PHYSICS

BOOKS - GR BATHLA & SONS PHYSICS (HINGLISH)

BASIC MATHEMATICS

Solved Problems

1. Differentiate the following functions with respect to x :

(i) $x^3 + 2x^2 + 1$ (ii) $e^{(3x+2)}$ (iii) $\cos^2 x$

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2. A particle moves so that its position vector varies with time as $\vec{r} = A\cos\omega t\hat{i} + A\sin\omega t\hat{j}$. If $\frac{d\vec{r}}{dt}$ gives instantaneous velocity. Find the initial velocity of particle.

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3. If $x = at^3, y = bt^2$, then find $\frac{dy}{dx}$.

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4. The volume of sphere is given by $V = \frac{4}{3}\pi R^3$ where R is the radius of sphere. Find the rate of change of volume with respect to R.

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5. The angular displacement of particle (in radian) is given by $\theta = t^2 + t$. Calculate angular velocity at t=2 second.

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6. The height reached in time t by a particle thrown upward with a speed u is given by

$h = ut - \frac{1}{2}\mathrm{gt}^2$

where g is acceleration due to gravity. Find the time taken in reaching the maximum height.

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7. A mass M is split into two parts m and (M-m) which are then separated by certain distance. Find ratio (m/M) to maximise the gravitational force $F = \frac{Gm(M-m)}{r^2}$ between the parts. Here G = gravitational constant and r is the distance between m and (M-m).

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8. Find the maximum or minimum values of the function.

$y = 9 - (x - 3)^2$

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9. Integrate the following w.r.t.x :

(a) $x^3 + 2x + 1$ (b) $\frac{1}{2x + 3}$

(c) $\cos^2 x$

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10. Evaluate $\int_{\infty}^{R} \frac{GMm}{x^2} dx$

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11. A force F=(a+bx) acts on a particle in x direction where a and b are constants . Find the work done by this force during displacement from x_1 to x_2 .

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12. A rod of length L is placed along the x-axis between x=0 and x=L. The linear mass density is λ such that $\lambda = a + bx$. Find the mass of the rod.

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13. Using the method of integration show that area of circle is πR^2 .

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14. Find value of $(104)^{1/2}$ using binomial approx.

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15. The value of acceleration due to gravity (g) at height h above the surface of earth is given by $g' = \frac{gR^2}{(R+h)^2}$. If $h << R$, then prove that $g' = g\left(1 - \frac{2h}{R}\right)$.

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16. Solve $x^2 + x - 2 = 0$

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17. Find the sum of

$h + 2e^2h + 2e^4h + 2e^6h + \ldots .$

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18. Find the sum of the series

$1 + 4 + 7 + 10 + \ldots\ldots$ to 40 terms.

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19. (a) Draw the graph for line $y=3x$.

(b) $y=-x$

(c) $y=2x+4$

(d) $y=-x+2$

(e) $y=2x-4$

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20. Draw the circle which touches x-axis and its centre is (α, β) .

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21. Draw the circle . When circle touches both axes. Take (α, α) as centre.

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22. (a) Find the value of followings :

(i) $\sin 150^\circ$, (ii) $\cos 135^\circ$, (iii) $\cos(-30^\circ)$, (iv) $\tan 225^\circ$.

(b) If $\cos\theta = \frac{4}{5}$. Find the value of $\sin\theta$ and $\tan\theta$.

(c) Find the value of $\sin^{-1}(1)$.

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Problems For Practice

1. Given, $y = (ax + b)^2$, evaluate $\frac{dy}{dx}$.

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2. Given, $y = \sin 2x$. Then find $\frac{dy}{dx}$.

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3. Given , y=ln(ax+b), then find $\frac{dy}{dx}$

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4. $y = x^3 - 4x^2 + 5$. Find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$

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5. Find value of $\int (ax + b)^{3/2} dx.$

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6. Find value of $\int \frac{dx}{(ax + b)}$.

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7. Find value of $\int \sin 2x dx$.

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8. Given , $\varepsilon - L\frac{di}{dt} = iR$, find the value of i at any time t in terms of constant ε, L and R. At t=0 , $i = 0$.

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9. Plot the line $-3x - 5y = 15$.

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10. A particle starts with some initial velocity with an acceleration along the direction of motion. Draw a graph depicting the variation of velocity (v) along y-axis with the variation of displacement(s) along x-axis.

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11. Plot a graph for the equation $y = ax - bx^2$, where a and b are positive constants.

