MATHS

BOOKS - MAXIMUM PUBLICATION

PRINCIPLE OF MATHEMATICAL INDUCTION

Example

1. For all $n \geq 1$, prove that

$$1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2 > \frac{n^3}{3}$$

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2. For all $n \geq 1$,prove that

$$1 + 2 + 3 + \ldots\ldots + n < \frac{1}{8}(2n+1)^2$$

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3. For all $n \geq 1$, prove that $p(n): 2^{3n} - 1$ is divisible by 7.

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4. For all $n \geq 1$, prove that

$p(n): n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.

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5. For all $n \geq 1$, prove that

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots\dots\dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

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6. For all $n \geq 1$, prove that

$$1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots\dots + \frac{1}{(1+2+3+\dots.+n)} = \frac{2n}{(n+1)}$$

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7. For all $n \geq 1,$ prove that $1.2.3 + 2.3.4 + \ldots\ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

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8. For all $n \geq 1$, prove that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

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9. For all $n \geq 1$, prove that $p(n): n(n+1)(n+5)$ is divisible by 3.

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10. For all $n \geq 1$, prove that $p(n): 2.7^n + 3.5^n - 5$ is divisible by 24.

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11. Consider the statement "$p(n) : 9^n - 1$ is a multiple of 8". Where n is a natural number.

Is $p(1)$ true?

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12. Consider the statement "$p(n) : 9^n - 1$ is a multiple of 8". Where n is a natural number.

Assuming $p(k)$ is true, show that $p(k+1)$ is true.

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13. Consider the statement "$P(n) : x^n - y^n$ is divisible by $x - y$".

Show that $P(1)$ is true.

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14. Consider the statement "$P(n) : x^n - y^n$ is divisible by $x - y$".

Using the principal of Mathematical induction verify that $P(n)$ is true for all natural numbers.

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15. Consider the statement "$7^n - 3^n$ is divisible by 4"

Verify the result for $n = 2$.

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16. Consider the statement "$7^n - 3^n$ is divisible by 4"

Prove the statement using mathematical induction.

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17. Which among the following is the least number that will divide $7^{2n} - 4^{2n}$ for every positive integer n? [4,7,11,13]

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18. Prove by mathematical induction.

$(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$,

where $i = \sqrt{-1}$

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19. Given $P(n): 3^{2n} - 1$ is divisible by 8 Check whether P(1) is true.

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20. Given $P(n): 3^{2n} - 1$ is divisible by 8.

If $P(k)$ is true then prove $P(k+1)$ is true.

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21. Given $P(n) : 3^{2n}$ 1 is divisible by 8.

Is the statement $P(n)$ true for all natural numbers? Justify your answer.

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22. Prove that by $1.2 + 2.3 + 3.4 + \ldots\ldots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}$ by using the principle of mathematical induction for all $n \in N$.

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23. By the Principal of mathematical induction, prove that $1 + 5 + 5^2 + \ldots\ldots + 5^{n-1} = \dfrac{5^n - 1}{4}$

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24. Consider the statement $P(n): n(n+1)(2n+1)$ is divisible by 6.

Verify the statement for n=2.

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25. Consider the statement $P(n): n(n+1)(2n+1)$ is divisible by 6.

By assume that $P(k)$ is true for a natural number k, Verify that $P(k+1)$ is true.

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26. Consider the statement

$$P(n): 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Check whether $P(1)$ is true.

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27. Consider the statement

$$P(n): 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 - \frac{n(n+1)(2n+1)}{6}$$

By assume that $P(k)$ is true, prove that $P(k+1)$ is true.

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28. Consider the statement

$$P(n): 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Is $P(n)$ true for all natural number n? Justify your answer.

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29. Consider the statement

$$P(n) = 1 + 3 + 3^2 + \ldots\ldots + 3^{n-1} = \frac{3^{n-1}}{2}$$

Check `P(1) is true.

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30. Consider the statement

$$P(n) = 1 + 3 + 3^2 + \ldots\ldots + 3^{n-1} = \frac{3^n - 1}{2}$$

If $P(k)$ is true, prove that $P(k+1)$ is true.

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31. Consider the statement

$$p(n) : 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Verify the result for n=2.

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32. Consider the statement

$$p(n) : 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^{-2}$$

Prove the statement using mathematical induction.

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33. Consider the statement

$$P(n): 1.2 + 2.3 + 3.4 + \ldots\ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

Prove that $P(1)$ is true.

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34. Consider the statement

$$P(n): 1.2 + 2.3 + 3.4 + \ldots\ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

Assume that $P(k)$ is true for a natural number k, verify that $P(k+1)$ is true.

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35. Consider the statement

$P(n) = 3^{2n+2} - 8n - 9$ is divisible by 8

Verify the statement for n=1.

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36. Consider the statement

$P(n) = 3^{2n+2} - 8n - 9$ is divisible by 8

Prove the statement using the principle of mathematical induction for all natural numbers.

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37. Consider the statement:

$$P(n) = 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Prove that $P(1)$ is true.

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38. Consider the statement:

$$P(n) = 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

If $P(k)$ is true, prove that $P(k+1)$ is true.

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39. Consider the statement:

$$P(n) = 1^3 + 2^3 + 3^3 + \ldots\ldots.. + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Is $P(n)$ true for all natural number n? Why?

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40. Using the principal of Mathematical induction, prove that

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

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41. A statement $p(n)$ for a natural number n is given by

$$p(n): \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

Verify that $p(1)$ is true.

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42. A statement $p(n)$ for a natural number n is given by

$$p(n): \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

By assuming that $p(k)$ is true for a natural number k, show that $p(k+1)$ is true.

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43. Consider the statement

$P(n): 7^n - 3^n$ is divisible by 4.

Show that $P(1)$ is true.

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44. Consider the statement

$P(n): 7^n - 3^n$ is divisible by 4.

Verify, by the method of mathematical induction, that $P(n)$ is true for all natural numbers.

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45. Consider the following statement:

$$P(n): a + ar + ar^2 + \dots\dots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$$

Prove that $P(1)$ is true.

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46. Consider the following statement:

$$P(n): a + ar + ar^2 + \dots\dots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$$

Hence by using the principle of mathematical induction, prove that $P(n)$ is true for all natural numbers n.

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47. Consider the statement "$10^{2n-1} + 1$ is divisible by 11". Verify that $P(1)$ is true and prove the statement by using mathematical induction.

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