MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

BINOMIAL THEOREM

Others

1. Prove that

$$\left({}^{2n}C_0\right)^3 - \left({}^{2n}C_1\right)^3 - \left({}^{2n}C_2\right)^3 - \ldots . + (-1)^n \left({}^{2n}C_{2n}\right)^2 = (-1)^n . {}^{2n}C_n$$

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2. Find the largest term in the expansion of $(3+2x)^{50}, where x = 1/5.$

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3. Find the following sum: $\frac{1}{n!}+\frac{1}{2!(n-2)!}+\frac{1}{4!(n-4)!}+\ldots$

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4. Find the sum of the last 30 coefficients in the expansion of $(1+x)^{59}$, when expanded in ascending powers of x.

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5. If $x=1/3$, find the greatest tem in the expansion of $(1+4x)^{8}$.

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6. If the sum of coefficients in the expansion of $(x-2y+3z)^{n}$ is 128, then find the greatest coefficient in the expansion of $(1+x)^{n}$.

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7. Find the sum of the coefficients in the expansion of $\left(1+2x+3x^2+nx^n\right)^2$.

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8. The number of terms in the expansion of $(1+x)^{101}\left(1+x^2-x\right)^{100}$ in powers of x is

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9. Find the sum of coefficients in $\left(1+x-3x^2\right)^{4163}$.

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10. Find the middle term in the expansion of $\left(x^2+\frac{1}{x^2}+2\right)^n$.

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11. In the expansion of $(1+x)^{50}$, find the sum of coefficients of odd powers of $x\cdot$

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12. If $(1+x-2x^2)^6 = 1 + a_1x + a_2x^2 + \; + a_{12}x^{12}$, then find the value of $a_2 + a_4 + a_6 + \; + a_{12}\cdot$

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13. If the middle term in the binomial expansion of $\left(\frac{1}{x} + x\sin x\right)^{10}$ is equal to $\frac{63}{8}$, find the value of $x\cdot$

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14. Find the sum $C_0 + 3C_1 + 3^2C_2 + \; + 3^nC_n\cdot$

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15. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r$, then prove that $C_1 + 2C_2 + 3C_3 + ... + nC_n = n2^{n-1}$. .

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16. If $T_0, T_1, T_2, , T_n$ represent the terms in the expansion of $(x+a)^n$, then find the value of $(T_0 - T_2 + T_4 -)^2 + (T_1 - T_3 + T_5 -)^2 n \in N$.

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17. If $\left(1+x+x^2\right)^n = a_0 + a_1 x + a_2 x^2 + + a_{2n} x^{2n}$, find the value of $a_0 + a_3 + a_6 + + , n \in N$.

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18. Find the sum $C_0 - C_2 + C_4 - C_6 +$ ....,where $C_r =^n C_r$.

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19. Prove that $\hat{} \ nC_0 +^n C_3 +^n C_6 + \ = \frac{1}{3}\left(2^n + 2\cos\left(\frac{n\pi}{3}\right)\right)$.

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20. Given that the 4th term in the expansion of $[2 + (3x/8)]^{10}$ has the maximum numerical value. Then find the range of value of $x\cdot$

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21. Find the greatest coefficient in the expansion of $(1 + 2x/3)^{15}\cdot$.

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22. Find the greatest term in the expansion of $\sqrt{3}\left(1 + \frac{1}{\sqrt{3}}\right)^{20}\cdot$

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23. Find the numerically greatest term in the expansion of $(3-5x)^{15} when x = 1/5.$

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24. Let n be an odd natural number greater than 1. Then , find the number of zeros at the end of the sum $99^n + 1.$

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25. Find the remainder when 27^{40} is divided by 12.

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26. In the expansion of $(1+x)^n,$ 7th and 8th terms are equal. Find the value of (7/x $+6)^2$.

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27. Find the sum $\sum_{j=0}^{n} \left(\hat{} (4n+1) C_j + ^{4n+1} C_{2n-j} \right)$.

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28. Show that no three consecutive binomial coefficients can be in G.P.

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29. Find the sum $\sum_{r=1}^{n} r^n \frac{\hat{} nC_r}{\hat{} nC_{r-1}}$.

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30. Show that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

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31. If the 3rd, 4th , 5th and 6th term in the expansion of $(x + \alpha)^n$ be, respectively, $a, b, candd,$ prove that $\frac{b^2 - ac}{c^2 - bd} = \frac{5a}{3c}$.

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32. Find the remainder when 7^{98} is divided by 5.

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33. Show that $2^{4n+4} - 15n - 16, where$ n $\in N$ is divisible by 225.

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34. If $(2 + \sqrt{3})^n = I + f,$ where I and n are positive integers and $0 < f < 1,$

show that I is an odd integer and $(1 - f)(1 + f) = 1$

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35. Find the degree of the polynomial

$$\frac{1}{\sqrt{4x+1}}\left\{\left(\frac{1+\sqrt{4x+1}}{2}\right)^7-\left(\frac{1+\sqrt{4x+1}}{2}\right)^7\right\}$$

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36. If 9^7+7^9 is divisible b 2^n, then find the greatest value of $n, where n \in N$.

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37. Prove that $\sqrt{10}\left[\left(\sqrt{10}+1\right)^{100}-\left(\sqrt{10}-1\right)^{100}\right]$ is an even integer .

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38. Find the remainder when $x=5^{5^{5^{5\cdots}}}$ (24 times 5) is divided by 24.

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39. Find the remainder when $1690^{2608} + 2608^{1690}$ is divided by 7.

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40. Find the value of $\{3^{2003} / 28\}, where\{.\}$ denotes the fractional part.

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41. Find the remainder when 5^{99} is divided by 13.

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42. Find the remainder when 7^{103} is divided by 25.

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43. Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.

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44. If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is α and the coefficients of middle terms in the expansion of $(1+x)^{2n+1}$ are β and γ then relate $\alpha, \beta \text{ and } \gamma$.

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45. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1:7:42, then find the value of n.

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46. In the coefficients of rth, $(r+1)th, and(r+2)th$ terms in the binomial expansion of $(1+y)^m$ are in A.P., then prove that $m^2 - m(4r+1) + 4r^2 - 2 = 0$.

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47. Prove that $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)(C_3 + C_4)\ldots\ldots\ldots.(C_{n-1} + C_n)$ = $\dfrac{C_0 C_1 C_2 \ldots. C_{n-1}(n+1)^n}{n!}$

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48. If a_1, a_2, a_3, a_4 be the coefficient of four consecutive terms in the expansion of $(1+x)^n$, then prove that: $\dfrac{a_1}{a_1 + a_2} + \dfrac{a_3}{a_3 + a_4} = \dfrac{2a_2}{a_2 + a_3}$.

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49. Find the sum of $\sum_{r=1}^{n} \frac{r^n C_r}{\hat{} \; nC_{r-1}}$.

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50. Find the positive integer just greater than $(1 + 0.0001)^{10000}$.

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51. Find (i) the last digit, (ii) the last two digits, and (iii) the last three digits of 17^{256}.

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52. If 10^m divides the number $101^{100} - 1$ then, find the greatest value of m.

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53. Using the principle of mathematical induction, prove that $(2^{3n} - 1)$ is divisible by 7 for all $n \in N$

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54. If x is very large as compare to y, then prove that $\sqrt{\frac{x}{x+y}}\sqrt{\frac{x}{x-y}} = 1 + \frac{y^2}{2x^2}$.

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55. Find the coefficient of x^n in the expansion of $(1 - 9x + 20x^2)^{-1}$.

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56. Prove that the coefficient of x^r in the expansion of $(1 - 2x)^{-\frac{1}{2}}$ is $\frac{2r!}{(2^r)(r!)^2}$

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57. Find the sum: $1-\frac{1}{8}+\frac{1}{8}\times\frac{3}{16}-\frac{1\times3\times5}{8\times16\times24}+\ldots$

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58. Show that $\sqrt{3}$ =1+$\frac{1}{3}$+($\frac{1}{3}$).($\frac{3}{6}$)+(1/3)*((3/6)*(5/9)*(7/12)+.......

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59. Assuming x to be so small that x^2 and higher power of x can be neglected, prove that $\frac{\left(1+\frac{3x}{4}\right)^{-4}(16-3x)^{\frac{1}{2}}}{(8+x)^{\frac{2}{3}}}=1-\left(\frac{305}{96}\right)x$

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60. Find the sum $\sum\sum_{0\leq i<j\leq n-1} j^{n}C_{i}.$

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61. Find the condition for which the formula $(a+b)^m = a^m + ma^{m-1}b + \frac{m(m-1)}{1 \times 2}a^{m-2}b^2 +$ holds.

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62. Find the value of x, for which $\frac{1}{\sqrt{5+4x}}$ can be expanded as infinite series.

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63. Find the fourth term in the expansion of $(1-2x)^{3/2}$.

