MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

CONIC SECTIONS

Others

1. Let $A(0, 1), B(1, 1), C(1, -1), D(-1, 0)$ be four points. If P is any other point, then $PA + PB + PC + PD \geq d$, where $[d]$ represents greatest integer.

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2. If $\left(a\cos\theta_1, a\sin\theta_1\right), \left(a\cos\theta_2, a\sin\theta_2\right)$ and $\left(a\cos\theta_3, a\sin\theta_3\right)$ represent the vertices of an equilateral triangle inscribed in a circle, then (a)

$\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$ (b) $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 0$ (c)

$\tan\theta_1 + \tan\theta_2 + \tan\theta_3 = 0$ (d) $\cot\theta_1 + \cot\theta_2 + \cot\theta_3 = 0$

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3. The area of triangle ABC is $20cm^2$ The coordinates of vertex A are $-5, 0)$

and those of B are $(3, 0)$ The vertex C lies on the line $x - y = 2$. The

coordinates of C are

(a)$(5, 3)$ (b) $(- 3, - 5)$ $(- 5, - 7)$ (d) $(7, 5)$

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4. If a, b, c are the pth, qth, rth terms, respectively, of an HP , show that the

points $(bc, p), (ca, q),$ and (ab, r) are collinear.

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5. Let $ABCD$ be a rectangle and P be any point in its plane. Show that

$AP^2 + PC^2 = PB^2 + PD^2\.$

6. A rod of length k slides in a vertical plane, its ends touching the coordinate axes. Prove that the locus of the foot of the perpendicular from the origin to the rod is $\left(x^2 + y^2\right)^3 = k^2 x^2 y^2\.$

7. Prove that the circumcenter, orthocentre, incenter, and centroid of the triangle formed by the points $A(-1, 11), B(-9, -8),$ and $C(15, -2)$ are collinear, without actually finding any of them.

8. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in GP with the same common ratio, then the points $\left(x_1, y_1\right), \left(x_2, y_2\right),$ and $\left(x_3, y_3\right)$ (a)lie on a straight line (b)lie on an ellipse (c)lie on a circle (d) are the vertices of a triangle.

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9. Statement 1 :If the lines $2x + 3y + 19 = 0$ and $9x + 6y - 17 = 0$ cut the x-axis at A, B and the y-axis at $C, D,$ then the points, A, B, C, D are concyclic. Statement 2 : Since $OAxOB = OCxOD,$ where O is the origin, A, B, C, D are concyclic.

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10. If the points $\left(x_1, y_1\right), \left(x_2, y_2\right),$ and $\left(x_3, y_3\right)$ are collinear show that $\dfrac{y_2 - y_3}{x_2 x_3} + \dfrac{y_3 - y_1}{x_3 x_1} + \dfrac{y_1 - y_2}{x_1 x_2} = 0$

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11. Find $\dfrac{dy}{dx}$, **if** $y = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$.

12. The coordinates of A, B, C are $(6, 3), (- 3, 5), (4, - 2)$, respectively, and P is any point (x, y) . Show that the ratio of the area of PBC to that of ABC is $\dfrac{|x + y - 2| \cdot}{7}$

13. A line cuts the x-axis at $A(7, 0)$ and the y-axis at $B(0, - 5)$ A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R.

14. For points $P \equiv \left(x_1, y_1\right)$ and $Q \equiv \left(x_2, y_2\right)$ of the coordinate plane, a new distance $d(P, Q) = \left|x_1 - x_1\right| + \left|y_1 - y_2\right|$. Let $O = (0, 0)$ and $A = (3, 2)$. Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram.

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15. Statement 1 : Let the vertices of a ABC be $A(-5, -2), B(7, 6)$, and $C(5, -4)$. Then the coordinates of the circumcenter are $(1, 2)$ Statement 2 : In a right-angled triangle, the midpoint of the hypotenuse is the circumcenter of the triangle.

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16. If (x, y) and (x, y) are the coordinates of the same point referred to two sets of rectangular axes with the same origin and it $ux + vy$, where u and v are independent of x and y, becomes $VX + UY$, show that $u^2 + v^2 = U^2 + V^2$.

17. OX and OY are two coordinate axes. On OY a fixed point $P(0, c)$ is taken and on OX any point Q is taken. On PQ, an equilateral triangle is described, its vertex R being on the side of PQ away from O. Then prove that the locus of R is $y = \sqrt{3}x - c$

18. Two vertices of a triangle are $(5, -1)$ and $(-2, 3)$ If the orthocentre of the triangle is the origin, find the coordinates of the third point.

19. The vertices of a triangle are $\left[at_1t_2, a\left(t_1 + t_2\right)\right]$, $\left[at_2t_3, a\left(t_2 + t_3\right)\right]$, $\left[at_3t_1, a\left(t_3 + t_1\right)\right]$ Then the orthocenter of the triangle is (a) $\left(-a, a\left(t_1 + t_2 + t_3\right) - at_1t_2t_3\right)$ (b) $\left(-a, a\left(t_1 + t_2 + t_3\right) + at_1t_2t_3\right)$ (c) $\left(a, a\left(t_1 + t_2 + t_3\right) + at_1t_2t_3\right)$ (d) $\left(a, a\left(t_1 + t_2 + t_3\right) - at_1t_2t_3\right)$

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20. If (- 6, - 4), (3, 5), (- 2, 1) are the vertices of a parallelogram, then the remaining vertex can be (a)(0, - 1) (b) 7, 9) (c)(- 1, 0) (d) (- 11, - 8)

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21. The maximum area of the triangle whose sides a,b and c satisfy $0 \le a \le 1, 1 \le b \le 2$ and $2 \le c \le 3$ is

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22. If (- 4, 0) and (1, - 1) are two vertices of a triangle of area $4 sq units$, then its third vertex lies on (a)$y = x$ (b) $5x + y + 12 = 0$ (c)$x + 5y - 4 = 0$ (d) $x + 5y + 12 = 0$

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23. Let $O \equiv (0, 0), A \equiv (0, 4), B \equiv (6, 0)$ Let P be a moving point such that the area of triangle POA is two times the area of triangle POB . The locus of P will be a straight line whose equation can be

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24. Through the point $P(\alpha, \beta)$, where $\alpha\beta > 0,$ the straight line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is drawn so as to form a triangle of area S with the axes. If $ab > 0,$ then the least value of S is(a) $\alpha\beta$ (b) $2\alpha\beta$ (c) $3\alpha\beta$ (d) none

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25. The vertices A and D of square $ABCD$ lie on the positive sides of x and y axis , respectively. If the vertex C is the point $(12, 17)$, then the coordinates of vertex B are (a)$(14, 16)$ (b)$(15, 3)$ (c)$(17, 5)$ (d) $(17, 12)$

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26. A light ray emerging from the point source placed at $P(2, 3)$ is reflected at a point Q on the y-axis. It then passes through the point $R(5, 10)$ The coordinates of Q are (a) $(0, 3)$ (b) $(0, 2)$ $(0, 5)$ (d) none of these

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27. If the origin is shifted to the point $\left(\dfrac{ab}{a - b}, 0\right)$ without rotation, then the equation $(a - b)\left(x^2 + y^2\right) - 2abx = 0$ becomes

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28. In $ABC,$ the coordinates of B are $(0, 0)$, $AB = 2$, $\angle ABC = \dfrac{\pi}{3}$, and the middle point of BC has coordinates $(2, 0)$ The centroid o the triangle is

$\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$ (b) $\left(\dfrac{5}{3}, \dfrac{1}{\sqrt{3}}\right)$ $\left(4 + \dfrac{\sqrt{3}}{3}, \dfrac{1}{3}\right)$ (d) none of these

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29. If a triangle $ABC, A \equiv (1, 10),$ circumcenter $\equiv \left(-\dfrac{1}{3}, \dfrac{2}{3}\right),$ and orthocentre $\equiv \left(\dfrac{11}{3}, \dfrac{4}{3}\right)$, then the coordinates of the midpoint of the side opposite to A are $\left(1, -\dfrac{11}{3}\right)$ (b) $(1, 5)$ $(1, -3)$ (d) $(1, 6)$

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30. A triangle ABC with vertices A(-1,0),B(-2,3/4) & C(-3,-7/6) has its orthocentre H, then the orthocentre of triangle BCH will be

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31. In ABC, if the orthocentre is $(1, 2)$ and the circumcenter is $(0, 0)$, then centroid of ABC) is (a) $\left(\dfrac{1}{2}, \dfrac{2}{3}\right)$ (b) $\left(\dfrac{1}{3}, \dfrac{2}{3}\right)$ (c) $\left(\dfrac{2}{3}, 1\right)$ (d) none of these

32. If the vertices of a triangle are $\left(\sqrt{5}, 0\right)$, $\left(\sqrt{3}, \sqrt{2}\right)$, and $(2, 1)$, then the orthocentre of the triangle is (a) $\left(\sqrt{5}, 0\right)$ (b) $(0, 0)$ $\left(\sqrt{5} + \sqrt{3} + 2, \sqrt{2} + 1\right)$ (d) none of these

33. The vertices of a triangle are $\left(pq, \dfrac{1}{pq}\right)$, (pq), $\left(qr, \dfrac{1}{qr}\right)$, and $\left(rq, \dfrac{1}{rp}\right)$, where p, q and r are the roots of the equation $y^3 - 3y^2 + 6y + 1 = 0$. The coordinates of its centroid are $(1, 2)$ (b) $(2, -1)$ (c) $(1, -1)$ (d) $(2, 3)$

34. If two vertices of a triangle are $(-2, 3)$ and $(5, -1)$ the orthocentre lies at the origin, and the centroid on the line $x + y = 7$, then the third vertex lies at $(a)(7, 4)$ (b) $(8,14)$(c) $(12 ,21)$` (d) none of these

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35. P and Q are points on the line joining $A(-2, 5)$ and $B(3, 1)$ such that $AP = PQ = QB$. Then the distance of the midpoint of PQ from the origin is (a)3 (b) $\dfrac{\sqrt{37}}{2}$ (c) 4 (d) 3.5

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36. The point $(4, 1)$ undergoes the following three transformations successively: (a) Reflection about the line $y = x$ (b) Translation through a distance 2 units along the positive direction of the x-axis. (c) Rotation through an angle $\dfrac{\pi}{4}$ about the origin in the anti clockwise direction. The final position of the point is given by the co-ordinates.

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37. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle POR is (are) always rational point(s) ?

38. If $P(1, 2), Q(4, 6), R(5, 7),$ and $S(a, b)$ are the vertices of a parallelogram $PQRS,$ then (a)$a = 2, b = 4$ (b) $a = 3, b = 4$ (c)$a = 2, b = 3$ (d) $a = 1, b = -1$

39. If the area of the triangle formed by the points $(2a, b)(a + b, 2b + a),$ and $(2b, 2a)$ is $2q units,$ then the area of the triangle whose vertices are $(a + b, a - b), (3b - a, b + 3a),$ and $(3a - b, 3b - a)$ will be____

40. The incenter of the triangle with vertices $\left(1, \sqrt{3}\right)$, $(0, 0)$, and $(2, 0)$ is

(a) $\left(1, \dfrac{\sqrt{3}}{2}\right)$ (b) $\left(\dfrac{2}{3}, \dfrac{1}{\sqrt{3}}\right)$ (c) $\left(\dfrac{2}{3}, \dfrac{\sqrt{3}}{2}\right)$ (d) $\left(1, \dfrac{1}{\sqrt{3}}\right)$

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41. The locus of the moving point whose coordinates are given by $\left(e^t + e^{-t}, e^t - e^{-t}\right)$ where t is a parameter, is $xy = 1$ (b) $x + y = 2$ $x^2 - y^2 = 4$ (d) $x^2 - y^2 = 2$

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42. The distance between the circumcenter and the orthocentre of the triangle whose vertices are $(0, 0)$, $(6, 8)$, and $(-4, 3)$ is L Then the value of $\dfrac{2}{\sqrt{5}}L$ is_____

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43. A man starts from the point $P(-3, 4)$ and reaches the point $Q(0, 1)$ touching the x-axis at $R(\alpha, 0)$ such that $PR + RQ$ is minimum. Then $5|\alpha|$ (A) 3 (B) 5 (C) 4 (D) 2

44. Statement 1 : The area of the triangle formed by the points $A(1000, 1002)$, $B(1001, 1004)$, $C(1002, 1003)$ is the same as the area formed by the point $A'(0, 0)$, $B'(1, 2)$, $C'(2, 1)$ Statement 2 : The area of the triangle is constant with respect to the translation of axes.

45. Consider three points $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$, and $R = ((\cos(\beta - \alpha + \theta), \sin(\beta - \theta)))$, where $0 < \alpha, \beta, \theta < \dfrac{\pi}{4}$ Then

46. abc

47. Each equation contains statements given in two columns which have to be matched. Statements (a,b,c,d) in column I have to be matched with Statements (p, q, r, s) in column II. If the correct match are $a\vec{p}, a\vec{s}, b\vec{q}, b\vec{r}, c\vec{p}, c\vec{q}$, and $d\vec{s}$, then the correctly bubbled 4x4 matrix should be as follows: Figure Consider the lines represented by equation $\left(x^2 + xy - x\right)x(x - y) = 0$, forming a triangle. Then match the following: Column I|Column II Orthocenter of triangle |p. $\left(\dfrac{1}{6}, \dfrac{1}{2}\right)$ Circumcenter|q. $\left(1\left(2 + 2\sqrt{2}\right), \dfrac{1}{2}\right)$ Centroid|r. $\left(0, \dfrac{1}{2}\right)$ Incenter|s. $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$

48. A straight line passing through $P(3, 1)$ meets the coordinate axes at $A and B$. It is given that the distance of this straight line from the origin O

is maximum. The area of triangle OAB is equal to $\dfrac{50}{3}$ squnits (b) $\dfrac{25}{3}$ squnits $\dfrac{20}{3}$ squnits (d) $\dfrac{100}{3}$ squnits

49. Let $A \equiv (3, -4), B \equiv (1, 2)$ Let $P \equiv (2k - 1, 2k + 1)$ be a variable point such that $PA + PB$ is the minimum. Then k is 7/9 (b) 0 (c) 7/8 (d) none of these

50. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$

then the two triangles with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and $(a_1, b_1), (a_2, b_2), (a_3, b_3)$ are

51. *OPQR* is a square and *M, N* are the middle points of the sides *PQandQR* , respectively. Then the ratio of the area of the square to that of triangle *OMN* is (a) 8:1 (b) 2:1 (c) 8:3 (d) 7:3

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52. Which of the following sets of points form an equilateral triangle? $(a)(1, 0), (4, 0), (7, -1)$ $(b)(0, 0), \left(\dfrac{3}{2}, \dfrac{4}{3}\right), \left(\dfrac{4}{3}, \dfrac{3}{2}\right)$ $(c)\left(\dfrac{2}{3}, \right), \left(0, \dfrac{2}{3}\right), (1, 1)$ (d) None of these

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53. A particle *p* moves from the point $A(0, 4)$ to the point $10, -4)$. The particle *P* can travel the upper-half plane $\{(x, y) \mid y \geq \}$ at the speed of $1m/s$ and the lower-half plane $\{(x, y) \mid y \leq 0\}$ at the speed of 2 m/s. The coordinates of a point on the x-axis, if the sum of the squares of the

travel times of the upper- and lower-half planes is minimum, are (a) (1, 0) (b) (2, 0) (c) (4, 0) (d) (5, 0)

54. ABC is an isosceles triangle. If the coordinates of the base are $B(1, 3)$ and $C(-2, 7)$, the coordinates of vertex A can be $(1, 6)$ (b) $\left(-\dfrac{1}{2}, 5\right)$ $\left(\dfrac{5}{6}, 6\right)$ (d) none of these

55. If two vertices of a triangle are (1,3) and (4,-1) and the area of triangle is 5 sq. units, then the angle at the third vertex lies in :

56. Number of points with integral co-ordinates that lie inside a triangle whose co-ordinates are (0, 0), (0, 21) and (21,0).

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57. Let $O(0, 0)$, $P(3, 4)$, and $Q(6, 0)$ be the vertices of triangle OPQ . The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are a. $\left(\frac{4}{3}, 3\right)$ b. $\left(3, \frac{2}{3}\right)$ c. $\left(3, \frac{4}{3}\right)$ d. $\left(\frac{4}{3}, \frac{2}{3}\right)$

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58. The orthocentre of the triangle with vertices $(0, 0), (3, 4)$, and $(4, 0)$ is

(a) $\left(3, \frac{5}{4}\right)$ (b) $(3, 12)$ (c) $\left(3, \frac{3}{4}\right)$ (d) $(3, 9)$

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59. The area of a triangle is 5. Two of its vertices are $A(2, 1)$ and $B(3, -2)$.

The third vertex C is on $y = x + 3$. Find C

60. Statement 1 : If the vertices of a triangle are having rational coordinates, then its centroid, circumcenter, and orthocentre are rational.

Statement 2 : In any triangle, orthocentre, centroid, and circumcenter are collinear, and the centroid divides the line joining the orthocentre and circumcenter in the ratio 2:1.

61. If $A\left(1, p^2\right)$, $B(0, 1)$ and $C(p, 0)$ are the coordinates of three points, then the value of p for which the area of triangle ABC is the minimum is (a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{2}}$ (d) none of these

62. If the point $\left(x_1 + t\left(x_2 - x_1\right), y_1 + t\left(y_2 - y_1\right)\right)$ divides the join of $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ internally, then $t < 0$ (b) `01(d)t=1`

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63. $OPQR$ is a square and M, N are the midpoints of the sides PQ and QR, respectively. If the ratio of the area of the square to that of triangle OMN is $\lambda : 6$, then $\dfrac{\lambda}{4}$ is equal to 2 (b) 4 (c) 2 (d) 16

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64. If $\displaystyle\sum_{i-1}^{4}\left(x1^2 + y1^2\right) \le 2x_1x_3 + 2x_2x_4 + 2y_2y_3 + 2y_1y_4,$ the points $\left(x_1, y_1\right), \left(x_2, y_2\right), \left(x_3, y_3\right), \left(x_4, y_4\right)$ are the vertices of a rectangle collinear the vertices of a trapezium

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65. In an acute triangle ABC , if the coordinates of orthocentre H are $(4, b)$, of centroid G are $(b, 2b - 8)$ and of circumcenter S are $(- 4, 8)$, then b cannot be (a) 4 (b) 8 (c) 12 (d) -12

66. Consider the points O(0,0),A(0,1), and B(1,1) in the x-y plane. Suppose that points C(x,1)andD(1,y) are chosen such that 0 < x < 1. And such that O,C,andD are collinear. Let the sum of the area of triangles OAC and BCD be denoted by S. Then which of the following is/are correct?.

67. If all the vertices of a triangle have integral coordinates, then the triangle may be (a) right-angle (b) equilateral (c) isosceles (d) none of these

68. The locus of a point represent by

$$x = \frac{a}{2}\left(\frac{t+1}{t}\right), y = \frac{a}{2}\left(\frac{t-1}{t}\right), \text{where } t = \ \in R - \{0\}, \text{is}$$

69. The points $A(0, 0)$, $B(\cos\alpha, \sin\alpha)$ and $C(\cos\beta, \sin\beta)$ are the vertices of a right-angled triangle then

70. The ends of a diagonal of a square are $(2, -3)$ and $(-1, 1)$ Another vertex of the square can be (a) $\left(-\frac{3}{2}, -\frac{5}{2}\right)$ (b) $\left(\frac{5}{2}, \frac{1}{2}\right)$ (c) $\left(\frac{1}{2}, \frac{5}{2}\right)$ (d) none of these

71. Point $P(p, 0)$, $Q(q, 0)$, $R(0, p)$, $S(0, q)$ form (a) parallelogram (b) rhombus (c) cyclic quadrilateral (d) none of these

72. A rectangular billiard table has vertices at $P(0, 0)$, $Q(0, 7)$, $R(10, 7)$, and $S(10, 0)$ A small billiard ball starts at $M(3, 4)$, moves in a straight line to the top of the table, bounces to the right side of the table, and then comes to rest at $N(7, 1)$. The y - coordinate of the point where it hits the right side is (a)3.7 (b) 3.8 (c)3.9(d) 4

73. If one side of a rhombus has endpoints (4, 5) and (1, 1), then the maximum area of the rhombus is 50 sq. units   (b) 25 sq. units 30 sq. units   (d) 20 sq. units

74. A rectangle $ABCD$, where $A \equiv (0, 0), B \equiv (4, 0), C \equiv (4, 2)D \equiv (0, 2)$, undergoes the following transformations successively: $f_1(x, y) \xrightarrow{} y, x$

$f_2(x, y) \xrightarrow{} x + 3y, y$ $f_3(x, y)\left(x - y)/2, (x + y)/2\right)$ The final figure will be square (b) a rhombus a rectangle (d) a parallelogram

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75. If a straight line through the origin bisects the line passing through the given points $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$, then the lines (a) are perpendicular (b) are parallel (c) have an angle between them of $\dfrac{\pi}{4}$ (d) none of these

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76. Let $A_r, r = 1, 2, 3,$, be the points on the number line such that OA_1, OA_2, OA_3 are in $GP,$ where O is the origin, and the common ratio of

the *GP* be a positive proper fraction. Let M, be the middle point of the line segment $A_r A_{r+1}$. Then the value of $\sum_{r=1}^{\infty} OM_r$ is equal to

77. The vertices of a parallelogram *ABCD* are $A(3, 1)$, $B(13, 6)$, $C(13, 21)$, and $D(3, 16)$ If a line passing through the origin divides the parallelogram into two congruent parts, then the slope of the line is (a) $\frac{11}{12}$ (b) $\frac{11}{8}$ (c) $\frac{25}{8}$ (d) $\frac{13}{8}$

78. Point A and B are in the first quadrant; point O is the origin. If the slope of OA is 1, the slope of OB is 7, and $OA = OB$, then the slope of AB is $-\frac{1}{5}$ (b) $-\frac{1}{4}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{2}$

79. In a $ABC, A \equiv (\alpha, \beta), B \equiv (1, 2), C \equiv (2, 3)$, point A lies on the line $y = 2x + 3$, where α, β are integers, and the area of the triangle is S such that $[S] = 2$ where $[\,.\,]$ denotes the greatest integer function. Then the possible coordinates of A can be $(-7, -11)$ $(-6, -9)$ $(2, 7)$ $(3, 9)$

80. If $y = ae^{mx} + be^{-mx}$, then $\dfrac{d^2y}{dx^2} - m^2y$ is equals to (a).$m^2\left(ae^{mx} - be^{mx}\right)$
(b).1 (c).0 (d).None of these

81. The vertices of a triangle are $A(-1, -7), B(5, 1) and C(1, 4)$ If the internal angle bisector of $\angle B$ meets the side AC in D, then find the length AD

82. The points $\left(0, \dfrac{8}{3}\right)$, $(1, 3)$ and $(82, 30)$ are the vertices of (A) an obtuse-angled triangle (B) an acute-angled triangle (C) a right-angled triangle (D) none of these

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83. Consider the triangle whose vertices are $(0, 0)$, $(5, 12)$ and $(16, 12)$ Find coordinates of (i) Centroid (ii) Incentre (iii) Circumcentre

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84. A Point A divides the join of P(-5,1) and Q(3.5) in the ratio $k:1$. Then the integral value of K for which the area of $\triangle ABC$. Where B is (1,5) and C is $(7, -2)$ is equal to 2 units in magnitude is

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85. Find the equation of the circle having center at (2,3) and which touches $x + y = 1$.

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86. If the lines $x + y = 6$ and $x + 2y = 4$ are diameters of the circle which passes through the point $(2, 6)$, then find its equation.

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87. Find the equation of the circle with radius 5 whose center lies on the x-axis and passes through the point $(2, 3)$.

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88. The line $2x - y + 1 = 0$ is tangent to the circle at the point $(2, 5)$ and the center of the circle lies on $x - 2y = 4$. Then find the radius of the

circle.

89. Find the image of the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ in the line $2x - 3y + 5 = 0$

90. If $x^2 + y^2 - 2x + 2ay + a + 3 = 0$ represents the real circle with nonzero radius, then find the values of a.

91. Find the equation of the circle having radius 5 and which touches line $3x + 4y - 11 = 0$ at point (1, 2).

92. If the equation $px^2 + (2 - q)xy + 3y^2 - 6qx + 30y + 6q = 0$ represents a circle, then find the values of p and q.

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93. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.

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94. Find the area of the triangle formed by the tangents from the point $(4, 3)$ to the circle $x^2 + y^2 = 9$ and the line joining their points of contact.

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95. Tangents are drawn to $x^2 + y^2 = 1$ from any arbitrary point P on the line $2x + y - 4 = 0$. The corresponding chord of contact passes through a

fixed point whose coordinates are(a) $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ (b) $\left(\dfrac{1}{2}, 1\right)$(c) $\left(\dfrac{1}{2}, \dfrac{1}{4}\right)$ (d) $\left(1, \dfrac{1}{2}\right)$

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96. Find the length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + c_1 = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + c_2 = 0$

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97. Find the locus of a point which moves so that the ratio of the lengths of the tangents to the circles $x^2 + y^2 + 4x + 3 = 0$ and $x^2 + y^2 - 6x + 5 = 0$ is $2:3$.

