



MATHS

BOOKS - OSWAAL PUBLICATION MATHS (KANNADA ENGLISH)

CONTINUITY AND DIFFERENTIABILITY

Continuity Very Short Answer Type Questions

1. The function $f(x) = \frac{1}{x-5}$ is not continuous at $x=5$. Justify the statement.

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2. Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$, at $x = -3$ and at $x = 5$.

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3. Examine that $f(x) = \sin|x|$ is a continuous function.

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Continuity Short Answer Type Questions I

1. Check the continuity of the function $f(x)$ given $f(x) = 2x + 3$ at $x = 1$.

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2. Find all points of discontinuity of f , where f is defined by

$$f(x) = \begin{cases} 2x + 3, & \text{if } x \leq 2 \\ 2x - 3, & \text{if } x > 2 \end{cases}$$

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3. if a function $f(x)$ is differentiable at $x = c$ then it is also continuous at $x = c$

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Continuity Long Answer Type Questions I

1. If the function $f(x)$ given by $f(x) = \begin{cases} 3ax + b & \text{if } x > 1 \\ 11 & \text{if } x = 1 \\ 5ax - 2b & \text{if } x < 1 \end{cases}$ is

continuous at $x = 1$ then find the values of a and b .

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2. Find the relationship between a and b so that the function f

defined by: $f(x) = \begin{cases} ax + 1 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$

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3. Find the value of k if $f(x) = \begin{cases} kx + 1 & \text{if } x \leq 5 \\ 3x - 1 & \text{if } x > 5 \end{cases}$ is continuous at $x=5$.

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4. The function is defined by $f(x) = \begin{cases} kx + 1 & \text{if } x \leq 5 \\ 3x - 5 & \text{if } x > 5 \end{cases}$ is continuous at $x = 5$. Find k.

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5. For what value of k , function $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$?

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6. Find the values of a and b such that the function defined by $f(x) = \begin{cases} 5, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \end{cases}$

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7. Find the value of the constant k so that the function f , defined below, is continuous at $x = 0$, where $f(x) = \begin{cases} \left(\frac{1 - \cos 4x}{8x^2} \right) k, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

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$$= \frac{1 - \cos 4x}{x^2}, \quad \dots \quad x < 0$$

8. Let : $f(x) = a, \quad \dots \quad x = 0$

$$= \frac{\sqrt{x}}{\sqrt{16 - \sqrt{x} - 4}}, \quad \dots \quad x > 0$$

If $f(x)$ is continuous at $x = 0$, then : $a =$

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9. If $f(x) = \begin{cases} 1 & \text{if } x \leq 3 \\ ax + b & \text{if } 3 < x < 5 \\ 7 & \text{if } x \geq 5 \end{cases}$

Find a and b , so that $f(x)$ is a continuous function.

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10. $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1} & \text{if } 0 \leq x \leq 1 \end{cases}$ at $x = 0$.

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11. $f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x - 2)^2}, & x \neq 2 \\ k, & x = 2 \end{cases}$. Find the value of k , so that the following function is continuous at $x=2$.

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12. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ at $x = \frac{\pi}{2}$

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13. If $f(x) = \begin{cases} x^2 + ax + b, & \text{if } 0 \leq x < 2 \\ 3x + 2, & \text{if } 2 \leq x \leq 4 \\ 2ax + 5b, & \text{if } 4 < x \leq 8 \end{cases}$ is continuous on $[0, 8]$ then $a - b = \dots\dots$

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14. Discuss the continuity of the function $f(x)$ at $x = \frac{1}{2}$, when $f(x)$ is defined as follows :

$$f(x) = \begin{cases} \frac{1}{2} + x & 0 \leq x < \frac{1}{2} \\ 1 & x = \frac{1}{2} \\ \frac{3}{2} - x & \frac{1}{2} < x \leq 1 \end{cases} .$$

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15. Find the value of ' a ' if the function $f(x)$ defined by $f(x) = \begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ 2x + 1, & x > 2 \end{cases}$ is continuous at $x = 2$

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16. Find the value of ' a ' for which the function f defined by $f(x) = \begin{cases} a \frac{\sin \pi}{2} (x + 1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$.

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17. Find all the points for discontinuity of the function $f(x) = [x^2]$ on $[1,2]$, where $[x]$ denotes the greatest integer function.

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$$18. f(x) = \begin{cases} |x| + 3 & \text{if } x \leq -3 \\ -2x & \text{if } -3 < x < 3 \\ 6x + 2 & \text{if } x \geq 3 \end{cases}$$

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19. For what value of k is the function defined by

$$f(x) = \begin{cases} k(x^2 + 2) & \text{if } x \leq 0 \\ 3x + 1 & \text{if } x > 0 \end{cases}$$

continuous at $x=0$? Also write whether the function is continuous at $x=1$.

