MATHS

NCERT - NCERT MATHEMATICS(GUJRATI)

APPLICATION OF DERIVATIVES

Example

1. Find the rate of change of the area of a circle per second with respect to its radius r when r = 5 cm.

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2. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When $x = 10\text{cm}$ and $y = 6\text{cm}$, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

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3. Show that the function f given by $f(x) = x^3 - 3x^2 + 4x, x \in R$is strictly increasing on R.

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4. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$is (a) strictly increasing (b) strictly decreasing

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5. Find the intervals in which f(x) = sin 3x, $x \in \left[0, \frac{\pi}{2}\right]$ is (i) increasing, (ii) decreasing.

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6. Example 13 Find the intervals in which the function f tives f(x) = sin x + cos x,0 $\leq x \leq$ 2π is strictly increasing or strictly decreasing.

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7. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

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8. Find the equation of the tangent to the curve $y = \frac{x - 7}{(x - 2(x - 3)}$ at the point where it cuts the x-axis.

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9. Find local maximum and local minimum values of the function f given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12.$

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10. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5.$

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11. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

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12. Find the shortest distance of the point (0, c) from the parabola $y = x^2$, where $0 \leq c \leq 5$.

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13. Let AP and BQ be two vertical poles at points A and B, respectively. If

$AP = 16m, BQ = 22m and AB = 20m$, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.

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14. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

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15. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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16. Find absolute maximum and minimum values of a function f given by $f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}, x \in [-1, 1]$.

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17. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

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18. A water tank has the shape of an inverted righ circular cone with its axis vertical and vertex lowermost . Its semi-vertical angle is

$\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 4 cubic meter per hour . Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 2 m.

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19. A man 2 metres high walks at a uniform speed of 5 km/hr away from a lamp-post 6 metres high. Find the rate at which the length of his shadow increases.

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20. Find the equation of the normal to the curve $x^2 = 4y$which passes through the point (1, 2).

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21. The equation of tangents to the curve $y = \cos(x + y), \ -2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$, is

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22. Find intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5} + 11$is (a) strictly increasing (b) strictly decreasing.

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23. $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$ is always and increasing function on the interval

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24. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/sec. Find the rate at which its area is increasing when radius is 3.2 cm.

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25. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

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26. Manufacturer can sell x items at a price of rupees $\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is Rs $\left(\frac{x}{5} + 500\right)$. Find the number of items he should sell to earn maximum profit

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Exercise 6 1

1. Find the rate of change of the area of a circle with respect to its radius r when

(a) r=3 cm (b) r=4 cm

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2. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of an edge is 12 cm?

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3. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

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4. An edge of a variable cube is increasing at the rate of $3cm/s$. How fast is the volume of the cube increasing when the edge is $10cm$ long?

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5. A stone is dropped into a quiet lake and waves move in circles at the speed of $5cm/s$. At the instant when the radius of the circular wave is $8cm$, how fast is the enclosed area increasing?

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6. 6. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

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7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle

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8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

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9. A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm

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10. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

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11. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate

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12. The radius of an air bubble is increasing at the rate of $\frac{1}{2}cm/s$. At what rate is the volume of the bubble increasing when the radius is 1 cm?

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13. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$.Find the rate of change of its volume with respect to x.

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14. Sand is pouring from a pipe at the rate of $12\,cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

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15. The total cost C (x) in Rupees associated with the production of x units of an item is given by

$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000.$

Find the marginal cost when 17 units are produced

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16. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

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17. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is:

A. 10π

B. 12π

C. 8π

D. 11π

Answer: B

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18. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5.$ The marginal revenue, when $x = 15$is (A) 116 (B) 96 (C) 90 (D) 126

A. 116

B. 96

C. 90

D. 126

Answer: D

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Exercise 6 2

1. Show that the function given by $f(x) = 3x + 17$is strictly increasing on R.

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2. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

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3. Show that the function given by f $f(x) = 3x + 17(x) = s \in x$is (a) strictly increasing in $\left(0, \frac{\pi}{2}\right)$ (b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$(c) neither increasing nor decreasing in $(0, \pi)$

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4. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$is(a) strictly

increasing (b) strictly decreasing

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5. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$is (a) strictly increasing (b) strictly decreasing

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6. Find the intervals in which the following functions are strictly increasing or decreasing:

(a x^2+2x-5 (b) $10-6x-2x^2$ (c) $-2x^3-9x^2-12x+1$ (d) $6-9x-x^2$ (e) $(x+1)^3(x-3)^3$

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7. Find the interval in which the following function are strictly increasing or decreasing ?