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98. The tangent at any point P on the circle $x^2 + y^2 = 4$ meets the coordinate axes at A and B. Then find the locus of the midpoint of AB.

99. If a line passing through the origin touches the circle $(x - 4)^2 + (y + 5)^2 = 25$, then find its slope.

100. If the chord of contact of the tangents drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$ subtends a right angle at the center, then prove that $h^2 + k^2 = 2a^2$.

101. If the straight line $x - 2y + 1 = 0$ intersects the circle $x^2 + y^2 = 25$ at points P and Q, then find the coordinates of the point of intersection of the tangents drawn at P and Q to the circle $x^2 + y^2 = 25$.

102. If the chord of contact of the tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$, then prove that a, b and c are in GP.

103. The lengths of the tangents from any point on the circle

$15x^2 + 15y^2 - 48x + 64y = 0$ to the two circles

$5x^2 + 5y^2 - 24x + 32y + 75 = 0$

$5x^2 + 5y^2 - 48x + 64y = 0$ are in the ratio

104. Find the equation of the normal to the circle $x^2 + y^2 = 9$ at the point $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}} \right)$.

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105. Find the equations of tangents to the circle $x^2 + y^2 - 22x - 4y + 25 = 0$ which are perpendicular to the line $5x + 12y + 8 = 0$

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106. If the length tangent drawn from the point (5, 3) to the circle $x^2 + y^2 + 2x + ky + 17 = 0$ is 7, then find the value of k

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107. A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$. Then find its equations.

108. Find the equation of the normal to the circle $x^2 + y^2 - 2x = 0$ parallel to the line $x + 2y = 3$.

109. Find the equation of the tangent to the circle $x^2 + y^2 + 4x - 4y + 4 = 0$ which makes equal intercepts on the positive coordinates axes.

110. If the distances from the origin of the centers of three circles $x^2 + y^2 + 2\lambda x - c^2 = 0, (i = 1, 2, 3)$, are in GP, then prove that the lengths of the tangents drawn to them from any point on the circle $x^2 + y^2 = c^2$ are in GP.

111. Find the equation of the normals to the circle $x^2 + y^2 - 8x - 2y + 12 = 0$ at the point whose ordinate is -1

112. An infinite number of tangents can be drawn from $(1, 2)$ to the circle $x^2 + y^2 - 2x - 4y + \lambda = 0$. Then find the value of λ.

113. If the circle $x^2 + y^2 - 4x - 8y - 5 = 0$ intersects the line $3x - 4y = m$ at two distinct points, then find the values of m.

114. The line $Ax + By + C = 0$ cuts the circle $x^2 + y^2 + ax + by + c = 0$ at P and Q. The line $A'x + B'x + C' = 0$ cuts the circle $x^2 + y^2 + a'x + b'y + c' = 0$ at R and S. If $P, Q, R,$ and S are concyclic, then show that $|a - a' \ b - b' \ c - c' \ A B C A' B' C'| = 0$

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115. Find the equation of the circle which cuts the three circles $x^2 + y^2 - 3x - 6y + 14 = 0, x^2 + y^2 - x - 4y + 8 = 0,$ and $x^2 + y^2 + 2x - 6y + 9 = 0$ orthogonally.

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116. Find the equations to the common tangents of the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$

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117. Equation of the smaller circle that touches the circle $x^2 + y^2 = 1$ and passes through the point (4,3) is

118. Show that the circles $x^2 + y^2 - 10x + 4y - 20 = 0$ and $x^2 + y^2 + 14x - 6y + 22 = 0$ touch each other. Find the coordinates of the point of contact and the equation of the common tangent at the point of contact.

119. If the radical axis of the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x - 2y + 1 = 0$, show that either $g = \dfrac{3}{4}$ or $f = 2$

120. The equation of three circles are given $x^2 + y^2 = 1, x^2 + y^2 - 8x + 15 = 0, x^2 + y^2 + 10y + 24 = 0$. Determine the coordinates of the point P such that the tangents drawn from it to the circle are equal in length.

121. If the circles $x^2 + y^2 + 2a'x + 2b'y + c' = 0$ and $2x^2 + 2y^2 + 2ax + 2by + c = 0$ intersect othrogonally, then prove that $aa' + bb' = c + \dfrac{c'}{2}$.

122. A circle passes through the origin and has its center on $y = x$ If it cuts $x^2 + y^2 - 4x - 6y + 10 = 0$ orthogonally, then find the equation of the circle.

123. Prove that the equation of any tangent to the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ is of the form $y = m(x - 1) + 3\sqrt{1 + m^2} - 2$.

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124. The tangent to the circle $x^2 + y^2 = 5$ at $(1, -2)$ also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$. Find the coordinats of the corresponding point of contact.

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125. If $S_1 = \alpha^2 + \beta^2 - a^2$, then angle between the tangents from (α, β) to the circle $x^2 + y^2 = a^2$, is

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126. If $a > 2b > 0$, then find the positive value of m for which $y = mx - b\sqrt{1 + m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and

$(x - a)^2 + y^2 = b^2\,^{.}$

127. Find the angle between the two tangents from the origin to the circle

$(x - 7)^2 + (y + 1)^2 = 25$

128. Two circles C_1 and C_2 intersect at two distinct points P and Q in a line passing through P meets circles C_1 and C_2 at A and B , respectively. Let Y be the midpoint of AB and QY meets circles C_1 and C_2 at X a n d Z respectively. Then prove that Y is the midpoint of XZ

129. Find the equation of the tangent at the endpoints of the diameter of circle $(x - a)^2 + (y - b)^2 = r^2$ which is inclined at an angle θ with the

positive x-axis.

130. Find the equations of the tangents to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the straight line $4x + 3y + 5 = 0$

131. If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0,$ tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + \left(g^2 + f^2\right)\cos^2\alpha = 0$, then find the angle between the tangents.

132. The lengths of the tangents from $P(1, -1)$ and $Q(3, 3)$ to a circle are $\sqrt{2}$ and $\sqrt{6}$, respectively. Then, find the length of the tangent from $R(-1, -5)$ to the same circle.

133. Which of the following is a point on the common chord of the circle $x^2 + y^2 + 2x - 3y + 6 = 0$ and $x^2 + y^2 + x - 8y - 13 = 0$?

(a)(1, - 2) (b) (1, 4)(c) (1, 2) (d) (1, - 4)

134. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersects at points P and Q , then find the values of a for which the line $5x + by - a = 0$ passes through $PandQ$.

135. Find the angle at which the circles $x^2 + y^2 + x + y = 0$ and $x^2 + y^2 + x - y = 0$ intersect.

136. Find the angle which the common chord of $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ subtends at the origin.

137. If the tangents are drawn to the circle $x^2 + y^2 = 12$ at the point where it meets the circle $x^2 + y^2 - 5x + 3y - 2 = 0$, then find the point of intersection of these tangents.

138. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of the circle $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ then prove that

$$2g'\left(g - g'\right) + 2f'\left(f - f'\right) = c - c'$$

139. Find the length of the common chord of the circles $x^2 + y^2 + 2x + 6y = 0$ and $x^2 + y^2 - 4x - 2y - 6 = 0$

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140. If the circle $x^2 + y^2 = 1$ is completely contained in the circle $x^2 + y^2 + 4x + 3y + k = 0$, then find the values of k.

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141. Prove that the pair of straight lines joining the origin to the points of intersection of the circles $x^2 + y^2 = a$ and $x^2 + y^2 + 2(gx + fy) = 0$ is $a'\left(x^2 + y^2\right) - 4(gx + fy)^2 = 0$

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142. The circles $x^2 + y^2 - 12x - 12y = 0$ and $x^2 + y^2 + 6x + 6y = 0$. a.touch each other externally b.touch each other internally c.intersect at two points d.none of these

143. If θ is the angle between the two radii (one to each circle) drawn from one of the point of intersection of two circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$, then prove that the length of the common chord of the two circles is $\dfrac{2ab\sin\theta}{\sqrt{a^2 + b^2 - 2ab\cos\theta}}$

144. If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ cut the coordinae axes at concyclic points, then prove that $\left|a_1a_2\right| = \left|b_1b_2\right|$

145. A line is drawn through a fix point $P(\alpha, \beta)$ to cut the circle $x^2 + y^2 = r^2$ at A and B. Then PA.PB is equal to :

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146. Circles are drawn through the point $(2, 0)$ to cut intercept of length 5 units on the x-axis. If their centers lie in the first quadrant, then find their equation.

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147. Find the equation of the circle passing through the origin and cutting intercepts of lengths 3 units and 4 units from the positive axes.

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148. Find the point of intersection of the circle $x^2 + y^2 - 3x - 4y + 2 = 0$ with the x-axis.

149. Find the values of k for which the points $(2k, 3k)$, $(1, 0)$, $(0, 1)$, $and (0, 0)$ lie on a circle.

150. If one end of the diameter is $(1, 1)$ and the other end lies on the line $x + y = 3$, then find the locus of the center of the circle.

151. Tangent drawn from the point $P(4, 0)$ to the circle $x^2 + y^2 = 8$ touches it at the point A in the first quadrant. Find the coordinates of another

point B on the circle such that $AB = 4$.

152. If the join of $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ makes on obtuse angle at $\left(x_3, y_3\right)$,

then prove that $\left(x_3 - x_1\right)\left(x_3 - x_2\right) + \left(y_3 - y_1\right)\left(y_3 - y_2\right) < 0$

153. Find the range of values of m for which the line $y = mx + 2$ cuts the

circle $x^2 + y^2 = 1$ at distinct or coincident points.

154. Centre of the circle whose radius is 3 and which touches internally

the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ at the point (-1 -1) is

155. Find the number of common tangents that can be drawn to the circles $x^2 + y^2 - 4x - 6y - 3 = 0$ and $x^2 + y^2 + 2x + 2y + 1 = 0$

156. Find the radical center of the circles $x^2 + y^2 + 4x + 6y = 19$, $x^2 + y^2 = 9$, $x^2 + y^2 - 2x - 4y = 5$,

157. Two circles C_1 and C_2 intersect in such a way that their common chord is of maximum length. The center of C_1 is (1, 2) and its radius is 3 units. The radius of C_2 is 5 units. If the slope of the common chord is $\frac{3}{4}$, then find the center of C_2

158. The equation of a circle is $x^2 + y^2 = 4$. Find the center of the smallest circle touching the circle and the line $x + y = 5\sqrt{2}$

159. Consider four circles $(x \pm 1)^2 + (y \pm 1)^2 = 1$. Find the equation of the smaller circle touching these four circles.

160. Consider the circles $x^2 + (y - 1)^2 = 9$, $(x - 1)^2 + y^2 = 25$. They are such that these circles touch each other one of these circles lies entirely inside the other each of these circles lies outside the other they intersect at two points.

161. If the circles of same radius a and centers at $(2, 3)$ and $(5, 6)$ cut orthogonally, then find a.

162. If the two circles $2x^2 + 2y^2 - 3x + 6y + k = 0$ and $x^2 + y^2 - 4x + 10y + 16 = 0$ cut orthogonally, then find the value of k .

163. Find the condition that the circle $(x - 3)^2 + (y - 4)^2 = r^2$ lies entirely within the circle $x^2 + y^2 = R^2$.

164. Find the locus of the center of the circle which cuts off intercepts of lengths $2a$ and $2b$ from the x-and the y-axis, respectively.

165. Find the equation of a circle whose centre is (3,-1) and which cuts off a chord 6 length 6 units on the line 2x-5y + 18 = 0.

166. Find the equation of the circle which touches both the axes and the line $x = c$.

167. Find the equation of the circle which touches the x-axis and whose center is (1, 2).

168. Find the equations of the circles which pass through the origin and cut off chords of length a from each of the lines $y = x$ and $y = -x$

169. Find the radius of the circle $(x - 5)(x - 1) + (y - 7)(y - 4) = 0$.

170. Find the equation of the circle which passes through the points $(3, -2) and (-2, 0)$ and the center lies on the line $2x - y = 3$

171. Prove that the locus of the centroid of the triangle whose vertices are $(a cos t, a sin t)$, $(b sin t, -b cos t)$, and $(1, 0)$, where t is a parameter, is circle.

172. If one end of the diameter of the circle $2x^2 + 2y^2 - 4x - 8y + 2 = 0$ is (3,2), then find the other end of the diameter.

173. If a circle whose center is $(1, -3)$ touches the line $3x - 4y - 5 = 0$, then find its radius.

174. Prove that the locus of the point that moves such that the sum of the squares of its distances from the three vertices of a triangle is constant is a circle.

175. Column I|Column II The length of the common chord of two circles of radii 3 units and 4 units which intersect orthogonally is $\dfrac{k}{5}$. Then k is equal to|p. 1 The circumference of the circle $x^2 + y^2 + 4x + 12y + p = 0$ is bisected by the circle $x^2 + y^2 - 2x + 8y - q = 0$. Then $p + q$ is equal to|q. 24 The number of distinct chords of the circle $2x\left(x - \sqrt{2}\right) + y(2y - 1) = 0$, where the chords are passing through the point $\left(\sqrt{2}, \dfrac{1}{2}\right)$ and are bisected on the x-axis, is|r. 32 One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y = x + 7$. If $A and B$ are the poinst (- 3, 4) and (5, 4), respectively, then the area of the rectangle is|s. 36

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176. The number of integral values of λ for which the equation $x^2 + y^2 + \lambda x + (1 - \lambda)y + 5 = 0$ is the equation fo a circle whose radius cannot exceed 5, is 14 (b) 18 (c) 16 (d) none of these

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177. Let C_1 and C_2 be two circles whose equations are $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 2x = 0$ and $P(\lambda, \lambda)$ is a variable point . List 1 a) P lies inside C1 but outside C2 2)P lies inside C2 but outside C1 c)P lies outside C1 but outside C2 d)P does not lie inside C2 List 2 p)$\lambda \in (-\infty, -1) \cup (0, \infty)$ q)$\lambda \in (-\infty, -1) \cup (1, \infty)$ r)$\lambda \in (-1, 0)$ s)$\lambda \in (0, 1)$

178. Find the points on the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ which are the farthest and nearest to the point $(-5, 6)$

179. If the line $x\cos\theta + y\sin\theta = 2$ is the equation of a transverse common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6\sqrt{3}x - 6y + 20 = 0$, then the value of θ is (a) $\dfrac{5\pi}{6}$ (b) $\dfrac{2\pi}{3}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{6}$

180. Find the values of α for which the point $(\alpha - 1, \alpha + 1)$ lies in the larger segment of the circle $x^2 + y^2 - x - y - 6 = 0$ made by the chord whose equation is $x + y - 2 = 0$

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181. Statement 1 : The equation of chord through the point $(-2, 4)$ which is farthest from the center of the circle $x^2 + y^2 - 6x + 10y - 9 = 0$ is $x + y - 2 = 0$. Statement 2 : In notations, the equation of such chord of the circle $S = 0$ bisected at (x_1, y_1) must be $T = \overset{.}{S}$

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182. $\int \dfrac{dx}{x\left(x^5 + 1\right)}$

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183. Statement 1 : If two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other, then $f'g = fg'$ Statement 2 : Two circles touch other if the line joining their centers is perpendicular to all possible common tangents.

184. Find the greatest distance of the point $P(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$

185. Statement 1 : If the circle with center $P(t, 4 - 2t), t \in R,$ cut the circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2x - y - 12 = 0$, then both the intersections are orthogonal. Statement 2 : The length of tangent from P for $t \in R$ is the same for both the given circles.

186. Find the area of the region in which the points satisfy the inequaties $4 < x^2 + y^2 < 16$ and $3x^2 - y^2 \geq 0$.

187. If points A and B are (1, 0) and (0, 1), respectively, and point C is on the circle $x^2 + y^2 = 1$, then the locus of the orthocentre of triangle ABC is $\quad x^2 + y^2 = 4 \qquad x^2 + y^2 - x - y = 0 \qquad x^2 + y^2 - 2x - 2y + 1 = 0$ $x^2 + y^2 + 2x - 2y + 1 = 0$

188. If the line $x + 2by + 7 = 0$ is a diameter of the circle $x^2 + y^2 - 6x + 2y = 0$, then find the value of b.

189. Column I|Column II If the circle lies in the first quadrant, then,|p. $g < 0$ If the circle lies above the x-axis, then|q. $g > 0$ If the circle lies on the left of the y-axis, then|r. $g^2 - c < 0$ If the circle touches the positive x-axis and does not intersect the y-axis, then|s. $c > 0$

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190. Find the number of point (x, y) having integral coordinates satisfying the condition $x^2 + y^2 < 25$

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191. The circle $x^2 + y^2 - 6x - 10y + k = 0$ does not touch or intersect the coordinate axes, and the point (1, 4) is inside the circle. Find the range of value of k

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192. Statement 1 :The circles $x^2 + y^2 + 2px + r = 0$ and $x^2 + y^2 + 2qy + r = 0$

touch if $\dfrac{1}{p^2} + \dfrac{1}{q^2} = \dfrac{1}{r}$. Statement 2 : Two centers $C_1 and C_2$ and radii

$r_1 and r_2$, respectively, touch each other if $\left| r_1 + r_2 \right| = c_1 c_2$

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193. If the circle $x^2 + y^2 + 2x + 3y + 1 = 0$ cuts $x^2 + y^2 + 4x + 3y + 2 = 0$ at

A and B, then find the equation of the circle on AB as diameter.

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194. If the radii of the circles $(x - 1)^2 + (y - 2)^2 + (y - 2)^2 = 1$ and

$(-7)^2 + (y - 10)^2 = 4$ are increasing uniformly w.r.t. time as 0.3 units/s and

0.4 unit/s, respectively, then at what value of t will they touch each other?

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195. A and B are two points in the xy-plane, which are $2\sqrt{2}$ units distance apart and subtend an angle of 90^0 at the point $C(1, 2)$ on the line $x - y + 1 = 0$, which is larger than any angle subtended by the line segment AB at any other point on the line. Find the equation(s) of the circle through the points A, B and C

196. Two circles with radii a and b touch each other externally such that θ is the angle between the direct common tangents, $(a > b \geq 2)$. Then prove that $\theta = 2\sin^{-1}\left(\dfrac{a - b}{a + b}\right)$.

197. From the variable point A on circle $x^2 + y^2 = 2a^2$, two tangents are drawn to the circle $x^2 + y^2 = a^2$ which meet the curve at B and C Find the locus of the circumcenter of ABC

198. Two fixed circles with radii r_1 and r_2, $\left(r_1 > r_2\right)$, respectively, touch each other externally. Then identify the locus of the point of intersection of their direction common tangents.

199. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is touched by $y = x$ at P such that $OP = 6\sqrt{2}$, then the value of c is 36 (b) 144 (c) 72 (d) none of these

200. Find the radius of the smallest circle which touches the straight line $3x - y = 6$ at $(1, -3)$ and also touches the line $y = x$. Compute up to one place of decimal only.

201. The number of points $P(x, y)$ lying inside or on the circle $x^2 + y^2 = 9$ and satisfying the equation $\tan^4 x + \cot^4 x + 2 = 4\sin^2 y$ is_____

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202. C_1 and C_2 are circle of unit radius with centers at (0, 0) and (1, 0), respectively, C_3 is a circle of unit radius. It passes through the centers of the circles $C_1 and C_2$ and has its center above the x-axis. Find the equation of the common tangent to $C_1 and C_3$ which does not pass through C_2.

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203. The area of the triangle formed by the positive x - axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $\left(1, \sqrt{3}\right)$ is (a)$2\sqrt{3} squnits$. (b) $3\sqrt{2} squnits$ (c)$\sqrt{6} squnits$ (d) none of these

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204. Find the equation of the smallest circle passing through the point of intersection of the line $x + y = 1$ and the circle $x^2 + y^2 = 9$.

205. Let P be a point on the circle $x^2 + y^2 = 9$, Q a point on the line $7x + y + 3 = 0$, and the perpendicular bisector of PQ be the line $x - y + 1 = 0$. Then the coordinates of P are $(0, -3)$ (b) $(0, 3)$ (c) $\left(\dfrac{72}{25}, \dfrac{21}{35}\right)$ (d) $\left(-\dfrac{72}{25}, \dfrac{21}{25}\right)$

206. Show that the equation of the circle passing through $(1, 1)$ and the points of intersection of the circles $x^2 + y^2 + 13x - 13y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is $4x^2 + 4y^2 + 30x - 13y - 25 = 0$.

207. A straight line moves such that the algebraic sum of the perpendiculars drawn to it from two fixed points is equal to $2k$. Then, then straight line always touches a fixed circle of radius. $2k$ (b) $\dfrac{k}{2}$ (c) k (d) none of these

208. Let S_1 be a circle passing through $A(0, 1)$ and $B(-2, 2)$ and S_2 be a circle of radius $\sqrt{10}$ units such that AB is the common chord of $S_1 and S_2$. Find the equation of S_2

209. The coordinates of the middle point of the chord cut-off by $2x - 5y + 18 = 0$ by the circle $x^2 + y^2 - 6x + 2y - 54 = 0$ are $(1, 4)$ (b) $(2, 4)$ (c) $(4, 1)$ (d) $(1, 1)$

210. A variable circle which always touches the line $x + y - 2 = 0$ at (1, 1) cuts the circle $x^2 + y^2 + 4x + 5y - 6 = 0$. Prove that all the common chords of intersection pass through a fixed point. Find that points.

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211. The range of parameter $'a'$ for which the variable line $y = 2x + a$ lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 16x - 2y + 61 = 0$ without intersecting or touching either circle is (a)$a \in \left(2\sqrt{5} - 15, 0\right)$ (b) $a \in \left(-\infty, 2\sqrt{5} - 15, \right)$ (c)$a \in \left(2\sqrt{5} - 15, -\sqrt{5} - 1\right)$ (d) $a \in \left(-\sqrt{5} - 1, \infty\right)$

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212. Find the equation of the circle which is touched by $y = x$, has its center on the positive direction of the x=axis and cuts off a chord of length 2 units along the line $\sqrt{3}y - x = 0$

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213. Find the locus of the centers of the circles $x^2 + y^2 - 2ax - 2by + 2 = 0$, where a and b are parameters, if the tangents from the origin to each of the circles are orthogonal.

214. A circle touches the y-axis at the point (0, 4) and cuts the x-axis in a chord of length 6 units. Then find the radius of the circle.

215. Three concentric circles of which the biggest is $x^2 + y^2 = 1$, have their radii in A.P If the line $y = x + 1$ cuts all the circles in real and distinct points. The interval in which the common difference of the A.P will lie is:

216. Tangents PA and PB are drawn to $x^2 + y^2 = a^2$ from the point $P\left(x_1, y_1\right)$ Then find the equation of the circumcircle of triangle PAB

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217. Let $A = (-1, 0)$, $B = (3, 0)$ and PQ be any line passing through $(4, 1)$ having slope m Find the range of m for which there exist two points on PQ at which AB subtends a right angle.

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218. If the abscissa and ordinates of two points P and Q are the roots of the equations $x^2 + 2ax - b^2 = 0$ and $x^2 + 2px - q^2 = 0$, respectively, then find the equation of the circle with PQ as diameter.

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219. The equation of radical axis of two circles is $x + y = 1$. One of the circles has the ends ofa diameter at the points $(1, -3)$ and $(4, 1)$ and the other passes through the point $(1, 2)$.Find the equations of these circles.

220. Find the parametric form of the equation of the circle $x^2 + y^2 + px + py = 0$.

221. S is a circle having the center at $(0, a)$ and radius $b(b < a)$ A variable circle centered at $(\alpha, 0)$ and touching the circle S meets the X-axis at $MandN$. Find the a point P on the Y-axis, such that $\angle MPN$ is a constant for any choice of α

222. The point on a circle nearest to the point $P(2, 1)$ is at a distance of 4 units and the farthest point is $(6, 5)$. Then find the equation of the circle.

223. $S(x, y) = 0$ represents a circle. The equation $S(x, 2) = 0$ gives two identical solutions: $x = 1$. The equation $S(1, y) = 0$ given two solutions: $y = 0, 2$. Find the equation of the circle.

224. Find the length of intercept, the circle $x^2 + y^2 + 10x - 6y + 9 = 0$ makes on the x-axis.

225. Find the equation of the family of circle which touch the pair of straight lines $x^2 - y^2 + 2y - 1 = 0$.

226. Find the center of the circle $x = -1 + 2\cos\theta, y = 3 + 2\sin\theta$.

227. Column I|Column II Two intersecting circle|p. have a common tangent Two mutually external circles|q. have a common normal Two circles, one strictly inside the other|r. do not have a common tangent Two branches of a hyperbola|s. do not have a common normal

228. Find the equation of the circle which touches both the axes and the straight line $4x + 3y = 6$ in the first quadrant and lies below it.

229. If the intercepts of the variable circle on the x- and y-axis are 2 units and 4 units, respectively, then find the locus of the center of the variable circle.