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20. Find the values of a and b such that the function defined as follows is continuous :

$$f(x) = \begin{cases} x + 2 & x \leq 2 \\ ax + b & 2 < x < 5 . \\ 3x - 2 & x \geq 5 \end{cases}$$

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21. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ at $x = \frac{\pi}{2}$

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22. If the functions $f(x)$, defined below is continuous at $x = 0$, find the value of k :

$$f(x) = \begin{cases} \frac{1 - \cos 2x}{2x^2} & , \quad x < 0 \\ 0 & , \quad x = 0 \\ \frac{x}{|x|} & , \quad x > 0 \end{cases}$$

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23. Show that the function $f(x)=|x-2|$ is continuous but not differentiable at $x=2$.

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24. Show that the function f defined as follows, is continuous at $x=2$, but not differentiable at $x=2$.

$$f(x) = \begin{cases} 3x - 2 & 0 < x \leq 1 \\ 2x^2 - x & 1 < x \leq 2 \\ 5x - 4 & x > 2 \end{cases}$$

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25. Show that the function $f(x) = |x - 3|$, $x \in \mathbb{R}$, is continuous but not differentiable at $x = 3$.

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Differentiability Very Short Answer Type Questions

1. Write the derivative of $\sin x$ with respect to $\cos x$.

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2. Differentiate $e^{\log(e^x)}$, $x > 0$ with respect to x .

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3. If $y = e^{3 \log x}$, then show that $\frac{dy}{dx} = 3x^2$.

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4. Differentiate $\log(\cos e^x)$ w. r. to x .

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5. Differentiate $\sin \sqrt{x}$ with respect to x .

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6. $y = \sin(x^2 + 5)$, Find $\frac{dy}{dx}$

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7. If $y = \log(\sin x)$, find $\frac{dy}{dx}$.

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8. Find the derivation of $\cos(x^2)$ with respect to x .

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9. If $y = \tan(2x + 3)$ find $\frac{dy}{dx}$

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10. Find $\frac{dy}{dx}$, if $y = \cos(1 - x)$.

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11. The derivative of $(\log)_{10}x$ w.r.t. x^2 is equal to $\frac{1}{2x^2}\log_e 10$ (b) $\frac{2}{x^2}(\log)_{10}e$ $\frac{1}{2x^2}(\log)_{10}e$ (d) none of these

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12. If $y = \sin x^\circ$, find $\frac{dy}{dx}$.

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13. Find $\frac{dy}{dx}$, if $x - y = 2\pi$.

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14. Find $\frac{dy}{dx}$, if $y = a^x$, where a is a positive constant.

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15. Find $\frac{dy}{dx}$, if $y = e^{x^3}$.

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Differentiability Short Answer Type Questions I

1. If $\sqrt{x} + \sqrt{y} = \sqrt{5}$, Prove that $\frac{dy}{dx} = \frac{-3}{2}$ when $x = 4$ and $y = 9$.

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2. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below:(i) by using product rule(ii) by expanding the product to obtain a single polynomial.(iii) by logarithmic differentiation.Do they all give the same answer?

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3. Find $\frac{dy}{dx}$ of $2x + 3y = \sin y$

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4. $x = at^2, y = 2at,$

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5. $y = \tan^{-1} \left[\frac{3x - x^3}{1 - 3x^2} \right], -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

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6. Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 y = k$, where k is constant.

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7. Find the derivative of $\sqrt{x} + \sqrt{y} = 9$ at $(4, 9)$.

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8. If $y = \log_7(\log_7 x)$, find $\frac{dy}{dx}$

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9. Differentiate $x^{\sin x}$, $x > 0$ with respect to x .

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10. if $y = \tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$ prove that $\frac{dy}{dx} = \frac{1}{2}$

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11. $x = 4t$, $y = \frac{4}{t}$

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12. Differentiate $\left(x + \frac{1}{x}\right)^x$ w.r.t.x

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13. $x^2 + xy + Y^2 = 100$

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14. If $\sqrt{x} + \sqrt{y} = \sqrt{10}$, show that $\frac{dy}{dx} + \sqrt{\frac{y}{x}} = 0$

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15. differentiate $\sec(\tan(\sqrt{x}))$

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16. Find $\frac{dy}{dx}$ if $y = (\log x)^{\cos x}$

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17. If $y = x^{x^x}$, then find $\frac{dy}{dx}$.