A. x^2+2x+5 B. $10-6x-2x^2$

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8. Find the intervals in which the given functions are strictly increasing decreasing:

$-2x^3 - 9x^2 - 12x + 1$

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9. Find the intervals in which the functions are strictly increasing or decreasing:

$6 - 9x - x^2$

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10. Find the intervals in which the functions are strictly increasing or decreasing:

$(x+1)^3(x-3)^3$

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11. Show that $y = \log(1+x) - \frac{2x}{2+x}, x \succ 1$, is an increasing function of x throughout its domain.

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12. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function

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13. Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

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14. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

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15. Prove that the function f given by f(x) = $x^2 - x + 1$ is neither strictly increasing nor decreasing on (– 1, 1).

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16. Which of the following functions is not decreasing on $(0, \pi/2)$?

A. cos x

B. cos 2x

C. cos 3x

D. tan x

Answer: A::B

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17. On which of the following intervals is the function f given by f(x) = x^{100} + sin x –1 decreasing ?

A. (0,1)

B. $\frac{\pi}{2}, \pi$

C. $0, \frac{\pi}{2}$

D. None of these

Answer: D

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18. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$is strictly increasing on $(1, 2)$.

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19. Let P be an interior point of triangle ABC. Let Q and R be the reflections of P in AB and AC, respectively. If Q,A,R are collinear then $\angle A$ equals

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20. Prove that the function f given by $f(x) = \log \cos x$ is strictly

decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

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21. Prove that the function f given by f (x) = log |cos x| is decreasing on $\left(0, \frac{\pi}{2}\right)$ and increasing on $\left(\frac{3\pi}{2}, 2\pi\right)$.

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22. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$is increasing in R.

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23. The interval in which $y = x^2 e^{-x}$ is increasing is (A) $(-\infty, \infty)$ (B) (2, 0) (C) $(2, \infty)$ (D) (0, 2)

A. $(-\infty, \infty)$

B. (-2,0)

C. (2,00)

D. (0,2)

Answer: D

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Exercise 6 3

1. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

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2. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}, x \neq 2$at $x = 10.$

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3. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

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4. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

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5. Find the slope of the normal to the curve $x = a\cos^3\theta, y = \sin^3\theta$at $\theta = \frac{\pi}{4}$.

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6. Find the slope of the normal to the curve $x = 1 - a\sin\theta, y = b\cos^2\theta$at $\theta = \frac{\pi}{2}$.

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7. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis

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8. Find a point on the curve y $=(x-2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

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9. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y \quad = \quad x \qquad 11$.

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10. Find the equation of all lines having slope 1 that are tangents to the curve $y = \frac{1}{x-1}, x \neq 1.$

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11. Find the equation of all lines having slope 1 that are tangents to the curve $y = \frac{1}{x-1}, x \neq 1.$

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12. Find the equations of all lines having slope 0 which are tangent to the curve $y = \frac{1}{x^2 - 2x + 3}$.

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13. Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are (i) parallel to x-axis (ii) parallel to y-asis

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14. Find the equations of the tangent and normal to the given curves at the indicated points: (i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0, \quad 5)$ (ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(1, \quad 3)$ (iii) $y = x^3$ at $(1, \quad 1)$ (iv) $y = x^2$ at $(0, \quad 0)$

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15. find the equation of the tangent and normal to the given curves at the given points

(i) $y = x^4 - 6x^3 - 10x + 5$ at $(1, 3)$

(ii) $y^2 = \dfrac{x^3}{4 - x}$ at $(2 - 2)$

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16. Find a point on the curve $y = x^3$, where the tangent to the curve is parallel to the chord joining the points (1, 1) and (3, 27)

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17. Find the equations of the tangent and the normal to the curve $y = x^2$ at $(0, 0)$ at the indicated points

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18. Find the equations of the tangent to the given curves at the indicated points:

x= cos t , y =sin t at t = $\frac{\pi}{4}$

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19. Find the equation of the tangent line to the curve y = x^2 – 2x +7 which is

(a) parallel to the line 2x – y + 9 = 0

(b) perpendicular to the line 5y – 15x = 13.

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20. Show that the tangents to the curve $y = 7x^3 + 11$at the points where $x = 2$and $x = 2$are parallel.

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21. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

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22. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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23. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$at which the tangents are parallel to the x-axis.

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24. Find the equation of the normal at the point $(am^2, am^3)$for the curve $ay^2 = x^3$.

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25. Find the equation of the normals to the curve $y = x^3 + 2x + 6$which are parallel to the line $x + 14y + 4 = 0$.

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26. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$at the point $(at^2, 2at)$.

