MATHS

BOOKS - SUNSTAR MATHS (KANNADA ENGLISH)

SUPPLEMENTARY EXAM QUESTION PAPER JULY - 2016

Part A

1. An operation $*$ on Z^* (the set of all non-negative integers) is defined as

$a * b = a - b, \forall a, b \varepsilon Z^+$. Is $*$ binary operation on Z^+?

2. Write the domain of $f(x) = \cos^{-1} x$.

3. define a scalar matrix.

4. Find values of x, if

$(i) \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$(ii) \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

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5. If $y = \tan(2x + 3)$ find $\dfrac{dy}{dx}$

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6. Find $\displaystyle\int (2x^2 + e^x)\,dx$.

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7. Find the unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$

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8. Write the direction cosines of z-axis.

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9. Define optimal solution in Linear Programming problem.

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10. If $P(A) = \dfrac{7}{13}$, $P(B) = \dfrac{9}{13}$ and $P(A \cap B) = \dfrac{4}{13}$ find $P(A/B)$

Part B

1. If $f: R \to R$ and $g: R \to R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Find gof and fog.

2. prove that

$$3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right), x \in \left[\frac{-1}{2}, \frac{1}{2}\right]$$

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3. Find the value of $\sin^{-1}\left(\sin\dfrac{2\pi}{3}\right)$

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4. Find the area of the triangle with vertices $(2, 8), (-4, 2)$ and $(5, 1)$ using determinats

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5. If $y = \cos^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right) 0 < x < 1$ find $\dfrac{dy}{dx}$

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6. Differentiate $x^{\sin x}, x > 0$ with respect to x.

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7. Find the interval in which the function f given $f(x) = 2x^2 - 3x$ is stricitly increasing

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8. Find $\int x^2 . \log x \, dx$

9. Evaluate: $\int_0^1 \dfrac{dx}{\sqrt{1 - x^2}}$

10. Find the order and degree of the differential equation, $\dfrac{d^3 y}{dx^3} + 2\dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} = 0$

11. If two vectors $\vec{a}$ and $\vec{b}$ such that $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3$ and $\vec{a} \cdot \vec{b} = 4,$ find $\left|\vec{a} - \vec{b}\right|$

12. Find the area of the parallelogram whose adjacent sides are given by vectors.

13. Show that the lines $\dfrac{x-5}{7} = \dfrac{y+2}{-5} = \dfrac{z}{1}$ and $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ are perpendicular to each other.

14. Find the probability distribution of number of heads in two tosses of a coin .

Part C

1. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is even} is an equivalence relation.

2. Solve : $\tan^{-1} 2x + \tan^{-1} 3x = \dfrac{\pi}{4}$

3. Using elementary transformations, find the inverse of the matrices

$$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

4. If $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$ then prove that $\dfrac{dy}{dx} = -\cot\left(\dfrac{\theta}{2}\right)$

5. Verify mean value theorem for the function $f(x) = x^2$ in the interval [2,4].

6. Using differentials find the approximate value of

$(25)^{1/3}$

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7. Evaluate : $\int e^{2x} \left[\dfrac{1 - \sin 2x}{1 - \cos 2x} \right] dx.$

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8. evaluate : $\int \dfrac{x}{(x + 1)(x + 2)} dx.$

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9. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x- axis in the first qudrant .

10. Form the differential equation representing family of curve $\dfrac{x}{a} + \dfrac{y}{b}$ =1 where a and b are arbitrary constants .

11. Prove that

$$\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right].$$

12. Show that the position vector of the point P, which divides the line joining the points A and B having position vector a and b internally in the ratio:

m:n is $\dfrac{m\vec{b} + n\vec{a}}{m + n}$

13. Find the vector equation of the line passing through the point $(-1, 0, 2)$ and $(3, 4, 6)$

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14. A die is tossed thrice. Find the probability of getting an odd number at least once.

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Part D

1. Let R+ be the set of all non negative real numbers. Show that the function $f: R_+ \to [4, \infty]$ given by $f(x) = x^2 + 4$ is invertible and write inverse of 'f'.

2. If

$$A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

calculate AC, BC and (A+B)C. Also verify that (A+B)C=AC+BC

3. Solve the following system of equations by matrix method.

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

4. If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2 y_2 + x y_1 + y = 0$

5. A ladder 5cm long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 2cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?
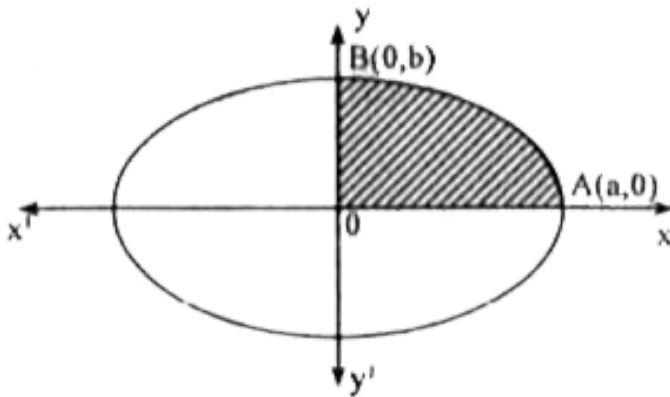
6. evaluate $\int \sqrt{5 + x^2 + 2x}\,dx$.

7. Find the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

8. Find the general solution of the differential equation $x\dfrac{dy}{dx} + 2y = x^2 \, (x \neq 0)$

9. Derive the equation of a plane in normal form both in the vector and Cartesian form .

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10. If a fair coin is tossed 10 times, find the probability of

Exactly six heads

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Part E

1. $\displaystyle\int_{0}^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ equals:

2. Prove that

$$\begin{vmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{vmatrix} = (a + b + c)^3$$