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64. Prove that $.^n C_0 .^{2n} C_n -^n C_1 .^{2n-2} C_n +^n C_2 .^{2n-4} C_n \equiv 2^n$.

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65. Prove that $\hat{} \, nC_0 .^n C_0 -^{n+1} C_1 .^n C_1 +^{n+2} C_2 .^n C_2 \equiv (-1)^n.$

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66. Find the sum of the coefficients of all the integral powers of x in the expansion of $\left(1+2\sqrt{x}\right)^{40}$.

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67. If the sum of the coefficient in the expansion of $\left(\alpha^2x^2-2\alpha x+1\right)^{51}$ vanishes, then find the value of α

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68. Prove that $\sum_{\alpha+\beta+\gamma=10} \frac{10!}{\alpha!\beta!\gamma!}=3^{10}.$

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69. If $\left(1+x-2x^2\right)^{20}=a_0+a_1x+a_2x^2+a_3x^3+\ldots+a_{40}x^{40}$, then find the value of $a_1+a_3+a_5+\ldots+a_{39}$.

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70. Find the sum of the series $.^{15}C_0+^{15}C_1+^{15}C_2+\ldots\ldots\ldots\ldots+^{15}C_7$

.

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71. Find the sum $\sum_{k=0}^{10}.^{20}C_k$.

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72. Find the sum of all the coefficients in the binomial expansion of $\left(x^2+x-3\right)^{319}$.

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73. If the sum of coefficient of first half terms in the expansion of $(x+y)^n$ is 256 , then find the greatest coefficient in the expansion.

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74. Find the value of $\sum_{p=1}^{n}\left(\sum_{m=p}^{n} .^{n} C_{m} .^{m} C_{p}\right)$. And hence, find the value of $\lim_{n \to \infty} \frac{1}{3^n} \sum_{p=1}^{n}\left(\sum_{m=p}^{n} .^{n} C_{m} .^{m} C_{p}\right)$.

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75. Show that the middle term in the expansion of $(1+x)^{2n} is \frac{(1.\,3.\,5(2n-1))}{n!} 2^n x^n, where n$ is a positive integer.

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76. If the middle term in the expansion of $\left(x^2 + 1/x\right)^n$ is $924\, x^6$, then find the value of n.

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77. The first three terms in the expansion of $(1+ax)^n (n \neq 0)$ are $1,\ 6x and 16x^2$. Then find the value of $a and n$.

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78. If x^4 occurs in the rth term in the expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$, then find the value of r.

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79. Find the coefficient of x^{-10} in the expansion of $\left(\frac{a}{x} + bx\right)^{12}$.

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80. Find the constant term in the expansion of $(x - 1/x)^6$.

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81. If the coefficients of $(r - 5)thand(2r - 1)th$ terms in the expansion of $(1 + x)^{34}$ are equal, find r.

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82. In $\left(2^{\frac{1}{3}} + \frac{1}{3^{\frac{1}{3}}}\right)^n$ if the ratio of 7th term from the beginning to the 7th term from the end is 1/6, then find the value of n.

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83. If the coefficient of 4th term in the expansion of $(a + b)^n$ is 56, then n is

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84. If $p and q$ are positive, then prove that the coefficients of $x^p and x^q$ in the expansion of $(1+x)^{p+q}$ will be equal.

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85. Find the number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$.

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86. If x^p occurs in the expansion of $\left(x^2+1/x\right)^{2n}$, prove that its coefficient is $\dfrac{(2n)!}{\left[\frac{1}{3}(4n-p)\right]!\left[\frac{1}{3}(2n+p)\right]!}$.

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87. Find the coefficient of $a^3b^4c^5$ in the expansion of $(bc + ca + ab)^6$.

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88. Find the coefficient of x^7 in the expansion of $\left(1 - x - x^2 + x^3\right)^6$.

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89. If the number of terms in the expansion of $(x + y + z)^n$ are 36, then find the value of n.

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90. Find the coefficient of a^3b^4c in the expansion of $(1 + a + b - c)^9$.

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91. Find the coefficient of x^4 in the expansion of $\left(1+x+x^2+x^3\right)^{11}$.

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92. Find the number of terms which are free from radical signs in the expansion of $\left(y^{1/5}+x^{1/10}\right)^{55}$.

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93. Find the coefficient of x^5 in the expansion of $\left(1+x^2\right)^5(1+x)^4$.

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94. Find the coefficient of x^{13} in the expansion of $(1-x)^5\times\left(1+x+x^2+x^3\right)^4$.

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95. Find the sum $.^{10}C_1 + ^{10}C_3 + ^{10}C_5 + ^{10}C_7 + ^{10}C_9$

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96. Find the sum of $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots,$

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97. If n is an even positive integer, then find the value of x if the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also.

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98. If $|x| < 1$, then find the coefficient of x^n in the expansion of $(1 + 2x + 3x^2 + 4x^3 +)^{1/2}$.

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99. If $(r+1)th$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then find the value of r.

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100. If $|x|<1$, then find the coefficient of x^n in the expansion of $(1+x+x^2+......)^2$.

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101. If $|x|>1$, then expand $(1+x)^{-2}$.

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102. Find the cube root of 217, correct to two decimal places.

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103. Find the coefficient of x^2 in $\left(\frac{a}{a+x}\right)^{1/2}+\left(\frac{a}{a-x}\right)^{1/2}$

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104. Prove that

$\hat{} \, 10C_1(x-1)^2 -^{10} C_2(x-2)^2 +^{10} C_3(x-3)^2 \pm^{10} C_{10}(x-10)^2 = x^2$

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105. If the third term in the expansion of $(1+x)^m is -\frac{1}{8}x^2,$ then find the value of $m\cdot$

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106. Prove that $\sum_{r=0}^{n} r(n-r)(\hat{} \, nC_r)^2 = n^2(\hat{} \, (2n-2)C_n)\cdot$

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107. Prove that $1 -^{n} C_1 \frac{1+x}{1+nx} +^{n} C_2 \frac{1+2x}{(1+nx)^2} -^{n} C_3 \frac{1+3x}{(1+nx)^3} + \dots (n+1) terms =$

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108. Find the coefficient of x^{20} in $\left(x^2 + 2 + \frac{1}{x^2}\right)^{-5} \left(1 + x^2\right)^{40}$.

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109. The number of terms in the expansion of $(a+b+c)^n, where n \in N$.

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110. Find the coefficient of x^{50} in the expansion of $(1+x)^{101} \times \left(1 - x + x^2\right)^{100}$.

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111. Find the coefficient of x^4 in the expansion of $\left(2 - x + 3x^2\right)^6$.

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112. Find the coefficient of $x^k \in 1 + (1 + x) + (1 + x)^2 + \; + (1 + x)^n (0 \leq k \leq n)$.

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113. Find the term independent of x in the expansion of $\left(1 + x + 2x^3\right)\left[\left(3x^2/2\right) - (1/3x)\right]^9$

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114. If $aandb$ are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, wherever n is a positive integer.

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115. Find the $a, b, and n$ in th expansion of $(a+b)^n$ if the first three terms of the expansion are 729, 7290, and 30375, respectively.

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116. Find the coefficient of x^{25} in expansion of expression $\sum_{r=0}^{50} \hat{} (50)C_r(2x-3)^r(2-x)^{50-r}$.

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117. If the sum of the coefficients of the first, second, and third terms of the expansion of $\left(x^2+\frac{1}{x}\right)^m$ is $46,$ then find the coefficient of the term that does not contain $x\cdot$

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118. If $p+q=1$, then show that $\sum_{r=0}^{n} r^2\ \hat{}\ nC_r p^r q^{n-r}=npq+n^2p^2.$

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119. If $(18x^2+12x+4)^n=a_0+a_{1x}+a2x2+\ +a_{2n}x^{2n}$, prove that $a_r=2^n3^r\left(\ \hat{}\ (2n)C_r+^n C_1^{2n-2}C_r+^n C_2^{2n-4}C_r+\right).$

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120. Prove that $\hat{}\ mC_1^nC_m-^m C_2^{2n}C_m+^m C_3^{3n}C_m\equiv(-1)^{m-1}n^m.$

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121. Prove that

$^nC_0^{2n}C_n-^n C_1^{2n-1}C_n+^n C_2\times^{2n-2}C_n+\ +(-1)^n\ \hat{}\ nC_n^nC_n=1.$

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122. Find the sum $\sum_{r=0}^{n} \hat{} (n+r)C_r$.

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123. Find the value of $\sum \sum_{0 \leq i \leq j \leq n} (i+j)\left(nC_i + nC_j\right) \cdot$

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124. Find the value of $\sum \sum_{0 \leq i \leq j \leq n} c_i^n c_j^n \cdot$

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125. Find the value of $\sum \sum_{0 \leq i < j \leq n} \left(.^n C_i + .^n C_j\right)$.

