MATHS

NCERT - NCERT MATHEMATICS(GUJRATI)

CONTINUITY AND DIFFERENTIABILITY

Examples

1. Check the continuity of the function f given by

$f(x) = 2x + 3 \quad \text{at} \quad x = 1.$

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2. Examine whether the function f given by $f(x) = x^2$ is continuous at x= 0.

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3. Discuss the continuity of the function f given by $f(x) = |x| \quad \text{at} \quad x = 0.$

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4. Is the function defined by $f(x) = |x|$, a continuous function?

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5. Discuss the continuity of the function f given by $f(x) = x^3 + x^2 - 1.$

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6. Discuss the continuity of the function f defined by $f(x) = \frac{1}{x}, x \neq 0.$

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7. Discuss the continuity of the function f defined by $\begin{cases} x+2 & \text{if } x \leq 1 \\ x-2 & \text{if } x > 1 \end{cases}$.

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8. Find all the points of discontinuity of the function f defined by $\begin{cases} x+2 & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x-2 & \text{if } x > 1 \end{cases}$.

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9. Discuss the continuity of the function f given by

$$\begin{cases} x & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}.$$

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10. Show that a function p is a polynomial function is continuous.

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11. Find all the points of discontinuity of the greatest interger function defined by $f(x) = [x]$,

where [x] denote the greatest integer less than or equal to x.

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12. Prove that every rational function is continuous.

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13. Discuss the continuity of sine function.

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14. Find the derivative of the function given by $f(x) = \sin\left(x^2\right)$.

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15. Find the derivative of $\tan(2x + 3)$.

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16. Differentiate $\sin\left(\cos\left(x^2\right)\right)$ with respect to x.

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17. Find $\frac{dy}{dx}$ if $x - y = \pi$.

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18. Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$.

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19. Find the derivative of f given by $f(x) = \sin^{-1} x$assuming it exists.

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20. Find the derivative of / given by $f(x) = \tan^{-1} x$assuming it exists.

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21. Is it true that $x = e^{\log x}$ for all real

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22. Differentiate the following w.r.t. x :

e^{-x}

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23. Differentiate each of the following w.r.t. x :

(i) $\sin(\log x), x > 0$

(ii) $\log(\log x), x > 1$

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24. Differentiate the following w.r.t. x :

$\cos^{-1}(e^x)$.

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25. Differentiate the following w.r.t. x :

$e^{\cos x}$

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26. Differentiate $\sqrt{\left((x-3)\frac{x^2+4}{(3x^2+4x+5)}\right.}$ w.r.t x.

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27. Differentiate a^x w.r.t. x, where a is a positive constant.

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28. Differentiate $x^{\sin x}$, $x > 0$w.r.t. x.

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29. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

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30. Find $\frac{dy}{dx}$, if $x = a\cos\theta, y = a\sin\theta$

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31. $x = at^2, y = 2at,$

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32. Find $\frac{dy}{dx}$, when $x = a(\theta + \sin\theta) and y = a(1 - \cos\theta)$

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33. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

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34. Find $\frac{d^2y}{dx^2}$, if $y = x^3 + \tan x$.

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35. If $y = \quad A \quad s \in \quad x + \quad B \quad \cos \quad x \quad ,$ then prove that $\frac{d^2y}{dx^2} + y = 0$.

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36. If $y = 3\,e^{2x} + 2\,e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

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37. If $y = \sin^{-1} x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$

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38. Verify Rolles theorem for the function $y = x^2 + 2, a = -2$and $b = 2.$

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39. Verify Mean Value Theorem for the function $f(x) = x^2$ in the interval [2, 4].

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Miscellaneous Examples

1. Differentiate the following w.r.t x. (i) $\cos^{-1}(\sin x)$ (ii) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ (iii) $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

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2. Differentiate w.r.t.x, the following functions :

$e^{\sec^2 x} + 3\cos^{-1} x.$

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3. Differentiate w.r.t.x, the following functions :

$\log_7(\log x).$

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4. Differentiate the following w.r.t.x.