230. The angle between the pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α . then the equation of the locus of the point P is   a. $x^2 + y^2 + 4x - 6y + 4 = 0$ b. $x^2 + y^2 + 4x - 6y - 9 = 0$ c. $x^2 + y^2 + 4x - 6y - 4 = 0$ d, $x^2 + y^2 + 4x - 6y + 9 = 0$

231. Two rods of lengths $a and b$ slide along the x - and y-axis , respectively, in such a manner that their ends are concyclic. Find the locus of the center of the circle passing through the endpoints.

232. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of λ is..........

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233. A circle with center at the origin and radius equal to a meets the axis of x at $A and B P(\alpha)$ and $Q(\beta)$ are two points on the circle so that $\alpha - \beta = 2\gamma$, where γ is a constant. Find the locus of the point of intersection of AP and BQ

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234. Two vertices of an equilateral triangle are $(-1, 0)$ and $(1, 0)$, and its third vertex lies above the x-axis. The equation of its circumcircel is _____

235. The locus of the point of intersection of the tangents to the circle $x^2 + y^2 = a^2$ at points whose parametric angles differ by $\dfrac{\pi}{3}$.

236. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq q$) are bisected by the x-axis, then $p^2 = q^2$ (b) $p^2 = 8q^2$ $p^2 < 8q^2$ (d) $p^2 > 8q^2$

237. Find the locus of the center of the circle touching the circle $x^2 + y^2 - 4y - 2x = 4$ internally and tangents on which from (1, 2) are making of 60^0 with each other.

238. If the line $ax + by = 2$ is a normal to the circle $x^2 + y^2 + 4x + 4y = 0$ and a tangent to the circle $x^2 + y^2 = 1$, then a and b are

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239. If a line segement $AM = a$ moves in the plane XOY remaining parallel to OX so that the left endpoint A slides along the circle $x^2 + y^2 = a^2$, then the locus of $\dot{M}$

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240. The ends of a quadrant of a circle have the coordinates (1, 3) and (3, 1). Then the center of such a circle is

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241. The tangents to $x^2 + y^2 = a^2$ having inclinations α and β intersect at P. If $\cot\alpha + \cot\beta = 0$, then find the locus of P

242. If the length of a common internal tangent to two circles is 7, and that of a common external tangent is 11, then the product of the radii of the two circles is (A) 36 (B) 9 (C) 18 (D) 4

243. If $C_1, C_2,$ and C_3 belong to a family of circles through the points $\left(x_1, y_2\right)$ and $\left(x_2, y_2\right)$ prove that the ratio of the length of the tangents from any point on C_1 to the circles C_2 and C_3 is constant.

244. Two circle are externally tangent. Lines PAB and P A 'B ' are common tangents with A and A ' on the smaller circle and B and B ' on the larger circle. If PA=AB=4, then the square of the radius of the circle is_____

245. Prove that quadrilateral $ABCD$, where $AB \equiv x + y - 10$, $BC \equiv x - 7y + 50 = 0$, $CD \equiv 22x - 4y + 125 = 0$, $and DA = 2x - 4y$ is concyclic. Also find the equation of the circumcircle of $ABCD$.

246. Statement 1 : Let $S_1 : x^2 + y^2 - 10x - 12y - 39 = 0$, $S_2 x^2 + y^2 - 2x - 4y + 1 = 0$ and $S_3 : 2x^2 + 2y^2 - 20x = 24y + 78 = 0$. The radical center of these circles taken pairwise is $(-2, -3)$ Statement 2 : The point of intersection of three radical axes of three circles taken in pairs is known as the radical center.

247. Find the locus of the midpoint of the chords of the circle $x^2 + y^2 - ax - by = 0$ which subtend a right angle at the point $\left(\dfrac{a}{2}, \dfrac{b}{2}\right)$. is

248. Let the lines $(y - 2) = m_1(x - 5)$ and $(y + 4) = m_2(x - 3)$ intersect at right angles at P (where m_1 and m_2 are parameters). If the locus of P is $x^2 + y^2 + gx + fy + 7 = 0$, then the value of $|f + g|$ is_____

249. A variable circle passes through the point $A(a, b)$ and touches the x-axis. Show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$

250. Find the equation of the circle if the chord of the circle joining (1, 2) and (- 3, 1) subtents 90^0 at the center of the circle.

251. Find the equation of the circle which passes through (1, 0) and (0, 1) and has its radius as small as possible.

252. Tangents are drawn from the origin to the circle $x^2 + y^2 - 2hx - 2hy + h^2 = 0, (h \geq 0)$ Statement 1 : Angle between the tangents is $\dfrac{\pi}{2}$ Statement 2 : The given circle is touching the coordinate axes.

253. Let A (-2,-2)and B (2,2) be two points AB subtends an angle of 45 ° at any points P in the plane in such a way that area of $\triangle PAB$ is 8 square unit, then number of possibe position(s) of P is

254. Consider the family of circles $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ passing through two fixed points $A and B$. Then the distance between the points $A and B$ is____

255. If a circle passes through the point $(0, 0), (a, 0) and (0, b)$, then find its center.

256. The line $3x + 6y = k$ intersects the curve $2x^2 + 3y^2 = 1$ at points A and B. The circle on AB as diameter passes through the origin. Then the value of k^2 is_____

257. Find the equation of the circle which passes through the points $(1, -2)$, $(4, -3)$ and whose center lies on the line $3x + 4y = 7$.

258. If real numbers x and y satisfy $(x + 5)^2 + (y - 12)^2 = (14)^2$, then the minimum value of $\sqrt{x^2 + y^2}$ is_____

259. Show that a cyclic quadrilateral is formed by the lines $5x + 3y = 9$, $x = 3y$, $2x = y$ and $x + 4y + 2 = 0$ taken in order. Find the equation of the circumcircle.

260. A circle $x^2 + y^2 + 4x - 2\sqrt{2}y + c = 0$ is the director circle of the circle $S_1 and S_1$ is the director circle of circle S_2, and so on. If the sum of radii of all these circles is 2, then the value of c is $k\sqrt{2}$, where the value of k is_____

261. A point P moves in such a way that the ratio of its distance from two coplanar points is always a fixed number ($\neq 1$) . Then, identify the locus of the point.

262. The sum of the slopes of the lines tangent to both the circles $x^2 + y^2 = 1$ and $(x - 6)^2 + y^2 = 4$ is_____

263. Prove that the maximum number of points with rational coordinates on a circle whose center is $\left(\sqrt{3}, 0\right)$ is two.

264. Let C_1 and C_2 are circles defined by $x^2 + y^2 - 20x + 64 = 0$ and $x^2 + y^2 + 30x + 144 = 0$. The length of the shortest line segment PQ that is tangent to C_1 at P and to C_2 at Q is

265. Prove that for all values of θ, the locus of the point of intersection of the lines $x\cos\theta + y\sin\theta = a$ and $x\sin\theta - y\cos\theta = b$ is a circle.

266. The chord of contact of tangents from a point P to a circle passes through Q. If $l_1 and l_2$ are the length of the tangents from $P and Q$ to the circle, then PQ is equal to $\dfrac{l_1 + l_2}{2}$ (b) $\dfrac{l_1 - l_2}{2}$ $\sqrt{l12 + l22}$ (d) $2\sqrt{l12 + l22}$

267. Find the length of the chord $x^2 + y^2 - 4y = 0$ along the line $x + y = 1$. Also find the angle that the chord subtends at the circumference of the larger segment.

268. The chords of contact of tangents from three points A, B and C to the circle $x^2 + y^2 = a^2$ are concurrent. Then A, B and C will (a)be concyclic (b) be collinear (c)form the vertices of a triangle (d)none of these

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269. Tangents are drawn to the circle $x^2 + y^2 = a^2$ from two points on the axis of $x,$ equidistant from the point $(k, 0).$ Show that the locus of their intersection is $ky^2 = a^2(k - x).$

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270. The common chord of the circle $x^2 + y^2 + 6x + 8y - 7 = 0$ and a circle passing through the origin and touching the line $y = x$ always passes through the point. (a) $\left(-\frac{1}{2}, \frac{1}{2} \right)$ (b) (1, 1) (c) $\left(\frac{1}{2}, \frac{1}{2} \right)$ (d) none of these

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271. P is the variable point on the circle with center at C CA and CB are perpendiculars from C on the x- and the y-axis, respectively. Show that the locus of the centroid of triangle PAB is a circle with center at the centroid of triangle CAB and radius equal to the one-third of the radius of the given circle.

272. If the angle between the tangents drawn to $x^2 + y^2 + 2gx + 2fy + c = 0$ from $(0, 0)$ is $\dfrac{\pi}{2}$, then $g^2 + f^2 = 3c$ $g^2 + f^2 = 2c$ $g^2 + f^2 = 5c$ $g^2 + f^2 = 4c$

273. Find the locus of center of circle of radius 2 units, if intercept cut on the x-axis is twice of intercept cut on the y-axis by the circle.

274. Any circle through the point of intersection of the lines $x + \sqrt{3}y = 1$ and $\sqrt{3}x - y = 2$ intersects these lines at points $P and Q$. Then the angle subtended by the arc PQ at its center is 180^0 (b) 90^0 (c) 120^0 depends on center and radius

275. A straight line moves so that the product of the length of the perpendiculars on it from two fixed points is constant. Prove that the locus of the feet of the perpendiculars from each of these points upon the straight line is a unique circle.

276. The number of such points $\left(a + 1, \sqrt{3}a\right)$, where a is any integer, lying inside the region bounded by the circles $x^2 + y^2 - 2x - 3 = 0$ and $x^2 + y^2 - 2x - 15 = 0$, is

277. A tangent is drawn to each of the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$. Show that if the two tangents are mutually perpendicular, the locus of their point of intersection is a circle concentric with the given circles.

278. Perpendiculars are drawn, respectively, from the points $P and Q$ to the chords of contact of the points $Q and P$ with respect to a circle. Prove that the ratio of the lengths of perpendiculars is equal to the ratio of the distances of the points $P and Q$ from the center of the circles.

279. Find the locus of the midpoint of the chord of the circle $x^2 + y^2 - 2x - 2y = 0$, which makes an angle of 120^0 at the center.

280. Find the center of the smallest circle which cuts circles $x^2 + y^2 = 1$ and $x^2 + y^2 + 8x + 8y - 33 = 0$ orthogonally.

281. A point moves so that the sum of the squares of the perpendiculars let fall from it on the sides of an equilateral triangle is constant. Prove that its locus is a circle.

282. From a point P on the normal $y = x + c$ of the circle $x^2 + y^2 - 2x - 4y + 5 - \lambda^2 - 0$, two tangents are drawn to the same circle touching it at point $BandC$. If the area of quadrilateral $OBPC$ (where O is the center of the circle) is 36 sq. units, find the possible values of λ It is given that point P is at distance $|\lambda|\left(\sqrt{2} - 1\right)$ from the circle.

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283. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a variable triangle OAB Sides OA and OB lie along the x- and y-axis, respectively, where O is the origin. Find the locus of the midpoint of side AB

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284. Consider three circles C_1, C_2 and C_3 such that C_2 is the director circle of C_1, and C_3 is the director circle of C_2. Tangents to C_1, from any point on C_3 intersect C_2, at P and Q. Find the angle between the

tangents to C_2 at P and Q. Also identify the locus of the point of intersection of tangents at P and Q.

285. The line $9x + y - 18 = 0$ is the chord of contact of the point $P(h, k)$ with respect to the circle $2x^2 + 2y^2 - 3x + 5y - 7 = 0$, for (a) $\left(\dfrac{24}{5}, -\dfrac{4}{5}\right)$ (b) $P(3, 1)$ (c) $P(-3, 1)$ (d) $\left(-\dfrac{2}{5}, \dfrac{12}{5}\right)$

286. A circle $x^2 + y^2 + 4x - 2\sqrt{2}y + c = 0$ is the director circle of circle S_1 and S_1 is the director circle of circle S_2, and so on. If the sum of radii of all these circles is 2, then find the value of c.

287. Tangents are drawn to the circle $x^2 + y^2 = 9$ at the points where it is met by the circle $x^2 + y^2 + 3x + 4y + 2 = 0$. Find the point of intersection of these tangents.

288. Find the length of the chord of contact with respect to the point on the director circle of circle $x^2 + y^2 + 2ax - 2by + a^2 - b^2 = 0$.

289. The distance between the chords of contact of tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin & the point (g,f) is

290. If $3x + y = 0$ is a tangent to a circle whose center is $(2, -1)$, then find the equation of the other tangent to the circle from the origin.

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291. Find the number of common tangent to the circles $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y + 9 = 0$

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292. Two variable chords $ABandBC$ of a circle $x^2 + y^2 = r^2$ are such that $AB = BC = r$. Find the locus of the point of intersection of tangents at $AandC$.

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293. Find the equation of the chord of the circle $x^2 + y^2 = 9$ whose middle point is $(1, -2)$

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294. Find the circle of minimum radius which passes through the point $(4, 3)$ and touches the circle $x^2 + y^2 = 4$ externally.

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295. A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. Find the locus of the center of the circle drawn on this chord as diameter.

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296. The radius of the tangent circle that can be drawn to pass through the point $(0, 7)$ and $(0, 6)$ and touching the x-axis is $(a)\dfrac{5}{2}$ $(b)\dfrac{13}{2}$ $(c)\dfrac{7}{2}$ $(d)\dfrac{19}{2}$

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297. Find the equation of the chord of the circle $x^2 + y^2 = a^2$ passing through the point $(2, 3)$ farthest from the center.

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298. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 sq. units. Then the equation of the circle is $x^2 + y^2 + 2x - 2y = 62$ $x^2 + y^2 + 2x - 2y = 47$ $x^2 + y^2 - 2x + 2y = 47$ $x^2 + y^2 - 2x + 2y = 62$

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299. Find the middle point of the chord of the circle $x^2 + y^2 = 25$ intercepted on the line $x - 2y = 2$

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300. Find the area of the triangle formed by the tangents from the point $(4, 3)$ to the circle $x^2 + y^2 = 9$ and the line joining their points of contact.

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301. Find the equation of a circle with center $(4, 3)$ touching the circle $x^2 + y^2 = 1$

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302. Find the equation of the tangent to the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ which makes with the coordinate axes a

triangle of area a^2.

303. Find the condition if the circle whose equations are $x^2 + y^2 + c^2 = 2ax$ and $x^2 + y^2 + c^2 - 2by = 0$ touch one another externally.

304. Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Then the locus of the mid-points of the secants by the circle is

305. A variable chord of the circle $x^2 + y^2 = 4$ is drawn from the point $P(3, 5)$ meeting the circle at the point A and B A point Q is taken on the

chord such that $2PQ = PA + PB$. The locus of Q is $x^2 + y^2 + 3x + 4y = 0$

$x^2 + y^2 = 36$ $x^2 + y^2 = 16$ $x^2 + y^2 - 3x - 5y = 0$

306. In triangle ABC , the equation of side BC is $x - y = 0$. The circumcenter and orthocentre of triangle are (2, 3) and (5, 8), respectively. The equation of the circumcirle of the triangle is a) $x^2 + y^2 - 4x + 6y - 27 = 0$ b) $x^2 + y^2 - 4x - 6y - 27 = 0$ c) $x^2 + y^2 + 4x - 6y - 27 = 0$ d) $x^2 + y^2 + 4x + 6y - 27 = 0$

307. Let a and b represent the lengths of a right triangles legs. If d is the diameter of a circle inscribed into the triangle, and D is the diameter of a circle circumscribed on the triangle, the $d + D$ equals. (a)$a + b$ (b) $2(a + b)$ (c)$\frac{1}{2}(a + b)$ (d) $\sqrt{a^2 + b^2}$

308. If the chord $y = mx + 1$ of the circles $x^2 + y^2 = 1$ subtends an angle of 45^0 at the major segment of the circle, then the value of m is 2 (b) -2 (c) +1 (d) none of these

309. $(6, 0), (0, 6), and(7, 7)$ are the vertices of a ABC . The incircle of the triangle has equation. a $x^2 + y^2 - 9x - 9y + 36 = 0$ b $x^2 + y^2 + 9x - 9y + 36 = 0$ c $x^2 + y^2 + 9x + 9y - 36 = 0$ d $x^2 + y^2 + 18x - 18y + 36 = 0$

310. If O is the origin and $OPandOQ$ are the tangents from the origin to the circle $x^2 + y^2 - 6x + 4y + 8 - 0$, then the circumcenter of triangle OPQ is (3, - 2) (b) $\left(\frac{3}{2}, -1\right)$ $\left(\frac{3}{4}, -\frac{1}{2}\right)$ (d) $\left(-\frac{3}{2}, 1\right)$

311. The range of values of r for which the point $\left(-5 + \dfrac{r}{\sqrt{2}}, -3 + \dfrac{r}{\sqrt{2}}\right)$ is an interior point of the major segment of the circle $x^2 + y^2 = 16$, cut-off by the line $x + y = 2$, is:

312. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y - 93 = 0$ with its sides parallel to the coordinate axes. The coordinates of its vertices are

$(-6, -9), (-6, 5), (8, -9), (8, 5)$ $(-6, -9), (-6, -5), (8, -9), (8, 5)$

$(-6, -9), (-6, 5), (8, 9), (8, 5)$ $(-6, -9), (-6, 5), (8, -9), (8, -5)$

313. Statement 1 : The least and greatest distances of the point $P(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ are 6 units and 15 units, respectively. Statement 2 : A point (x_1, y_1) lies outside the circle

$S = x^2 + y^2 + 2gx + 2fy + c = 0$ if $S_1 > 0$, where

$S_1 = x1^2 + y1^2 + 2gx_1 + 2fy_1 + \cdot$

314. Statement 1 : The number of circles passing through (1, 2), (4, 8) and (0, 0) is one. Statement 2 : Every triangle has one circumcircle

315. The locus of the midpoint of a line segment that is drawn from a given external point P to a given circle with center O (where O is the orgin) and radius r is a straight line perpendiculat to PO a circle with center P and radius r a circle with center P and radius $2r$ a circle with center at the midpoint PO and radius $\dfrac{r \cdot}{2}$

316. The difference between the radii of the largest and smallest circles which have their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$ and passes through point (a,b) lying outside the circle is :

317. The centre of a circle passing through (0,0), (1,0) and touching the Circle $x^2 + y^2 = 9$ is a. $\left(\frac{1}{2}, \sqrt{2}\right)$ b. $\left(\frac{1}{2}, \frac{3}{\sqrt{2}}\right)$ c. $\left(\frac{3}{2}, \frac{1}{\sqrt{2}}\right)$ d. $\left(\frac{1}{2}, -\frac{1}{\sqrt{2}}\right)$.
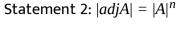
318. Each question has four choices a, b, c and d, out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. (a)Both the statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT1. (b)Both the statements are TRUE but STATEMENT 2 is NOT the correct explanation of STATEMENT 1. (c) STATEMENT 1 is TRUE and

STATEMENT 2 is FALSE. (d) STATEMENT 1 is FALSE and STATEMENT 2 is TRUE.

Statement 1: $|adj(adj(adjA))| - |A|^{n-1}\wedge 3$, where n is order of matrix A

Statement 2: $|adjA| = |A|^n$

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319. Statement 1 : If the chords of contact of tangents from three points A, B and C to the circle $x^2 + y^2 = a^2$ are concurrent, then A, B and C will be collinear. Statement 2 : Lines $\left(a_1 x + b_1 y + c_1\right) + k\left(a_2 x + b_2 y + c_2\right) = 0$ alwasy pass through a fixed point for $k \in R$.

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320. Statement 1 : Circles $x^2 + y^2 = 144$ and $x^2 + y^2 - 6x - 8y = 0$ do not have any common tangent. Statement 2 : If two circles are concentric, then they do not hav common tangents.

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321. The locus of the point from which the lengths of the tangents to the

circles $x^2 + y^2 = 4$ and $2\left(x^2 + y^2\right) - 10x + 3y - 2 = 0$ are equal to (a) a

straight line inclined at $\dfrac{\pi}{4}$ with the line joining the centers of the circles

(b) a circle (c) an ellipse (d)a straight line perpendicular to the line joining

the centers of the circles.

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322. The locus of the center of the circle touching the line $2x - y = 1$ at

(1, 1) is (a)$x + 3y = 2$ (b) $x + 2y = 2$ (c)$x + y = 2$ (d) none of these

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323. The distance from the center of the circle $x^2 + y^2 = 2x$ to the

common chord of the circles $x^2 + y^2 + 5x - 8y + 1 = 0$ and

$x^2 + y^2 - 3x + 7y - 25 = 0$ is 2 (b) 4 (c) $\dfrac{34}{13}$ (d) $\dfrac{26}{17}$

324. The circle passing through the point (-1,0) and touching the y-axis at (0,2) also passes through the point (A) $\left(-\frac{3}{2}, 0 \right)$ (B) $\left(-\frac{5}{2}, 2 \right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2} \right)$ (D) (- 4, 0).

325. The equation of the circumcircle of an equilateral triangle is $x^2 + y^2 + 2gx + 2fy + c = 0$ and one vertex of the triangle in (1, 1). The equation of the incircle of the triangle is $a.\ 4\left(x^2 + y^2\right) = g^2 + f^2$

$b.\ 4\left(x^2 + y^2\right) + 8gx + 8fy = (1 - g)(1 + 3g) + (1 - f)(1 + 3f)$

$c.\ 4\left(x^2 + y^2\right) + 8gx + 8fy = g^2 + f^2$ d. None of These

326. A circle with radius $|a|$ and center on the y-axis slied along it and a variable line through (a, 0) cuts the circle at points P and Q . The region in

which the point of intersection of the tangents to the circle at points P and Q lies is represented by $y^2 \geq 4\left(ax - a^2\right)$ (b) $y^2 \leq 4\left(ax - a^2\right)$ $y \geq 4\left(ax - a^2\right)$ (d) $y \leq 4\left(ax - a^2\right)$

327. If the angle of intersection of the circle $x^2 + y^2 + x + y = 0$ and $x^2 + y^2 + x - y = 0$ is θ , then the equation of the line passing through (1, 2) and making an angle θ with the y-axis is (A) $x = 1$ (B) $y = 2$ (C) $x + y = 3$ (D) $x - y = 3$

328. The range of values of α for which the line $2y = gx + \alpha$ is a normal to the circle $x^2 + y^2 + 2gx + 2gy - 2 = 0$ for all values of g is (a)$[1, \infty)$ (b) $[-1, \infty)$ (c)$(0, 1)$ (d) $(-\infty, 1]$

329.) Six points$(x, y_i), i = 1, 2, ..., 6$ are taken on the circle x4 such that the circle $x^2 + y$ 4 such that 6 6 J:1-8 and Σ, 4 . The line segment $X, = 8$ and and 2-yi = 4. The line segment x1 i=1 joining orthocentre of a triangle formed by any three points and centroid of a triangle formed by other three points passes through a fixed i=1 points (h,k), then h+k is A) 1 B) 2 C) 3 D) 4

330. Consider a circle $x^2 + y^2 + ax + by + c = 0$ lying completely in the first quadrant. If $m_1 and m_2$ are the maximum and minimum values of $\dfrac{y}{x}$ for all ordered pairs (x, y) on the circumference of the circle, then the value of $\left(m_1 + m_2\right)$ is (a)$\dfrac{a^2 - 4c}{b^2 - 4c}$ (b) $\dfrac{2ab}{b^2 - 4c}$ (c)$\dfrac{2ab}{4c - b^2}$ (d) $\dfrac{2ab}{b^2 - 4ac}$

331. The equation of the circle passing through the point of intersection of the circle $x^2 + y^2 = 4$ and the line $2x + y = 1$ and having minimum possible radius is (a)$5x^2 + 5y^2 + 18x + 6y - 5 = 0$ (b)

$5x^2 + 5y^2 + 9x + 8y - 15 = 0$ (c) $5x^2 + 5y^2 + 4x + 9y - 5 = 0$ (c)

$5x^2 + 5y^2 - 4x - 2y - 18 = 0$

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332. The centers of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is $4 \le x^2 + y^2 \le 64$ $x^2 + y^2 \le 25\ x^2 + y^2 \ge 25$ (d) $3 \le x^2 + y^2 \le 9$

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333. The coordinates of two points $PandQ$ are $(x_1, y_1)and(x_2, y_2)andO$ is the origin. If the circles are described on $OPandOQ$ as diameters, then the length of their common chord is (a)$\dfrac{|x_1 y_2 + x_2 y_1|}{PQ}$ (b) $\dfrac{|x_1 y_2 - x_2 y_1|}{PQ}$ $\dfrac{|x_1 x_2 + y_1 y_2|}{PQ}$ (d) $\dfrac{|x_1 x_2 - y_1 y_2|}{PQ}$

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334. The area of the triangle formed by the positive x-axis with the normal and the tangent to the circle $x^2 + y^2 = 4$ at $\left(1, \sqrt{3}\right)$ is

335. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that,the common chord is of maximum length and has a slope equal to $\dfrac{3}{4}$, then the co-ordinates of the centre of C_2 are:

336. A circle touches the line $y = x$ at point P such that $OP = 4\sqrt{2}$, Circle contains (-10,2) in its interior & length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of the circle

337. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the center. Then the locus of the centroid of the $\triangle PAB$ as P moves on the circle is (1) A parabola (2) A circle (3) An ellipse (4) A pair of straight lines

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338. Let PQ and RS be tangent at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then prove that $2r = \sqrt{PQ \times RS}$.

