MATHS

BOOKS - OSWAAL PUBLICATION

MATHS (KANNADA ENGLISH)

APPLICATIONS OF DERIVATIVES

Topic I Rate Of Change Of Quantities Short Answer Type Questions I

1. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.

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2. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then how fast is the slope of the curve changing when $x = 3$?

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3. The total cost associated with provision of free mid-day meals to x students of a school in primary classes is given by C(x) = $0.005x^3 - 0.02x^2 + 30x + 50$. If the marginal cost is given by rate of change $\frac{dc}{dx}$ of total cost, write, the marginal cost of food for 300 students.

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4. The total expenditure(in ₹) regquiredfor providing the cheap edition of a book for poor

and deservin students is given by R(x) = $3x^2$ + 36x, where x is the number of set of books. If the marginal expenditure is defined as $\frac{dR}{dx}$, write the marginal expenditure required for 1200 such sets.

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5. The amount of pollution content added in air in a city due to $x - diesel$ vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the arginal increase in pollution content when

3 diesel vehicles are added and write which value is indicated in the above question.

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6. The money to be spend for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (Marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, find the

marginal revenue, when $x = 5,$ and write which value does the question indicate.

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7. $If\ C = 0.003x^3 + 0.02x^2 + 6x + 250$ gives the amount of carbon pollution in air in an area on the entry of x number of vehicles, then find the marginal carbon pollution in the air, when 3 vehicles have entered in the area and write which value does the question indicate.

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8. The contentment obtained after eating X-units of a new dish at a trial function is given by the function f(x) $= x^3 + 6x^2 + 5x + 3$. If the marginal contentment is defined as the rate of change f(X) with respect to the number of units consumed at an instant, then find the marginal contentment when three units of dish are consumed.

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Topic I Rate Of Change Of Quantities Long Answer Type Questions Ii

1. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When $x = 10\text{cm}$ and $y = 6\text{cm}$, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

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2. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres?

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3. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate

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4. A ladder 24 ft long leans against a vertical wall. The lower end is moving away at rate of 3 ft/sec find the rate at which the top of the ladder is moving downwards. If its foot is 8ft from the wall.

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5. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is

increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle

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6. The total cost C(x) in Rupees, associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are

produced, where by marginal cost we mean the instantaneous rate of cha

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7. Sand is pouring from a pipe at the rate of $12\,cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

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8. Two equal sides of an isosceles triangle with fixed base 'a' are decreasing at the rate of 9 cm/second. How fast is the area of the riangle decreasing, when the two sides are equal to 'a' ?

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Topic 2 Tangents And Normals Short Answer Type Questions I

1. Find the point on the curve y = $x^3 - 11x + 5$

at which the tangent is y = x – 11.

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2. Find the slope of the tangent to the curve

$y = \frac{x-1}{x-2}, x \neq 2$at $x = 10$.

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Topic 2 Tangents And Normals Short Answer Type Questions Ii

1. Show that the curves $x = y^2$ and $xy = k$ cut at right angles; if $8k^2 = 1$

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2. Find the condition for the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to interest orthogonally.

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3. Find the equation of the tangent to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $\left(\sqrt{2}a, b\right)$.

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4. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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5. The equation of normal to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is

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6. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

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7. Show that the equation of tangent to the parabola

$y^2 = 4ax$ at (x_1, y_1) is $yy_1 = 2a(x + x_1)$

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8. Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$

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9. Find the equations of the tangent and the normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point $(1, 3)$

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10. Find the equations of the tangent and the normal to the curve $4x^2 + 9y^2 = 36$ at $(3\cos\theta,\ 2\sin\theta)$ at indicated points.

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11. At what point will be tangents to the curve $y = 2x^3 - 15x^2 + 36x - 21$ by parallel to x=axis? Also, find the equations of the tangents to the curve at these points.

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12. For the curve x =2 cos $\theta - \cos 2\theta$ and y = 2 sin $\theta - \sin 2\theta$, find the equation of tangent at $\theta = \frac{\pi}{4}$.

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13. Find the equation of the tangent to the curve $y = \left(x^3 - 1\right)(x - 2)$ at the points where the curve cuts the x-axis.