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12. If a particle starts moving along x-axis from origin with initial velocity u=2m/s and acceleration $4m/s^2$ the relation between displacement and time is given as $x = 2t + 2t^2$. Draw the displacement time graph for $t \geq 0$.

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13. A particle starts with uniform acceleration. Draw a graph taking the displacement(s) of the particle along y-axis and time(t) along x-axis. What is the curve known as?

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14. If $y = \left[\frac{x^2+1}{x+1}\right]$, then find $\frac{dy}{dx}$.

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15. If $y = \dfrac{\sin x}{x + \cos x}$, then find $\dfrac{dy}{dx}$.

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16. If $y = \cos^2 x$, then find $\dfrac{dy}{dx}$.

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17. If $y = \cos x^2$, then find $\dfrac{dy}{dx}$.

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18. If $x = at^4, y = bt^3$, then find $\frac{dy}{dx}$.

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19. The velocity v of a particle is given by the equation $v = 6t^2 - 6t^3$, where v is in ms^{-1}, t is the instant of time in seconds while 6 and 6 are suitable dimensional constants. At what values of t will the velocity be maximum and minimum ? Determine these maximum and minimum values of the velocity.

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20. Evaluate $\int \sqrt{1+y^2}.2ydy$

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21. Evaluate : $\int \frac{2xdx}{(x^2+1)^{3/2}}$

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22. How rapidly will the fluid level inside a vertical cylindrical tank drop if we pump the fluid out at the rate of 3000 L/min ? Take area of base of cylinder=A.

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23. A hot air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the

range finder's elevation angle is $\frac{\pi}{4}$. The angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at the moment ?

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24. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight cast. When the cruiser is 0.6 mile north of the intersection and the car

is 0.8 mile to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car ?

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25. Water runs into a conical tank at the rate of $9ft^3/\min$. The tank stands point down and has a height of 10 ft and a base radius of 5 ft.

How fast is the water level rising when the water is 6 ft deep ?

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26. Heating a plate. When a circular plate of metal is heated in an oven, its radius increases at the rate of $0.01 cm / \min$. At what rate is the plate's area increasing when the radius is 50 cm ?

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27. Changing dimensions in a rectangular box. Suppose that the edge lengths x, y and z of a closed rectangular box are changing at the following rates :

$\frac{dx}{dt} = 1m/\sec, \frac{dy}{dt} = -2m/\sec, \frac{dy}{dt} = 1/\sec$

,

Find the rates at which the box's (a) volume, (b) surface area and (c) diagonal length $s = \sqrt{x^2 + y^2 + z^2}$ are changing at the instant when x=4, y=3 and z=2 .

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28. A 13 ft leader is leaning against a house when its base starts to slide away. When the base is 12 ft from the house, then base is moving at the rate of 5 ft/sec.

(a) How fast is the top of the ladder sliding down the wall then?

(b) At what rate is the area of the triangle formed by the ladder, wall and ground changing then ?

(c) At what rate is the angle θ between the

ladder and the ground changing then ?

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29. Flying a kite . A girl flies a kite at a height of 3000 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her ?

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30. A growing sand pile . Sand falls from a conveyor belt at the rate of $10m^3/\min$ on to the top of a conical pile. The height of the pile is always three-eights of the base diameter. How fast are the (a) height and (b) radius changing when the pile is 4 m high ? Answer in cm/min.

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31. A growing raindrop. Suppose that a drop of mist is a perfect sphere and that, through

condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the drop's radius increases at a constant rate.

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32. A balloon and a bicycle. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground , a bicycle moving at a constant rate of 17 ft/sec passes under it .

How fast is the distance between the bicycle and balloon increasing 3 sec later ?

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33. If two resistors of R_1 and R_2 ohms are connected in parallel in an electric circuit to make an R ohm resistor, the value of R can be found from the equation,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 is decreasing at the rate of 1 ohm/sec

and R_2 i increasing at the rate of 0.5 ohm/sec, at what rate is R changing when $R_1 = 75$ ohm and $R_2 = 50$ ohm ?

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34. Suppose that at time $t \geq 0$ the position of a particle moving on the x-axis is $x = (t-1)(t-4)^4 m.$

(a) When is the particle at rest ?

(b) During what time interval does the particle

move to the left ?

(c) Find maximum velocity of particle while moving to the left.

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35. A draining conical reservoir. Water is flowing at the rate of $50m^3/\min$ from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m.

(a) How fast is the water level falling when the water is 5 m deep.

(b) How fast is the radius of the water's surfce changing then ? Answer in cm/min.

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