MATHS

BOOKS - SUNSTAR MATHS (KANNADA ENGLISH)

P.U BOARD LATEST MODEL QUESTION PAPER - 3

Part A

1. Let   * be a binary operation on N given by $a*b$=LCM of a and b. Find 20*16.

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2. What is the reflection of the graph of the function y=sin x along the y=x.

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3. What is the number of possible square matrices of order 3 with each entry 0 or 1?

4. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ are singular.

5. Write the derivative of $\sin^{-1}(\cos x)$ with respect to x.

6. Evaluate $\int_2^3 \frac{1}{x} dx$

7. Find λ if the vector $\hat{i} - \lambda\hat{j} + 2\hat{k}$ and $2\hat{i} + 4\hat{j}$ are perpendicular to each other .

8. Write the vector form of the equation of the line $\dfrac{x-3}{3} = \dfrac{y+4}{7} = \dfrac{z-6}{2}$

9. Define optimal solution in linear programming problem.

10. If $P(A) = 0.3$, $P(\text{not}\, B) = 0.4$, $P\left(A \big/ B'\right) = 0.5$ and A and B are independent events, find P(A and not B)

Part B

1. Show the signum function $f: R \to R$ given by

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$ is neither one-one nor onto

2. Find the value of $\tan\left[\sin^{-1}\dfrac{3}{5} + \cot^{-1}\dfrac{3}{2}\right]$

3. If the matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k

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4. If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, then show that $\dfrac{dy}{dx} = -\dfrac{y}{x}$

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5. Write the inverse trignometric function $\tan^{-1}\left(\dfrac{x}{\sqrt{a^2 - x^2}}\right) |x| < a$, in simplest form.

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6. If $\sin^2 x + \cos^2 y = 1$, show that $\dfrac{dy}{dx} = \dfrac{\sin 2x}{\sin 2y}$

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7. If the radius of a sphere is measured as 9cm with an error of 0.03m, find the approximate error in calculating its surface area.

8. Evaluate $\int \sin^{-1}(\cos x)dx$

9. Find $\int \dfrac{1}{\sqrt{7-6x-x^2}}dx$

10. Verify that $y = \sqrt{a^2 - x^2}$ is a solution of the differential equation $x + y\dfrac{dy}{dx} = 0$

11. Find the magnitude of two vector $\vec{a}$ and $\vec{b}$ having same magnitude such that the angle between them is 60 and their scalar product is $\dfrac{1}{2}$

12. Show that $\left(\vec{a} - \vec{b} \right) \times \left(\vec{a} + \vec{b} \right) = 2\left(\vec{a} \times \vec{b} \right)$

13. If the lines

$$\dfrac{x-1}{-3} = \dfrac{y-2}{2k} = \dfrac{z-3}{2} \quad \text{and} \quad \dfrac{x-1}{3k} = \dfrac{y-1}{1} = \dfrac{z-6}{-3}$$

are perpendicular, find the value of k.

14. A die is tossed thrice. Find the probability of getting an odd number at least once.

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Part C

1. Show that the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$ is given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation .

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2. Prove that $\tan^{-1}\left(\dfrac{\cos x}{1 + \sin x}\right) = \dfrac{\pi}{4} - \dfrac{x}{2} x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

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3. Using elementary operations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

4. If $y = (\sin x)^{\cos x} + x^{\sin x}$ find dy/dx.

5.   If

$$x = a\left(\cos\theta + \log\tan\left(\frac{\theta}{2}\right)\right) \quad \text{and} \quad y = a\sin\theta, \quad \text{show that} \quad \frac{dy}{dx} = \tan\theta$$

6. At what points, the function $f(x) = \sin x - \cos x, 0 < x < 2\pi,$ attains local maxima and minima.

7. Evaluate $\int e^x \sin x \, dx$

8. Find $\int \dfrac{\sin x}{\sin(x-a)} dx$

9. Find the area of the region enclosed by the circle $x^2 + y^2 = a^2$ by integration method.

10. Solve $\dfrac{2dy}{dx} = \dfrac{y}{x} + \dfrac{y^2}{x^2}$

11. Show that the points $A\left(2\hat{i} - \hat{j} + \hat{k}\right), B\left(\hat{i} - 3\hat{j} - 5\hat{k}\right)$ and $C\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right)$ are the vertices of right angled triangle.

12. Three vectors satisfy the condition $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{O}$.

13. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$.

14. A random variable x has the following probability distribution.

X	0	1	2	3	4 ·	5	6	7
P(x)	0	k	2k	2k	3k	K²	2k²	7k² + k

Determine (i) k (ii) $p(x < 3)$

15. If $f: R \rightarrow R$ defined by $f(x) = (4x + 3)$, show that f is invertible and find f^{-1}.

16. If $A = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 \\ -3 & 4 \end{bmatrix}$ verify that AB-BA is a skew symmetric matrix and AB+BA is symmetric matrix

17. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations.

$$2x - 3y + 5z = 11, \ 3x + 2y - 4z = -5 \text{ and } x + y - 2z = -3$$

18. If $y = e^{a\cos^{-1}x}, \ -2 \le x \le 1,$ show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$$

19. Sand is pouring from a pipe at the rate of $12cm^3/\sec.$ The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

20. Find the integral of $\dfrac{1}{\sqrt{x^2 - a^2}}$ with respect to x and evaluate

$$\int \frac{1}{\sqrt{x^2 + 6x - 7}}$$

21. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

22. Derive the equation of a plane in normal form both in the vector and Cartesian form .

23. Solve $\dfrac{dy}{dx} + \dfrac{2xy}{1+x^2} = \dfrac{4x^2}{1+x^2}$

24. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

(i) None (ii) Not more than one

(iii) more than one (iv) at least one will fuse after 150 days of use?

Part E

1. Prove that $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

2. Show that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx)$

3. Find the realtionship between a and b so that the function f defined

by $f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$ is continuous at x=3