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27. Show that the curves $x = y^2$ and $xy = k$ cut at right angles; if $8k^2 = 1$

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28. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. at the point (x_0, y_0)

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29. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

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30. The slope of the normal to the curve $y = 2x^2 + 3\sin$ x at $x = 0$is(A) 3 (B) $\frac{1}{3}$ (C)-3 (D) $-\frac{1}{3}$

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$

Answer: D

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31. The line y = x + 1 is a tangent to the curve y^2 = 4x at the poin

A. (1, 2)

B. (2, 1)

C. (1, – 2)

D. (– 1, 2)

Answer: A

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Exercise 6 4

1. $\sqrt{25.3}$

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2. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$\sqrt{49.5}$

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3. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$\sqrt{0.6}$

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4. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(0.009)^{\frac{1}{3}}$

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5. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(0.0999)^{\frac{1}{10}}$

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6. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(15)^{\frac{1}{4}}$

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7. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(26)^{\frac{1}{3}}$

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8. Find approminate value of $(255)^{\frac{1}{4}}$.

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9. Using differentials, find the approximate value of $(82)^{\frac{1}{4}}$ upto 3 places of decimal .

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10. Using differentials, find the approximate value of $\sqrt{401}$

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11. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(0.0037)^{\frac{1}{2}}$

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12. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(26.57)^{\frac{1}{3}}$

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13. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(81.5)^{\frac{1}{4}}$

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14. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(3.968)^{\frac{3}{2}}$

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15. Using differentials, find the approximate value of each of the up to 3 places of decimal.

$(32.15)^{\frac{1}{5}}$

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16. Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$.

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17. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

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18. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

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19. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

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20. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

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21. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area.

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22. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is :

A. 47.66

B. 57.66

C. 67.66

D. 77.66

Answer: D

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23. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is(A) 0.06 x^3m^3 (B) 0.6 x^3m^3 (C) 0.09 x^3m^3 (D) 0.9 x^3m^3

A. $0.06x^3$ m

B. $0.6x^3m^3$

C. $0.09x^3m^3$

D. $0.9x^3m^3$

Answer: C

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Exercise 6 5

1. Find the maximum and minimum values, if any, of the functions given by

f (x) = $(2x-1)^2+3$

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2. Find the maximum and minimum values, if any, of the functions given by

f(x) = $9x^2+12x+2$

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3. Find the maximum and minimum values, if any, of the functions given by

$f(x) = -(x-1)^2 + 10$

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4. Find the maximum and minimum values, if any, of the functions given by

g(x) =x^3 + 1

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5. Find the maximum and minimum values, if any, of the functions given by

$f(x) = |x + 2| - 1$

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6. Find the maximum and minimum values, if any, of the functions given by

$g(x) = -|x + 1| + 3$

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7. Find the maximum and minimum values, if any, of the functions given by

h(x) = sin(2x) + 5

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8. Find the maximum and minimum values, if any, of the following functions given by (i) $f(x) = |x + 2| \quad 1$ (ii) $g(x) = |x + 1| + 3$ (iii)

$h(x) = s \in (2x) + 5$ (iv) `f" "(x)"

"=" "|" "s in" "4x" "+"

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9. Find the maximum and minimum values, if any, of the functions given by

h(x) = x + 1, x $\in$ (– 1, 1)

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10. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

f(x) $= x^2$

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11. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as

the case may be:

g(x) = $x^3 - 3x$

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12. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

h(x) = sin x+ cos x , 0 $< x < \frac{\pi}{2}$

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13. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

f(x) = sin x - cos x , 0 $< x < 2\pi$

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14. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as

the case may be:

f(x) $= x^3 - 6x^2 + 9x + 15$

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15. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

g(x) $= \frac{x}{2} + \frac{2}{x} x > 0$

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16. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as the case may be:

g(x) $= \frac{1}{x^2 + 2}$

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17. Find the local maxima and local minima, if any, of the functions. Find also the local maximum and the local minimum values, as

the case may be:

f(x) = $x\sqrt{1-x}, 0 < \ < 1$

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18. Prove that the following functions do not have maxima or minima:(i) $f(x) = ex$ (ii) $g(x) = \log x$(iii) $h(x) = x^3 + x^2 + x + 1$

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19. Prove that the functions do not have maxima or minima:

g(x) = log x

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20. Prove that the functions do not have maxima or minima:

h(x) = $x^3 + x^2 + x + 1$

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21. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f(x) $= x^3, x \in [-2, 2]$

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22. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f (x) = sin x + cos x , $x \in [0, \pi]$

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23. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f(x) = $4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

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24. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f(x) =$(x-1)^2+3, x \in$ [-3 ,1)

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25. Find the maximum profit that a company can make, if the profit function isgiven by $p(x)=41-24x-18x^2$

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26. Find the maximum value and the minimum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$.

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27. At what points in the interval $[0, 2\pi]$, does the function $s \in \quad 2x$ attain its maximum value?

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28. What is the maximum value of the function

$s \in \quad x \quad + \quad \cos \quad x$?