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339. Find the coordinates of the point at which the circles $x^2 + y^2 - 4x - 2y + 4 = 0$ and $x^2 + y^2 - 12x - 8y + 36 = 0$ touch each other. Also, find equations of common tangents touching the circles the distinct points.

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340. Let AB be chord of contact of the point $(5, -5)$ w.r.t the circle $x^2 + y^2 = 5$. Then find the locus of the orthocentre of the triangle PAB, where P is any point moving on the circle.

341. Let P be any moving point on the circle $x^2 + y^2 - 2x = 1$. AB be the chord of contact of this point w.r.t. the circle $x^2 + y^2 - 2x = 0$. The locus of the circumcenter of triangle $CAB(C$ being the center of the circle) is

$a.\ 2x^2 + 2y^2 - 4x + 1 = 0$ $b.\ x^2 + y^2 - 4x + 2 = 0$ $c.\ x^2 + y^2 - 4x + 1 = 0$

$d.\ 2x^2 + 2y^2 - 4x + 3 = 0$

342. If eight distinct points can be found on the curve $|x| + |y| = 1$ such that from eachpoint two mutually perpendicular tangents can be drawn

to the circle $x^2 + y^2 = a^2$, then find the range of a

343. A circle of radius 5 units has diameter along the angle bisector of the lines $x + y = 2$ and $x - y = 2$. If the chord of contact from the origin makes an angle of 45^0 with the positive direction of the x-axis, find the equation of the circle.

344. A circle of radius 1 unit touches the positive x-axis and the positive y-axis at A and B, respectively. A variable line passing through the origin intersects the circle at two points D and E. If the area of triangle DEB is maximum when the slope of the line is m, then find the value of m^{-2}

345. The number of rational point(s) [a point (a, b) is called rational, if a and b both are rational numbers] on the circumference of a circle having center (π, e) is at most one (b) at least two exactly two (d) infinite

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346. AB is a diameter of a circle. CD is a chord parallel to AB and $2CD = AB$. The tangent at B meets the line AC produced at E then AE is equal to -

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347. Two parallel tangents to a given circle are cut by a third tangent at the point R and Q . Show that the lines from R and Q to the center of the circle are mutually perpendicular.

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348. If the equation of any two diagonals of a regular pentagon belongs to the family of lines $(1 + 2\lambda)\lambda - (2 + \lambda)x + 1 - \lambda = 0$ and their lengths are $\sin 36^0$, then the locus of the center of circle circumscribing the given pentagon (the triangles formed by these diagonals with the sides of pentagon have no side common) is (a) $x^2 + y^2 - 2x - 2y + 1 + \sin^2 72^0 = 0$ (b)$x^2 + y^2 - 2x - 2y + \cos^2 72^0 = 0$ (c)$x^2 + y^2 - 2x - 2y + 1 + \cos^2 72^0 = 0$ (d) $x^2 + y^2 - 2x - 2y + \sin^2 72^0 = 0$

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349. If $OA and OB$ are equal perpendicular chords of the circles $x^2 + y^2 - 2x + 4y = 0$, then the equations of $OA and OB$ are, where O is the origin. $3x + y = 0$ and $3x - y = 0$ $3x + y = 0$ and $3y - x = 0$ $x + 3y = 0$ and $y - 3x = 0$ $x + y = 0$ and $x - y = 0$

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350. *ABCD* is a square of unit area. A circle is tangent to two sides of *ABCD* and passes through exactly one of its vertices. The radius of the circle is (a) $2 - \sqrt{2}$ (b) $\sqrt{2} - 1$ (c) $\sqrt{2} - \dfrac{1}{2}$ (d) $\dfrac{1}{\sqrt{2}}$

351. *BandC* are fixed points having coordinates $(3, 0)$ and $(-3, 0)$, respectively. If the vertical angle *BAC* is 90^0, then the locus of the centroid of *ABC* has equation. (a)$x^2 + y^2 = 1$ (b) $x^2 + y^2 = 2$ (c) $9\left(x^2 + y^2\right) = 1$ (d) $9\left(x^2 + y^2\right) = 4$

352. A straight line with slope 2 and y-intercept 5 touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then the coordinates of Q are (a) $(-6, 11)$ (b) $(-9, -13)$ (c)$(-10, -15)$ (d) $(-6, -7)$

353. A pair of tangents is drawn to a unit circle with center at the origin and these tangents intersect at A enclosing an angle of 60^0 . The area enclosed by these tangents and the arc of the circle is (a) $\dfrac{2}{\sqrt{3}} - \dfrac{\pi}{6}$ (b) $\sqrt{3} - \dfrac{\pi}{3}$ (c) $\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{6}$ (d) $\sqrt{3}\left(1 - \dfrac{\pi}{6}\right)$

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354. A line meets the coordinate axes at A and B . A circle is circumscribed about the triangle OAB If $d_1 and d_2$ are distances of the tangents to the circle at the origin O from the points $A and B$, respectively, then the diameter of the circle is (a) $\dfrac{2d_1 + d_2}{2}$ (b) $\dfrac{d_1 + 2d_2}{2}$ (c)$d_1 + d_2$ (d) $\dfrac{d_1 d_2}{d_1 + d_2}$

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355. A circle of constant radius a passes through the origin O and cuts the axes of coordinates at points P and Q . Then the equation of the

locus of the foot of perpendicular from O to PQ is (A)

$$\left(x^2 + y^2\right)\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 4a^2 \qquad \text{(B)} \left(x^2 + y^2\right)^2\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = a^2 \qquad \text{(C)}$$

$$\left(x^2 + y^2\right)^2\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 4a^2 \text{ (D) } \left(x^2 + y^2\right)\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = a^2$$

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356. The equation of the line inclined at an angle of $\frac{\pi}{4}$ to the x-axis ,such that the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 10x - 14y + 65 = 0$ intercept equal length on it, is (A) $2x - 2y - 3 = 0$ (B) $2x - 2y + 3 = 0$ (C) $x - y + 6 = 0$ (D) $x - y - 6 = 0$

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357. If a circle of constant radius $3k$ passes through the origin O and meets the coordinate axes at $A and B$, then the locus of the centroud of triangle OAB is (a)$x^2 + y^2 = (2k)^2$ (b)$x^2 + y^2 = (3k)^2$ (c)$x^2 + y^2 = (4k)^2$ (d) $x^2 + y^2 = (6k)^2$

358. A straight line l_1 with equation $x - 2y + 10 = 0$ meets the circle with equation $x^2 + y^2 = 100$ at B in the first quadrant. A line through B perpendicular to l_1 cuts the y-axis at $P(0, t)$. The value of t is (a)12 (b) 15 (c) 20 (d) 25

359. Let C be a circle with two diameters intersecting at an angle of 30^0 A circle S is tangent to both the diameters and to C and has radius unity. The largest radius of C is (a) $1 + \sqrt{6} + \sqrt{2}$ (b) $1 + \sqrt{6} - \sqrt{2}$ (c) $\sqrt{6} + \sqrt{2} - 11$ (d) none of these

360. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally then k equals (A) 2 or $-\dfrac{3}{2}$ (B) -2 or $-\dfrac{3}{2}$ (C) 2 or $\dfrac{3}{2}$ (D) -2 or $\dfrac{3}{2}$

361. An acute triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have coordinates (3, 4) and (-4, 3) respectively, then find $\angle QPR$.

362. A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of center is:

363. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of circle is:

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364. The centre of circle inscribed in a square formed by lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ is a.(4, 7) b.(7, 4) c.(9, 4) d.(4, 9)

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365. If the tangent at the point on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight ine $5x - 2y + 6 = 0$ at a point Q on the y- axis then the length of PQ is

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366. Consider square $ABCD$ of side length 1. Let P be the set of all segments of length 1 with endpoints on the adjacent sides of square $ABCD$. The midpoints of segments in P enclose a region with area A The value of A is (a) $\dfrac{\pi}{4}$ (b) $1 - \dfrac{\pi}{4}$ (c) $4 - \dfrac{\pi}{4}$ (d) none of these

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367. The number of intergral value of y for which the chord of the circle $x^2 + y^2 = 125$ passing through the point $P(8, y)$ gets bisected at the point $P(8, y)$ and has integral slope is (a)8 (b) 6 (c) 4 (d) 2

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368. Statement 1 : The circle having equation $x^2 + y^2 - 2x + 6y + 5 = 0$ intersects both the coordinate axes. Statement 2 : The lengths of x and y intercepts made by the circle having equation $x^2 + y^2 + 2gx + 2fy + c = 0$ are $2\sqrt{g^2 - c}$ and $2\sqrt{f^2 - c}$, respectively.

369. Statement 1 : The center of the circle having $x + y = 3$ and $x - y = 1$ as its normals is (1, 2) Statement 2 : The normals to the circle always pass through its center

370. Statement 1 : The equations of the straight lines joining the origin to the points of intersection of $x^2 + y^2 - 4x - 2y = 4$ and $x^2 + y^2 - 2x - 4y - 4 = 0$ is $x - y = 0$. Statement 2 : $y + x = 0$ is the common chord of $x^2 + y^2 - 4x - 2y = 4$ and $x^2 + y^2 - 2x - 4y - 4 = 0$

371. Statement 1 : Points $A(1, 0), B(2, 3), C(5, 3), and D(6, 0)$ are concyclic. Statement 2 : Points $A, B, C, and D$ form an isosceles trapezium or $ABandCD$ meet at E Then $EA \cdot EB = EC \cdot ED$

372. Statement I The chord of contact of tangent from three points A, B and C to the circle $x^2 + y^2 = a^2$ are concurrent, then A, B and C will be collinear. Statement II A, B and C always lie on the normal to the circle $x^2 + y^2 = a^2$.

373. Statement 1 : The equation $x^2 + y^2 - 2x - 2ay - 8 = 0$ represents, for different values of a, a system of circles passing through two fixed points lying on the x-axis. Statement 2 : $S = 0$ is a circle and $L = 0$ is a straight line. Then $S + \lambda L = 0$ represents the family of circles passing through the points of intersection of the circle and the straight line (where λ is an arbitrary parameter).

374. The circles having radii r_1 and r_2 intersect orthogonally. Find the length of their common chord.

375. Tangents PA and PB are drawn to $x^2 + y^2 = 9$ from any arbitrary point P on the line $x + y = 25$. The locus of the midpoint of chord AB is

$25\left(x^2 + y^2\right) = 9(x + y)$ $\qquad$ $25\left(x^2 + y^2\right) = 3(x + y)$ $\qquad$ $5\left(x^2 + y^2\right) = 3(x + y)$

$noneofthese$

376. The two circles which pass through $(0, a) and (0, -a)$ and touch the line $y = mx + c$ will intersect each other at right angle if (A) $a^2 = c^2(2m + 1)$ $\qquad$ (B) $\qquad$ $a^2 = c^2\left(2 + m^2\right)$ $\qquad$ (C)$c^2 = a^2\left(2 + m^2\right)$ $\qquad$ (D) $c^2 = a^2(2m + 1)$

377. If the pair of straight lines $xy\sqrt{3} - x^2 = 0$ is tangent to the circle at

PandQ from the origin O such that the area of the smaller sector formed

by *CPandCQ* is $3\pi squnit,$ where C is the center of the circle, the *OP*

equals $\dfrac{\left(3\sqrt{3}\right)}{2}$ (b) $3\sqrt{3}$ (c) 3 (d) $\sqrt{3}$

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378. The locus of the midpoint of a chord of the circle $x^2 + y^2 = 4$ which

subtends a right angle at the origins is (a) $x + y = 2$ (b) $x^2 + y^2 = 1$

$x^2 + y^2 = 2$ (d) $x + y = 1$

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379. The condition that the chord $x\cos\alpha + y\sin\alpha - p = 0$ of $x^2 + y^2 - a^2 = 0$

may subtend a right angle at the center of the circle is _

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380. Let the base AB of a triangle ABC be fixed and the vertex C lies on a fixed circle of radius r Lines through $A and B$ are drawn to intersect $CB and CA$, respectively, at $E and F$ such that $CE : EB = 1 : 2 and CF : FA = 1 : 2$. If the point of intersection P of these lines lies on the median through AB for all positions of AB, then the locus of P is a circle of radius $\dfrac{r}{2}$ a circle of radius $2r$ a parabola of latus rectum $4r$ a rectangular hyperbola

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381. If the chord of contact of tangents from a point P to a given circle passes through Q, then the circle on PQ as diameter. cuts the given circle orthogonally touches the given circle externally touches the given circle internally none of these

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382. Statement : Points (1, 1), (2, 3), and (3, 5) are collinear.

383. Statement 1 : The number of circles touching lines $x + y = 1, 2x - y = 5,$ and $3x + 5y - 1 = 0$ is four Statement 2 : In any triangle, four circles can be drawn touching all the three sides of the triangle.

384. The line $2x - y + 1 = 0$ is tangent to the circle at the point (2, 5) and the center of the circle lies on $x - 2y = 4.$ The radius of the circle is (a) $3\sqrt{5}$ (b) $5\sqrt{3}$ (c) $2\sqrt{5}$ (d) $5\sqrt{2}$

385. The equation of the chord of the circle $x^2 + y^2 - 3x - 4y - 4 = 0$, which passes through the origin such that the origin divides it in the ratio 4:1, is

386. A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centers of the circles. The are of the rhombus is (A) $8\sqrt{3}$ sq.units (B) $4\sqrt{3}$ sq.units (C) $6\sqrt{3}$ sq.units (D) none of these

387. In a triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. If a chord joins A with the point of intersection D of the hypotenuse and the semicircle, then the length of AC is equal to

$\dfrac{AB \dot{A}D}{\sqrt{AB^2 + AD^2}}$ (b) $\dfrac{AB \dot{A}D}{AB + AD}$ $\sqrt{AB \dot{A}D}$ (d) $\dfrac{AB \dot{A}D}{\sqrt{AB^2 - AD^2}}$

388. Two congruent circles with centered at (2, 3) and (5, 6) which intersect at right angles, have radius equal to (a) $2\sqrt{3}$ (b) 3 (c) 4 (d) none of these

389. The locus of the center of the circle such that the point (2,3) is the midpoint of the chord $5x + 2y = 16$ is (a)$2x - 5y + 11 = 0$ (b) $2x + 5y - 11 = 0$ (c)$2x + 5y + 11 = 0$ (d) none of these

390. The value of 'c' for which the set $\left\{(x, y) \mid x^2 + y^2 + 2x \leq 1\right\} \cap \{(x, y) \mid x - y + c \leq 0\}$ contains only one point in common is

391. A circle of radius unity is centered at thet origin. Two particles tart moving at the same time from the point $(1, 0)$ and move around the circle in opposite direction. One of the particle moves anticlockwise with constant speed v and the other moves clockwise with constant speed $3v$. After leaving $(1, 0)$, the two particles meet first at a point P, and continue until they meet next at point Q. The coordinates of the point Q are

392. Two circles with radii a and b touch each other externally such that θ is the angle between the direct common tangents, $(a > b \geq 2)$. Then prove that $\theta = 2\sin^{-1}\left(\dfrac{a - b}{a + b}\right)$.

393. A circle is inscribed into a rhombous ABCD with one angle 60. The distance from the centre of the circle to the nearest vertex is equal to 1. If P is any point of the circle then $|PA|^2 + |PB|^2 + |PC|^2 + |PD|^2$ is equal to:

394. Consider: $L_1: 2x + 3y + p - 3 = 0$ $L_2: 2x + 3y + p + 3 = 0$ where p is a real number and $C: x^2 + y^2 + 6x - 10y + 30 = 0$ Statement 1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle $\overset{.}{C}$ Statement 2 : If line L_1 is a a diameter of circle C, then line L_2 is not a chord of circle $\overset{.}{C}$ (A) Both the statement are True and Statement 2 is the correct explanation of Statement 1. (B) Both the statement are True but Statement 2 is not the correct explanation of Statement 1. (C) Statement 1 is True and Statement 2 is False. (D) Statement 1 is False and Statement 2 is True.

395. The straight line 2x-3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts. If S = { $\left(2, \dfrac{3}{4}\right), \left(\dfrac{5}{2}, \dfrac{3}{4}\right), \left(\dfrac{1}{4}, -\dfrac{1}{4}\right), \left(\dfrac{1}{8}, \dfrac{1}{4}\right)$ }, then the number of point(s) in S lying inside the smaller part is

396. Let $ABCD$ be a quadrilateral with area 18 , side AB parallel to the side CD, $and AB = 2CD$. Let AD be perpendicular to $ABandCD$. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is 3 (b) 2 (c) $\dfrac{3}{2}$ (d) 1

397. Consider a family of circles which are passing through the point $(- 1, 1)$ and are tangent to the x-axis. If (h, k) are the coordinates of the center of the circles, then the set of values of k is given by the interval.

$(a) k \geq \dfrac{1}{2}$ (b) $-\dfrac{1}{2} \leq k \leq \dfrac{1}{2}$ $k \leq \dfrac{1}{2}$ (d) $0 < k < \dfrac{1}{2}$

398. If the conics whose equations are

$S \equiv \sin^2\theta x^2 + 2hxy + \cos^2\theta y^2 + 32x + 16y + 19 = 0, S' \equiv \cos^2\theta x^2 + 2h'xy + s \in^2$

intersect at four concyclic points, then, (where $\theta \in R$) $h + h' = 0$ (b) $h = h'$ $h + h' = 1$ (d) none of these

399. The range of values of λ, $(\lambda > 0)$ such that the angle θ between the pair of tangents drawn from $(\lambda, 0)$ to the circle $x^2 + y^2 = 4$ lies in $\left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$ is (a) $\left(\dfrac{4}{\sqrt{3}}, \dfrac{2}{\sqrt{2}}\right)$ (b) $\left(0, \sqrt{2}\right)$ (c) $(1, 2)$ (d) none of these

400. The equation of the incircle of equilateral triangle ABC where $B \equiv (2, 0), C \equiv (4, 0)$, and A lies in the fourth quadrant is: (a) $x^2 + y^2 - 6x + \dfrac{2y}{\sqrt{3}} + 9 = 0$ (b) $x^2 + y^2 - 6x - \dfrac{2y}{\sqrt{3}} + 9 = 0$ (c) $x^2 + y^2 + 6x + \dfrac{2y}{\sqrt{3}} + 9 = 0$ (d) none of these

401. $f(x, y) = x^2 + y^2 + 2ax + 2by + c = 0$ represents a circle. If $f(x, 0) = 0$ has equal roots, each being 2, and $f(0, y) = 0$ has 2 and 3 as its roots, then the center of the circle is (a) $\left(2, \dfrac{5}{2}\right)$ (b) Data is not sufficient (c) $\left(-2, -\dfrac{5}{2}\right)$ (d) Data is inconsistent

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402. The area bounded by the curves $x^2 + y^2 = 1$, $x^2 + y^2 = 4$ and the pair of lines $\sqrt{3}x^2 + \sqrt{3}y^2 = 4xy$, in the first quadrant is (1) $\dfrac{\pi}{2}$ (2) $\dfrac{\pi}{6}$ (3) $\dfrac{\pi}{4}$ (4) $\dfrac{\pi}{3}$

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403. The straight line $x\cos\theta + y\sin\theta = 2$ will touch the circle $x^2 + y^2 - 2x = 0$ if (a)$\theta = n\pi, n \in IQ$ (b) $A = (2n + 1)\pi, n \in I$ $\theta = 2n\pi, n \in I$ (d) none of these

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404. The centre of a circle passing through (0,0), (1,0) and touching the

Circle $x^2 + y^2 = 9$ is a. $\left(\frac{1}{2}, \sqrt{2}\right)$ b. $\left(\frac{1}{2}, \frac{3}{\sqrt{2}}\right)$ c. $\left(\frac{3}{2}, \frac{1}{\sqrt{2}}\right)$ d. $\left(\frac{1}{2}, -\frac{1}{\sqrt{2}}\right)$.

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405. The locus of the centre of a circle which touches externally the circle

$x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches Y-axis, is given by the equation

(a) x2-6x-10y+14 = 0 (b) x2-10x-6y + 14 = 0 (c) yr_6x-10y+14-0 (d) y,2-10x-6y +

14 = 0

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406. If the two circles $(x + 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$

intersect at two distinct point,then (A) $r > 2$ (B) $2 < r < 8$ (C) $r < 2$ (D)

$r = 2$

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407. Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangents is $4x + 3y = 10$, find the equations of the circles.

408. The circle which can be drawn to pass through (1, 0) and (3, 0) and to touch the y-axis intersect at angle θ Then $\cos\theta$ is equal to $(a)\dfrac{1}{2}$ (b) $-\dfrac{1}{2}$ (c) $\dfrac{1}{4}$ (d) $-\dfrac{1}{4}$

409. The locus of the midpoints of the chords of contact of $x^2 + y^2 = 2$ from the points on the line $3x + 4y = 10$ is a circle with center P If O is the origin, then OP is equal to 2 (b) 3 (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{3}$

410. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$. Its sides are parallel to the coordinate axes. One vertex of the square is $\left(1 + \sqrt{2}, -2\right)$ (b) $\left(1 - \sqrt{2}, -2\right)$ $\left(1, -2 + \sqrt{2}\right)$ (d) none of these

411. Two circle $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1, 1) is $x^2 + y^2 - 6x + 4 = 0$ $x^2 + y^2 - 3x + 1 = 0$ $x^2 + y^2 - 4y + 2 = 0$ none of these

412. The equation of the tangent to the circle $x^2 + y^2 = 25$ passing through (- 2, 11) is (a) $4x + 3y = 25$ (b) $3x + 4y = 38$ (c) $24x - 7y + 125 = 0$ (d) $7x + 24y = 250$

413. If the area of the quadrilateral by the tangents from the origin to the circle $x^2 + y^2 + 6x - 10y + c = 0$ and the radii corresponding to the points of contact is 15, then the value of c is (a) 9 (b) 4 (c) 5 (d) 25

414. If the circles $x^2 + y^2 - 9 = 0$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touch each other, then α is (a) $-\dfrac{4}{3}$ (b) 0 (c) 1 (d) $\dfrac{4}{3}$

415. Point M moves on the circle $(x - 4)^2 + (y - 8)^2 = 20$. Then it brokes away from it and moving along a tangent to the circle, cuts the x-axis at the point (-2,0). The co-ordinates of a point on the circle at which the moving point broke away is

416. The points on the line $x = 2$ from which the tangents drawn to the circle $x^2 + y^2 = 16$ are at right angles is (are) (a) $\left(2, 2\sqrt{7}\right)$ (b) $\left(2, 2\sqrt{5}\right)$ (c) $\left(2, -2\sqrt{7}\right)$ (d) $\left(2, -2\sqrt{5}\right)$

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417. Co-ordinates of the centre of a circle, whose radius is 2 unit and which touches the pair of lines $x^2 - y^2 - 2x + 1 = 0$ is (are)

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418. Three sided of a triangle have equations $L_1 \equiv y - m_i x = o; i = 1, 2$ and 3. Then $L_1 L_2 + \lambda L_2 L_3 + \mu L_3 L_1 = 0$ where $\lambda \neq 0, \mu \neq 0$, is the equation of the circumcircle of the triangle if (a)$1 + \lambda + \mu = m_1 m_2 + \lambda m_2 m_3 + \lambda m_3 m_1$ (b)$m_1(1 + \mu) + m_2(1 + \lambda) + m_3(\mu + \lambda) = 0$ (c)$\dfrac{1}{m_3} + \dfrac{1}{m_1} + \dfrac{1}{m_1} = 1 + \lambda + \mu$ (d)none of these

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419. If the equation $x^2 + y^2 + 2hxy + 2gx + 2fy + c = 0$ represents a circle, then the condition for that circle to pass through three quadrants only but not passing through the origin is $f^2 > c$ (b) $g^2 > 2c > 0$ (d) $h = 0$

420. Consider two circles $x^2 + y^2 - 4x - 6y - 8 = 0$ and $x^2 + y^2 - 2x - 3 = 0$

Statement 1 : Both the circles intersect each other at two distinct points.

Statement 2 : The sum of radii of the two circles is greater than the distance between their centers.

421. Statement-1: The point $(\sin\alpha, \cos\alpha)$ does not lie outside the parabola

$y^2 + x - 2 = 0$ when $\alpha \in \left[\dfrac{\pi}{2}, \left(5\dfrac{\pi}{6}\right]\right] \cup \left[\pi, \dfrac{3\pi}{2}\right]$ Statement-2: The point $\left(x_1, y_1\right)$ lies outside the parabola $y^2 = 4ax$ if $y_1^2 - 4ax_1, 0$.

422. The equation of the circle which touches the axes of coordinates and the line $\dfrac{x}{3} + \dfrac{y}{4} = 1$ and whose center lies in the first quadrant is $x^2 + y^2 - 2cx - 2cy + c^2 = 0$, where c is (a) 1 (b) 2 (c) 3 (d) 6

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423. The equations of tangents to the circle $x^2 + y^2 - 6x - 6y + 9 = 0$ drawn from the origin in (a).$x = 0$ (b) $x = y$ (c) $y = 0$ (d) $x + y = 0$

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424. Statement 1 : Two orthogonal circles intersect to generate a common chord which subtends complimentary angles at their circumferences.