$\cos^{-1}(\sin x)$.

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5. $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

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6. Differentiate the following w.r.t.x.

$\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

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7. For a positive constant a find $\frac{dy}{dx}$, where $y - a^{1+\frac{1}{t}}$ and $x - \left(t + \frac{1}{t}\right)^a$.

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8. For a positive constant a find $\frac{dy}{dx}$, where $y = a^{t+\frac{1}{t}}$ and $x = \left(t + \frac{1}{t}\right)^a$.

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9. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

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Exercise 5 1

1. Prove that the function $f(x) = 5x - 3$is continuous at $x = 0$, at $x = -3$and at $x = 5$.

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2. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

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3. Examine the following functions for continuity.

(a) $f(x) = x - 5$ (b) $f(x) = \frac{1}{x - 5}$ (c) $f(x) = \frac{x^2 - 25}{x + 5}$ (d) $f(x) = |x - 5|$

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4. Examine the following functions for continuity.

$$f(x) = \frac{1}{x-5}, x \neq 5$$

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6. Examine the following functions for continuity.

(a) $f(x) = x - 5$ (b) $f(x) = \frac{1}{x-5}$ (c) $f(x) = \frac{x^2 - 25}{x + 5}$ (d) $f(x) = |x - 5|$

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7. Prove that the function $f(x) = x^n$ is continuous at $x = n$, where n is a positive integer.

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8. Is the function f defined by

$$f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 5 & \text{if } x > 1 \end{cases}$$

continuous at x = 0? At x = 2 ?

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9. Find all points of discontinuity of f, where f is defined by

$$f(x) = \{2x + 3, \quad \text{if} \quad x \leq 22x - 3, \quad \text{if} \quad x > 2$$

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10. $$f(x) = \begin{cases} |x| + 3 & \text{if} \quad x \leq -3 \\ -2x & \text{if} \quad -3 < x < 3 \\ 6x + 2 & \text{if} \quad x \geq 3 \end{cases}$$

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11. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 00, \quad \text{if } x = 0 \end{cases}$$

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12. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 01, \quad \text{if } x \geq 0 \end{cases}$$

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13. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} x+1, & \text{if } x \geq 1 \\ x^2+1, & \text{if } x < 1 \end{cases}$$

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14. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} x^3-3, & \text{if } x \leq 2 \\ x^2+1, & \text{if } x < 2 \end{cases}$$

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15. Find all points of discontinuity of f, where f is defined by

$$f(x) = \{x^{10} - 1, \quad \text{if} \quad x \leq 1 x^2, \quad \text{if} \quad x > 1$$

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16. Is the function defined by

$$f(x) = \{x + 5, \quad \text{if} \quad x \leq 1 x - 5, \quad \text{if} \quad x > 1$$

a continuous function?

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17. Discuss the continuity of the function f, where f is defined by $f(x)\begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$

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18. $f(x)\begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases}$

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19. $f(x)\begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$

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20. Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax + 1 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$$ is continuous at x=3.

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21. For what value of $\lambda$is the function defined by

$$f(x) = \{\lambda(x^2 - 2x), \quad \text{if } x \leq 04x + 1, \quad \text{if } x > 0$$

continuous at $x = 0$? What about continuity at $x = 1$?

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22. Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points which [x] denotes the greatest integer function.

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23. Is the function defined by $f(x) = x^2 - \sin x + 5$continuous at $x = \pi$?

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24. Discuss the continuity of the following functions a) $f(x) = \sin x + \cos x$

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25. Discuss the continuity of the following functions a) $f(x) = \sin x + \cos x$

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26. Discuss the continuity of the following functions a) $f(x) = \sin x + \cos x$

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27. Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

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28. Find all points of discontinuity of f, where

$$f(x) = \begin{cases} \dfrac{\sin x}{x}, & \text{if } x < 0 \\ x + 1, & \text{if } x \geq 0 \end{cases}$$

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29. Determine if f defined by

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is a continuous function?