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126. Find the sum $\sum \sum_{0 \leq i < j \leq n} {}^nC_i {}^nC_j$

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127. Prove that $\sum_{r=0}^{s} \sum_{s=1}^{n} \hat{} \, nC_s^n C_r = 3^n - 1.$

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128. Find the sum $\sum \sum_{0 \leq i < j \leq n} {}^nC_i$

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129. Find the coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$.

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130. Find the term in $\left(3\sqrt{\left(\frac{a}{\sqrt{b}}\right)}+\left(\sqrt{\frac{b}{\hat{}}3\sqrt{a}}\right)\right)^{21}$ which has the same power of a and b.

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131. Using the binomial theorem, evaluate $(102)^{5}$.

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132. Find the 6th term in expansion of $\left(2x^{2}-1/3x^{2}\right)^{10}$.

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133. Find a if the 7th and 18th terms of the expansion $(2+a)^{50}$ are equal.

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134. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^{n}$ is $\sqrt{6}:1$.

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135. Simplify: $x^{5}+10x^{4}a+40x^{3}a^{2}+80x^{2}a^{3}+80xa^{4}+32a^{5}$.

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136. Find the value of

$$\frac{18^{3}+7^{3}+3\times 18\times 7\times 25}{3^{6}+6\times 243\times 2+15\times 18\times 4+20\times 27\times 8+15\times 9\times 16}$$

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137. Find the approximation of $(0.99)^{5}$ using the first three terms of its expansion.

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138. If for $n \in N, \sum_{k=0}^{2n} (-1)^k (\hat{} (2n)C_k)^2 = A,$ then find the value of $\sum_{k=0}^{2n} (-1)^k (k-2n)(\hat{} (2n)C_k)^2.$

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139. There are two bags each of which contains n balls. A man has to select an equal number of balls from both the bags. Prove that the number of ways in which a man can choose at least one ball from each bag $is^{2n}C_n - 1.$

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140. Find the sum $\sum_{i=0}^{r} .^{n_1} C_{r-i} .^{n_2} C_i$.

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141. Prove that $\sum_{r=0}^{2n}\left(r.^{2n}C_r\right)^2 = n^{4n}C_{2n}$.

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142. If $kandn$ are positive integers and $s_k = 1^k + 2^k + 3^k + \; + n^k$, then prove that $\sum_{r=1}^{m} \hat{} \; (m+1)C_r s_r = (n+1)^{m+1} - (n+1)$.

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143. Prove that $\sum_{r=1}^{n}(-1)^{r-1}\left(1+\frac{1}{2}+\frac{1}{3}+\;+\frac{1}{r}\right)^n C_r = \frac{1}{n}$.

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144. Prove that

$$\frac{C_1}{1} - \frac{C_2}{2} + \frac{C_3}{3} - \frac{C_4}{4} + \; + \frac{(-1)^{n-1}}{n}C_n = 1 + \frac{1}{2} + \frac{1}{3} + \; + \frac{1}{n}.$$

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145. Prove that $\sum_{r=0}^{n} \hat{} \; nC_r \sin rx \cos(n-r)x = 2^{n-1}\sin(nx)$.

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146. Find the last two digits of the number $(23)^{14}$.

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147. Find the last two digits of the number 27^{27}.

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148. Find the number of nonzero terms in the expansion of $(1+3\sqrt{2}x)^9 + (1-3\sqrt{2}x)^9$.

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149. Find the value of $\left(\sqrt{2}+1\right)^{6}-\left(\sqrt{2}-1\right)^{6}$.

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150. Using binomial theorem (without using the formula for $.^{n} C_{r}$) , prove that

$.^{n} C_{4} +^{m} C_{2} -^{m} C_{1}.^{n} C_{2} = .^{m} C_{4} -^{m+n} C_{1}.^{m} C_{3} +^{m+n} C_{2}.^{m} C_{2} -^{m+n} C_{3}^{m}$

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151. Prove that

$.(r+1)\cdot^{n} C_{r} - r\cdot^{n} C_{r} + ... + (-1)^{r}.^{n} C_{r} = (-1)^{r}.^{n-2} C_{r}.$

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152. Find the sum $.^{n} C_{0} +^{n} C_{4} +^{n} C_{8} + \ldots$

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153. Find the value of ^ $4nC_0 +^{4n} C_4 +^{4n} C_8 + \ + {}^{4n}C_{4n}$.

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154. Find the coefficient of x^n in the polynomial $(x +^n C_0)(x + 3^n C_1) \times (x + 5^n C_2) \left[x + (2n+1)^n C_n\right]\cdot$

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155. If $(1+x)^{15} = C_0 + C_1 x + C_2 x^2 + \ + C_{15} x^{15}$, then find the value of $C_2 + 2C_3 + 3C_4 + \ + 14C_{15}\cdot$

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156. Prove that $\frac{.^n C_0}{1} + \frac{.^n C_2}{3} + \frac{.^n C_4}{5} + \frac{.^n C_6}{7} + \ldots = \frac{2^n}{n+1}\cdot$

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157. Find the sum $\sum \sum_{0 \le i \le j \le n}^{n} C_i^n C_j$

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158. Find the sum $\sum \sum_{i \ne j}^{n} \hat{} \, n C_i^n C_j$

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159. Show that the integer next above $\left(\sqrt{3}+1\right)^{2m}$ contains 2^{m+1} as a factor.

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160. Prove that $\frac{1^2}{3}{}^nC_1 + \frac{1^2+2^2}{5^n}C_2\frac{1^1+2^2+3^2}{7^n}C_3 + \; + \frac{1^2+2^2+\;+n^2}{(2n+1)^n}C_n = \frac{n(n+3)}{62^{n-2}}.$

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161. Prove that

$$\frac{1}{n+1} = \frac{.^n C_1}{2} - \frac{2(.^n C_2)}{3} + \frac{3(.^n C_3)}{4} - \ldots + (-1^{n+1}) \frac{n \cdot (.^n C_n)}{n+1}$$

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162. Find the sum $2C_0 + \frac{2^3}{2}C_1 + \frac{2^3}{3}C_2 + \frac{2^4}{4}C_3 + \ + \frac{2^{11}}{11}C_{10}.$

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163. If in the expansion of $(2x+5)^{10}$, the numerically greatest tem in equal to the middle term, then find the values of $x.$

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164. Find the value of $\frac{1}{81^n} - \frac{10}{(81^n)^{2n}}C_1 + \frac{10^2}{(81^n)^{2n}}C_2 - \frac{10^3}{(81^n)^{2n}}C_3 + \quad + \frac{10^{2n}}{81^n}$.

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165. Find the value of $5C_3 + 4C_2$

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166. Find the sum $1C_0 + 2C_1 + 3C_2 + \quad + (n+1)C_n, whereC_r =^n C_r$.

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167. If $\left(1 + x + x^2 + \quad + x^p\right)^n = a_0 + a_1x + a_2x^2 + \quad + a_{np}x^{np}$, then find the value of $a_1 + 2a_2 + 3a_3 + \ddot{+} npa_{np}$.

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168. If $n > 2,$ then prove that $C_1(a-1) - C_2 \times (a-2) + \; + (-1)^{n-1} C_n(a-n) = a, where C_r = ^n C_r$

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169. Find the sum $C_0 - C_2 + C_4 - C_6 + \ldots\ldots\ldots$,where $C_r = ^n C_r$.

A. $n(n+1)2^n - 1$

B. $n(n+3)2^n - 2$

C. $2n.^{2n} C_n$

D. none of these

Answer: null

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170. If $x+y=1,$ prove that $\sum_{r=0}^{n} .^n C_r x^r y^{n-r} = 1.$

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171. Find the sum $3C_1 + 5C_2$

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172. Prove that $\frac{.^n C_1}{2} + \frac{.^n C_3}{4} + \frac{.^n C_5}{6} + \ldots . = \frac{2^n - 1}{n+1}$.

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173. If $(1+x)^n = \sum_{r=0}^{n} \hat{} \, nC_r$, show that $C_0 + \frac{C_1}{2} + + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$.

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174. If $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r (x-3)^r and a_k = 1$ for all $k \geq n$, then show that $b_n =^{2n+1} C_{n+1}$.

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175. Statement 1: $3^{2n+2} - 8n - 9$ is divisible by 64, $\forall n \in N$. Statement 2: $(1+x)^n - nx - 1$ is divisible by x^2, $\forall n \in N$.

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176. Statement 1: The number of distinct terms in $(1+x+x^2+x^3+x^4)^{1000} is 4001.$ Statement 2: The number of distinct terms in expansion $(a_1 + a_2 + \quad + a_m)^n is^{n+m-1} C_{m-1}^{\cdot}$

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177. Statement1: if $n \in N and n$ is not a multiple of 3 and $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then the value of $\sum_{r=0}^{n} (-1)^r a r^n C_r$ is zero Statement 2: The coefficient of x^n in the expansion of $(1-x^3)^n$ is zero, if $n = 3k+1 \text{ or } n = 3k+2.$

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178. Statement 1:Three consecutive binomial coefficients are always in A.P.

