rod of length r. Calculate the

moment of inertia of the system about an axis passing through its centre of mass and perpendicular to the rod.

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10. A thin uniform rod AB of mass m and length l is hinged at one end to the level floor and stands vertically. If it s allowed to fall, with what angular velocity will it strike the floor?

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11. A hoop of radius 3m weighs 150 kg. It rolls along a horizontal floor so that its center of mass has a speed of $15cm/\sec$. How much work has to be done to stop it? ltbrlt [Hint : A hoop is a ring]

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12. A sphere rolls up an inclined plane whose inclination is 30°. At the bottom of the inclined plane, the center of mass of the sphere has a translational speed of $5ms^{-1}$ (a)

How far does the sphere travel up the plane? (b) How long does it take to return to the bottom?

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13. A student sits on a stool that is free to rotate about a vertical axis. He holds out his arms horizontally, with a 4-kg weight in each hand. The stool is set in rotation with angular speed of 0.5 revolution per second. Calcualate the angular speed of the student is 90 cm and

his rotational inertia is 7kg m^2.

[Hint: Use conservation principle]

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14. A solid sphere of 1kg reaches the bottom of an inclined plane after rolling without slipping over a horizontal plane with velocity of $20ms^{-1}$. The inclination of the plane with the horizontal is 30°. What is the velocity of the center of mass of the sphere immediately after it starts moving up the plane? Up to what

distance along the plane will it rise? Assume sufficient friction to prevent slipping.

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15. A thin uniform rod of mass m and length l, pivoted freely at its base, is allowed to fall from a vertical position. Calcualte its angular velocity and acceleration when it has turned through 60°.

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16. What must be the relation between length and radius of a cylinder of given mass and density so that its moment of inertia about an axis through its center of mass and perpendicular to its length may be minimum?

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17. A circular disc is of mass M and radius r. From it a circualr piece is cut out with a radius of the disc as its diameter. Find the moment of

inertia of the remainder about the axix of the disc

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18. A string is wrapped around a cylinder of mass M and radius r. The string is pulled vertically upward to prevent the center of mass from falling as the cylinder unwinds the string. (a) What is the tension in the string? B) How much work has been done on the cylinder once it has reached an angular speed ω? c)

what is the length of string unwound in the time its acquires angular speed ω?

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19. A small sphere of mass m and radius r rolls without slipping on the insides of a large hemisphere of radius R whose axis of symmetry is vertical. Its starts at the top from rest. What is the kinetic energy of the small sphere at the bottom? What fraction is rotational and what transiational? What

normal force does the small spheres exert on the hemisphere at the bottom?

[Hint: consider circular motion of center of mass at the bottom to calcualte normal reaction. For others consider conservation of energy.?

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20. A billiard ball is struck by a cue when it starts moving with velocity v_0 without rolling. How far will the ball move before it ceases to

slide on the table if μ is the coefficient of friction between ball and table?

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21. a) Explain why the cushion of a billard table is made to receive the impact of the billard ball at height of 0.7 d where d is the diameter of the ball.

b) With what part of a sabre should an object be slashed in order that the user does not feel the impact on the hand? Consider the sabre as

a uniform lamina.

[Hint: In either case if the striking point is at such a distance that the lower point remains at rest, there will a rise no force. Such a point is called center of impact].

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22. A right circular cylinder of radius r and mass m is suspended by a cord that is wound around its surface. It is allowed to fall, prove that at its center of gravity will follow a

vertical rectilinear path and find the accleration along this path. Determine also the tensile force in the cord.

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23. The mass of the earth is increasing at the rate 1 part in 5×20^{19} per day by the accretion of meteors falling normally upon the earth's surface. Find the corresponding rate of change of the period of rotation of the earth supporting the earth to be a sphere of

uniform density.

[Hint: Consider the deposit as a spherical shell and apply principle of conservation of angular momentum]

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24. A meter stick of length l and mass M lies on a frictionlesss table. The stick is free to move in any way on the table. A hockey ball of mass m moving perpendicuarly to the mass of the ball so that it remains at rest immediately

after the collision?