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30. Examine the continuity of f, where f is defined by

$$f(x) = \{\sin x - \cos x, \quad \text{if} \quad x \neq 0 - 1, \quad \text{if} \quad x = 0$$

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31. Find the values of k so that the function f is continuous at the indicated point in

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if} \quad x \neq \frac{\pi}{2} 3, \quad \text{if} \quad x = \frac{\pi}{2} \end{cases} \text{ at } x = \frac{\pi}{2}$$

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32. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \{kx^2, \quad \text{if} \quad x \leq 23, \quad \text{if} \quad x > 2$ at $x = 2$.

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33. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \{kx + 1, \quad \text{if} \quad x \leq \pi \cos x, \quad \text{if} \quad x > \pi$ at $x = \pi$

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34. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \{kx + 1, \quad \text{if} \quad x \leq 53x - 5, \quad \text{if} \quad x > 5$ at $x = 5$

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35. Find the values of a and b such that the function defined by `f(x)={5, ifxlt=2a x+b , if2

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36. Show that the function defined by $f(x) = \cos\left(x^2\right)$ is a continuous function.

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37. Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

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38. Examine that $\sin|x|$ is a continuous function.

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39. Find all the points of discontinuity of f defined by $f(x) = |x| |x+1|$.

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Exercise 5 2

1. $\sin\left(x^2+5\right)$

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2. Differentiate the functions with respect to x

$\cos(\sin x)$

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3. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\sin(ax + b)$

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4. Differentiate the functions with respect to x in

Exerecises 1 to 8.

$\sec\left(\tan\left(\sqrt{x}\right)\right)$

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5. Differentiate the functions with respect to x in

Exerecises 1 to 8.

$$\frac{\sin(ax+b)}{\cos(cx+d)}$$

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6. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\cos x^3 . \sin^2\left(x^5\right).$

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7. $2\sqrt{\cot\left(x^2\right)}$

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8. Differentiate the functions with respect to x

$\cos\left(\sqrt{x}\right)$

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9. Prove that the function f given by $f(x) = |x \quad 1|, \quad x \in R$ is not differentiable at $x \quad = \quad 1$

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10. Prove that the greatest integer function defined by$f(x) = [x], 0 < x < 3$is not differentiable at $x = 1 and x = 2.$

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Exercise 5 3

1. Find $\frac{dy}{dx}$ in the following

$2x + 3y = \sin x$

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2. Find $\frac{dy}{dx}$ in the following

$2x + 3y = \sin x$

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3. $ax + by^2 = \cos y$

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4. Find $\frac{dy}{dx}$ in the following: $xy + y^2 = tanx + y$

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5. Find $\frac{dy}{dx}$ in the following: $x^2 + xy + y^2 = 100$

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6. $x^3 + x^2y + xy^2 + Y^3 = 81$

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7. Find $\frac{dy}{dx}$ in the following:$\sin^2 y + \cos xy = \pi$

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8. Find $\frac{dy}{dx}$ in the following:$\sin^2 x + \cos^2 y = 1$

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9. Graph of $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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10. $y = \tan^{-1}\frac{3x - x^3}{1 - 3(x^2)}, -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

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11. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) 0 < x < 1$

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12. Find $\frac{dy}{dx}$ in the following: `y=sin^(-1)((1-x^2)/(1+x^2)),0

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13. Find $\frac{dy}{dx}$ in the following:`y=cos^(-1)((2x)/(1+x^2)),-1

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14. Find $\frac{dy}{dx}$ in the following: `y=sin^(-1)(2xsqrt(1-x^2)),-1/(sqrt(2))

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15. $y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right), 0 < x < \frac{1}{\sqrt{2}}$

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Exercise 5 4

1. Differentiate the following w.r.t. x: $\frac{e^x}{\sin x}$

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2. Differentiate the following w.r.t. x: $\frac{e^x}{\sin x}$

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3. Differentiate the following w.r.t. x :

e^{x^3}.