Statement 2: Three consecutive binomial coefficients are not in H.P.

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179. The value of $\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + (302)(3012) + + (3020)(3030) =$ a. $\hat{}\ 60C20$ b. $\hat{}\ 30C10$ c. $\hat{}\ 60C30$ d. $\hat{}\ 40C30$

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180. If $f(x) = x^n, f(1) + \frac{f^1(1)}{1} + \frac{f^2(1)}{2!} + \frac{f^n(1)}{n!}, where f^r(x)$ denotes the rth order derivative of $f(x)$ with respect to $x,$ is a. n b. 2^n c. 2^{n-1} d. none of these

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181. The fractional part of $\frac{2^{4n}}{15}$ is $(n \in N)$ (A) $\frac{1}{15}$ (B) $\frac{2}{15}$ (C) $\frac{4}{15}$ (D) none of these

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182. The value of $.^{15}C_0^2 - .^{15}C_1^2 + .^{15}C_2^2 - .... - .^{15}C_{15}^2$ is

a. 15

b. -15

c. 0

d. 51

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183. If the sum of the coefficients in the expansion of $\left(1 - 3x + 10x^2\right)^n isa$ and if the sum of the coefficients in the expansion of $\left(1 + x^2\right)^n isb$, then a. $a = 3b$ b. $a = b^3$ c. $b = a^3$ d. none of these

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184. If $\left(1 + x - 2x^2\right)^6 = 1 + a_1x + a_2x^2 + \quad + a_{12}x^{12}$, then find the value of $a_2 + a_4 + a_6 + \quad + a_{12}$.

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185. Maximum sum of coefficient in the expansion of $\left(1 - x\sin\theta + x^2\right)^n$ is 1 b. 2^n c. 3^n d. 0

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186. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is a. 924 b. 792 c. 1594 d. none of these

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187. The number of distinct terms in the expansion of $\left(x+\frac{1}{x}+x^2+\frac{1}{x^2}\right)^{15}$ is/are (with respect to different power of x) 255 b. 61 c. 127 d. none of these

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188. The sum of the coefficients of even power of x in the expansion of $(1+x+x^2+x^3)^5$ is 256 b. 128 c. 512 d. 64

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189. Column I, Column II The coefficient of the two consecutive terms in the expansion of $(1+x)^n$ will be equal, then n can be, p. 9 If 15^n+23^n is divided, by 19, then n can be, q. 10 $\hat{}\ 10C_0^{20}C_{10}-^{10}C_1^{18}C_{10}+^{10}C_2^{16}C_{10}-$ is divisible by 2^n, $the \cap$ can be, r. 11 If the coefficients of T_r, T_{r+1}, T_{r+2} terms of $(1+x)^{14}$ are in A.P., then r is less than, s. 12

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190. If the coefficient of $x^7 \in \left[ax^2 - \left(\frac{1}{bx^2}\right)\right]^{11}$ equal the coefficient of x^{-7} in satisfy the $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}, then a and b$ satisfy the relation $a + b = 1$ b. $a - b = 1$ c. $b = 1$ d. $\frac{a}{b} = 1$

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191. If the coefficients of the $(2r+4)th, (r-2)th$ term in the expansion of $(1+x)^{18}$ are equal, then the value of r is.

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192. If the coefficients of the rth, $(r+1)th, (r+2)th$ terms is the expansion of $(1+x)^{14}$ are in A.P, then the largest value of r is.

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193. If the three consecutive coefficients in the expansion of $(1+x)^n$ are 28, 56, and 70, then the value of n is.

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194. Degree of the polynomial $\left[\sqrt{x^2+1}+\sqrt{x^2-1}\right]^8+\left[\frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}}\right]^8$ is.

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195. Least positive integer just greater than $(1+0.00002)^{50000}$ is.

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196. If $U_n=\left(\sqrt{3}+1\right)^{2n}+\left(\sqrt{3}-1\right)^{2n}$, then prove that $U_{n+1}=8U_n-4U_{n-1}$.

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197. Prove that the coefficient of x^n in the expansion of $\frac{1}{(1-x)(1-2x)(1-3x)}$ is $\frac{1}{2}(3^{n+2}-2^{n+3}+1)$

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198. The value of $(30, 0)(30, 10) - (30, 1)(30, 11) + (30, 2)(30, 12) - \ldots\ldots\ldots\ldots + (30, 20)(3$

, where $(n, r) = nC_r$ is a. $(30, 10)$ b. $(30, 15)$ c. $(60, 30)$ d. $(31, 10)$

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199. Prove that $\hat{\ } nC_1(\hat{\ } nC_2)(\hat{\ } nC_3)^3(\hat{\ } nC_n)^n \leq \left(\frac{2^n}{n+1}\right)^{n+1_C()_2}, \forall n \in N.$

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200. Prove that $\frac{1}{(m!)^n}C_0 + \frac{n}{((m+1)!)^n}C_1 + \frac{n(n-1)}{((m+2)!)^n}C_2 + + \frac{n(n-1)2\times 1}{((m+n)!)^n}C_n =$

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201. If $n = 12m(m \in N)$, prove that $\hat{}\ nC_0 - \frac{\hat{}\ nC_2}{\left(2+\sqrt{3}\right)^2} + \frac{\hat{}\ nC_4}{\left(2+\sqrt{3}\right)^4} - \frac{\hat{}\ nC_6}{\left(2+\sqrt{3}\right)^6} + =$
$(-1)^m\left(\frac{2\sqrt{2}}{1+\sqrt{3}}\right)^n.$

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202. In the expansion of $(1+x)^n(1+y)^n(1+z)^n$, the sum of the co-efficients of the terms of degree 'r' is (a) $.^{n^3}C_r$ (b) $.^nC_{r^3}$ (c) $.^{3n}C_r$ (d) $3.^{2n}C_r$

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203. Prove that

$$\hat{}\ 100C_0^{100}C_2 +^{100} C_2^{100}C_4 +^{100} C_4^{100}C_6 + \ +^{100} C_{98}^{100}C_{100} = \frac{1}{2}\left[\ \hat{}\ (200)C_{98}\right.$$

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204. Prove that $\sum_{r=1}^{m-1} \frac{2r^2 - r(m-2) + 1}{(m-r)^m C_r} = m - \frac{1}{m}$.

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205. Find the coefficients of x^{50} in the expression $(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + ... + 1001x^{1000}$.

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206. If $b_1, b_2 b_n$ are the nth roots of unity, then prove that $\hat{}\ nC_1\dot{b}_1 +^n C_2\dot{b}_2 + \ +^n C_n\dot{b}_n - \frac{b_1}{b_2}\{(1+b_2)^n - 1\}^{\cdot}$

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207. If $.^{n+1}C_{r+1} : ^nC_r : ^{n-1}C_{r-1} = 11 : 6 : 3,$ then $nr = ?$ a. 20 b. 30 c. 40 d. 50

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208. If the last tem in the binomial expansion of $\left(2^{\frac{1}{3}} - \frac{1}{\sqrt{2}}\right)^n is \left(\frac{1}{3^{\frac{5}{3}}}\right)^{\log_3 8}$, then 5th term from the beginning is 210 b. 420 c. 105 d. none of these

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209. Find the last two digits of the number $(23)^{14}$.

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210. The value of x for which the sixth term in the expansion of $\left[2^{\log}2\sqrt{9^{x-1}+7}+\frac{1}{2^{\frac{1}{5}}(\log)_2\left(3^{(x-1)+1}\right)}\right]^7$ is 84 is a. 4 b. 1 or 2 c. 0 or 1 d. 3

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211. If the 6th term in the expansion of $\left(\frac{1}{x^{\frac{8}{3}}}+x^2(\log)_{10}x\right)^8$ is 5600, then x equals 1 b. $(\log)_e 10$ c. 10 d. x does not exist

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212. The total number of terms which are dependent on the value of x in the expansion of $\left(x^2-2+\frac{1}{x^2}\right)^n$ is equal to $2n+1$ b. $2n$ c. n d. $n+1$

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213. In the expansion of $\left(3^{-x/4}+3^{5x/4}\right)^{n}$ the sum of binomial coefficient is 64 and term with the greatest binomial coefficient exceeds the third by $(n-1)$, the value of x must be 0 b. 1 c. 2 d. 3

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214. If n is an integer between 0 and 21, then the minimum value of $n!(21-n)!$ is attained for $n=\ 1$ b. 10 c. 12 d. 20

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215. If R is remainder when $6^{83}+8^{83}$ is divided by 49, then the value of $R/5$ is.

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216. Let $a and b$ be the coefficients of x^3 in $(1+x+2x^2+3x^3)^4 and (1+x+2x^2+3x^3+4x^4)^4$, then respectively. Then the value of $4a/b$ is.

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217. Let $1+\sum_{r=1}^{10}\left(3^r.{}^{10}C_r+r.{}^{10}C_r\right)=2^{10}\left(\alpha.4^5+\beta\right)$ where $\alpha, \beta \in N$ and $f(x)=x^2-2x-k^2+1$. If α, β lies between the roots of $f(x)=0$, then find the smallest positive integral value of k.