Statement 2 : Two orthogonal circles intersect to generate a common chord which subtends supplementary angles at their centers.

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425. Two circles $C_1 and C_2$ both pass through the points $A(1, 2) and E(2, 1)$ and touch the line $4x - 2y = 9$ at B and D, respectively. The possible coordinates of a point C, such that the quadrilateral $ABCD$ is a parallelogram, are (a, b) Then the value of $|ab|$ is_____

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426. A circle C_1 of radius b touches the circle $x^2 + y^2 = a^2$ externally and has its centre on the positiveX-axis; another circle C_2 of radius c touches the circle C_1, externally and has its centre on the positive x-axis. Given $a < b < c$ then three circles have a common tangent if a,b,c are in

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427. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its center is

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428. Difference in the values of the radius of a circle whose center is at the origin and which touches the circle $x^2 + y^2 - 6x - 8y + 21 = 0$ is_____

429. A triangle is inscribed in a circle of radius 1. The distance between the orthocentre and the circumcentre of the triangle cannot be

430. Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ externally at the point (5, 5).

431. Let $2x^2 + y^2 - 3xy = 0$ be the equation of pair of tangents drawn from the origin to a circle of radius 3, with center in the first quadrant. If A is the point of contact. Find OA

432. Find the equation of a circle which passes through the point $(2, 0)$ and whose centre is the point of intersection of the lines $3x + 5y = 1$, $(2 + c)x + 5c^2y = 1$ where limit of c tends to 1.

433. Let T_1, T_2 and be two tangents drawn from (-2, 0) onto the circle $C: x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles when taken two at a time

434. Let C_1 be the circle with center $O_1(0, 0)$ and radius 1 and C_2 be the circle with center $O_2\left(t, t^2 + 1\right)$, $(t \in R)$, and radius 2. Statement 1 : Circles $C_1 and C_2$ always have at least one common tangent for any value of t Statement 2 : For the two circles $O_1 O_2 \geq |r_1 - r_2|$, where $r_1 and r_2$ are their radii for any value of t

435. From the point $P\left(\sqrt{2}, \sqrt{6}\right)$, tangents $PA and PB$ are drawn to the circle $x^2 + y^2 = 4$ Statement 1 :The area of quadrilateral $OAPB(O$ being the origin) is 4. Statement 2 : The area of square is a^2, where a is the length of side.

436. C_1 is a circle of radius 1 touching the x- and the y-axis. C_2 is another circle of radius greater than 1 and touching the axes as well as the circle

C_1 . Then the radius of C_2 is (a)$3 - 2\sqrt{2}$ (b) $3 + 2\sqrt{2}$ $3 + 2\sqrt{3}$ (d) none of these

437. There are two circles whose equation are $x^2 + y^2 = 9$ and $x^2 + y^2 - 8x - 6y + n^2 = 0, n \in Z$ If the two circles have exactly two common tangents, then the number of possible values of n is (a)2 (b) 8 (c) 9 (d) none of these

438. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 - 6x + 2y = 0$

439. Prove That : No tangent can be drawn from the point $\left(\dfrac{5}{2}, 1\right)$ to the circumcircle of the triangle with vertices $\left(1, \sqrt{3}\right), \left(1, -\sqrt{3}\right), \left(3, -\sqrt{3}\right)$.

440. A circle passes through the points $A(1, 0)$ and $B(5, 0)$, and touches the y-axis at $C(0, h)$. If $\angle ACB$ is maximum, then (a)$h = 3\sqrt{5}$ (b) $h = 2\sqrt{5}$ (c) $h = \sqrt{5}$ (d) $h = 2\sqrt{10}$

441. The locus of a point which moves such that the sum of the square of its distance from three vertices of a triangle is constant is a/an circle   (b) straight line   (c) ellipse   (d) none of these

442. The equation of four circles are $(x \pm a)^2 + \left(y \pm a2 = a^2\right.$. The radius of a circle touching all the four circles is $\left(\sqrt{2} + 2\right)a$ (b) $2\sqrt{2}a$ $\left(\sqrt{2} + 1\right)a$ (d) $\left(2 + \sqrt{2}\right)a$

443. An isosceles triangle ABC is inscribed in a circle $x^2 + y^2 = a^2$ with the vertex A at $(a, 0)$ and the base angle B and C each equal 75^0 . Then the coordinates of an endpoint of the base are (a) $\left(-\dfrac{\sqrt{3}a}{2}, \dfrac{a}{2}\right)$ (b) $\left(-\dfrac{\sqrt{3}a}{2}, a\right)$ (c) $\left(\dfrac{a}{2}, \dfrac{\sqrt{3}a}{2}\right)$ (d) $\left(\dfrac{\sqrt{3}a}{2}, -\dfrac{a}{2}\right)$

444. A region in the x - y plane is bounded by the curve $y = \sqrt{25 - x^2}$ and the line $y = 0$. If the point $(a, a + 1)$ lies in the interior of the region, then (a)$a \in (-4, 3)$ (b) $a \in (-\infty, -1) \cup (3, \infty)$ (c)$a \in (-1, 3)$ (d) none of these

445. If (α, β) is a point on the circle whose center is on the x-axis and which touches the line $x + y = 0$ at $(2, -2)$, then the greatest value of α is

(a)$4 - \sqrt{2}$ (b) 6 (c)$4 + 2\sqrt{2}$ (d) $+\sqrt{2}$

446. The area of the triangle formed by joining the origin to the point of intersection of the line $x\sqrt{5} + 2y = 3\sqrt{5}$ and the circle $x^2 + y^2 = 10$ is (a) 3 (b) 4 (c) 5 (d) 6

447. A circle with center (a, b) passes through the origin. The equation of the tangent to the circle at the origin is (a)$ax - by = 0$ (b) $ax + by = 0$ $bx - ay = 0$ (d) $bx + ay = 0$

448. A particle from the point $P\left(\sqrt{3}, 1\right)$ moves on the circle $x^2 + y^2 = 4$ and after covering a quarter of the circle leaves it tangentially. The

equation of a line along with the point moves after leaving the circle is

449. The circles $x^2 + y^2 + 2x + 4y - 20 = 0$ and $x^2 + y^2 + 6x - 8y + 10 = 0$ a) are such that the number of common tangents on them is 2 b) are orthogonal c) are such that the length of their common tangents is $5\left(\dfrac{12}{5}\right)^{\frac{1}{4}}$ d) are such that the length of their common chord is $5\dfrac{\sqrt{3}}{2}$

450. If the circles $x^2 + y^2 - 9 = 0$ and $x^2 + y^2 + 2ax + 2y + 1 = 0$ touch each other, then α is (a)- $\dfrac{4}{3}$ (b) 0 (c) 1 (d) $\dfrac{4}{3}$

451. The equation of a circle of radius 1 touching the circles $x^2 + y^2 - 2|x| = 0$ is (a) $x^2 + y^2 + 2\sqrt{2}x + 1 = 0$ (b) $x^2 + y^2 - 2\sqrt{3}y + 2 = 0$ (c)

$x^2 + y^2 + 2\sqrt{3}y + 2 = 0$ (d) $x^2 + y^2 - 2\sqrt{2} + 1 = 0$

452. Which of the following lines have the intercepts of equal lengths on the circle, $x^2 + y^2 - 2x + 4y = 0$ (A) $3x - y = 0$ (B) $x + 3y = 0$(C) $x + 3y + 10 = 0$ (D) $3x - y - 10 = 0$

453. If a circle passes through the point of intersection of the lines $x + y + 1 = 0$ and $x + \lambda y - 3 = 0$ with the coordinate axis, then find the value of λ .

454. The circles $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 + 4x + 4y - 1 = 0$ (a)touch internally (b)touch externally (c)have $3x + 4y - 1 = 0$ as the

common tangent at the point of contact have $3x + 4y + 1 = 0$ (d)as the common tangent at the point of contact

455. The equation of the line(s) parallel to $x - 2y = 1$ which touch(es) the circle $x^2 + y^2 - 4x - 2y - 15 = 0$ is (are) (a)$x - 2y + 2 = 0$ (b) $x - 2y - 10 = 0$ (c) $x - 2y - 5 = 0$ (d) $3x - y - 10 = 0$

456. If the conics whose equations are

$S_1 : \left(\sin^2\theta\right)x^2 + (2h\tan\theta)xy + \left(\cos^2\theta\right)y^2 + 32x + 16y + 19 = 0$

$S_1 : \left(\sin^2\theta\right)x^2 - \left(2h^{'}\cot\theta\right)xy + \left(\sin^2\theta\right)y^2 + 16x + 32y + 19 = 0$ intersect at four concyclic points, where $\theta\left[0, \dfrac{\pi}{2}\right]$, then the correct statement(s) can be $h + h^{'} = 0$ (b) $h - h^{'} = 0$ $\theta = \dfrac{\pi}{4}$ (d) none of these

457. The range of values of a such that the angle θ between the pair of tangents drawn from $(a, 0)$ to the circle $x^2 + y^2 = 1$ satisfies `pi/2

458. From the point A $(0, 3)$ on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn & extended to a M point such that AM=2AB. The equation of the locus of M is: (A)$x^2 + 8x + y^2 = 0$ (B)$x^2 + 8x + (y - 3)^2 = 0$ (C) $(x - 3)^2 + 8x + y^2 = 0$ (D)$x^2 + 8x + 8y = 0$

459. Tangents are drawn from external poinl $P(6, 8)$ to the circle $x^2 + y^2 = r^2$ find the radius r of the circle such that area of triangle formed by the tangents and chord of contact is maximum is (A) 25 (B) 15 (C) 5 (D) none

460. The radius of the circle touching the line $2x + 3y + 1 = 0$ at $(1,-1)$ and cutting orthogonally the circle having line segment joining $(0, 3)$ and $(-2,-1)$ as diameter is

461. If the abscissa and ordinates of two points P and Q are the roots of the equations $x^2 + 2ax - b^2 = 0$ and $x^2 + 2px - q^2 = 0$, respectively, then find the equation of the circle with PQ as diameter.

462. Line segments AC and BD are diameters of the circle of radius 1. If $\angle BDC = 60^0$, the length of line segment AB is_____

463. As shown in the figure, three circles which have the same radius r, have centres at $(0, 0); (1, 1)$ and $(2, 1)$. If they have a common tangentline, as shown then, their radius 'r' is -

464. The acute angle between the line $3x - 4y = 5$ and the circle $x^2 + y^2 - 4x + 2y - 4 = 0$ is θ. Then find the value of $9\cos\theta$.
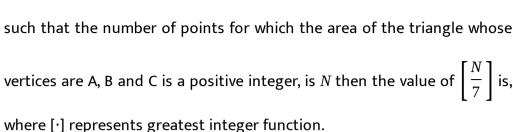
465. If two perpendicular tangents can be drawn from the origin to the circle $x^2 - 6x + y^2 - 2py + 17 = 0$, then the value of $|p|$ is___

466. Let $A(-4, 0)$, $B(4, 0)$ & $C(x, y)$ be points on the circle $x^2 + y^2 = 16$ such that the number of points for which the area of the triangle whose vertices are A, B and C is a positive integer, is N then the value of $\left[\dfrac{N}{7}\right]$ is, where $[\cdot]$ represents greatest integer function.

467. If the circle $x^2 + y^2 + (3 + \sin\beta)x + 2\cos\alpha y = 0$ and $x^2 + y^2 + 2\cos\alpha x + 2cy = 0$ touch each other, then the maximum value of c is

468. A tangent at a point on the circle $x^2 + y^2 = a^2$ intersects a concentric circle C at two points $P and Q$. The tangents to the circle X at $P and Q$ meet at a point on the circle $x^2 + y^2 = b^2$ Then the equation of the circle is $x^2 + y^2 = ab$ $x^2 + y^2 = (a - b)^2$ $x^2 + y^2 = (a + b)^2$ $x^2 + y^2 = a^2 + b^2$

469. Tangent are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$, λ being the variable. The locus of the point of intersection of these tangents is $2x - y + 10 = 0$ (b) $2x + y - 10 = 0$ $x - 2y + 10 = 0$ (d) $2x + y - 10 = 0$

470. From the points $(3, 4)$, chords are drawn to the circle $x^2 + y^2 - 4x = 0$. The locus of the midpoints of the chords is (a) $x^2 + y^2 - 5x - 4y + 6 = 0$ (b)$x^2 + y^2 + 5x - 4y + 6 = 0$ (c)$x^2 + y^2 - 5x + 4y + 6 = 0$ (d) $x^2 + y^2 - 5x - 4y - 6 = 0$

471. The angles at which the circles $(x - 1)^2 + y^2 = 10 and x^2 + (y - 2)^2 = 5$ intersect is $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

472. Two circles of radii 4cm and 1cm touch each other externally and θ is the angle contained by their direct common tangents. Then $\sin\theta$ is equal to (a)$\dfrac{24}{25}$ (b) $\dfrac{12}{25}$ (c)$\dfrac{3}{4}$ (d) none of these

473. The locus of the midpoints of the chords of the circle $x^2 + y^2 - ax - by = 0$ which subtend a right angle at $\left(\dfrac{a}{2}, \dfrac{b}{2}\right)$ is (a) $ax + by = 0$ (b) $ax + by = a^2 + b^2$ (c) $x^2 + y^2 - ax - by + \dfrac{a^2 + b^2}{8} = 0$ (d) $x^2 + y^2 - ax - by - \dfrac{a^2 + b^2}{8} = 0$

474. A is a point (a, b) in the first quadrant. If the two circles which passes through A and touches the coordinate axes cut at right angles then :

475. The number of common tangent(s) to the circles $x^2 + y^2 + 2x + 8y - 23 = 0$ and $x^2 + y^2 - 4x - 10y - 19 = 0$ is 1 (b) 2 (c) 3 (d) 4

476. If the tangents are drawn from any point on the line $x + y = 3$ to the circle $x^2 + y^2 = 9$, then the chord of contact passes through the point. (a) (3, 5) (b) (3, 3) (c) (5, 3) (d) none of these

477. If the radius of the circumcircle of the triangle TPQ, where PQ is chord of contact corresponding to point T with respect to circle $x^2 + y^2 - 2x + 4y - 11 = 0$, is 6 units, then minimum distances of T from the director circle of the given circle is

478. If the radius of the circumcircle of the triangle TPQ, where PQ is chord of contact corresponding to point T with respect to circle $x^2 + y^2 - 2x + 4y - 11 = 0$, is 6 units, then minimum distances of T from the director circle of the given circle is

479. The equation of the locus of the middle point of a chord of the circle $x^2 + y^2 = 2(x + y)$ such that the pair of lines joining the origin to the point of intersection of the chord and the circle are equally inclined to the x-axis is $x + y = 2$ (b) $x - y = 2$ $2x - y = 1$ (d) none of these

480. Two circles $C_1 and C_2$ intersect at two distinct points $P and Q$ in a line passing through P meets circles $C_1 and C_2$ at $A and B$, respectively. Let Y be

the midpoint of AB, *andQY* meets circles $C_1 andC_2$ at *XandZ* , respectively.

Then prove that Y is the midpoint of XZ

481. The two points A and B in a plane are such that for all points P lies on circle satisfied $\dfrac{PA}{PB} = k$, then k will not be equal to (a)0 (b)1 (c)2 (d)None

482. The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are _____ and _____

483. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.

484. find the area of the quadrilateral formed by a pair of tangents from the point (4,5) to the circle $x^2 + y^2 - 4x - 2y - 11 = 0$ and pair of its radii.

485. From the origin, chords are drawn to the circle $(x - 1)^2 + y^2 = 1$. The equation of the locus of the mid-points of these chords

486. The equation of the circle passing through the point of intersection of the circles $x^2 + y^2 - 4x - 2y = 8$ and $x^2 + y^2 - 2x - 4y = 8$ and the point $(-1, 4)$ is (a) $x^2 + y^2 + 4x + 4y - 8 = 0$ (b)$x^2 + y^2 - 3x + 4y + 8 = 0$ (c) $x^2 + y^2 + x + y = 0$ (d)$x^2 + y^2 - 3x - 3y - 8 = 0$

487. If the radii of the circle $(x - 1)^2 + (y - 2)^2 = 1$ and $(x - 7)^2 + (y - 10)^2 = 4$ are increasing uniformly w.r.t. times as 0.3 unit/s is and 0.4 unit/s, then they will touch each other at t equal to (a) 45s (b) 90s (c) 10s (d) 135s

488. The equation of the circle which has normals $x - 1). (y - 2) = 0$ and a tangent $3x + 4y = 6$ is $x^2 + y^2 - 2x - 4y + 4 = 0$ $x^2 + y^2 - 2x - 4y + 5 = 0$ $x^2 + y^2 = 5 (x - 3)^2 + (y - 4)^2 = 5$

489. A wheel of radius 8 units rolls along the diameter of a semicircle of radius 25 units; it bumps into this semicircle. What is the length of the portion of the diameter that cannot be touched by the wheel? 12   (b) 15   (c) 17   (d) 20

490. The point ([p+1],[p]) is lying inside the circle $x^2 + y^2 - 2x - 15 = 0$. Then the set of all values of p is (where [.] represents the greatest integer function) (a)[- 2, 3) (b) (- 2, 3) (c)[- 2, 0) ∪ (0, 3) (d) [0, 3)

491. The squared length of the intercept made by the line $x = h$ on the pair of tangents drawn from the origin to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $\dfrac{4ch^2}{\left(g^2 - c\right)^2}\left(g^2 + f^2 - c\right)$ $\dfrac{4ch^2}{\left(f^2 - c\right)^2}\left(g^2 + f^2 - c\right)$ $\dfrac{4ch^2}{\left(f^2 - f^2\right)^2}\left(g^2 + f^2 - c\right)$ (d) none of these

492. Two parallel tangents to a given circle are cut by a third tangent at the points A and B If C is the center of the given circle, then $\angle ACB$ (a)depends on the radius of the circle. (b)depends on the center of the circle. (c)depends on the slopes of three tangents. (d)is always constant

493. Three equal circles each of radius r touch one another. The radius of the circle touching all the three given circles internally is $\left(2 + \sqrt{3}\right)r$ (b) $\dfrac{\left(2 + \sqrt{3}\right)}{\sqrt{3}}r$ $\dfrac{\left(2 - \sqrt{3}\right)}{\sqrt{3}}r$ (d) $\left(2 - \sqrt{3}\right)r$

494. If $\left(m_i, \dfrac{1}{m_i}\right), i = 1, 2, 3, 4$ are concyclic points then the value of $m_1 m_2 m_3 m_4$ is

495. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle with AB as a diameter is (A)

$$\left(x - \frac{1}{2}\right)^3 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2} \quad \text{(B)} \quad \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4} \quad \text{(C)}$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{2} \text{ (D)} \left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{4}$$

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496. The equation of the locus of the mid-points of chords of the circle $4x^2 + 4y^2 - 12x + 4y + 1 = 0$ that subtends an angle of $\frac{2\pi}{3}$ at its centre is $x^2 + y^2 - kx + y + \frac{31}{16} = 0$ then k is

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497. The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point (h,k) then 4(h+k) is

498. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a given circle. Find the locus of the foot of the perpendicular drawn from the origin upon any chord of S which subtends a right angle at the origin.

499. The circle $x^2 + y^2$ - $4x$ - $4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcenter of the triangle is $x + y$ - $xy + k\left(x^2 + y^2\right)^{\frac{1}{2}} = 0$. Find k.

500. Let a given line L_1 intersect the X and Y axes at P and Q respectively. Let another line L_2 perpendicular to L_1 cut the X and Y-axes at R and S,

respectively. Show that the locus of the point of intersection of the line PS and QR is a circle passing through the origin

501. Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C1 of diameter 6. If the center of C1, lies in the first quadrant then the equation of the circle C2, which is concentric with C1, and cuts intercept of length 8 on these lines
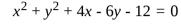
502. On the line segment joining (1, 0) and (3, 0) , an equilateral triangle is drawn having its vertex in the fourth quadrant. Then the radical center of the circles described on its sides. (a) $\left(3, -\dfrac{1}{\sqrt{3}}\right)$ (b) $\left(3, -\sqrt{3}\right)$ (c) $\left(2, -1\sqrt{3}\right)$ (d) $\left(2, -\sqrt{3}\right)$

503. From a point $R(5, 8)$, two tangents RP and RQ are drawn to a given circle $S = 0$ whose radius is 5. If the circumcenter of triangle PQR is $(2, 3)$, then the equation of the circle $S = 0$ is a $x^2 + y^2 + 2x + 4y - 20 = 0$ b $x^2 + y^2 + x + 2y - 10 = 0$ c $x^2 + y^2 - x + 2y - 20 = 0$ d $x^2 + y^2 + 4x - 6y - 12 = 0$

504. Find the equations of the circles passing through the point $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$

505. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ Suppose that the tangents at the points B(1,7) and D(4,-2) on the circle meet at the point C. Find the area of the quadrilateral ABCD

506. If $r_1 and r_2$ are the radii of the smallest and the largest circles, respectively, which pass though (5, 6) and touch the circle $(x - 2)^2 + y^2 = 4$, then $r_1 r_2$ is (a) $\dfrac{4}{41}$ (b) $\dfrac{41}{4}$ (c) $\dfrac{5}{41}$ (d) $\dfrac{41}{6}$

507. From an arbitrary point P on the circle $x^2 + y^2 = 9$, tangents are drawn to the circle $x^2 + y^2 = 1$, which meet $x^2 + y^2 = 9$ at A and B . The locus of the point of intersection of tangents at A and B to the circle $x^2 + y^2 = 9$ is

(a) $x^2 + y^2 = \left(\dfrac{27}{7}\right)^2$ (b) $x^2 - y^2\left(\dfrac{27}{7}\right)^2$ $y^2 - x^2 = \left(\dfrac{27}{7}\right)^2$ (d) none of these

508. If $C_1 : x^2 + y^2 = \left(3 + 2\sqrt{2}\right)^2$ is a circle and PA and PB are a pair of tangents on C_1, where P is any point on the director circle of C_1, then the radius of the smallest circle which touches c_1 externally and also the

two tangents PA and PB is

(a) $2\sqrt{3} - 3$ (b) $2\sqrt{2} - 1$ $2\sqrt{2} - 1$ (d) 1

509. The minimum radius of the circle which is orthogonal with both the circles $x^2 + y^2 - 12x + 35 = 0$ and $x^2 + y^2 + 4x + 3 = 0$ is (a) 4 (b) 3 (c) $\sqrt{15}$ (d) 1

510. If a circle of radius r is touching the lines $x^2 - 4xy + y^2 = 0$ in the first quadrant at points $A and B$, then the area of triangle $OAB(O$ being the origin) is (a)$3\sqrt{3}\dfrac{-r^2}{4}$ (b) $\dfrac{\sqrt{3}r^2}{4}$ (c)$\dfrac{3r^2}{4}$ (d) r^2

511. Suppose $ax + by + c = 0$, where $a, b and c$ are in AP be normal to a family of circles. The equation of the circle of the family intersecting the circle $x^2 + y^2 - 4x - 4y - 1 = 0$ orthogonally is (a)$x^2 + y^2 - 2x + 4y - 3 = 0$ (b) $x^2 + y^2 - 2x + 4y + 3 = 0$ (c)$x^2 + y^2 + 2x + 4y + 3 = 0$ (d) $x^2 + y^2 + 2x - 4y + 3 = 0$

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512. Two circles of radii a and b touching each other externally, are inscribed in the area bounded by $y = \sqrt{1 - x^2}$ and the x-axis. If $b = \dfrac{1}{2}$, then a is equal to (a) $\dfrac{1}{4}$ (b) $\dfrac{1}{8}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{\sqrt{2}}$

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513. Let P be any moving point on the circle $x^2 + y^2 - 2x = 1$. AB be the chord of contact of this point w.r.t. the circle $x^2 + y^2 - 2x = 0$. The locus of the circumcenter of triangle $CAB(C$ being the center of the circle) is

a. $2x^2 + 2y^2 - 4x + 1 = 0$ b. $x^2 + y^2 - 4x + 2 = 0$ c. $x^2 + y^2 - 4x + 1 = 0$

d. $2x^2 + 2y^2 - 4x + 3 = 0$

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514. C1 and C2 are two concentric circles, the radius of C2 being twice that of C1. From a point P on C2, tangents PA and PB are drawn to C1. Then the centroid of the triangle PAB (a) lies on C1 (b) lies outside C1 (c) lies inside C1 (d) may lie inside or outside C1 but never on C1

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515. Let C be any circle with centre $\left(0, \sqrt{2}\right)$ Prove that at most two rational points can be there on C (A rational point is a point both of whose coordinates are rational numbers)

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516. Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point P not on the curve. A line drawn from the point P intersect the curve at points Q and R. If he product $PQ. PR$ is (A) a pair of straight line (B) a circle (C) a parabola (D) an ellipse or hyperbola

517. Let a circle be given by $2x(x - a) + y(2y - b) = 0, (a \neq 0, b \neq 0)$. Find the condition on a and b if two chords each bisected by the x-axis, can be drawn to the circle from $\left(a, \dfrac{b}{2}\right)$

518. Consider a family of circles passing through the points $(3, 7)$ and $(6,5)$. Answer the following questions. Number of circles which belong to the family and also touchingx- axis are

519. Let *xandy* be real variables satisfying $x^2 + y^2 + 8x - 10y - 40 = 0$. Let $a = \max\left\{\sqrt{(x+2)^2 + (y-3)^2}\right\}$ and $b = \min\left\{\sqrt{(x+2)^2 + (y-3)^2}\right\}$.