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4. Differentiate the following w.r.t. x:

$\sin\left(\tan^{-1} e^{-x}\right)$

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5. Differentiate the following w.r.t. x:$\log(\cos e^x)$

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6. Differentiate the following w.r.t. x :

$e^x + e^{x^3} + \ldots\ldots\ldots\ldots + e^{x^3}.$

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7. Differentiate the following w.r.t. x :

$\sqrt{(3)^{\sqrt{x}}}, x > 0.$

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8. $\log(\log x),\ x > 1$

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9. Differentiate the following w.r.t. x: $\frac{\cos x}{\log x},\ x > 0$

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10. Differentiate the following w.r.t. x:

$\cos(\log x + e^x),\ x < 0$

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Exercise 5 5

1. Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

$\cos x . \cos 2x . \cos 3x .$

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2. Find the derivative of $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ w.r.t. x

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3. Differentiate the functions given w.r.t. x:

$(\log x)^{\cos x}$

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4. Differentiate the functions given w.r.t. x:

$x^x - 2^{\sin x}$

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5. Find the derivative of $(x+3)^2(x+4)^3(x+5)^4$ w.r.t. x

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6. Differentiate the functions given w.r.t. x: $\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

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7. Differentiate the following w.r.t. x: $(\log x)^x + x^{\log x}$

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8. Differentiate the following w.r.t. x:

$(\sin x)^x + \sin^{-1}\sqrt{x}$

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9. Differentiate the following w.r.t. x:

$x^{\sin x} + (\sin x)^{\cos x}$

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10. Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

$$x^{\cos x} + \frac{x^2 + 1}{x^2 - 1}.$$

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11. Differentiate $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$ w.r.t.x.

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12. Find $\frac{dy}{dx}$ of the functions given $x^y + y^x = 1$

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13. Find $\frac{dy}{dx}$ of the functions given $y^x = x^y$

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14. Find $\frac{dy}{dx}$ of the functions given $(\cos x)^y = (\cos y)^x$

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15. Find $\frac{dy}{dx}$ of the functions given in Exercises 12 to 15.

$xy = e^{(x-y)}$.

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16. Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

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17. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below:(i) by using product rule(ii) by expanding the product to obtain a

single polynomial.(iii) by logarithmic differentiation.Do they all give the same answer?

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18. If u, v and w are functions of x, then show that $\frac{d}{dx}(uvw) = \frac{du}{dx}vw + u\frac{dv}{dx}w + uv\frac{dw}{dx}$ in two ways - first by repeated application of product rule, second by logarithmic differentiation.

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Exercise 5 6

1. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}.x = 2at^2, y = at^4$

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2. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}.x = a\cos\theta, y = b\cos\theta$

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3. $x = \sin t, y = \cos 2t$

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4. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}. x = 4t, y = \frac{4}{t}$

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5. $x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

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6. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta).$

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7. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}.$

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8. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a\left(\cos t + \frac{\log \tan(t)}{2}\right), y = a \sin t.$

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9. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without

eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a\sec\theta, y = b\tan\theta.$

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10. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}.x = a\cos\theta, y = b\cos\theta$

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11. If

$x = \sqrt{a^{\sin\hat{}((-1)t)}}, y = \sqrt{a^{\cos\hat{}((-1)t)}},$

show that $\frac{dy}{dx} = -\frac{y}{x}$

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Exercise 5 7

1. $x^2 + 3x + 2$

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2. Find the second order derivatives of the functions given. x^{20}

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3. Find the second order derivatives of the functions given.$x\cos x$

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4. Find the second order derivatives of the functions given.$\log x$

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5. Find the second order derivatives of the functions given in Exercises 1 to 10.

$x^3 \log x$.

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6. Find the second order derivatives of the functions given.$e^x \sin 5x$.

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7. Find the second order derivatives of the functions given in Exercises 1 to 10.

$e^{6x} \cos 3x.$

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8. Find the second order derivatives of the functions given.$\tan^{-1} x.$

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9. Find the second order derivatives of the functions given.$\log(\log x)$

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10. Find the second order derivatives of the functions given. sin(log x)

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11. If $y = 5\cos x - 3s \in x$, prove that $\frac{d^2y}{dx^2} + y = 0$

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12. If $y = \cos^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

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13. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + x y_1 + y = 0$

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14. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

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15. If $y = 500\,e^{7x} + 600\,e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49\,y$.