[Hint: Elastic collision means that kinetic energy is conserved in the process. Consider conservation of kinetic energy, conservation of angualr momentum and linear momentum, i.e.,

$\frac{1}{2}mv^2 = \left(\frac{1}{2}Mv^{2'} + \frac{1}{2}I\varepsilon^2, \quad mv\frac{1}{2} = I\varepsilon,\right.$

$mv = Mv'.$

Eliminate $v^{2'}$ and w^2 from the first with the help of the other two. Use the expression $I = \frac{1}{12}MI^2\Big]$

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25. A long light thread is wond partly around cylinder of radius r and mass m partly on a simialr cylinder but free to turn about a fixed axis. The movable one is allowed to fall from rest. Find the velocity of the cylinder as a functiono of the height h through which it has fallen. Does the cylinder fall without slipping?

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26. A tim,ber of mas m rests on two rollers, each of mass $\frac{m}{2}$ and radius r and is pulled

along a horizontal plane by a force P. Assuming there is no slipping and treating the rollers as right circular cylinders, find the acceleration of the timber.

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27. Show that in order to get a billiard ball to roll without sliding from the start the cue must hit the ball not at the center (that is, a height above the table equal to the ball's radius R) but exactly at a height $\frac{2}{5}$R above the

center

[Hint, Consider linear momentum and angular motion of the ball. Apply the condtion for rolling without sliding. F = ma and $\tau = F \times h = \frac{2}{5} mR^2 \times \alpha$. Condition rolling without a sliding it a $= \alpha R$]

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28. Two masses 500 g and 460g are suspended from the ends of a light string passing over a frictionless heavy pulley of radius 5cm. When

released from rest the heavier mass is observed to fall 75cm in 5s. What is the rotational inertia of the pulley? ltbr. [Hint: When the pulley is heavy the tensions on the two slides are different.]

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29. A solid cylinder of weight 20 kg and radius 7.5 cm is placed on a inclined plane with inclination 30° to the horizontal. A light thin tape is wound around the cylinder and is

taken tangentially over the upper surface parallel to the plane and finally attached to a 5-kg body after passing over a light smooth pulley. Find the tension in the tape and linear acceleration of the cylinder up the plance, assuming no slip.

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30. A point A is located on the rim of a wheel of radius R which rolls witout slipping along a horizontal surface witih velocity V. Find the

total distance traversed by the point A between successive moments at which it touches the surface.

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31. A plank of mass m_1 with a sphere of mass m_2 on it rests on a smooth horizontal surface. A canstant horizontal force F is applied to the plank. With what sliding between the plank and the sphere?

[hint: Rel vel. Of the lowest point $v-(2)+\varepsilon R-v_1$=0 for no sliding)

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32. If in the loop-the-loop track of figure a small spherical ball of mass m is released and it rolls without slipping down the loop, calcualte the normal force exerted by the track on the ball when it is at the highest position. Take R: r=$6:1$

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33. A uniform rod of length 2a is held at inclination α with the horizontla with its lower end at a height h above a horizontal plane surface and is let fall. It rebounds elastically from the plane. If you just before the other end touches the plane the rod makes the same angle with the horizontal, find the value of h in terms of a and α.

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34. An experimentor stands on a stool capable of rotating about a vertical axis and holds a rotating wheel of moment of intertia I_1 about its axis and angualr velocity ε_1 with its axis coinciding with the vertical axis of the stool. The moment of inertia of the stool and the experiment about the vertical axis is I_2. Calculate the change in kinetic energy of the system if the experiment rotates the wheel through

i) 180° and ii) 90°

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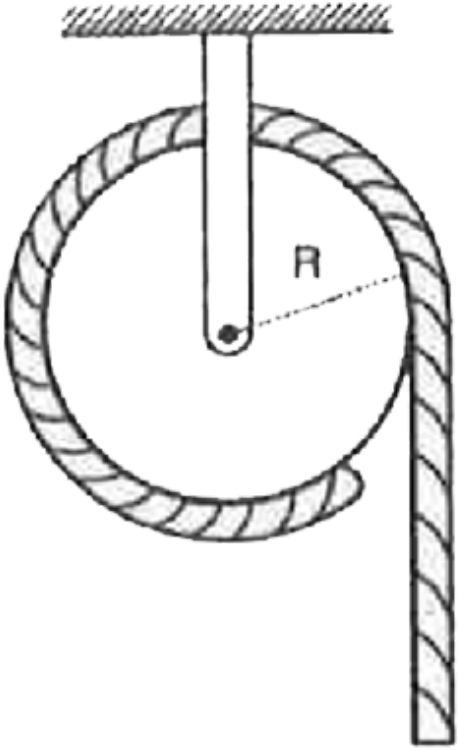
35. A uniform cylinder of radius R, and mass M rotates freely about a stationary horizontal axix O. A thin cord of length l and mass m is wound on the cylinder in a single layer and its free end is falling vertically. Find the acceleration as a function of the length x of the hanging part of the cord. Assume the center of mass of the wound part of the cord