Then (a)$a + b = 18$ (b) $a + b = \sqrt{2}$ (c) $a - b = 4\sqrt{2}$ (d) $ab = 73$

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520. $A\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ is a point on the circle $x^2 + y^2 = 1$ and B is another

point on the circle such that are length $AB = \dfrac{\pi}{2}$ units. Then, the

coordinates of B can be (a) $\left(\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}\right)$ (b) $\left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ (c)

$\left(-\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}\right)$ (d) none of these

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521. Tangent drawn from the point $(a, 3)$ to the circle $2x^2 + 2y^2 = 25$ will be perpendicular to each other if a equals (a) 5 (b) -4 (c) 4 (d) -5

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522. Consider the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. Let O be the centre of the circle and tangent at A(7,3) and B(5, 1) meet at C. Let S=0 represents family of circles passing through A and B, then

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523. If the circle $x^2 + y^2 + 2a_1x + c = 0$ lies completely inside the circle $x^2 + y^2 + 2a_2x + c = 0$ then

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524. Let C, C_1, C_2 be circles of radii 5,3,2 respectively. C_1 and C_2, touch each other externally and C internally. A circle C_3 touches C_1 and C_2 externally and C internally. If its radius is $\dfrac{m}{n}$ where m and n are relatively prime positive integers, then 2n-m is:

525. Let ABC be a triangle right-angled at A and S be its circumcircle. Let S_1 be the circle touching the lines AB and AC and the circle S internally. Further, let S_2 be the circle touching the lines AB and AC produced and the circle S externally. If r_1 and r_2 are the radii of the circles S_1 and S_2, respectively, show that $r_1 r_2 = 4$ area (ABC)

526. ABCD is a rectangle. A circle passing through vertex C touches the sides AB and AD at M and N respectively. If the distance lof the line MN

from the vertex C is P units then the area of rectangle ABCD is

527. If the length of the common chord of two circles $x^2 + y^2 + 8x + 1 = 0$ and $x^2 + y^2 + 2\mu y - 1 = 0$ is $2\sqrt{6}$, then the values of μ are ± 2 (b) ± 3 (c) ± 4 (d) none of these

528. The equation of circle of minimum radius which contacts the three circle

$x^2 + y^2 - 4y - 5 = 0$, $x^2 + y^2 + 12x + 4y + 31 = 0$, $x^2 + y^2 + 6x + 12y + 36 = 0$

then the radius of given circle is $\left(l + \dfrac{m}{36}\sqrt{949} \right)$ then the value of $l + m$ is :

529. The locus of the midpoint of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origins is (a) $x + y = 2$ (b) $x^2 + y^2 = 1$ (c) $x^2 + y^2 = 2$ (d) $x + y = 1$

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530. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$, Statement I The tangents are mutually perpendicular Statement, IIs The locus of the points frorn which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$ (a) Statement I is correct, Statement II is correct; Statement II is a correct explanation for Statementl (b(Statement I is correct, Statement I| is correct Statement II is not a correct explanation for Statementl (c)Statement I is correct, Statement II is incorrect (d) Statement I is incorrect, Statement II is correct

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531. The equation of the line passing through the points of intersection of the circles $3x^2 + 3y^2 - 2x + 12y - 9 = 0$ and $x^2 + y^2 + 6x + 2y - 15 = 0$ is

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532. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is : (A) $20\left(x^2 + y^2\right) - 36x + 45y = 0$ (B) $20\left(x^2 + y^2\right) + 36x - 45y = 0$ (C) $20\left(x^2 + y^2\right) - 20x + 45y = 0$ (D) $20\left(x^2 + y^2\right) + 20x - 45y = 0$

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533. If the tangent at the point $P(2, 4)$ to the parabola $y^2 = 8x$ meets the parabola $y^2 = 8x + 5$ at Q and $R,$ then find the midpoint of chord QR.

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534. Find the locus of the midpoints of the portion of the normal to the parabola $y^2 = 4ax$ intercepted between the curve and the axis.

535. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, such that one vertex of this triangle coincides with the vertex of the parabola. Then find the side length of this triangle.

536. M is the foot of the perpendicular from a point P on a parabola $y^2 = 4ax$ to its directrix and SPM is an equilateral triangle, where S is the focus. Then find SP.

537. Find the locus of the middle points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola.

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538. A quadrilateral is inscribed in a parabola $y^2 = 4ax$ and three of its sides pass through fixed points on the axis. Show that the fourth side also passes through a fixed point on the axis of the parabola.

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539. A right-angled triangle ABC is inscribed in parabola $y^2 = 4x$, where A is the vertex of the parabola and $\angle BAC = \frac{\pi}{2}$. If $AB = \sqrt{5}$, then find the area of ABC

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540. Let there be two parabolas $y^2 = 4ax$ and $y^2 = -4bx$ (where $a \neq b$ and $a, b > 0$) . Then find the locus of the middle points of the intercepts between the parabolas made on the lines parallel to the common axis.

541. The equation of aparabola is $y^2 = 4x P(1, 3)$ and $Q(1, 1)$ are two points in the $xy - plane$ Then, for the parabola. (a)P and Q are exterior points. (b) P is an interior point while Q is an exterior point (c)P and Q are interior points. (d)P is an exterior point while Q is an interior point

542. AP is perpendicular to PB , where A is the vertex of the parabola $y^2 = 4x$ and P is on the parabola. B is on the axis of the parabola. Then find the locus of the centroid of PAB

543. Find the value of P such that the vertex of $y = x^2 + 2px + 13$ is 4 units above the x-axis. (a) ± 2 (b) 4 (c) ± 3 (d) 5

544. The point $(a, 2a)$ is an interior point of the region bounded by the parabola $y^2 = 16x$ and the double ordinate through the focus. then find the values of a

545. Find the point where the line $x + y = 6$ is a normal to the parabola $y^2 = 8x$.

546. Find the equation of the tangent to the parabola $9x^2 + 12x + 18y - 14 = 0$ which passes through the point $(0, 1)$.

547. Find the angle between the tangents drawn to $y^2 = 4x$, where it is intersected by the line $y = x - 1$.

548. How many distinct real tangents that can be drawn from $(0, -2)$ to the parabola $y^2 = 4x$?

549. Find the angle at which the parabolas $y^2 = 4x$ and $x^2 = 32y$ intersect.

550. If the tangents at the points $P and Q$ on the parabola $y^2 = 4ax$ meet at T, $and S$ is its focus, the prove that $SP, ST, and SQ$ are in GP.

551. The tangents to the parabola $y^2 = 4x$ at the points (1, 2) and (4,4) meet on which of the following lines? (A) x = 3 (B) y = 3 (C) x + y = 4 (D) none of these

552. From an external point P, a pair of tangents is drawn to the parabola $y^2 = 4x$ If $\theta_1 and th\eta_2$ are the inclinations of these tangents with the x-axis such that $\theta_1 + \theta_2 = \frac{\pi}{4}$, then find the locus of P

553. If the line $x + y = a$ touches the parabola $y = x - x^2$, then find the value of a.

554. Find the slopes of the tangents to the parabola $y^2 = 8x$ which are normal to the circle $x^2 + y^2 + 6x + 8y - 24 = 0$.

555. Find the angle between the tangents drawn from (1, 3) to the parabola $y^2 = 4x$

556. Find the values of α so that the point $P\left(\alpha^2, \alpha\right)$ lies inside or on the triangle formed by the lines $x - 5y + 6 = 0, x - 3y + 2 = 0 and x - 2y - 3 = 0$.

557. The locus of the centre of a circle the touches the given circle externally is a _____

558. If on a given base BC, a triangle is described such that the sum of the tangents of the base angles is m, then prove that the locus of the opposite vertex A is a parabola.

559. The parametric equation of a parabola is $x = t^2 + 1, y = 2t + 1.$ Then find the equation of the directrix.

560. If the focus of a parabola is (2, 3) and its latus rectum is 8, then find the locus of the vertex of the parabola.

561. $y^2 + 2y - x + 5 = 0$ represents a parabola. Find its vertex, equation of axis, equation of latus rectum, coordinates of the focus, equation of the directrix, extremities of the latus rectum, and the length of the latus rectum.

562. Find the equation of the parabola which has axis parallel to the y-axis and which passes through the points (0, 2), (- 1, 0), and (1, 6).

563. Prove that the focal distance of the point (x, y) on the parabola $x^2 - 8x + 16y = 0$ is $|y + 5|$

564. Find the points on the parabola $y^2 - 2y - 4x = 0$ whose focal length is 6.

565. If the length of the common chord of circle $x^2 + y^2 = 4$ and $y^2 = 4(x - h)$ is maximum, then find the value of h.

566. From a variable point p on line $2x - y - 1 = 0$ pair of tangents are drawn to parabola $x^2 = 8y$ then chord of contact passes through a fixed

point.

567. The locus of the middle points of the focal chords of the parabola,

$y^2 = 4x$ is:

568. If the distance of the point $(\alpha, 2)$ from its chord of contact w.r.t. the

parabola $y^2 = 4x$ is 4, then find the value of α

569. TP and TQ are tangents to the parabola $y^2 = 4ax$ at P and Q,

respectively. If the chord PQ passes through the fixed point $(-a, b)$, then

find the locus of T

570. Find the locus of the midpoint of normal chord of parabola $y^2 = 4ax$

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571. If normal to the parabola $y^2 - 4ax = 0$ at α point intersects the parabola again such that the sum of ordinates of these two points is 3, then show that the semi-latus rectum is equal to -1.5α

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572. If the parabolas $y^2 = 4ax$ and $y^2 = 4c(x - b)$ have a common normal other than the x-axis $(a, b, c$ being distinct positive real numbers), then prove that $\dfrac{b}{a - c} > 2$.

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573. Find the angle made by a double ordinate of length $8a$ at the vertex of the parabola $y^2 = 4ax$.

574. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100m long is supported by vertical wires attached to the cable, the longest wire being 30m and the shortest being 6m. Find the length of the supporting wire attached to the roadway 18m from the middle.

575. If the chord of contact of tangents from a point P to the parabola $y^2 = 4ax$ touches the parabola $x^2 = 4by,$ then find the locus of P.

576. Tangents are drawn from any point on the line $x + 4a = 0$ to the parabola $y^2 = 4ax$ Then find the angle subtended by the chord of contact at the vertex.

577. If a normal to a parabola $y^2 = 4ax$ makes an angle ϕ with its axis, then it will cut the curve again at an angle

578. Tangents are drawn to the parabola $y^2 = 4ax$ at the point where the line $lx + my + n = 0$ meets this parabola. Find the point of intersection of these tangents.

579. Find the vertex of the parabola $x^2 = 2(2x + y)$.

580. Find the length of the common chord of the parabola $y^2 = 4(x + 3)$ and the circle $x^2 + y^2 + 4x = 0$.

581. Find the coordinates of any point on the parabola whose focus is (0, 1) and directrix is $x + 2 = 0$

582. If the focus and vertex of a parabola are the points (0, 2) and (0, 4), respectively, then find the equation

583. Find the length of the latus rectum of the parabola whose focus is at (2, 3) and directrix is the line $x - 4y + 3 = 0$.

584. The focal chord of the parabola $y^2 = ax$ is $2x - y - 8 = 0$. Then find the equation of the directrix.

585. The vertex of a parabola is (2, 2) and the coordinats of its two extremities of latus rectum are (- 2, 0) and (6, 0). Then find the equation of the parabola.

586. Find the equation of the directrix of the parabola $x^2 - 4x - 3y + 10 = 0$.

587. Find the locus of the midpoint of chords of the parabola $y^2 = 4ax$ that pass through the point $(3a, a)$

588. In the parabola $y^2 = 4ax$, then tangent at P whose abscissa is equal to the latus rectum meets its axis at T, and normal P cuts the curve again at Q Show that $PT : PQ = 4 : 5$.

589. If the normal to the parabola $y^2 = 4ax$ at point t_1 cuts the parabola again at point t_2, then prove that $(t_2)^2 \geq 8$.

590. If the normals from any point to the parabola $y^2 = 4x$ cut the line $x = 2$ at points whose ordinates are in AP, then prove that the slopes of tangents at the co-normal points are in GP.

591. If (h,k) is a point on the axis of the parabola $2\{(x - 1)^2 + (y - 1)^2\} = (x + y)^2$ from where three distinct normal can be drawn, then the least integral value of h is :

592. A ray of light moving parallel to the X-axis gets reflected from a parabolic mirror whose equation is $(y - 2)^2 = 4(x + 1)$. After reflection , the ray must pass through the point

593. A circle and a parabola $y^2 = 4ax$ intersect at four points. Show that the algebraic sum of the ordinates of the four points is zero. Also show that the line joining one pair of these four points is equally inclined to the axis.

594. A parabola mirror is kept along $y^2 = 4x$ and two light rays parallel to its axis are reflected along one straight line. If one of the incident light rays is at 3 units distance from the axis, then find the distance of the other incident ray from the axis.

595. If incident from point $(-1, 2)$ parallel to the axis of the parabola $y^2 = 4x$ strike the parabola, then find the equation of the reflected ray.

596. Find the equation of parabola having focus at (1,1) and vertex at (-3,-3).

597. Find the equation of the parabola with focus $f(4, 0)$ and directrix $x = -4$.

598. Find the value of λ if the equation $(x - 1)^2 + (y - 2)^2 = \lambda(x + y + 3)^2$ represents a parabola. Also, find its focus, the equation of its directrix, the equation of its axis, the coordinates of its vertex, the equation of its latus rectum, the length of the latus rectum, and the extremities of the latus rectum.

599. The equation of the latus rectum of a parabola is $x + y = 8$ and the equation of the tangent at the vertex is $x + y = 12$. Then find the length of the latus rectum.

600. Prove that the locus of the center of a circle, which intercepts a chord of given length $2a$ on the axis of x and passes through a given point on the axis of y distant b from the origin, is a parabola.

601. Find the value of λ if the equation $9x^2 + 4y^2 + 2\lambda xy + 4x - 2y + 3 = 0$ represents a parabola.

602. Find the range of values of λ for which the point $(\lambda, -1)$ is exterior to both the parabolas $y^2 = |x|$.

603. Prove that the locus of a point, which moves so that its distance from a fixed line is equal to the length of the tangent drawn from it to a given circle, is a parabola.

604. LOL' and MOM' are two chords of parabola $y^2 = 4ax$ with vertex A passing through a point O on its axis. Prove that the radical axis of the circles described on LL' and MM' as diameters passes though the vertex of the parabola.

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605. If (a, b) is the midpoint of a chord passing through the vertex of the parabola $y^2 = 4(x + 1)$, then prove that $2(a + 1) = b^2$.

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606. If two of the three feet of normals drawn from a point to the parabola $y^2 = 4x$ are $(1, 2)$ and $(1, -2)$, then find the third foot.

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607. If three distinct normals can be drawn to the parabola $y^2 - 2y = 4x - 9$ from the point $(2a, b)$, then find the range of the value of a

608. Find the locus of thepoint of intersection of two normals to a parabolas which are at right angles to one another.

609. $P\left(t_1\right)$ and $Q\left(t_2\right)$ are the point $t_1 and t_2$ on the parabola $y^2 = 4ax$. The normals at $PandQ$ meet on the parabola. Show that the middle point PQ lies on the parabola $y^2 = 2a(x + 2a)$

610. Prove that the locus of the point of intersection of the normals at the ends of a system of parallel cords of a parabola is a straight line which is a normal to the curve.

611. Find the number of distinct normals that can be drawn from $(-2, 1)$ to the parabola $y^2 - 4x - 2y - 3 = 0$

612. If the line passing through the focus S of the parabola $y = ax^2 + bx + c$ meets the parabola at $PandQ$ and if $SP = 4$ and $SQ = 6$, then find the value of a

613. If a focal chord of $y^2 = 4ax$ makes an angle $\alpha \in \left[0, \dfrac{\pi}{4}\right]$ with the positive direction of the x-axis, then find the minimum length of this focal chord.

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614. Find the length of the normal chord which subtends an angle of 90 ° at the vertex of the parabola $y^2 = 4x$.

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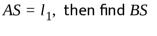
615. Find the locus of the point of intersection of the normals at the end of the focal chord of the parabola $y^2 = 4ax$

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616. The abscissa and ordinates of the endpoints A and B of a focal chord of the parabola $y^2 = 4x$ are, respectively, the roots of equations $x^2 - 3x + a = 0$ and $y^2 + 6y + b = 0$. Then find the equation of the circle with AB as diameter.

617. If AB is a focal chord of $x^2 - 2x + y - 2 = 0$ whose focus is S and $AS = l_1$, then find $\overset{.}{BS}$

618. A circle is drawn to pass through the extremities of the latus rectum of the parabola $y^2 = 8\overset{.}{x}$ It is given that this circle also touches the directrix of the parabola. Find the radius of this circle.

619. Circles drawn on the diameter as focal distance of any point lying on the parabola $x^2 - 4x + 6y + 10 = 0$ will touch a fixed line whose equation is

a. y=1 b. y=-1 c. y=2 d. y=-2

620. If the length of a focal chord of the parabola $y^2 = 4ax$ at a distance b from the vertex is $c,$ then prove that $b^2c = 4a^3.$

621. Find the equation of the parabola whose focus is $S(-1, 1)$ and directrix is $4x + 3y - 24 = 0$.

622. If $x^2 + y^2 = \log(xy)$, find $\dfrac{dy}{dx}$.

623. If $(2, -8)$ is at an end of a focal chord of the parabola $y^2 = 32x$, then find the other end of the chord.

624. Prove that the length of the intercept on the normal at the point $P\left(at^2, 2at\right)$ of the parabola $y^2 = 4ax$ made by the circle described on the line joining the focus and P as diameter is $a\sqrt{1 + t^2}$.

625. Find the minimum distance between the curves $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$

626. If $y = 2x + 3$ is a tangent to the parabola $y^2 = 24x,$ then find its distance from the parallel normal.

627. Three normals to $y^2 = 4x$ pass through the point (15, 12). Show that one of the normals is given by $y = x - 3$ and find the equation of the other.

628. Find the locus of the point from which the two tangents drawn to the parabola $y^2 = 4ax$ are such that the slope of one is thrice that of the other.

629. Find the angle between the tangents drawn from the origin to the parabolas $y^2 = 4a(x - a)$ (a) $90°$ (b) $30°$ (c) $\tan^{-1}\left(\dfrac{1}{2}\right)$ (d) $45°$

630. Find the locus of the point of intersection of the perpendicular tangents of the curve $y^2 + 4y - 6x - 2 = 0$.

631. Three normals are drawn from the point (7, 14) to the parabola $x^2 - 8x - 16y = 0$. Find the coordinates of the feet of the normals.

632. Find the equation of normal to the parabola $y = x^2 - x - 1$ which has equal intercept on the axes. Also find the point where this normal meets

the curve again.

633. If $y = x + 2$ is normal to the parabola $y^2 = 4ax$, then find the value of a.

634. Find the equations of normal to the parabola $y^2 = 4ax$ at the ends of the latus rectum.

635. The coordinates of the ends of a focal chord of the parabola $y^2 = 4ax$ are $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$. Then find the value of $x_1 x_2 + y_1 y_2$.

636. If $t_1 and t_2$ are the ends of a focal chord of the parabola $y^2 = 4ax$, then prove that the roots of the equation $t_1x^2 + ax + t_2 = 0$ are real.

637. If the length of focal chord of $y^2 = 4ax$ is l, then find the angle between the axis of the parabola and the focal chord.

638. If length of focal chord PQ is l, and p is the perpendicular distance of PQ from the vertex of the parabola, then prove that $l \propto \dfrac{1}{p^2}$.

639. Find the equation of the tangent to the parabola $y^2 = 8x$ having slope 2 and also find the point of contact.

640. Find the equation of tangents of the parabola $y^2 = 12x$, which passes through the point (2, 5).

641. If the line $y = 3x + c$ touches the parabola $y^2 = 12x$ at point P , then find the equation of the tangent at point Q where PQ is a focal chord.

642. Find the equation of the tangent to the parabola $y = x^2 - 2x + 3$ at point (2, 3).

643. Find the equation of the tangent to the parabola $x = y^2 + 3y + 2$ having slope 1.

644. Find the equation of tangents drawn to the parabola $y = x^2 - 3x + 2$ from the point $(1, -1)$.

645. If a tangent to the parabola $y^2 = 4ax$ meets the x-axis at T and intersects the tangents at vertex A at P, and rectangle $TAPQ$ is completed, then find the locus of point Q

646. The parabola $y^2 = 4x$ and the circle having its center at (6, 5) intersect at right angle. Then find the possible points of intersection of these curves.

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647. The tangents to the parabola $y^2 = 4ax$ at the vertex V and any point P meet at Q . If S is the focus, then prove that $SP, SQ,$ and SV are in GP.

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648. Show that $x\cos\alpha + y\sin\alpha = p$ touches the parabola $y^2 = 4ax$ if $p\cos\alpha + a\sin^2\alpha = 0$ and that the point of contact is $\left(a\tan^2\alpha, \ -2a\tan\alpha\right).$

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649. A tangent to the parabola $y^2 = 8x$ makes an angle of 45^0 with the straight line $y = 3x + 5$. Then find one of the points of contact.

650. Find the equation of the common tangent of $y^2 = 4ax$ and $x^2 = 4ay$.

651. If the lines L_1 and L_2 are tangents to $4x^2 - 4x - 24y + 49 = 0$ and are normals for $x^2 + y^2 = 72$, then find the slopes of L_1 and L_2

652. Find the shortest distance between the line $y = x - 2$ and the parabola $y = x^2 + 3x + 2$.

653. If two tangents drawn from the point (α, β) to the parabola $y^2 = 4x$ are such that the slope of one tangent is double of the other, then prove that $\alpha = \dfrac{2}{9}\beta^2$.

654. Find $\dfrac{dy}{dx}$, $\tan^{-1}\left\{\dfrac{\cos x - \sin x}{\cos x + \sin x}\right\}$.

655. Find the angle at which normal at point $P\left(at^2, 2at\right)$ to the parabola meets the parabola again at point Q.

656. If tangents are drawn to $y^2 = 4ax$ from any point P on the parabola $y^2 = a(x + b)$, then show that the normals drawn at their point for contact meet on a fixed line.

657. Find the equation of a parabola having its focus at $S(2, 0)$ and one extremity of its latus rectum at $(2, 2)$

658. Find the equation of a parabola having focus at $(0, -3)$ and directrix $y = 3$.

659. Find the equation of a parabola having its vertex at $A(1, 0)$ and focus at $S(3, 0)$

660. A beam is supported at its ends by supports which are 12 metres apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection 1 cm?

661. Find the coordinates of points on the parabola $y^2 = 8x$ whose focal distance is 4.

662. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus.

663. An arch is in the form of a parabola with its axis vertical. The arc is 10m high and 5m wide at the base. How wide is it 2m from the vertex of the parabola?

664. If the vertex of a parabola is the point (- 3, 0) and the directrix is the line $x + 5 = 0$, then find its equation.

665. The chord AB of the parabola $y^2 = 4ax$ cuts the axis of the parabola at $\overset{.}{C}$ If $A \equiv \left(at_1^2, 2at_1\right), B \equiv \left(at_2^2, 2at_2\right)$, and $AC:AB = 1:3$, then prove that $t_2 + 2t_1 = 0$.

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666. Prove that the chord $y - x\sqrt{2} + 4a\sqrt{2} = 0$ is a normal chord of the parabola $y^2 = 4ax$. Also find the point on the parabola when the given chord is normal to the parabola.

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667. Find the point on the curve $y^2 = ax$ the tangent at which makes an angle of $45°$ with the x-axis.

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668. Find the equation of the straight lines touching both $x^2 + y^2 = 2a^2$ and $y^2 = 8ax$.

669. Find the points of contact Q and R of a tangent from the point $P(2, 3)$ on the parabola $y^2 = 4x$.

670. Two straight lines $(y - b) = m_1(x + a)$ and $(y - b) = m_2(x + a)$ are the tangents of $y^2 = 4ax$ Prove $m_1 m_2 = -1$.

671. A pair of tangents are drawn to the parabola $y^2 = 4ax$ which are equally inclined to a straight line $y = mx + c$, whose inclination to the

axis is α . Prove that the locus of their point of intersection is the straight line $y = (x - a)\tan 2\alpha$

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672. Tangent are drawn from the point (- 1, 2) on the parabola $y^2 = 4x$. Find the length that these tangents will intercept on the line $x = 2$.

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673. Tangents are drawn to the parabola $(x - 3)^2 + (y - 4)^2 = \dfrac{(3x - 4y - 6)^2}{25}$ at the extremities of the chord $2x - 3y - 18 = 0$. Find the angle between the tangents.