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18. Find the derivative of $x^x - 2^{\sin x}$ w.r.t. x

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19. Find derivative of $\frac{e^x}{\sin x}$ w.r.t. to x

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20. If $y = \tan(x + y)$, find $\frac{dy}{dx}$.

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21. Find the derivative of $(3x^2 - 7x + 3)^{5/2}$ with respect to x .

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22. If $y + \sin y = \cos x$, find $\frac{dy}{dx}$.

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23. If $y = (\sin^{-1} x)^x$ find $\frac{dy}{dx}$

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24. If $y = \sin(\log_e x)$ prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{x}$

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Differentiability Short Answer Type Questions II

1. If $y = \sin^{-1} \left[\frac{2^{x+1}}{1+4^x} \right]$ find $\frac{dy}{dx}$.

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2. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$ with respect to x .

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3. Find the derivative of $x^{\sin x} + (\sin x)^{\cos x}$ w.r.t. x

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4. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

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5. If $x = a^{\sqrt{\sin^{-1} t}}$ and $y = a^{\sqrt{\cos^{-1} t}}$, then show that $\frac{dy}{dx} = -\frac{y}{x}$

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6. If $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$ then find $\frac{dy}{dx}$

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7. If $x = at^2$ and $y=2$ at then find $\frac{dy}{dx}$

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8. If $y = \tan^{-1}(\sqrt{1+x^2} - x)$ then, prove that

$$\frac{dy}{dx} = -\frac{1}{2(x^2 + 1)}$$

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9. If $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), 1/\sqrt{3}$

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10. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) 0 < x < 1$ then find $\frac{dy}{dx}$

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11. if $y = \tan x + \sec x$, prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$.

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12. Find $\frac{dy}{dx}$, when $x = a(\cos \theta + \theta \sin \theta)$ and $y = a(\sin \theta - \theta \cos \theta)$

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Differentiability Long Answer Type Questions I

1. If $x = a\left(\cos \theta + \log \tan\left(\frac{\theta}{2}\right)\right)$ and $y = a \sin \theta$ then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$ and $\theta = \frac{\pi}{4}$

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2. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

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3. If $y = 3 e^{2x} + 2 e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

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4. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that $x^2 y_2 + x y_1 + y = 0$

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5. If $y = (\tan^{-1} x^2)$, show that

$$(x^2 + 1)^2 \frac{d^2 y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2.$$

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6. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

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7. If $(x - y)e^{\frac{x}{x - y}} = a$, prove that $\frac{dy}{dx} + x = 2y$.

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8. If $x = a\left(\cos t + \frac{\log \tan t}{2}\right)$, $y = a \sin t$, evaluate $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$.

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9. If $y = \tan^{-1}\left(\frac{a}{x}\right) + \log \sqrt{\frac{x - a}{x + a}}$, Prove that $\frac{dy}{dx} = \frac{2a^3}{x^4 - a^4}$

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10. If $(\tan^{-1} x)^y + y^{\cot x} = 1$, then find $\frac{dy}{dx}$.

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11. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}(2x\sqrt{1-x^2})$, when $x \neq 0$.

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12. Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}(2x\sqrt{1-x^2})$, if $1/(\sqrt{2})$

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13. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, if $x > 0$

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14. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

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15. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^\theta(\sin\theta - \cos\theta)$ and $y = ae^\theta(\sin\theta + \cos\theta)$.

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16. If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, show that at $\frac{\pi}{4}$, $\frac{dy}{dx} = \frac{b}{a}$.

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17. If $x = \cos t(3 - 2 \cos^2 t)$ and $y = \sin t(3 - 2 \cos^2 t)$ find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

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18. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$.

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19. If $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, then prove that

$$y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$$

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20. If $x^m y^n = (x + y)^{m+n}$ prove that $\frac{dy}{dx} = \frac{y}{x}$

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21. Differentiate $\log(x^{\sin x} + \cot^2 x)$ w.r.tx.

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22. If $x = 2 \cos \theta - \cos 2\theta$ and $y = 2 \sin \theta - \sin 2\theta$ then prove that

$$\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$

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23. If $y = (\sin x)^x + \sin^{-1} \sqrt{x}$ then find $\frac{dy}{dx}$.