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218. Let $a=3^{1/224}+1$ and for all $n \geq 3$,

let

$f(n)={}^nC_0a^{n-1}-{}^nC_1a^{n-2}+{}^nC_2a^{n-3}+\ldots+(-1)^{n-1}\cdot{}^nC_{n-1}\cdot a^0$.

If the value of f(2016)+f(2017)=3^k, the value of K is

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219. If the constant term in the binomial expansion of $\left(x^2 - \frac{1}{x}\right)^n$, $n \in N$ is 15, then find the value of n.

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220. The largest value of x for which the fourth tem in the expansion $\left(5^{\left(\frac{2}{5}\right)(\log)_5\sqrt{4^x+44}} + \frac{1}{5^{\log_5\left(2^{(x-1)}+7\right)^{\frac{1}{3}}}}\right)^8$ is 336 is.

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221. The number of values in set of values of r for which $\hat{}\ 23C_r + 2.^{23} C_{r+1} +^{23} C_{r+2} \geq^{25} C_{15}$ is

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222. If the second term of the expansion $\left[a^{\frac{1}{13}} + \frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$, then the value of $\frac{\hat{\ } nC_3}{\hat{\ } nC_2}$ is.

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223. Given $(1 - 2x + 5x^2 - 10x^3)(1 + x)^n = 1 + a_1x + a_2x^2 +$ and that$a1^2 = 2a_2$ then the value of n is.

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224. Sum of last three digits of the number $N = 7^{100} - 3^{100}$ is.

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225. Let n be a positive integer and $(1 + x + x^2)^n = a_0 + a_1x + \dots. + a_{2n}x^{2n}$.

Show that $a_{02} - a_{12} + a_{22} + \dots + a_{2n2} = a_n$

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226. $\sum_{r=1}^{k}(-3)^{r-1}.^{3n}C_{2r-1}=0,$ where $k=\frac{3n}{2}$ and n is an even integer

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227. The coefficient of the middle term in the binomial expansion in powers of x of$(1+\alpha x)^4$ and of$(1-\alpha x)^6$ is the same, if α equals $-\frac{5}{3}$ b. $\frac{10}{3}$ c. $-\frac{3}{10}$ d. $\frac{3}{5}$

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228. If in the expansion of $(1+x)^n, a, b, c$ are three consecutive coefficients, then $n=$ a. $\frac{ac+ab+bc}{b^2+ac}$ b. $\frac{2ac+ab+bc}{b^2-ac}$ c. $\frac{ab+ac}{b^2-ac}$ d. none of these

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229. If n and k are positive integers, show that

$2^k(.^n C_0)(.^n C_k) - 2^{k-1}(.^n C_1)\left(.^{n-1} C_k - 1\right) + 2^{k-2}(.^n C_2)((n - 2k - 2))$

stands for $.^n C_k$.

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230. Prove that $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$ for all $n = 1, 2, \ldots$

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231. The coefficient of $1/x$ in the expansion of $(1+x)^n(1+1/x)^n$ is (a). $\frac{n!}{(n-1)!(n+1)!}$ (b). $\frac{(2n)!}{(n-1)!(n+1)!}$ (c). $\frac{(2n)!}{(2n-1)!(2n+1)!}$ (d). none of these

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232. The coefficient x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}++(1+x)^{30}$ is a. ${}^{51}C_5$ b. ${}^{9}C_5$ c. ${}^{31}C_6-{}^{21}C_6$ d. ${}^{30}C_5+{}^{20}C_5$

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233. If x^m occurs in the expansion $\left(x+1/x^2\right)^{2n}$, then the coefficient of x^m is a. $\dfrac{(2n)!}{(m)!(2n-m)!}$ b. $\dfrac{(2n)!3!3!}{(2n-m)!}$ c. $\dfrac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$ d. none of these

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234. If the coefficients of 5th, 6th , and 7th terms in the expansion of $(1+x)^n$ are in A.P., then $n=$ a. 7 only b. 14 only c. 7 or 14 d. none of these

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235. If $\left(1+2x+x^2\right)^n=\sum_{r=0}^{2n} a_r x^r$,then a_r is a.$\left(.^n C_2\right)^2$ b. $.^n C_r.^n C_{r+1}$ c. $.^{2n} C_r$ d. $.^{2n} C_{r+1}$

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236. In the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n \in N$ if sum of the coefficients of x^5 and x^{10} is 0 then n is

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237. If the coefficients of rth and $(r+1)th$ terms in the expansion of $(3+7x)^{29}$ are equal, then r is equals to a. 15 b. 21 c. 14 d. none of these

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238. In the expansion of $\left(1+3x+2x^2\right)^6$, the coefficient of x^{11} is a. 144 b. 288 c. 216 d. 576

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239. If $n-1C_r=\left(k^2-3\right)^n C_{r+1},$ then (a) $(-\infty,-2]$ (b) $[2,\infty)$ (c) $\left[-\sqrt{3},\sqrt{3}\right]$ (d) $\left(\sqrt{3},2\right]$

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240. Prove that $\dfrac{3!}{2(n+3)}=\sum_{r=0}^{n}(-1)^r\left(\dfrac{\hat{}\ nC_r}{\hat{}\ (r+3)C_r}\right)$

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241. If $a_n=\sum_{r=0}^{n}\dfrac{1}{nC_r},$ then $\sum_{r=0}^{n}\dfrac{r}{nC_r}$ equals

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242. The expression $\left(x+\dfrac{\left(x^3-1\right)^{\frac{1}{2}}}{2}\right)^5+\left(x-\dfrac{\left(x^3-1\right)^{\frac{1}{2}}}{2}\right)^5$ is a polynomial of degree

a. 5 b. 6 c. 7 d. 8

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243. Find $\left(\frac{dy}{dx}\right)$ of $\sin(\cos\theta)$ is

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244. In the binomial expansion of $(a-b)^n, n \geq 5,$ the sum of the 5th and 6th term is zero. Then a/b equals $(n-5)/6$ b. $(n-4)/5$ c. $n/(n-4)$ d. $6/(n-5)$

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245. Coefficient of x^{11} in the expansion of $(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$ is 1051 b. 1106 c. 1113 d. 1120

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246. Given positive integers $r > 1, n > 2$ and that the coefficient of $(3rd)th$ and $(r+2)th$ terms in the binomial expansion of $(1+x)^{2n}$ are equal. Then (a) $n = 2r$ (b) $n = 2r + 1$ (c) $n = 3r$ (d) non of these

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247. The coefficient of x^4 in $\left(x/2 - 3/x^2\right)^{10}$ is a.$\frac{405}{256}$ b. $\frac{504}{259}$ c. $\frac{450}{263}$ d. none of these

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248. If C_r stands for nC_r, then the sum of the series $\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - \ldots\ldots. + (-1)^n(n+1)C_n^2\right]$,where n is an even positive integer, is

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249. The sum $\sum_{m}^{i=0} \binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q} = 0$, if $p < q$) is maximum when $'m'$ is

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250. The coefficient of X^{24}in the expansion of $\left(1+X^2\right)^{12}\left(1+X^{12}\right)\left(1+X^{24}\right)$

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251. The term independent of a in the expansion of $\left(1+\sqrt{a}+\frac{1}{\sqrt{a}-1}\right)^{-30}$ is (a) $30C_{20}$ (b) 0 (c) $30C_{10}$ (d) non of these

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252. The coefficient of x^{53} in the expansion $\sum_{m=0}^{100} \hat{} \ 100C_m(x-3)^{100-m}2^m$ is (a) $100C_{47}$ (b.) $100C_{53}$ (c.) $-100C_{53}$ (d.) none of these

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253. The coefficient of the term independent of x in the exampansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is 210 b. 105 c. 70 d. 112

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254. In the expansion of $(1+x+x^3+x^4)^{10}$, the coefficient of x^4 is $\hat{} \ 40C_4$ b. $\hat{} \ 10C_4$ c. 210 d. 310

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255. If coefficient of $a^2b^3c^4 \in (a+b+c)^m$ (where $n \in N$) is $L(L \neq 0)$, then in same expansion coefficient of $a^4b^4c^1$ will be (A) L (B) $\frac{L}{3}$ (C) $\frac{mL}{4}$ (D) $\frac{L}{2}$

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256. The last two digits of the number 3^{400} are:

(A) 81 (B) 43 (C) 29 (D) 01

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257. The expression $\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}\right)^6$ is polynomial of degree

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258. The coefficient of $x^r[0 \le r \le (n-1)]$ in the expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ldots.$

$+ (x+2)^{n-1}$ is $a.^n C_r(3^r - 2^n)$ $b.^n C_r\left(3^{n-r} - 2^{n-r}\right)$ $c.^n C_r\left(3^r + 2^{n-r}\right)$

d. none of these

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259. If $\left(1 + 2x + 3x^2\right)^{10} = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{20} x^{20}$ then a_1 = ?