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16. If $e^y(x+1) = 1$, $showthat \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

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17. If $y = \left(\tan^{-1} x\right)^2$, show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right) y_1 = 2$

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Exercise 5 8

1. Verify Rolles theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$.

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2. Examine if Rolles theorem is applicable to any of the following functions. Can you say something about the converse of Rolles theorem from these example?(i) $f(x) = [x]$ for $x \in [5, 9]$ (ii) $f(x) = [x]$ for $x \in [-2, 2]$ (iii) `f(x)=x^

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3. Examine if Rolles theorem is applicable to any of the following functions. Can you say something about the converse of Rolles theorem from these

example?(i) $f(x) = [x]$ for $x \in [5, 9]$(ii)

$f(x) = [x]$ for $x \in [-2, 2]$(iii) `f(x)=x^

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4. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these example?

$f(x) = x^2 - 1$ for $x \in [1, 2]$.

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5. If $f:[-5,5]\overrightarrow{R}$ is differentiable function and if $f'(x)$ does not vanish anywhere, then prove that $f(-5) \neq f(5)$.

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6. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$in the interval [a, b], where $a = 1 and db = 4.$

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7. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$in the interval [a, b], where $a = 1 and b = 3$. Find all $c \in (1, 3)$for which $f'(c) = 0$.

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8. Examine the applicability of Mean Value Theorem for all three functions given in the above exercise 2.

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Miscellaneous Exercise On Chapter 5

1. Differentiate w.r.t.x the function in Exercises 1 to 11.

$$\left(3x^2 - 9x + 5\right)^9$$

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2. Differentiate w.r.t. x the function$\sin^3 x + \cos^6 x$

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3. Differentiate w.r.t. x the function $(5x)^{3\cos 2x}$.

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4. $\sin^{-1}\left(x\sqrt{x}\right), 0 \leq x \leq 1$

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5. Differentiate w.r.t.x the function in Exercises 1 to 11.

$$\frac{\cos^{-1}\frac{1}{2}}{\sqrt{2x+7}}, \quad -7 < x < 2.$$

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6. $\frac{\cot^{-1}\left\{\sqrt{1+\sin x}+\sqrt{1-\sin x}\right\}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$

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7. Differentiate w.r.t. x the function $(\log x)^{\log x}, x > 1$

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8. Differentiate w.r.t.x the function in Exercises 1 to 11.

$\cos(a\cos x + b\sin x)$, for some constant a and b.

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9. $(\sin x - \cos x)^{(\sin x - \cos x)}$, $\frac{\pi}{4} < x < \frac{3\pi}{4}$

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10. Differentiate w.r.t.x the function in Exercises 1 to 11.

$x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and $x > 0$.

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11. $x^{x^2-3} + (x-3)^{x^2}$, for $x > 3$.

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12. Find $\frac{dy}{dx}$, if `y=12(1-cost), x=10(t-sint),-pi/2

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13. Find $\frac{dy}{dx}$, if $y = \sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}, \ -1 \leq x \leq 1.$

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14. $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx} =$

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15. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant

independent of a and b.

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16. If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

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17. If $x = a(\cos t + t\, s \in t)$ and $y = a\,(\sin t - t\cos t)$, then find $\frac{d^2y}{dx^2}$.

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18. If $f(x) = |x|^3$, show that $f(x)$ exists for all real x and find it.

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19. Using mathematical induction prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all positive integers n.

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20. Using the fact that $s \in (A + B) = s \in A \cos B + \cos A s \in B$and the differentiation, obtain the sum formula for cosines.

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21. Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer.

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22. If $y = |f(x)g(x)h(x)lmnabc|$, prove that $\frac{dy}{dx} = |f'(x)g'(x)h'(x)lmnabc|$

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23. If $y = e^{a\cos^{-1}x}$, $-1 \leq x \leq 1$ then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

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