MATHS

NCERT - NCERT MATHEMATICS(GUJRATI)

LINEAR PROGRAMMING

Example Type

1. Solve the following linear programming problem graphically:

Maximise $Z = 4x + y$ 1

subject to the constraints:

$$x + y \leq 50 \;\ldots\ldots\ldots\ldots.2$$

$$3x + y \leq 90\ldots\ldots\ldots\ldots.3$$

$$x \geq 0, y \geq 0\ldots\ldots\ldots\ldots.4$$

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2. Solve the following linear programming problem graphically:

Minimise $Z = 200x + 500y\ldots\ldots\ldots\ldots.1$

subject to the constraints:

$$x + 2y \geq 10\ldots\ldots\ldots\ldots.2$$

$$3x + 4y \leq 24\text{.............}3$$

$$x \geq 0, y \geq 0\text{.............}4$$

3. Solve the following problem graphically.

Maximize $Z = 3x + 9y\text{.............}1$

subject to the constraints $x + 3y \leq 60\text{.............}2$

$$x + y \geq 10\text{.............}3$$

$$x \leq y\text{.............}4$$

$$x \geq 0, y \geq 0\text{.............}5$$

4. Determine graphically the minimum value of the objective function

$$Z = -50x + 20y ....1$$

subject to the constraints:

$$2x - y \geq -5 ....2$$

$$3x + y \geq 3 ....3$$

$$2x - 3y \leq 12 ....4$$

$$x \geq 0, y \geq 0 ....5$$

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5. Minimise $Z = 3x + 2y$

subject to the constraints,

$x + y \geq 8$..........1

$3x + 5y \leq 15$..........2

$x \geq 0, y \geq 0$..........3

6. (Diet Problem): A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamins C. Food I

contains 2 units/ kg of vitamins A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamine C. It costs Rs. 50 per kg to purchase Food I Rs. 70 per kg to purchase Food II. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.

7. (Allocation problem) A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per

hectare are estimated as rs. 10,500 and Rs. 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres pre hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximise the total profit of the society?

8. (Manufacturing problem) A manufacturing company makes two models A and B of product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model A and Rs. 12000 on each piece of Model B. How many pieces of model A and model B should be manufactured

per week to realise a maximum profit ? What is the maximum profit per week ?

Miscellaneous Examples

1. (Diet problem): A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg. of vitamin A and 1 unit/kg. of vitamin C. Food 'II' contains 1 unit/kg. of

vitamin A and 2 units/kg of vitamin C. It costs Rs. 50 per kg. to purchase Food 'I' and Rs. 70 per kg. to purchase Food 'II'. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.

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2. (Manufacturing problem) A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for atleast 5 hours a day. She

produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table :

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs. 600 and Rs. 400 on items M and N respectively. How many of each should she produce so as to maximise her profit assuming that she can sell all the items that she produced? What will be the maximum profit ?

Exercise 12 1 Solve The Following Linear Programming Problems Graphically

1. Maximise $Z = 3x + 4y$

Subject to the constraints

$x + y \leq 4, x \geq 0, y \geq 0$

2. Solve the following Linear Programming Problems graphically :

Minimise Z = -3x + 4y

subject   to

$$x + 2y \leq 8, \; 3x + 2y \leq 12, \; x \geq 0, \; y \geq 0$$

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3. Solve the following Linear Programming Problems graphically :

Maximise Z = 5x + 3y

subject to

$$3x + 5y \leq 15, 5x + 2y \leq 10, x \geq 0, y \geq 0$$

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4. Solve the following Linear Programming

Problems graphically :

Minimise Z = 3x + 5y

such that $x + 3y \geq 3, x + y \geq 2, x, y \geq 0$.

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5. Solve the following Linear Programming Problems graphically :

Maximise Z = 3x + 2y

subject to $x + 2y \leq 10, \; 3x + y \leq 15, \; x, y \geq 0$

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6. Solve the following Linear Programming Problems graphically :

Minimise Z = x + 2y

subject to $2x + y \geq 3, \; x + 2y \geq 6, \; x, y \geq 0$

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1. Minimise and Maximise $Z = 5x + 10y$

subject to $x + 2y \leq 120$, $x + y \geq 60$, $x, y \geq 0$

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2. Find the maximum and minimum value of Z = x + 2y, subject to the constraints

$x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y \geq 0$

.

3. Maximise $Z = -x + 2y$, subject to the

constraint,

$x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$

4. Minimize $Z = x + y$ subject to

$$x - y \leq 1, \; -x + y \leq 1, x, y \geq 0$$

Exercise 12 2

1. One kind of cake requires 200g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no

shortage of theother ingredients used in making the cakes.

2. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftman's time.In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftman's time.

(i) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectivelh, find the maximum profit of the factory when it works at full capacity.

3. A manufacturer produces nuts and bolts. It takes 1 hour of worl in machine A and 3 hours on machine B to produce a package of nuts. It take 3 hours on machien A and 1 hour on machine B to produce a package of bolts. He

earns a profit of Rs. 17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise bis profit, if he operates his machines for at the most 12 hours a day?

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4. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to

manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs. 5 and that from a shade is Rs. 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit ?

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5. A company manufacturers two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minute each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hour 20 minutes available for cutting and 4 hours for assembling. The profit is Rs. 5 each of type A and Rs. 5 each for type b souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

6. A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost Rs. 25000 and Rs. 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and if his profit on the desktop model is Rs. 4500 and on portable model is Rs. 5000.

7. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs. Rs. 4 per unit food and F_2 costs Rs. 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units, of vitamin A and 3 units of minerals, Formulate this as a linear programming problem. Find the minimum cost for diet theat consists of mixture of these two food and also meets the minimal nutritional requirements.

8. The corner points of the feasible region determined by the following sytem of linear inequalities:

$2x + y \leq 10, x + 3y \leq 15, x, y \geq 0$ are (0,0), (5,0),(3,4) and (0,5). Let $Z = px + qy$ where $p, q \geq 0$. Condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5) is

   A. p=q

   B. $p = 2q$

C. $p = 3q$

D. $q = 3p$

Answer:

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Miscellaneous Exercise

1. A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs. 250 per bag, contains 3 units of nutritional element A, 2.5 units of

element B and 2 units of element C. Brand Q costing Rs. 200 pre bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element c. The minimum requirements of nutrients A,B and C are 18 units, and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag.

2. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below :

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs. 16 and one kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet ?

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3. A manufacturer makes two type of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below :

Types of Toys	Machine		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs. 7.50 and that on each toy of type B is Rs. 5, whow that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

4. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit ?

5. A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

If the grower wants to minimise the amount of nitrogen added to the garden, how many bags

of each brand be used ? What is the minimum amount of nitrogen added in the garden ?

kg per bag	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

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6. A toy company manufacturers two types of dolls, A and B. Market research and available resources have indicated that the combined production level should not exceed 1200 dolls

per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company make profit of Rs. 12 ad Rs. 16 doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?