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260. In the expansion of $\left(5^{1/2} + 7^{1/8}\right)^{1024}$, the number of integral terms is 128 b. 129 c. 130 d. 131

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261. For which of the following value of x, 5^{th} term is the numerically greatest term in the expansion of $(1 + x/3)^{10}$:

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262. For natural numbers m, n, if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots$, and $a_1 = a_2 = 10$, t

a. $m < n$ b. $m > n$ c. $m + n = 80$ d. $m - n = 20$

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263. If the middle term in the expansion of $\left(\frac{x}{2} + 2\right)^8$ is 1120, then find the sum of possible real values of x.

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264. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$,

then $C_0 - (C_0 + C_1) + (C_0 + C_1 + C_2) - (C_0 + C_1 + C_2 + C_3) + \ldots + (-1)^{n-1}(C_0 + C_1 + C_{n-1})$, where n a) is even integer b) is a positive value c) a negative value d) divisible by2^{n-1} divisible by2^n

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265. In the expansion of $\left(x^2 + 1 + \frac{1}{x^2}\right)^n, n \in N,$

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266. The value of $\hat{} \ nC_1 +^{n+1} C_2 +^{n+2} C_3 + \ +^{n+m-1} C_m$ is equal to

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267. If

$(1+x)^n = C_0 + C_1x + C2x2 + \ + C_nx^n, n \in N, thenC_0 - C_1 + C_2 - \ +$

is equal to $(m < n)$

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268. The 10th term of $\left(3-\sqrt{\frac{17}{4}+3\sqrt{2}}\right)^{20}$ is (a) a irrational number (b) a rational number (c) a positive integer (d) a negative integer

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269. For the expansion $\left(x\sin p + x^{-1}\cos p\right)^{10}, (p \in R),$ The greatest value of the term independent of x is $(a)10!/2^5(5!)^2$ (b)the least value of sum of coefficient is zero (c)the greatest value of sum of coefficient is 32 (d)the least value of the term independent of x occurs when $p=(2n+1)\frac{\pi}{4}, n \in Z$

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270. Let $\left(1+x^2\right)^2(1+x)^n=\sum_{k=0}^{n+4} a_k x^k$. If a_1, a_2 and a_3 are in arithmetic progression, then the possible value/values of n is/are a. 5 b. 4 c. 3 d. 2

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271. The middle term in the expansion of $(x/2+2)^8$ is 1120, then $x \in R$ is equal to a. -2 b. 3 c. -3 d. 2

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272. If $\left(1+2x+x^2\right)^n=\sum_{r=0}^{2n} a_r x^r$,then a_r is a.$(.^n C_2)^2$ b. $.^n C_r .^n C_{r+1}$ c. $.^{2n} C_r$ d. $.^{2n} C_{r+1}$

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273.

$\left[\left(\hat{\ } nC_0+^n C_3+\right)-\frac{1}{2}\left(\hat{\ } nC_1+^n C_2+^n C_4+^n C_5\right]^2+\frac{3}{4}\left(\hat{\ } nC_1-^n C_2\right.\right.$

a.3 b. 4 c. 2 d. 1

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274. If $\sum_{r=0}^{n}\left(\frac{r+2}{r+1}\right).^{n}C_{r}=\frac{2^{8}-1}{6}$, then n is (A) 8 (B) 4 (C) 6 (D) 5

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275. Let $f(x)=a_0+a_1x+a_2x^2+\ldots+a_nx^n$ and $\frac{f(x)}{1-x}=b_0+b_1x+b_2x^2+\ldots+b_nx^n$, then a. $b_n+b_{n-1}=a_n$ b. $b_n-b_{n-1}=a_n$ c. $\frac{b_n}{b_{n-1}}=a_n$ d. none of these

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276. If $\left(1+x^2\right)^n=\sum_{r=0}^{n}a_rx^r=\left(1+x+x^2+x^3\right)^{100}$. If $a=\sum_{r=0}^{300}a_r$, then $\sum_{r=0}^{300}ra_r$ is

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277. The value of $\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n} {}^{k}C_{r-1}\right)$(where $r, k, n \in N$) is equal to a. $2^{n+1} - 2$ b. $2^{n+1} - 1$ c. 2^{n+1} d. none of these

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278. If $\frac{x^2 + x + 1}{1 - x} = a_0 + a_1x + a_2x^2 + , then \sum_{r=1}^{50} a_r$ is equal to 148 b. 146 c. 149 d. none of these

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279. Find $\frac{dy}{dt}$, if y = $\frac{1 - \cos t}{1 + \cos t}$ is

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280. The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3)\ldots.(1 + x^{100})$ is

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281. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5:10:14. Then $n =$ __________.

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282. If $(1-x)^{-n} = a_0 + a_1 x + a_2 x^2 + \ldots + a_r x^r + , then a_0 + a_1 + a_2 + \ldots + a_r$ is equal to $\frac{n(n+1)(n+2)(n+r)}{r!}$ $\frac{(n+1)(n+2)(n+r)}{r!}$ $\frac{n(n+1)(n+2)(n+r-1)}{r!}$ none of these

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283. The value of $\sum_{r=0}^{20} r(20-r)\left(.^{20}C_r\right)^2$ is equal to

a. $400^{39}C_{20}$ b. $400^{40}C_{19}$ c. $400^{39}C_{19}$ d. $400^{38}C_{20}$

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284. The coefficient of x^{10} in the expansion of $\left(1+x^2-x^3\right)^8$ is 476 b. 496 c. 506 d. 528

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285. If the term independent of x in the $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then k equals $2, -2$ b. $3, -3$ c. $4, -4$ d. $1, -1$

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286. The coefficient of x^2y^3 in the expansion of $(1-x+y)^{20}$ is $\frac{20!}{213!}$ b. $-\frac{20!}{213!}$ c. $\frac{20!}{5!2!3!}$ d. none of these

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287. The coefficient of x^5 in the expansion of $\left(x^2 - x - 2\right)^5$ is -83 b. -82 c. -86 d. -81

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288. The coefficient of $a^8 b^4 c^9 d^9$ in $(abc + abd + acd + bcd)^{10}$ is $10!$ b. $\frac{10!}{8!4!9!9!}$ c. 2520 d. none of these

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289. If the coefficient of $x^7 \in \left[ax^2 - \left(\frac{1}{bx^2}\right)\right]^{11}$ equal the coefficient of x^{-7} in satisfy the $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, $thena$ and b satisfy the relation a. $a + b = 1$ b. $a - b = 1$ c. $ab = 1$ d. $\frac{a}{b} = 1$

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290. If $(1+x)^5 = a_0 + a_1 x + a2x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$, then the value of $(a_0 - a_2 + a_4)^2 + (a_1 - a_3 + a_5)^2$ is equal to 243 b. 32 c. 1 d. 2^{10}

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291. The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is

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292. The coefficient of x^{28} in the expansion of $\left(1+x^3-x^6\right)^{30}$ is a 1 b. 0 c. 30^C_6 d. $\hat{}\ 30C_3$

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293. The coefficient of x^n in $(1+x)^{101}\left(1-x+x^2\right)^{100}$ is non zero, then n cannot be of the form a. $3r+1$ b. $3r$ c. $3r+2$ d. none of these

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294.

$$\sum_{r=0}^{n}(-1)^{r}\ \hat{}\ nC_r\left[\frac{1}{2^r}+\frac{3}{2^{2r}}+\frac{7}{2^{3r}}+\frac{15}{2^{4r}}+up\rightarrow mterms\right]=\frac{2^{mn}-}{2^{mn}(2^n-}$$

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295. In the expansion of $\left(7^{1/3}+11^{1/9}\right)^{6561}$, (a)there are exactly 730 rational term (b)there are exactly 5831 irrational terms (c)the term which involves greatest binomial coefficients is irrational (d)the term which involves greatest binomial coefficients is rational

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296. If for z as real or complex, $\left(1+z^2+z^4\right)^8=C_0+C1z2+C2z4++C_{16}z^{32}then$ (a) $C_0-C_1+C_2-C_3++C_{16}=1$ (b) $C_0+C_3+C_6+C_9+C_{12}+C_{15}=3^7$ (c)

$C_2 + C_5 + C_6 + C_{11} + C_{14} = 3^6$ (d)

$C_1 + C_4 + C_7 + C_{10} + C_{13} + C_{16} = 3^7$

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297. If

$f(m) = \sum_{i=0}^{m} (30(\hat{\ })30 - i)(20(\hat{\ })m - i) where (pq) =^p C_q, then$ (a)

maximum value of $f(m) is^{50} C_{25}$ (b)$f(0) + f(1) + \ldots f(50) = 2^{50}$ (c)$f(m)$

is always divisible by $50(1 \leq m \leq 49)$ (d)The value of

$\sum_{m=0}^{50} (f(m))^2 =^{100} C_{50}$

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298. The sum of coefficient in the expansion of $\left(1 + ax - 2x^2\right)^n$ is