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29. Find the maximum value of $2x^3 - 24x + 107$in the interval [1, 3]. Find the maximum value of the same function in $[3, 1]$.

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30. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$. Find the value of a .

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31. Find the maximum and minimum values of the function $f(x) = x + \sin 2x, (0 < x < \pi)$.

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32. Find two numbers whose sum is 24 and whose product is as large as possible.

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33. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

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34. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.

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35. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

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36. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maxi

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37. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off squares from each corners and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum possible?

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38. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

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39. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

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40. Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimetres, find the dimensions of the can which has the minimum surface area?

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41. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

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42. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

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43. Show that the right-circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

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44. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

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45. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

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46. The point on the curve $x^2 = 2y$which is nearest to the point (0, 5) is(A) $\left(2\sqrt{2}, 4\right)$ (B) $\left(2\sqrt{2}, 0\right)$ (C) (0, 0) (D) (2, 2)

A. $\left(2\sqrt{2}, 4\right)$

B. $\left(2\sqrt{2}, 0\right)$

C. $(0, 0)$

D. (2,2)

Answer: A

47. For all real values of x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is(A) 0 (B) 1 (C) 3 (D) $\frac{1}{3}$

A. 0

B. 1

C. 3

D. $\frac{1}{3}$

Answer: D

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48. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$is(A) $\left(\frac{1}{3}\right)^{\frac{1}{3}}$ (B) $\frac{1}{2}$ (C) 1 (D) 0

A. $\left(\frac{1}{3}\right)^{\frac{1}{3}}$

B. $\frac{1}{2}$

C. 1

D. 0

Answer: C

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Exercise 6 6

1. Using differentials, find the approximate value of each of the following:

$(a)\left(\frac{17}{81}\right)^{\frac{1}{4}}$ (b) $(33)^{-\frac{1}{5}}$

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2. Show that the function given by $f(x)=\frac{\log x}{x}$ has maximum at $x=e$.

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3. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

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4. Find the equation of the normal to curve $x^2 = 4y$which passes through the point (1, 2).

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5. Show that the normal at any point θ to the curve

$$x = a\cos\theta + a\theta\sin\theta, \; y = a\sin\theta - a\,\theta\cos\theta$$

is at a constant distance from the origin.

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6. Find the intervals in which the function f given by $f(x) = \frac{4\sin x - 2x - xc\otimes}{2 + \cos x}$ is (i) increasing (ii) decreasing.

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7. Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}, \ x \neq 0$ is (i) increasing (ii) decreasing.

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8. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

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9. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m3. If building of tank costs Rs 70 per square metre for the base and Rs 45 per square metre for sides, what is the cost of least expensive tank?

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10. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

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11. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

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12. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the maximum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

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13. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has local maxima local minima point of inflexion

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14. Find the absolute maximum and minimum values of the function f given by $f(x) = \cos^2 x + \sin x , x \in [0, \pi]$.

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15. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

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16. Let f be a function defined on [a, b] such that $f'(x) > 0$, for all $x \in (a, b)$. Then prove that f is an increasing function on (a, b).

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17. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

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18. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

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19. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of(A) 1 m^3/h (B) 0.1 m^3/h (C) 1.1 m^3/h (D) 0.5 m^3/h

A. 1 m/h

B. 0.1 m/h

C. 0.1 m/h

D. 0.5 m/h

Answer: A

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20. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2, -1), is

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $\frac{-6}{7}$

Answer: B

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21. The line $y = mx + 1$is a tangent to the curve $y^2 = 4x$if the value of m is(A) 1 (B) 2 (C) 3 (D) $\frac{1}{2}$

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer: A

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22. The normal at the point (1,1) on the curve $2y+x^2=3$is(A) $x+y=0$ (B) $xy=0$ (C) $x+y+1=0$(D) $xy=0$

A. x+y=0

B. x-y=0

C. x+y+1=0

D. x+y=1

Answer: B

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23. The normal to the curve $x^2 = 4y$passing (1,2) is(A) $x + y = 3$ (B) $xy = 3$ (C) $x + y = 1$ (D) $xy = 1$

A. x + y = 3

B. x – y = 3

C. x + y = 1

D. x – y = 1

Answer: A

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24. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are(A) $\left(4, \pm\frac{8}{3}\right)$ (B) $\left(4, \frac{-8}{3}\right)$ (C) $\left(4, \pm\frac{3}{8}\right)$ (D) $\left(\pm 4, \frac{3}{8}\right)$

A. $\left(4, \pm\frac{8}{3}\right)$

B. $\left(4, \frac{-8}{3}\right)$

C. $\left(4, \pm\frac{3}{8}\right)$

D. $\left(\pm 4, \frac{3}{8}\right)$

Answer: A

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