to be the axis of the cylinder.



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36. A carpet of mass M is rolled along its length so as to from a cylinder of radius R and is kept on a rough floor. When a negligibly small push is given to the cylindrical carpet, it stars unrolling itself without sliding on the floor. Calculate horizontal velocity of cylindrical part of the carpet when its radius reduces to $R/2$.

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37. A man of mass m stands on the edge of a horizontal unifom disc of mass M and radius R whichis capable of rotating freely about a stationary vertical axis passing through its center. At a certain moment the man starts moving along the edge of the disc, he shifts over angle θ relative to the disc and then stops. Find the angle through which the disc rotates.

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38. A uniform solid cylinder of radius R=20cm rolls over a horizontal plane and passes on to an inclined plane meeting the horizontal plane at angle $\alpha = 30^\circ$ with the horizontal. Find the maximum value of velocity v_0 which still permits the cylinder to roll into the inclined plane without a jump. Assume sliding to be absent.

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39. A particle at angular position θ_0 inside a fixed smooth hemispherical bowl of radius r is projected horizontally with velocity v_0. Calculate its value so that the particle may rise to the top.

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40. Two cylinders with radii r_1 and r_2 and rotational inertia I_1 and I_2 are supported on their horizontal axles. The first one is set in

rotation with angular velocity ε. The axle of the other cylinder (smaller) is moved until it touches the large cylinder and is caused to rotate by the frictional forces between the two. Find the angualr velocity of the two cylinders after slipping ceases between them. [Hint: Consider angular impulse received by the cylinders]

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41. Calculate the vertical and horizontal reactions of the pivot when the rod of the problem 15 has turned through $\theta = 60^\circ$.

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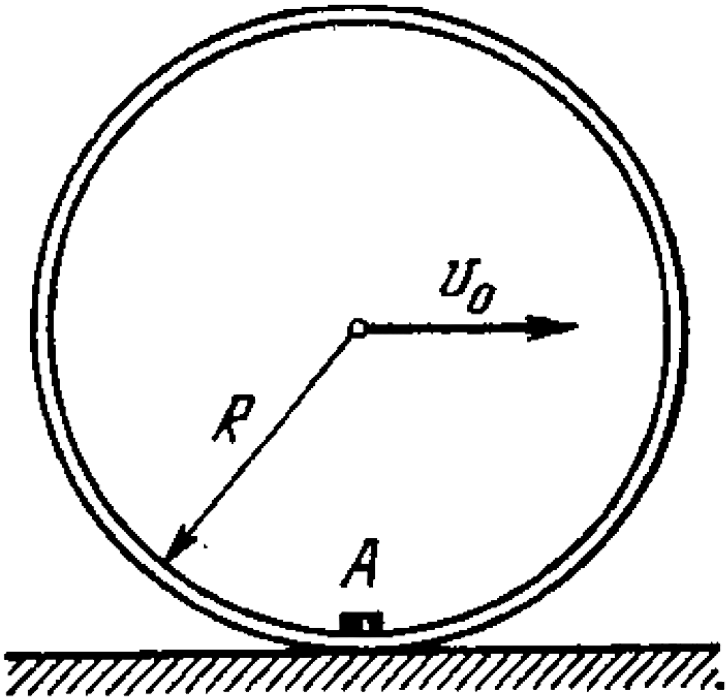
42. A small body A is fixed to the inside of a thin rigid hoop of radius R and mass equal to that of the body A. The hoop rolls without slipping over a horizontal plane, at the moments when the body A gets into the lower

position, the centre of the hoop moves with velocity v_0(figure). At what values of v_0 will the hoop move without bouncing?



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