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24. If $y = x \log \left\{ \frac{x}{(a + bx)} \right\}$, then show that

$$x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y \right)^2.$$

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25. If $y = \log \left\{ x + \sqrt{x^2 + a^2} \right\}$, prove that:

$$(x^2 + a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0.$$

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26. If $y = \left\{ x + \sqrt{x^2 + 1} \right\}^m$, then show that

$$(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - m^2 y = 0.$$

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27. If $x = \tan\left(\frac{1}{a} \log y\right)$, show that

$$(1 + x^2) \frac{d^2 y}{dx^2} + (2x - a) \frac{dy}{dx} = 0.$$

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28. If $x = \cos \theta, y = \sin^3 \theta$, prove that

$$y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 3 \sin^2 \theta (5 \cos^2 \theta - 1).$$

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29. Differentiate the following function with respect to x :

$$(\log x)^x + x^{\log x}$$

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30. If $x = a \sin t$ and $y = a \left(\cos t + \frac{\log \tan t}{2} \right)$, find $\frac{d^2 y}{dx^2}$

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31. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$

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32. If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

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33. If $x \sin(a + y) + \sin a \cos(a + y) = 0$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

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34. Differentiate the following with respect to x :

$$\sin^{-1} \left(\frac{2^{x+1} \cdot 3^x}{1 + (36)^x} \right)$$

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35. If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

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36. If $y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$, find $\frac{dy}{dx}$.

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37. If $y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$, find $\frac{dy}{dx}$.

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38. If $x = \cos t + \frac{\log \tan t}{2}$, $y = \sin t$, then find the value of $\frac{d^2y}{dt^2}$
and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

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39. If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$

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40. If $y = (\tan^{-1} x^2)$, show that

$$(x^2 + 1)^2 \frac{d^2 y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2.$$

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41. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, then show that

$$x^2 \frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = 0$$

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42. If $y = \sin^{-1} x$, show that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = 0$.

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43. Differentiate $\tan^{-1} \left[\frac{\sqrt{1+x^2}-1}{x} \right]$ with respect to x

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44. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2 y}{dx^2}$.

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45. Differentiate $\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\}$ with respect to $\cos^{-1} x^2$

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46. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2) \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} - y = 0$.

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47. If $x = a(\theta - \sin \theta)$ and $y = a(1 + \cos \theta)$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

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48. If $y = \log \sqrt{\frac{1 + \tan x}{1 - \tan x}}$, Prove that $\frac{dy}{dx} = \sec 2x$

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49. If $y = a \sin x + b \cos x$, show that $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2$

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50. Consider the curve $x = a(\cos \theta + \theta \sin \theta)$ and $y = a(\sin \theta - \theta \cos \theta)$. What is $\frac{dy}{dx}$ equal to? What is $\frac{d^2y}{dx^2}$ equal to?

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51. If $y = \cos^{-1} \left\{ \frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}} \right\}$, find $\frac{dy}{dx}$

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52. If

$y\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$, show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$

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53. If $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$, find $\frac{d^2y}{dx^2}$

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54. If $y = \log \sqrt{\frac{1+x \cos x}{1-x \cos x}}$, find $\frac{dy}{dx}$ in simplified form.

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55. If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, find the values of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$.

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56. Differentiate $x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$ with respect to x :

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57. If $x = \tan\left(\frac{1}{a} \log y\right)$, show that
 $(1 + x^2) \frac{d^2y}{dx^2} + (2x - a) \frac{dy}{dx} = 0$.

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58. If $\tan\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = a$ then prove that $\frac{dy}{dx} = \frac{y}{x}$

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59. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

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60. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

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61. If $y = (\cot^{-1} x)^2$, then show that

$$(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2.$$

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62. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

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63. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$

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64. If $y = e^a s \in^{(-1_x)}$, $-1 \leq x \leq 1$, then show that

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$$

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65. If $y = \cos^{-1} \left(\frac{3x + 4\sqrt{1-x^2}}{5} \right)$, find $\frac{dy}{dx}$

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66. If $y = \operatorname{cosec}^{-1}x$, $x > 1$, then show that

$$x(x^2 - 1) \frac{d^2y}{dx^2} + (2x^2 - 1) \frac{dy}{dx} = 0.$$

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67. If $y = (\sin x - \cos x)^{\sin x - \cos x}$, $\pi/4$

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Mvt And Rolle S Theorem Short Answer Type Questions li

1. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval $[a, b]$, where $a = 1$ and $b = 3$. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

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2. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$, where $a = 1$ and $b = 4$.

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3. Verify Rolles theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$.

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4. Verify Rolle's theorem for the function $y = x^2 + 2x \in [-2, 2]$.

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5. Verify Rolle's theorem for each of the following functions :

$$f(x) = e^{-x}(\sin x - \cos x) \quad \text{in} \quad \left[\frac{\pi}{4}, \frac{5\pi}{4} \right]$$



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