(a)positive, when $a < 1 and n = 2k, k \in N$ (b)negative, when

$a < 1 and n = 2k + 1, k \in N$ (c)positive, when $a < 1 and n \in N$ (d)zero,

when $a = 1$

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299. If the 4th term in the expansion of $(ax + 1/x)^n$ is 5/2, then a. $a = \frac{1}{2}$ b. $n = 8$ c. $a = \frac{2}{3}$ d. $n = 6$

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300. The number of values of r satisfying the equation 69 C 3r−1 − 69 C r 2 = 69 C r 2 −1 − 69 C 3r is:

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301. If $\left(4 + \sqrt{15}\right)^n = I + f$, where n is an odd natural number, I is an integer and ,then a.$I$is an odd integer b. $I$is an even integer c. $(I + f)(1 - f) = 1$ d. $1 - f = \left(4 - \sqrt{15}\right)^n$

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302. In the expansion of $(x+a)^n$ if the sum of odd terms is P and the sum of even terms is Q, then (a)$P^2 - Q^2 = (x^2 - a^2)^n$ (b) $4PQ = (x+a)^{2n} - (x-a)^{2n}$ (c) $2(P^2 + Q^2) = (x+a)^{2n} + (x-a)^{2n}$ (d)none of these

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303. If the coefficients of rth, $(r+1)th, and(r+2)th$ terms in the expansion of $(1+x)^{14}$ are in A.P., then r is/are a. 5 b. 11 c. 10 d. 9

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304. The value of x in the expression $\left(x + x^{(\log)_{10}x}\right)^5$ if third term in the expansion is 10,00,000 is/are

a. 10 b. 100 c. $10^{-5/2}$ d. $10^{-3/2}$

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305. Let $R = \left(5\sqrt{5} + 11\right)^{2n+1} and f = R - [R] where []$ denotes the greatest integer function, prove that $Rf = 4^{2n+1}$

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306. If $|x| < 1,$ then the coefficient of x^n in expansion of $\left(1 + x + x^2 + x^3 + \right)^2$ is a. n b. $n - 1$ c. $n + 2$ d. $n + 1$

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307. The coefficient of $x^5 \in \left(1 + 2x + 3x^2 + \right)^{-3/2} is(|x| < 1)$ 21 b. 25 c. 26 d. none of these

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308. If x is so small that x^3 and higher powers of x may be neglectd, then $\frac{(1+x)^{3/2} - \left(1 + \frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as a. $3x + \frac{3}{8}x^2$ b.

$1-\frac{3}{8}x^2$ c. $\frac{x}{2}-\frac{3}{\times^2}$ d. $-\frac{3}{8}x^2$

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309. If x is positive, the first negative term in the expansion of $(1+x)^{27/5} is(|x|<1)$ a.$5thterm$ b. $8thterm$ c. $6thterm$ d. $7thterm$

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310. Value of $\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3^k}(kC_r)$ is $\frac{2}{3}$ b. $\frac{4}{3}$ c. 2 d. 1

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311. If the expansion in powers of x of the function $1/[(1-ax)(1-bx)]$ is a$a_0+a_1x+a_2x^2+a_3x^3+, then a_n is$ a.$\frac{b^n-a^n}{b-a}$ b. $\frac{a^n-b^n}{b-a}$ c. $\frac{b^{n+1}-a^{n+1}}{b-a}$ d. $\frac{a^{n+1}-b^{n+1}}{b-a}$

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312. If $f(x) = 1 - x + x^2 - x^3 + \quad +^{15} + x^{16} - x^{17}$, then the coefficient of $x^2 \in f(x-1)$ is 826 b. 816 c. 822 d. none of these

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313. The sum of rational term in $\left(\sqrt{2} + \sqrt[3]{3} + \sqrt[6]{5}\right)^{10}$ is equal to 12632 b. 1260 c. 126 d. none of these

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314. The value of $\sum_{r=0}^{10} (r)^{20} C_r$ is equal to: a. $20\left(2^{18} +^{19} C_{10}\right)$ b. $10\left(2^{18} +^{19} C_{10}\right)$ c. $20\left(2^{18} +^{19} C_{11}\right)$ d. $10\left(2^{18} +^{19} C_{11}\right)$

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315. If $p = \left(8 + 3\sqrt{7}\right)^n and f = p - [p], where[.]$ denotes the greatest integer function, then the value of $p(1-f)$ is equal to

A. 1

B. 2

C. 2^n

D. 2^{2n}

Answer: A

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316. Statement 1: Greatest term in the expansion of $(1+x)^{12}$, when x=11//10 is 7th .

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317. Statement 1: Remainder $when 3456^{2222}$ is divided by 7 is 4. Statement 2: Remainder $when 5^{2222}$ is divided by 7 is 4.

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318. the value of x , for which the 6th term in the expansions of $\left[2^{\log_2}\left(\sqrt{9^{(x-1)+7}}\right)+\frac{1}{2^{\frac{1}{5}}(\log)_2\left(3^{x-1}+1\right)}\right]^7 is84$, is equal to a. 4 b. 3 c. 2 d. 1

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319. Each question has four choices a, b, c and d, out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. Both the statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT1. Both the statements are TRUE but STATEMENT 2 is NOT the correct explanation of STATEMENT 1. STATEMENT 1 is TRUE and STATEMENT 2 is FALSE. STATEMENT 1 is FALSE and STATEMENT 2 is TRUE. Statement 1: The value of $\left(\hat{}\,(10)^C{}_0\right)+\left(\hat{}\,(10)C_0+(10)C_1\right)+\left(\hat{}\,(10)C_0+(10)C_1+(10)C_2\right)$ is 102^9 . Statement 2: $\hat{}\,nC_1+2^nC_2+3^nC_3+n^nC_n=n2^{n-1}$.

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320. The number $51^{49}+51^{48}+51^{47}+\ldots\ldots.+51+1$ is divisible by a.

10 b. 20 c. 25 d. 50

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321. If $\sum_{r=0}^{n}\frac{r}{{}^{n}C_r}=\sum_{r=0}^{n}\frac{n^2-3n+3}{2.\,{}^{n}C_r}$, then

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322. If $(1+x)^n=C_0+C_1x+C_2x^2+\ldots\ldots+C_nx^n$, then show that the sum of the products of the coefficients taken two at a time, represented by $\sum\sum_{0\le i<j\le n}{}^{n}c_i{}^{n}c_j$ is equal to $2^{2n-1}-\frac{(2n)!}{2(n!)^2}$

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323. For any positive integer (m,n) (with $n\ge m$), Let $\binom{n}{m}=.^{n}C_m$

Prove that

$\binom{n}{m}+2\binom{n-1}{m}+3\binom{n-2}{m}+\ldots.+(n-m+1)\binom{m}{m}=$ $\binom{n+2}{m+2}$

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324. If $\sum_{r=0}^{n}\left\{a_r(x-\alpha+2)^r-b_r(\alpha-x-1)^r\right\}=0,$ then prove that $b_n-(-1)^n a_n=0.$

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325. Let $a=\left(2^{1/401}-1\right)$ and for each $n\geq 2, letb_n={}^nC_1+{}^nC_2\dot{a}+{}^nC_3a^2+\ldots\ldots+{}^nC_n\cdot a^{n-1}$. Find the value of $\left(b_{2006}-b_{2005}\right)$.

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326. Prove that

$$\sum_{r=0}^{n} \hat{}\ nC_r(-1)^r\left[i^r + i^{2r} + i^{3r} + i^{4r}\right] = 2^n + 2^{\frac{n}{2}+1}\cos(n\pi/4),$$

where $i = \sqrt{-1}$.

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327. Find the coefficient of x^n in $\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots\ldots. + \frac{x^n}{n!}\right)^2$.

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328. Prove that

$$\frac{\hat{}\ nC_0}{x} - \frac{\hat{}\ nC_0}{x+1} + \frac{\hat{}\ nC_1}{x+2} - +(-1)^n\frac{\hat{}\ nC_n}{x+n} = \frac{n!}{x(x+1)(x-n)},$$

where n is any positive integer and x is not a negative integer.