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14. Find the equations of the tangent and normal to the curve $x = a \sin 3t, y = \cos 2t$ at $t = \frac{\pi}{4}$

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15. Find the equations of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

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16. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

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Topic 3 Approximate Values Differentals Errors Short Answer Type Questions I

1. If the radius of a sphere is measured as 9cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

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2. Use differential to approximate $\sqrt{36.6}$

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3. Find the approximate value of $\sqrt{0.082}$ using differentials

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4. Using differentials, find the approximate value of $(3.968)^{3/2}$

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5. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area increases, when the side is 10 cm.

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6. Find the approximate value of $f(3.02)$, upto 2 places of decimal, where $f(x) = 3x^2 + 5x + 3$.

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7. Separate the interval $\left[0, \frac{\pi}{2}\right]$ into sub intervals in which function $f(x) = \sin^4(x) + \cos^4(x)$ is strictly increasing or decreasing.

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8. Find the intervals in which the function f given by $f(x) = \sin x + \cos x, \quad 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

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9. Show that $y = \log(1+x) - \frac{2x}{2+x}, x \succ 1,$ is an increasing function of x throughout its domain.

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10. Find the intervals in which the following is (a) increasing , (b) decreasing :

$f(x) = 2x^3 - 24x + 5$

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11. Find the intervals in which the function is (a) increasing, (b) decreasing:

f(x) = $(x+1)^3(x-3)^3$.

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12. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$is (a) strictly increasing (b) strictly decreasing

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13. Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)-\theta}$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

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14. Find the intervals in which the following function

f(x) = 20 - 9x + $6x^2 - x^3$

(a) strictly increasing, (b) strictly decreasing.

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Topic 5 Maxima And Minima Short Answer Type Questions I

1. Find the local maximum value of the function $g(x) = x^3 - 3x$

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Topic 5 Maxima And Minima Short Answer Type Questions Ii

1. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

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2. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

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3. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also, find the maximum volume.

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Topic 5 Maxima And Minima Long Answer Type Questions Ii

1. Show that of all the rectangles inscribed in a given circle, the square has the maximum area.

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2. A jet of enemy is along the curve $y = x^2 + 2$ and a soldier is placed at (3,2).Find the minimum distance between the jet and soldier.

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3. AB is a diameter of a circle and C is any point on the circle. Show that the area of ABC is maximum, when it is isosceles.

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4. Find the point P on the curve y^2 = 4ax which is nearest to the point (11a, 0).

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5. If the length of three sides of a trapezium other than the base is 10cm each, find the area of the trapezium, when it is maximum,

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6. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\frac{\cos^{-1} 1}{\sqrt{3}}$

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7. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface is $\cot^{-1}(\sqrt{2})$.

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8. From all the closed right circular closed cylindrical can of volume $128\pi cm^3$, Find the dimension of can which has minimum surface area.

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9. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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10. Prove that the height of the cylinder of maximum volume, that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume .

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11. If the sum of the lengths of the hypotenues and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi/3$.

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12. The sum of the perimeters of a circle and a square is k, where k is some constant. Prove that the sum of their areas is least when the

side of the square is double the radius of the circle.

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13. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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14. Prove that the radius of the right circular cylinder of greatest curved surface area which

can be inscribed in a given cone is half of that of the cone.

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15. Show that the right-circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

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16. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

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17. An open box, with a square base, is to be made out of a given quantity of metal sheet of area C^2. Show that the maximum volume of the box is $\frac{C^3}{6\sqrt{3}}$

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18. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

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19. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

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20. Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point $(1, 4)$

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21. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

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22. Show that all the rectangles with a given perimeter, the square has the largest area.

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23. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

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24. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α, is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

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25. Find the maximum area of an isosceles triangle inÅ›cribed in the ellipse

$\frac{x^2}{25} + \frac{y^2}{16} = = 1$, with its vertex atone end of the major axis.

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26. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when depth of the tank is half of its width.

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27. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the circle and the square is minimum?

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28. Two poles of height 16 m ans 22 m stand vertically on the ground 20 m apart. Find a

point on the ground, in between the poles, such that the sum of the square of the distances of this point from the tops of the poles is minimum.

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29. The lengths of the sides of an isosceles triangle are $9+x^2, 9+x^2$ and $18-2x^2$ units. Calculate the area of the triangle in terms of x and find the value of x which makes the area maximum.

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