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329. If n is a positive integer, prove that $1-2n+\frac{2n(2n-1)}{2!}-\frac{2n(2n-1)(2n-2)}{3!}++(-1)^{n-1}\frac{2n(2n-1)(}{(n-1}$

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330. Given, $s_n=1+q+q^2+\ldots..+q^n, S_n=1+\frac{q+1}{2}+\left(\frac{q+1}{2}\right)^2+\ldots+\left(\frac{q+}{2}\right.$ prove that ${}^{n+1}C_1+{}^{n+1}C_2s_1+{}^{n+1}C_3s_2+\ldots\ldots+{}^{n+1}C_{n+1}s_n=2^nS_n.$

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331. The sum of $1+n\left(1-\frac{1}{x}\right)+\frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2+\infty$ will be a. x^n b. x^{-n} c. $\left(1-\frac{1}{x}\right)^n$ d. none of these

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332. $\sum_{k=1}^{\infty} k\left(1-\frac{1}{n}\right)^{k-1} \Rightarrow ? a.$ n(n-1)$b.$ n(n+1)$c.$ n^2$d.$ (n+1)^2`

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333. The coefficient of x^4 in the expansion of $\left\{\sqrt{1+x^2}-x\right\}^{-1}$ in ascending powers of $x,$ when $|x|<1, is$ a. 0 b. $\frac{1}{2}$ c. $-\frac{1}{2}$ d. $-\frac{1}{8}$

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334. $1+\frac{1}{3}x+\frac{1\times 4}{3\times 6}x^2+\frac{1\times 4\times 7}{3\times 6\times 9}x^3+$ ------ is equal to a. x b. $(1+x)^{1/3}$ c. $(1-x)^{1/3}$ d. $(1-x)^{-1/3}$

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335. The value of $\sum_{r=1}^{15} \frac{r2^r}{(r+2)!}$ is (a). $\frac{(17)!-2^{16}}{(17)!}$ (b). $\frac{(18)!-2^{17}}{(18)!}$ (c). $\frac{(16)!-2^{15}}{(16)!}$ (d). $\frac{(15)!-2^{14}}{(15)!}$

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336. $(n+2)C_0\left(2^{n+1}\right)-(n+1)C_1\left(2^n\right)+(n)C_2\left(2^{n-1}\right)-\ldots.$ is equal to

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337. The value of $\sum_{r=0}^{50}(-1)^r\frac{(50)C_r}{r+2}$ is equal to $a.$ $\frac{1}{50\times 51}$ b. $\frac{1}{52\times 50}$ c. $\frac{1}{52\times 51}$ d. none of these

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338. In the expansion of $\left[(1+x)/(1-x)\right]^2,$ the coefficient of x^n will be a.$4n$ b. $4n-3$ c. $4n+1$ d. none of these

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339. Statement : The sum of coefficient in the expansion of $\left(3^{-x/4} + 3^{5x/4}\right)^{n} is 2^{n}$.

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340. Let n be a positive integer and k be a whole number, $k \leq 2n$. Statement 1: The maximum value of $\hat{}\ 2nC_k is^{2n}C_n$. Statement 2: $\frac{\hat{}\ (2n)C_{k+1}}{\hat{}\ (2n)C_k} \langle 1, f \text{ or } k = 0, 1, 2, , n-1 and \frac{\hat{}\ (2n)C_k}{\hat{}\ (2n)C_{k-1}\rangle} 1, f \text{ or } k = n$

.

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341. $Statement1: \sum \sum_{0 \leq i < j \leq n} \left(\frac{i}{.^{n}c_i} + \frac{j}{.^{n}c_j}\right)$ is equal to $\frac{n^2}{2}a$, where a , $\sum_{r=0}^{n} \frac{1}{.^{n}c_r} = a\ Statement2: \sum_{r=0}^{n} \frac{r}{.^{n}c_r} = \sum_{r=0}^{n} \frac{n-r}{.^{n}c_r}$

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342. Statement 1: $\hat{\ } mC_r +^m C_{r-1}(\hat{\ } nC_1) +^m C_{r-2}(\hat{\ } nC_2) + \dots. +^n C_r = 0,$ if $m + n < r$

Statement 2: $\hat{\ } nC_r = 0$, if $n < r$

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343. $1 + \left(\frac{1}{4}\right) + \left(\frac{1 \cdot 3}{4 \cdot 8}\right) + \left(\frac{1 \cdot 4 \cdot 7}{4 \cdot 8 \cdot 12}\right) + \dots. =$

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344. If $|x| < 1, then 1 + n\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \dots\dots$ is equal to a. $\left(\frac{2x}{1+x}\right)^n$ b. $\left(\frac{1+x}{2x}\right)^n$ c. $\left(\frac{1-x}{1+x}\right)^n$ d. $\left(\frac{1+x}{1-x}\right)^n$

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345. Statement 1: If p is a prime number $(p \neq 2)$, then $\left[\left(2+\sqrt{5}\right)^{p}\right]-2^{p+1}$ is always divisible by $p(where[.]$ denotes the greatest integer function). Statement 2: if n prime, then $\hat{} \ nC_1,^n C_2,^n C_2,^n C_{n-1}$ must be divisible by n.

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346. Statement 1: The total number of dissimilar terms in the expansion of $(x_1+x_2+\ +x_n)^3 is \dfrac{n(n+1)(n+2)}{6}$.

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347. Statement 1: In the expansion of $(1+x)^{41}\left(1-x+x^2\right)^{40}$, the coefficient of x^{85} is zero. Statement 2: In the expansion of $(1+x)^{41} and \left(1-x+x^2\right)^{40}, x^{85}$ term does not occur.

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348. Statement 1: The coefficient of x^n in $\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}++\frac{x^n}{n!}\right)^3$ is $\frac{3^n}{n!}$. Statement 2: The coefficient of x^n in e^{3x} is $\frac{3^n}{n!}$

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349. Evaluate $3C_2$

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350. Evaluate $5C_2$

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351. Find $\sum_{r=0}^{10} r^{10}C_r.3^r.(-2)^{10-r}$

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352. Find n if $nP_1 = 2$

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353. Evaluate $5P_2$

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354. The value of $\frac{{}^nC_0}{n} + \frac{{}^nC_1}{n+1} + \frac{{}^nC_2}{n+2} + ... + \frac{{}^nC_n}{2n}$ is equal to a. $\int_0^1 x^{n-1}(1-x)^n dx$ b. $\int_1^2 x^n(x-1)^{n-1} dx$ c. $\int_1^2 x^{n-1}(1+x)^n dx$ d. $\int_0^1 (1-x)^{n-1} dx$

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355. The value of ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + {}^{20}C_3 + {}^{20}C_4 + {}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15}$ is

a. $2^{19}-\dfrac{\left(\hat{}(20)\mathrm{C\_10}+\ {}^{20}C_9\right)}{2}$ b. $2^{19}-\dfrac{(\ \hat{}\ (20)C10+2\times^{20}C9)}{2}$ c. $2^{19}-\dfrac{\hat{}\ (20)C10}{2}$ d. none of these

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356. If $\left(3+x^{2008}+x^{2009}\right)^{2010}=a_0+a_1x+a_2x^2+\ +a_nx^n,$ then the value of $a_0-\frac{1}{2}a_1-\frac{1}{2}a_2+a_3-\frac{1}{2}a_4-\frac{1}{2}a_5+a_6-$ is a.3^{2010} b. 1 c. 2^{2010} d. none of these

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357.

$\hat{A}\hat{A}404C4\hat{A}\hat{a}'\hat{A}\hat{A}4C1\hat{A}\hat{A}\hat{A}303C4\hat{A}+\hat{A}\hat{A}4C2\hat{A}\hat{A}\hat{A}202C4\hat{A}\hat{a}'\hat{A}\hat{A}4C3\hat{A}\hat{A}\hat{A}$

is equal to a. (401)4 b. (101)4 c. 0 d. (201)4

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358. The sum of series $^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots\dots\dots\dots\dots\dots\dots\dots + {}^{20}C_{10}$ is

a. $\frac{1}{2}{}^{20}C10$

b. 0

c. $^{20}C10$

d. $^{20}C10$

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359. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, then $C_0C_2 + C_1C_3 + C_2C_4 + \dots + C_{n-2}C_n =$ a. $\frac{(2n)!}{(n!)^2}$ b. $\frac{(2n)!}{(n-1)!(n+1)!}$ c. $\frac{(2n)!}{(n-2)!(n+2)!}$ d. none of these

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360. The value of $\lim_{n\to\infty} \sum_{r=0}^{n} \left(\sum_{t=0}^{r-1} \frac{1}{5^n} \cdot {}^nC_r \cdot {}^rC_t . (3^t) \right)$ is equal to

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361. Prove that

$C_0 - 2^2 C_1 + 3^2 C_2 - 4^2 C_3 + \ + (- 1)^n (n + 1)^2 \times C_n = 0 where C_r =^n C_r$

.

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362. The remainder, if $1 + 2 + 2^2 + \ldots\ldots + 2^{1999}$ is divided by 5 is.

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363. Find the largest real value of x such that

$$\sum_{k=0}^{4} \left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right) = \frac{32}{3}.$$

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364. If in the expansion of $(a - 2b)^n$, the sum of 5th and 6th terms is 0, then the values of $\frac{a}{b}$ a. $\frac{n-4}{5}$ b. $\frac{2(n-4)}{5}$ c. $\frac{5}{n-4}$ d. $\frac{5}{2(n-4)}$

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365. The number of real negavitve terms in the binomial expansion of $(1 + ix)^{4n-2}, n \in N, n > 0, I = \sqrt{-1}$, is

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