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674. Find the locus of point of intersection of tangents to the parabola $y^2 = 4ax$

(i) which are inclined at an angle θ to each other

(ii) which intercept constant length c on the tangent at vertex

(iii) such that area of $\triangle ABR$ is constant c, where A and B are points of intersection of tangents with y-axis, R is point of intersection of tangents.

675. Mutually perpendicular tangents $TAandTB$ are drawn to $y^2 = 4ax$. Then find the minimum length of AB

676. Tangent $PAandPB$ are drawn from the point P on the directrix of the parabola $(x - 2)^2 + (y - 3)^2 = \dfrac{(5x - 12y + 3)^2}{160}$. Find the least radius of the circumcircle of triangle PAB

677. A square has one vertex at the vertex of the parabola $y^2 = 4ax$ and the diagonal through the vertex lies along the axis of the parabola. If the ends of the other diagonal lie on the parabola, the coordinates of the vertices of the square are (a)$(4a, 4a)$ (b) $(4a, -4a)$ (c)$(0, 0)$ (d) $(8a, 0)$

678. P, Q, and R are the feet of the normals drawn to a parabola ($(y - 3)^2 = 8(x - 2)$. A circle cuts the above parabola at points P, Q, R, and S . Then this circle always passes through the point. (a) (2, 3) (b) (3, 2) (c) (0, 3) (d) (2, 0)

679. The equation of the line that passes through (10, - 1) and is perpendicular to $y = \dfrac{x^2}{4} - 2$ is (a)$4x + y = 39$ (b) $2x + y = 19$ (c)$x + y = 9$ (d) $x + 2y = 8$

680. The axis of a parabola is along the line $y = x$ and the distance of its vertex and focus from the origin are $\sqrt{2}$ and $2\sqrt{2}$, respectively. If vertex and focus both lie in the first quadrant, then the equation of the parabola is

(a) $(x + y)^2 = (x - y - 2)$ (b)$(x - y)^2 = (x + y - 2)$ (c)$(x - y)^2 = 4(x + y - 2)$ (d) $(x - y)^2 = 8(x + y - 2)$

681. If the normal chord of the parabola $y^2 = 4x$ makes an angle $45°$ with the axis of the parabola, then its length, is

682. If the normals at points t_1 and t_2 meet on the parabola, then

(a) $t_1 t_2 = 1$ (b) $t_2 = -t_1 - \dfrac{2}{t_1}$ (c) $t_1 t_2 = 2$ (d) none of these

683. From a point $(\sin\theta, \cos\theta)$, if three normals can be drawn to the parabola $y^2 = 4ax$ then find the value of a .

684. If the normals to the parabola $y^2 = 4ax$ at the ends of the latus rectum meet the parabola at Q and $Q^{'}$, then $QQ^{'}$ is $(a)10a$ (b) $4a$ (c) $20a$ (d) $12a$

685. Tangent and normal drawn to a parabola at $A\left(at^2, 2at\right), t \ne 0$ meet the x-axis at point B and D , respectively. If the rectangle $ABCD$ is completed, then the locus of C is

$(a)y = 2a$ (b) $y + 2a = c$ $(c)x = 2a$ (d) none of these

686. PQ is a normal chord of the parabola $y^2 = 4ax$ at P, A being the vertex of the parabola. Through P, a line is drawn parallel to AQ meeting the x-axis at R. Then the line length of AR is

(A) equal to the length of the latus rectum (B)equal to the focal distance of the point P (C) equal to twice the focal distance of the point P (D) equal to the distance of the point P from the directrix.

687. If two normals to a parabola $y^2 = 4ax$ intersect at right angles then the chord joining their feet pass through a fixed point whose co-ordinates are:

688. If the normals to the parabola $y^2 = 4ax$ at P meets the curve again at Q and if PQ and the normal at Q make angle α andβ , respectively, with the x-axis, then $tan\alpha(tan\alpha + tan\beta)$ has the value equal to 0 (b) -2 (c) $-\dfrac{1}{2}$ (d) -1

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689. If a leaf of a book is folded so that one corner moves along an opposite side, then prove that the line of crease will always touch parabola.

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690. A parabola of latus rectum l touches a fixed equal parabola. The axes of two parabolas are parallel. Then find the locus of the vertex of the moving parabola.

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691. A movable parabola touches x-axis and y-axis at (0,1) and (1,0). Then the locus of the focus of the parabola is :

692. Let N be the foot of perpendicular to the x-axis from point P on the parabola $y^2 = 4ax$ A straight line is drawn parallel to the axis which bisects PN and cuts the curve at Q; if NO meets the tangent at the vertex at a point then prove that $AT = \dfrac{2}{3}PN$

693. Two lines are drawn at right angles, one being a tangent to $y^2 = 4ax$ and the other $x^2 = 4by$ Then find the locus of their point of intersection.

694. The area of the trapezium whose vertices lie on the parabola $y^2 = 4x$ and its diagonals pass through (1,0) and having length $\dfrac{25}{4}$ units each is

695. Find the range of parameter a for which a unique circle will pass through the points of intersection of the hyperbola $x^2 - y^2 = a^2$ and the parabola $y = x^2$ Also, find the equation of the circle.

696. Find the radius of the largest circle, which passes through the focus of the parabola $y^2 = 4(x + y)$ and is also contained in it.

697. A tangent is drawn to the parabola $y^2 = 4ax$ at P such that it cuts the y-axis at Q A line perpendicular to this tangents is drawn through Q which cuts the axis of the parabola at R . If the rectangle $PQRS$ is completed, then find the locus of S .

698. Tangents are drawn to the parabola at three distinct points. Prove that these tangent lines always make a triangle and that the locus of the orthocentre of the triangle is the directrix of the parabola.

699. Statement 1: The circumcircle of a triangle formed by the lines $x = 0$, $x + y + 1 = 0$ and $x - y + 1 = 0$ also passes through the point (1, 0).

Statement 2: The circumcircle of a triangle formed by three tangents of a parabola passes through its focus.

700. Statement 1: The point of intersection of the tangents at three distinct points $A, B, and C$ on the parabola $y^2 = 4x$ can be collinear. Statement 2: If a line L does not intersect the parabola $y^2 = 4x$, then from every point of the line, two tangents can be drawn to the parabola.

701. Statement 1: If the straight line $x = 8$ meets the parabola $y^2 = 8x$ at $P and Q$, then PQ substends a right angle at the origin. Statement 2: Double ordinate equal to twice of latus rectum of a parabola subtends a right angle at the vertex.

702. Statement 1: Normal chord drawn at the point (8, 8) of the parabola $y^2 = 8x$ subtends a right angle at the vertex of the parabola. Statement 2:

Every chord of the parabola $y^2 = 4ax$ passing through the point $(4a, 0)$ subtends a right angle at the vertex of the parabola.

703. Statement 1: The value of α for which the point $\left(\alpha, \alpha^2\right)$ lies inside the triangle formed by the lines $x = 0$, $x + y = 2$ and $3y = x$ is $(0, 1)$ Statement 2: The parabola $y = x^2$ meets the line$x + y = 2$ at$(1, 1)$

704. Statement 1: If there exist points on the circle $x^2 + y^2 = a^2$ from which two perpendicular tangents can be drawn to the parabola $y^2 = 2x$, then $a \geq \dfrac{1}{2}$ Statement 2: Perpendicular tangents to the parabola meet at the directrix.

705. Let L be a normal to the parabola $y^2 = 4x$ If L passes through the point (9, 6), then L is given by (a) $y - x + 3 = 0$ (b) $y + 3x - 33 = 0$ (c) $y + x - 15 = 0$ (d) $y - 2x + 12 = 0$

706. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P?

707. The tangent at any point P onthe parabola $y^2 = 4ax$ intersects the y-axis at Q Then tangent to the circumcircle of triangle PQS(S is the focus) at Q is a line parallel to x-axis y-axis a line parallel to y-axis (d) none of these

708. If $y = m_1 x + c$ and $y = m_2 x + c$ are two tangents to the parabola $y^2 + 4a(x + a) = 0$, then (a)$m_1 + m_2 = 0$ (b) $1 + m_1 + m_2 = 0$ (c) $m_1 m_2 - 1 = 0$ (d) $1 + m_1 m_2 = 0$

709. AB is a double ordinate of the parabola $y^2 = 4ax$ Tangents drawn to the parabola at $A and B$ meet the y-axis at $A_1 and B_1$, respectively. If the area of trapezium $\forall_1 B_1 B$ is equal to $12a^2$, then the angle subtended by $A_1 B_1$ at the focus of the parabola is equal to $2\tan^{-1}(3)$ (b) $\tan^{-1}(3)$ $2\tan^{-1}(2)$ (d) $\tan^{-1}(2)$

710. If $y + 3 = m_1(x + 2)$ and $y + 3 = m_2(x + 2)$ are two tangents to the parabola $y_2 = 8x$, then (a)$m_1 + m_2 = 0$ (b) $m_1 + m_2 = -1$ (c)$m_1 + m_2 = 1$

(d) none of these

711. A line of slope $\lambda(0 < \lambda < 1)$ touches the parabola $y + 3x^2 = 0$ at P. If S is the focus and M is the foot of the perpendicular of directrix from P, then $\tan \angle MPS$ equals (A) 2λ (B) $\dfrac{2\lambda}{-1 + \lambda^2}$ (C) $\dfrac{1 - \lambda^2}{1 + \lambda^2}$ (D) none of these

712. If $y = 2x - 3$ is tangent to the parabola $y^2 = 4a\left(x - \dfrac{1}{3}\right)$, then a is equal to (a) $\dfrac{22}{3}$ (b) -1 (c) $\dfrac{14}{3}$ (d) $\dfrac{-14}{3}$

713. The straight lines joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P intersect at

R Then the equation of the locus of R is

(a)$x^2 + 2y^2 - ax = 0$ (b)$2x^2 + y^2 - 2ax = 0$ (c)$2x^2 + 2y^2 - ay = 0$ (d)

$2x^2 + y^2 - 2ay = 0$

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714. Through the vertex O of the parabola $y^2 = 4ax$, two chords $OPandOQ$ are drawn and the circles on OP and OQ as diameters intersect at R If θ_1, θ_2 , and φ are the angles made with the axis by the tangents at P and Q on the parabola and by OR, then value of $\cot\theta_1 + \cot\theta_2$ is -2tanφ (b) -2tan(π - φ) 0 (d) 2cotφ

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715. A tangent is drawn to the parabola $y^2 = 4x$ at the point P whose abscissa lies in the interval (1, 4). The maximum possible area of the triangle formed by the tangent at P, the ordinates of the point P, and the x-axis is equal to (a)8 (b) 16 (c) 24 (d) 32

716. A parabola $y = ax^2 + bx + c$ crosses the x-axis at $(\alpha, 0)$ and $(\beta, 0)$ both to the right of the origin. A circle also pass through these two points. The length of a tangent from the origin to the circle is $\sqrt{\dfrac{bc}{a}}$ (b) ac^2 (c) $\dfrac{b}{a}$ (d) $\sqrt{\dfrac{c}{a}}$

717. From a point on the circle $x^2 + y^2 = a^2$, two tangents are drawn to the circle $x^2 + y^2 = b^2(a > b)$. If the chord of contact touches a variable circle passing through origin, show that the locus of the center of the variable circle is always a parabola.

718. A line AB makes intercepts of lengths a and b on the coordinate axes. Find the equation of the parabola passing through A, B, and the origin, if AB is the shortest focal chord of the parabola.

719. Prove that the line joining the orthocentre to the centroid of a triangle formed by the focal chord of a parabola and tangents drawn at its extremities is parallel to the axis of the parabola.

720. A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B If AB subtends a right angle at the vertex of the parabola, find the slope of AB

721. The equation of the line that touches the curves $y = x|x|$ and $x^2 + (y - 2)^2 = 4$, where $x \neq 0$, is:

722. Let PQ be a chord of the parabola $y^2 = 4x$. A circle drawn with PQ as a diameter passes through the vertex V of theparabola. If $ar(\Delta PVQ) = 20$ sq unit then the coordinates of P are

723. Statement 1: Through $(\lambda, \lambda + 1)$, there cannot be more than one normal to the parabola $y^2 = 4x$, if $\lambda < 2$. Statement 2 : The point $(\lambda, \lambda + 1)$ lies outside the parabola for all $\lambda \neq 1$.

724. Each question has four choices a, b, c and d, out of which only one is correct. Each question contains Statement 1 and Statement 2. Find the correct answer. Statement 1 : Slopes of tangents drawn from (4, 10) to theparabola $y^2 = 9x$ are and 1/4 and 9/4 . Statement 2 : Two tangents can be drawn to a parabola from any point lying outside the parabola.

725. Statement 1: The line joining the points $(8, -8) and \left(\dfrac{1}{2}, 2\right)$, which are on the parabola $y^2 = 8x$, press through the focus of the parabola. Statement 2: Tangents drawn at $(8, -8)$ and $\left(\dfrac{1}{2}, 2\right)$, on the parabola $y^2 = 4ax$ are perpendicular.

726. Statement 1: In the parabola $y^2 = 4ax$, the circle drawn the taking the focal radii as diameter touches the y-axis. Statement 2: The portion of

the tangent intercepted between the point of contact and directrix subtends an angle of 90^0 at focus.

727. The vertices A,B, and C of a variable right triangle lie on a parabola $y^2 = 4x$. If the vertex B containing the right angle always remains at the point (1,2), then find the locus of the centroid of triangle ABC.

728. Show that the common tangents to the parabola $y^2 = 4x$ and the circle $x^2 + y^2 + 2x = 0$ form an equilateral triangle.

729. Consider a curve $C : y^2 - 8x - 2y - 15 = 0$ in which two tangents $T_1 and T_2$ are drawn from $P(- 4, 1)$. Statement 1: $T_1 and T_2$ are mutually

perpendicular tangents. Statement 2: Point P lies on the axis of curve C

730. Statement 1: The line $ax + by + c = 0$ is a normal to the parabola $y^2 = 4ax$ Then the equation of the tangent at the foot of this normal is $y = \left(\dfrac{b}{a}\right)x + \left(\dfrac{a^2}{b}\right)$ Statement 2: The equation of normal at any point $P\left(at^2, 2at\right)$ to the parabola $y^2 = 4ax$ is $y = -tx + 2at + at^3$

731. Statement 1: The length of focal chord of a parabola $y^2 = 8x$ making on an angle of 60^0 with the x-axis is 32. Statement 2: The length of focal chord of a parabola $y^2 = 4ax$ making an angle with the x-axis is $4a\cosec^2\alpha$

732. Statement 1: $(5x - 5)^2 + (5y + 10)^2 = (3x + 4y + 5)^2$ is a parabola.

Statement 2: If the distance of the point from a given line and from a given point (not lying on the given line) is equal, then the locus of the variable point is a parabola.

733. If the bisector of angle APB, where $PA and PB$ are the tangents to the parabola $y^2 = 4ax$, is equally, inclined to the coordinate axes, then the point P lies on the a.tangent at vertex of the parabola b.directrix of the parabola c.circle with center at the origin and radius a d.the line of the latus rectum.

734. If d is the distance between the parallel tangents with positive slope to $y^2 = 4x$ and $x^2 + y^2 - 2x + 4y - 11 = 0$, then (a)$10 < d < 2$ (b)$4 < d < 6$(c) $d < 4$ (d) none of these

735. If $P\left(t^2, 2t\right)$, $t \in [0, 2]$, is an arbitrary point on the parabola $y^2 = 4x$, Q is the foot of perpendicular from focus S on the tangent at P, then the maximum area of PQS is (a) 1 (b) 2 (c) $\dfrac{5}{16}$ (d) 5

736. If the parabola $y = ax^2 - 6x + b$ passes through $(0, 2)$ and has its tangent at $x = \dfrac{3}{2}$ parallel to the x-axis, then (a) $a = 2, b = -2$ (b) $a = 2, b = 2$ a $= -2, b = 2$ (d) $a = -2, b = -2$

737. If the locus of the middle of point of contact of tangent drawn to the parabola $y^2 = 8x$ and the foot of perpendicular drawn from its focus to

the tangents is a conic, then the length of latus rectum of this conic is $\dfrac{9}{4}$

(b) 9 (c) 18 (d) $\dfrac{9}{2}$

738. The minimum area of circle which touches the parabolas $y = x^2 + 1$

and $y^2 = x - 1$ is $\dfrac{9\pi}{16} squnit$ (b) $\dfrac{9\pi}{32} squnit$ $\dfrac{9\pi}{8} squnit$ (d) $\dfrac{9\pi}{4} squnit$

739. Tangent is drawn to parabola $y^2 - 2y - 4x + 5 = 0$ at a point P which

cuts the directrix at the point Q. A point R is such that it divides QP

externally in the ratio 1/2:1. Find the locus of point R

740. The point of intersection of the tangents at the ends of the latus

rectum of the parabola $y^2 = 4x$ is_____

741. Â·If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2(y + 2)^2 = r^2$, then the value of r^2 is

742. From a point A, commont tangents are drawn to the circle $x^2 + y^2 = 8$ and parabola $y^2 = 16x$. The area of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola in squae units is...

743. The angle between a pair of tangents drawn from a point P to the hyperbola $y^2 = 4ax$ is $45°$. Show that the locus of the point P is hyperbola.

744. Three normals with slopes m_1, m_2 and m_3 are drawn from a point P not on the axis of the axis of the parabola $y^2 = 4x$. If $m_1 m_2 = \alpha$, results in the locus of P being a part of parabola, Find the value of α

745. Three normals are drawn from the point (c, 0) to the curve $y^2 = x$. Show that c must be greater than 1/2. One normal is always the axis. Find c for which the other two normals are perpendicular to each other.

746. Find the equation of the normal to curve $x^2 = 4y$ which passes through the point (1, 2).

747. Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio $1:2$ is parabola. Find the vertex of this parabola.

748. Points A, B, C lie on the parabola $y^2 = 4ax$ The tangents to the parabola at A, B and C, taken in pair, intersect at points P, Q and R. Determine the ratio of the areas of the $\triangle ABC$ and $\triangle PQR$

749. If the focus of the parabola $x^2 - ky + 3 = 0$ is (0,2), then a values of k is (are) (a) 4 (b) 6 (c) 3 (d) 2

750. Let P be a point whose coordinates differ by unity and the point does not lie on any of the axes of reference. If the parabola $y^2 = 4x + 1$ passes through P, then the ordinate of P may be (a) 3 (b) -1 (c) 5 (d) 1

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751. Statement 1: The line $x - y - 5 = 0$ cannot be normal to the parabola $(5x - 15)^2 + (5y + 10)^2 = (3x - 4y + 2)^2$. Statement 2: Normal to parabola never passes through its focus.

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752. If (h, k) is a point on the axis of the parabola $2(x - 1)^2 + 2(y - 1)^2 = (x + y + 2)^2$ from where three distinct normals can be drawn, then prove that $h > 2$.

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753. Column I, Column II Points from which perpendicular tangents can be drawn to the parabola $y^2 = 4x$, p. (- 1, 2) Points from which only one normal can be drawn to the parabola $y^2 = 4x$, q. (3, 2) Point at which chord $x - y - 1 = 0$ of the parabola $y^2 = 4x$ is bisected., r. (- 1, - 5) Points from which tangents cannot be drawn to the parabola $y^2 = 4x$, s. (5, - 2)

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754. Consider the parabola $y^2 = 12x$ Column I, Column II Equation of tangent can be, p. $2x + y - 6 = 0$ Equation of normal can be, q. $3x - y + 1 = 0$ Equation of chord of contact w.r.t. any point on the directrix can be, r. $x - 2y - 12 = 0$ Equation of chord which subtends right angle at the vertex can be, s. $2x - y - 36 = 0$

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755. If the tangent at the point $P(2, 4)$ to the parabola $y^2 = 8x$ meets the parabola $y^2 = 8x + 5$ at Q and $R,$ then find the midpoint of chord QR

756. Let P be the family of parabolas $y = x^2 + px + q, (q \neq 0)$, whose graphs cut the axes at three points. The family of circles through these three points have a common point (1, 0) (b) (0, 1) (c) (1, 1) (d) none of these

757. If normal at point P on the parabola $y^2 = 4ax, (a > 0)$, meets it again at Q in such a way that OQ is of minimum length, where O is the vertex of parabola, then OPQ is

(a)a right angled triangle (b)an obtuse angled triangle (c)an acute angle triangle (d)none of these

758. If line PQ , where equation is $y = 2x + k$, is a normal to the parabola whose vertex is ($-2, 3$) and the axis parallel to the x-axis with latus rectum equal to 2, then the value of k is (a) $\dfrac{58}{8}$ (b) $\dfrac{50}{8}$ (c) 1 (d) -1

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759. The parabola $y = x^2 + px + q$ cuts the straight line $y = 2x - 3$ at a point with abscissa 1. Then the value of $pandq$ for which the distance between the vertex of the parabola and the x-axis is the minimum is (a) $p = -1, q = -1$ (b) $p = -2, q = 0$ (c) $p = 0, q = -2$ (d) $p = \dfrac{3}{2}, q = -\dfrac{1}{2}$

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760. Tangent is drawn at any point (p, q) on the parabola $y^2 = 4ax$ .Tangents are drawn from any point on this tangant to the circle $x^2 + y^2 = a^2$, such that the chords of contact pass through a fixed point

(r, s). Then p, q, r ands can hold the relation (A) $r^2q = 4p^2s$ (B) $rq^2 = 4ps^2$ (C) $rq^2 = -4ps^2$ (D) $r^2q = -4p^2s$

761. The equation of the directrix of the parabola with vertex at the origin and having the axis along the x-axis and a common tangent of slope 2 with the circle $x^2 + y^2 = 5$ is (are) (a)$x = 10$ (b) $x = 20$ (c)$x = -10$ (d) $x = -20$

762. Tangent is drawn at any point $\left(x_1, y_1\right)$ other than the vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point $\left(x_2, y_2\right)$, then (a) x_1, a, x_2 in GP (b) $\dfrac{y_1}{2}, a, y_2$ are in GP (c)-4, $\dfrac{y_1}{y_2}, \dfrac{x_1}{x_2}$ are in GP (d) $x_1x_2 + y_1y_2 = a^2$

763. The angle between the tangents to the curve $y = x^2 - 5x + 6$ at the point $(2, 0)$ and $(3, 0)$ is (a)$\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{3}$ (c) π (d) $\dfrac{\pi}{4}$

764. Statement 1: If the parabola $y^2 = 4ax$ and the circle $x^2 + y^2 + 2bx = 0$ touch each other externally, then the roots of the equation $f(x) = x^2 - (b + a + 1)x + a = 0$ are real. Statement 2: For parabola and circle touching externally, $a and b$ must have the same sign.

765. If a line $y = 3x + 1$ cuts the parabola $x^2 - 4x - 4y + 20 = 0$ at A and B, then the tangent of the angle subtended by line segment AB at the origin is (a) $\dfrac{8\sqrt{3}}{205}$ (b) $\dfrac{8\sqrt{3}}{209}$ (c)$\dfrac{8\sqrt{3}}{215}$ (d) none of these

766. $P(x, y)$ is a variable point on the parabola $y^2 = 4ax$ and $Q(x + c, y + c)$ is another variable point, where c is a constant. The locus of the midpoint of PQ is (a) parabola (b) hyperbola (c) circle (d) none of these

767. If a and c are the lengths of segments of any focal chord of the parabola $y^2 = 2bx, (b > 0)$, then the roots of the equation $ax^2 + bx + c = 0$ are (a) real and distinct (b) real and equal (c) imaginary (d) none of these

768. AB is a chord of the parabola $y^2 = 4ax$ with its vertex at A. BC is drawn perpendicular to AB meeting the axis at C.The projecton of BC on the axis of the parabola is

769. The set of values of α for which the point $(\alpha, 1)$ lies inside the curves $c_1 : x^2 + y^2 - 4 = 0$ and $c_2 : y^2 = 4x$ is (a) $|\alpha| < \sqrt{3}$ (b) $|\alpha| < 2$ (c) $\frac{1}{4} < \alpha < \sqrt{3}$ (d) none of these

770. If P be a point on the parabola $y^2 = 3(2x - 3)$ and M is the foot of perpendicular drawn from the point P on the directrix of the parabola, then find length of each sides of an equilateral triangle SMP(where S is the focus of the parabola).

771. If $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$, then (a)$c = \dfrac{a}{m}$ (b) $c = am + \dfrac{a}{m}$ (c)$c = a + \dfrac{a}{m}$ (d) none of these

772. The angle between the tangents to the parabola $y^2 = 4ax$ at the points where it intersects with the line $x - y - a = 0$ is (a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{4}$ (c) π (d) $\dfrac{\pi}{2}$

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773. The area of the triangle formed by the tangent and the normal to the parabola $y^2 = 4ax$, both drawn at the same end of the latus rectum, and the axis of the parabola is $2\sqrt{2}a^2$ (b) $2a^2$ $4a^2$ (d) none of these

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774. Double ordinate AB of the parabola $y^2 = 4ax$ subtends an angle $\dfrac{\pi}{2}$ at the vertex of the parabola. Then the tangents drawn to the parabola at A. and B will intersect at (a) $(-4a, 0)$ (b) $(-2a, 0)$ $(-3a, 0)$ (d) none of these

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775. Statement 1: The normals at the points (4, 4) and $\left(\dfrac{1}{4}, -1\right)$ of the parabola $y^2 = 4x$ are perpendicular. Statement 2: The tangents to the parabola at the end of a focal chord are perpendicular.

776. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose:

777. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then find the slope of the line joining A and B .

778. The equations of the common tangents to the parabola $y = x^2$ and $y = -(x-2)^2$ is/are :

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780. Let (x,y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0,0) and (x,y) n the ratio 1:3. Then the locus of P is :

781. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at P, Q andthe parabola at R, S. Then area of

quadrilateral $PQRS$ is

782. If two distinct chords of a parabola $y^2 = 4ax$, passing through (a,2a) are bisected by the line x+y=1 ,then length of latus rectum can be a) 2 b) 7 c) 4 d) 5

783. The point of intersection of the tangents of the parabola $y^2 = 4x$ drawn at the endpoints of the chord $x + y = 2$ lies on

(a)$x - 2y = 0$ (b) $x + 2y = 0$ (c)$y - x = 0$ (d) $x + y = 0$

784. Which of the following line can be normal to parabola $y^2 = 12x$? (a) $x + y - 9 = 0$ (b) $2x - y - 32 = 0$ (c)$2x + y - 36 = 0$ (d) $3x - y - 72 = 0$

785. Which of the following line can be tangent to the parabola $y^2 = 8x$?

(a)$x - y + 2 = 0$ (b) $9x - 3y + 2 = 0$ (c)$x + 2y + 8 = 0$ (d) $x + 3y + 12 = 0$

786. The locus of the midpoint of the focal distance of a variable point moving on theparabola $y^2 = 4ax$ is a parabola whose (a)latus rectum is half the latus rectum of the original parabola (b)vertex is $\left(\dfrac{a}{2}, 0\right)$ (c)directrix is y-axis. (d)focus has coordinates (a, 0)

787. A quadrilateral is inscribed in a parabola. Then (a)the quadrilateral may be cyclic (b)diagonals of the quadrilateral may be equal (c)allpossible pairs of the adjacent side may be perpendicular (d)none of these

788. A normal drawn to the parabola $y^2 = 4ax$ meets the curve again at Q such that the angle subtended by PQ at the vertex is 90^0 Then the coordinates of P can be (a)$\left(8a, 4\sqrt{2}a\right)$ (b) $(8a, 4a)$ (c)$\left(2a, -2\sqrt{2}a\right)$ (d) $\left(2a, 2\sqrt{2}a\right)$

789. The parabola $y^2 = 4x$ and the circle having its center at (6, 5) intersect at right angle. Then find the possible points of intersection of these curves.

790. The extremities of latus rectum of a parabola are (1, 1) and (1, - 1) . Then the equation of the parabola can be (a)$y^2 = 2x - 1$ (b) $y^2 = 1 - 2x$ (c) $y^2 = 2x - 3$ (d) $y^2 = 2x - 4$

791. If $y = 2$ is the directrix and $(0, 1)$ is the vertex of the parabola $x^2 + \lambda y + \mu = 0$, then (a)$\lambda = 4$ (b) $\mu = 8$ (c) $\lambda = -8$ (d) $\mu = 4$

792. Through the vertex 'O' of parabola $y^2 = 4x$, chords OP and OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ.

793. If two chords drawn from the point $A(4, 4)$ to the parabola $x^2 = 4y$ are bisected by the line $y = mx$, the interval in which m lies is (a) $\left(-2\sqrt{2}, 2\sqrt{2}\right)$ (b) $\left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$ (c) $\left(-\infty, -2\sqrt{2} - 2\right) \cup \left(2\sqrt{2} - 2, \infty\right)$ (d) none of these

794. Statement 1: If the endpoints of two normal chords AB and CD (normal at A and C) of a parabola $y^2 = 4ax$ are concyclic, then the tangents at A and C will intersect on the axis of the parabola. Statement 2: If four points on the parabola $y^2 = 4ax$ are concyclic, then the sum of their ordinates is zero.

795. Consider the parabola $y^2 = 4x$ Let $A \equiv (4, -4)$ and $B \equiv (9, 6)$ be two fixed points on the parabola. Let C be a moving point on the parabola between A and B such that the area of the triangle ABC is maximum. Then the coordinates of C are (a) $\left(\frac{1}{4}, 1\right)$ (b) $(4, 4)$ (c) $\left(3, \frac{2}{\sqrt{3}}\right)$ (d) $\left(3, -2\sqrt{3}\right)$

796. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola at the point (1,2) is

797. Two straight lines are perpendicular to each other. One of them touches the parabola $y^2 = 4a(x + a)$ and the other touches $y^2 = 4b(x + b)$. Their point of intersection lies on the line. (a)$x - a + b = 0$ (b) $x + a - b = 0$ (c)$x + a + b = 0$ (d) $x - a - b = 0$

798. If the tangents and normals at the extremities of a focal chord of a parabola intersect at $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$, respectively, then (a)$x_1 = y_2$ (b) $x_1 = y_1$ (c)$y_1 = y_2$ (d) $x_2 = y_1$

799. Radius of the circle that passes through the origin and touches the parabola $y^2 = 4ax$ at the point $(a, 2a)$ is (a) $\frac{5}{\sqrt{2}}a$ (b) $2\sqrt{2}a$ (c) $\sqrt{\frac{5}{2}}a$ (d) $\frac{3}{\sqrt{2}}a$

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800. If A_1B_1 and A_2B_2 are two focal chords of the parabola $y^2 = 4ax$, then the chord A_1A_2 and B_1B_2 intersect on

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801. The tangent and normal at $P(t)$, for all real positive t, to the parabola $y^2 = 4ax$ meet the axis of the parabola in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle passing through the points P, T and G is

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802. $y = x + 2$ is any tangent to the parabola $y^2 = 8x$ The point P on this tangent is such that the other tangent from it which is perpendicular to it is (a)(2, 4) (b) (- 2, 0) (c)(- 1, 1) (d) (2, 0)

803. Two parabola have the same focus. If their directrices are the x-axis and the y-axis respectively, then the slope of their common chord is :

804. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex pf the parabola and base QR is a focal chord.The modulus of the difference of the ordinates of the points Q and R is :

805. The length of the chord of the parabola $y^2 = x$ which is bisected at the point (2, 1) is (a)$2\sqrt{3}$ (b) $4\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{5}$

806. The circle $x^2 + y^2 = 5$ meets the parabola $y^2 = 4x$ at P and Q . Then the length PQ is equal to (A) 2 (B) $2\sqrt{2}$ (C) 4 (D) none of these

807. A line is drawn form $A(-2, 0)$ to intersect the curve $y^2 = 4x$ at P and Q in the first quadrant such that $\dfrac{1}{AP} + \dfrac{1}{AQ} < \dfrac{1}{4}$ Then the slope of the line is always. (A) $> \sqrt{3}$ (B) $< \dfrac{1}{\sqrt{3}}$ (C) $> \sqrt{2}$ (D) $> \dfrac{1}{\sqrt{3}}$

808. Let $y = f(x)$ be a parabola, having its axis parallel to the y-axis, which is touched by the line $y = x$ at $x = 1$. Then, (a)$2f(0) = 1 - f'(0)$ (b) $f(0) + f'(0) + f^0 = 1$ (c)$f'(1) = 1$ (d) $f'(0) = f'(1)$

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809. Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet the axis at $P_1 and P_2$. If S is the focus of the parabola, then $\dfrac{1}{SP_1} + \dfrac{1}{SP_2}$ isequal $\rightarrow$ 4/a(b)2/1(c)1/a(d)1/(4a)`

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810. Let S be the focus of $y^2 = 4x$ and a point P be moving on the curve such that its abscissa is increasing at the rate of 4 units/s. Then the rate of increase of the projection of SP on $x + y = 1$ when P is at $(4, 4)$ is (a)$\sqrt{2}$ (b) -1 (c) $-\sqrt{2}$ (d) $-\dfrac{3}{\sqrt{2}}$

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811. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then (a) $d^2 + (2b + 3c)^2 = 0$ (b)$d^2 + (3b + 2c)^2 = 0$ (c)$d^2 + (2b - 3c)^2 = 0$ (d)none of these

812. If y_1, y_2, y_3 be the ordinates of a vertices of the triangle inscribed in a parabola $y^2 = 4ax$, then show that the area of the triangle is $\frac{1}{8a} \left| (y_1 - y_2)(y_2 - y_3)(y_3 - y_1) \right|$.

813. The circle $x^2 + y^2 + 2\lambda x = 0, \lambda \in R$, touches the parabola $y^2 = 4x$ externally. Then, (a)$\lambda > 0$ (b) $\lambda < 0$ $\lambda > 1$ (d) none of these

814. If PSQ is a focal chord of the parabola $y^2 = 8x$ such that $SP = 6$, then the length of SQ is (a)6 (b) 4 (c) 3 (d) none of these

815. Parabola $y^2 = 4a\left(x - c_1\right)$ and $x^2 = 4a\left(y - c_2\right)$, where $c_1 and c_2$ are variable, are such that they touch each other. The locus of their point of contact is (a)$xy = 2a^2$ (b) $xy = 4a^2$ (c)$xy = a^2$ (d) none of these

816. A circle touches the x-axis and also thouches the circle with center (0, 3) and radius 2. The locus of the center of the circle is (a) a circle (b) an ellipse (c) a parabola (d) a hyperbola

817. The locus of the vertex of the family of parabolas $y = \dfrac{a^3x^2}{3} + \dfrac{a^{2x}}{2} - 2a$

is $xy = \dfrac{105}{64}$ (b) $xy = \dfrac{3}{4}$ $xy = \dfrac{35}{16}$ (d) $xy = \dfrac{64}{105}$

818. Let P be the point (1,0) and Q be a point on the locus $y^2 = 8x$. The locus of the midpoint of PQ is

819. If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, then find the value of PA.PB(where $P = \left(\sqrt{3}, 0\right)$

820. The locus of a point on the variable parabola $y^2 = 4ax$, whose distance from the focus is always equal to k, is equal to (a is parameter)

(a) $4x^2 + y^2 - 4kx = 0$ (b) $x^2 + y^2 - 4kx = 0$ (c) $2x^2 + 4y^2 - 9kx = 0$ (d)

$4x^2 - y^2 + 4kx = 0$

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821. Tangent to the curve $y = x^2 + 6$ at a point $(1, 7)$ touches the circle

$x^2 + y^2 + 16x + 12y + c = 0$ at a point Q, then the coordinates of Q are

(A) $(-6, -11)$ (B) $(-9, -13)$ (C) $(-10, -15)$ (D) $(-6, -7)$

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822. The angle between the tangents drawn from the point $(1,4)$ to the

parabola $y^2 = 4x$ is (A) $\dfrac{\pi}{6}$ (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{2}$

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823. Statement 1: There are no common tangents between the circle

$x^2 + y^2 - 4x + 3 = 0$ and the parabola $y^2 = 2x$ Statement 2:Given circle and

parabola do not intersect.

824. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of k is (a)$\dfrac{1}{8}$ (b) 8 (c) 4 (d) $\dfrac{1}{4}$

825. C is the centre of the circle with centre (0, 1) and radius unity. $y = ax^2$ is a parabola. The set of the values of $'a'$ for which they meet at a point other than the origin, is

826. The set of points on the axis of the parabola $(x - 1)^2 = 8(y + 2)$ from where three distinct normals can be drawn to the parabola is the set (h, k) of points satisfying $(a)h > 2$ (b) $h > 1$ $(c)k > 2$ (d) none of these

827. The shortest distance between the parabolas $2y^2 = 2x - 1$ and $2x^2 = 2y - 1$ is: (a) $2\sqrt{2}$ (b) $\dfrac{1}{2\sqrt{2}}$ (c) 4 (d) $\sqrt{\dfrac{36}{5}}$

828. Normals at two points $(x_1 y_1)$ and (x_2, y_2) of the parabola $y^2 = 4x$ meet again on the parabola, where $x_1 + x_2 = 4$. Then $\left| y_1 + y_2 \right|$ is equal to $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) none of these

829. The endpoints of two normal chords of a parabola are concyclic. Then the tangents at the feet of the normals will intersect at

a. Tangent at vertex of the parabola

b. Axis of the parabola

c. Directrix of the parabola

d. None of these

830. From the point (15, 12), three normals are drawn to the parabola $y^2 = 4x$. Then centroid and triangle formed by three co-normals points is

(A) $\left(\dfrac{16}{3}, 0\right)$ (B) (4, 0) (C) $\left(\dfrac{26}{3}, 0\right)$ (D) (6, 0)

831. t 1 and t 2 are two points on the parabola $y^2 = 4ax$. If the focal chord joining them coincides with the normal chord, then

(a)$t1(t1 + t2) + 2 = 0$ (b) $t1+t2=0$ (c)$t1 \cdot t2 = -1$ (d) none of these

832. Tangent and normal are drawn at the point $P \equiv (16, 16)$ of the parabola $y^2 = 16x$ which cut the axis of the parabola at the points A and B, respectively. If the center of the circle through P, A and B is C, then the angle between PC and the axis of x is

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833. Length of the shortest normal chord of the parabola $y^2 = 4ax$ is

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834. The line $x - y - 1 = 0$ meets the parabola $y^2 = 4x$ at A and B. Normals at A and B meet at C. If D CD is normal at D, then the co-ordinates of D are

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835. If normal are drawn from a point $P(h, k)$ to the parabola $y^2 = 4ax$, then the sum of the intercepts which the normals cut-off from the axis of the parabola is $(h + c)$ (b) $3(h + a)$ $2(h + a)$ (d) none of these

836. If $x + y = k$ is normal to $y^2 = 12x$, then k is (a)3 (b) 9 (c) -9 (d) -3

837. An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having its focus at S. If chord AB lies towards the left of S, then the side length of this triangle is $2a\left(2 - \sqrt{3}\right)$ (b) $4a\left(2 - \sqrt{3}\right)$ $a\left(2 - \sqrt{3}\right)$ (d) $8a\left(2 - \sqrt{3}\right)$

838. $\min\left[(x_1 - x_2)^2 + \left(5 + \sqrt{1 - x_1^2} - \sqrt{4x_2}\right)^2\right]$, $\forall x_1, x_2 \in R$, is (a)$4\sqrt{5}$ - 1

(b) 3 - 2$\sqrt{2}$ (c)$\sqrt{5}$ + 1 (d) $\sqrt{5}$ - 1

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839. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is

(a)$x = -1$ (b) $x = 1$ $x = -\dfrac{3}{2}$ (d) $x = \dfrac{3}{2}$

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840. The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x-axis is $\sqrt{3}y = 3x + 1$

(b) $\sqrt{3}y = -(x + 3)$ (C)$\sqrt{3}y = x + 3$ (d) $\sqrt{3}y = -(3x - 1)$

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841. At what point on the parabola $y^2 = 4x$ the normal makes equal angle with the axes? (A) (4, 4) (B) (9, 6) (C) (4, - 4) (D) (1, ± 2)

842. The focal chord to $y^2 = 16x$ is tangent to $(x - 6)^2 + y^2 = 2$. Then the possible value of the slope of this chord is { - 1, 1} (b) { - 2, 2} $\left\{ -2, \frac{1}{2} \right\}$ (d) $\left\{ 2, -\frac{1}{2} \right\}$

843. If the normals to the parabola $y^2 = 4ax$ at three points $P, Q, and R$ meet at $A, and S$ is the focus, then $SP\overset{.}{S}Q\overset{.}{S}R$ is equal to $a^2 SA$ (b) SA^3 (c) aSA^2 (d) none of these

844. The locus of the midpoint of the segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix

(a) $y = 0$ (b) $x = -a$ (c) $x = 0$ (d) none of these

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845. The curve described parametrically by $x = t^2 + t + 1$, and $y = t^2 - t + 1$ represents. (a) a pair of straight lines (b) an ellipse (c) a parabola (d) a hyperbola

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846. Statement 1: The line $y = x + 2a$ touches the parabola $y^2 = 4a(x + a)$

Statement 2: The line $y = mx + am + \dfrac{a}{m}$ touches $y^2 = 4a(x + a)$ for all real values of m

values of m

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847. Consider a circle with its centre lying on the focus of the parabola, $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle & the parabola is:

848. Normal drawn to $y^2 = 4ax$ at the points where it is intersected by the line $y = mx + c$ intersect at P. The foot of the another normal drawn to the parabola from the point P is (a) $\left(\dfrac{a}{m^2}, -\dfrac{2a}{m} \right)$ (b) $\left(\dfrac{9a}{m}, -\dfrac{6a}{m} \right)$ (c) $\left(am^2, -2am \right)$ (d) $\left(\dfrac{4a}{m^2}, -\dfrac{4a}{m} \right)$

849. The radius of the circle touching the parabola $y^2 = x$ at $(1, 1)$ and having the directrix of $y^2 = x$ as its normal is (a)$\dfrac{5\sqrt5}{8}$ (b) $\dfrac{10\sqrt5}{3}$ (c)$\dfrac{5\sqrt5}{4}$ (d) none of these

850. Maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ is

____

851. If two different tangents of $y^2 = 4x$ are the normals to $x^2 = 4by$, then

(a)$|b| > \dfrac{1}{2\sqrt{2}}$ (b) $|b| < \dfrac{1}{2\sqrt{2}}$ (c)$|b| > \dfrac{1}{\sqrt{2}}$ (d) $|b| < \dfrac{1}{\sqrt{2}}$

852. The largest value of a for which the circle $x^2 + y^2 = a^2$ falls totally in the interior of the parabola $y^2 = 4(x + 4)$ is $4\sqrt{3}$ (b) 4 (c) $4\dfrac{\sqrt{6}}{7}$ (d) $2\sqrt{3}$

853. A ray of light travels along a line $y = 4$ and strikes the surface of curves $y^2 = 4(x + y)$. Then the equations of the line along which of reflected ray travels is $x = 0$ (b) $x = 2$ (c) $x + y$ (d) $2x + y = 4$

854. A set of parallel chords of the parabola $y^2 = 4ax$ have their midpoint on any straight line through the vertex any straight line through the focus a straight line parallel to the axis another parabola

855. A line L passing through the focus of the parabola $y^2 = 4(x - 1)$ intersects the parabola at two distinct points. If m is the slope of the line L, then (a) $-1 < m < 1$ (b) $m < -1$ or $m > 1$ (c) $m \in R$ (d) none of these

856. The ration in which the line segement joining the points $(4, -6)$ and $(3, 1)$ is divided by the parabola $y^2 = 4x$ is $\dfrac{-20 \pm \sqrt{155}}{11} : 1$ (b) $\dfrac{-20 \pm \sqrt{155}}{11} : 2$ $-20 \pm 2\sqrt{155} : 11$ (d) $-20 \pm \sqrt{155} : 11$

857. If (a, b) is the midpoint of a chord passing through the vertex of the parabola $y^2 = 4x$, then (a)$a = 2b$ (b) $a^2 = 2b$ (c)$a^2 = 2b$ (d) $2a = b^2$

858. A water jet from a function reaches it maximum height of 4 m at a distance 0.5 m from the vertical passing through the point O of water outlet. The height of the jet above the horizontal OX at a distance of 0.75 m from the point O is 5 m (b) 6 m (c) 3 m (d) 7 m

859. The vertex of the parabola whose parametric equation is

$x = t^2 - t + 1, y = t^2 + t + 1; t \in R,$ is (a) (1, 1) (b) (2, 2) (c) $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ (d) (3, 3)

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860. A point $P(x, y)$ moves in the xy-plane such that $x = a\cos^2\theta$ and $y = 2a\sin\theta,$ where θ is a parameter. The locus of the point P is a/an (A) circle (B) ellipse (C) unbounded parabola (D) part of the parabola

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861. The locus of the point $\left(\sqrt{3h}, \sqrt{3k + 2}\right)$ if it lies on the line $x - y - 1 = 0$ is (a) straight line (b) a circle (c) a parabola (d) none of these

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862. If the segment intercepted by the parabola $y^2 = 4ax$ with the line $lx + my + n = 0$ subtends a right angle at the vertex, then $4al + n = 0$ (b) $4al + 4am + n = 0$ $4am + n = 0$ (d) $al + n = 0$

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863. The graph of the curve $x^2 + y^2 - 2xy - 8x - 8y + 32 = 0$ falls wholly in the first quadrant (b) second quadrant third quadrant (d) none of these

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864. Consider two curves $C1 : y^2 = 4x$; $C2 = x^2 + y^2 - 6x + 1 = 0$. Then, a. C1 and C2 touch each other at one point b. C1 and C2 touch each other exactly at two point c. C1 and C2 intersect(but do not touch) at exactly two point d. C1 and C2 neither intersect nor touch each other

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865. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is

866. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is -

867. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P(\frac{1}{2}, 2)$ on the parabola and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of latus rectum. $\dfrac{\Delta_1}{\Delta_2}$ is :

868. Statement 1 : The curve $y = -\dfrac{x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$ Statement 2 : A parabola is symmetric about its axis. Both the statements are true and Statements 1 is the correct explanation of Statement 2. Both the statements are true but Statements 1 is not the correct explanation of Statement 2. Statement 1 is true and Statement 2 is false Statement 1 is false and Statement 2 is true

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869. A line L : y = mx + 3 meets y-axis at E (0, 3) and the arc of the parabola $y^2 = 16x\ 0 \leq y \leq 6$ at the point art $F\left(x_0, y_0\right)$. The tangent to the parabola at $F\left(X_0, Y_0\right)$ intersects the y-axis at $G(0, y)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum P) m= Q) = Maximum area of $\triangle EFG$ is (R) $y_0 = $ (S) $y_1 = $

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870. Match the following. Normals are drawn at points P Q and R lying on the parabola $y^2 = 4x$ which intersect at $(3,0)$

871. Tangents and normal drawn to the parabola $y^2 = 4ax$ at point $P\left(at^2, 2at\right), t \neq 0$, meet the x-axis at point $TandN$, respectively. If S is the focus of theparabola, then (a) $SP = ST \neq SN$ (b) $SP \neq ST = SN$ (c) $SP = ST = SN$ (d) $SP \neq ST \neq SN$

872. If the normals to the parabola $y^2 = 4ax$ at three points $\left(ap^2, 2ap\right)$ and $\left(aq^2, 2aq\right)$ and $\left(ar^2, 2ar\right)$ are concurrent, then the common root of equations $Px^2 + qx + r = 0$ and $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ is (a) p (b) q (c) r (d) 1

873. Normal AO, $\forall_1$, and $\forall_2$ are drawn to the parabola $y^2 = 8x$ from the point A(h,0). If triangle OA_1A_2 is equilateral, then possible value of h is

874. If $2x + y + \lambda = 0$ is a normal to the parabola $y^2 = -8x$, then λ is (a)12 (b) -12 (c) 24 (d) -24

875. The length of the latus rectum of the parabola whose focus is $\left(\dfrac{u^2}{2g}\sin2\alpha, \ -\dfrac{u^2}{2g}\cos2\alpha\right)$ and directrix is $y = \dfrac{u^2}{2g}$ is (a)$\dfrac{u^2}{g}\cos^2\alpha$ (b) $\dfrac{u^2}{g}\cos^2 2\alpha$ (c)$\dfrac{2u^2}{g}\cos^2 2\alpha$ (d) $\dfrac{2u^2}{g}\cos^2\alpha$

876. If parabolas $y^2 = \lambda x$ and $25\left[(x-3)^2 + (y+2)^2\right] = (3x - 4y - 2)^2$ are equal, then the value of λ is (a) 9 (b) 3 (c) 7 (d) 6

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877. The normal at the point $P\left(ap^2, 2ap\right)$ meets the parabola $y^2 = 4ax$ again at $Q\left(aq^2, 2aq\right)$ such that the lines joining the origin to P and Q are at right angle. Then (A) $p^2 = 2$ (B) $q^2 = 2$ (C) $p = 2q$ (D) $q = 2p$

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878. The set of points on the axis of the parabola $y^2 = 4x + 8$ from which the three normals to the parabola are all real and different is (a) $\{(k, 0)|k \leq -2\}$ (b) $\{(k, 0)|k \geq -2\}$ (c) $\{(0, k)|k \geq -2\}$ (d) none of these

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879. Which one of the following equation represent parametric equation to a parabolic curve? (a)$x = 3\cos t; y = 4\sin t$ (b)$x^2 - 2 = 2\cos t; y = 4\cos^2\left(\dfrac{t}{2}\right)$ (c)$\sqrt{x} = \tan t; \sqrt{y} = \sec t$ (d)$x = \sqrt{1 - \sin t}; y = \dfrac{\sin t}{2} + \dfrac{\cos t}{2}$

880. The vertex of a parabola is the point (a, b) and the latus rectum is of length l. If the axis of the parabola is parallel to the y-axis and the parabola is concave upward, then its equation is a. $(x + a)^2 = \dfrac{1}{2}(2y - 2b)$ b. $(x - a)^2 = \dfrac{1}{2}(2y - 2b)$ c. $(x + a)^2 = \dfrac{1}{4}(2y - 2b)$ d. $(x - a)^2 = \dfrac{1}{8}(2y - 2b)$

881. The curve represented by the equation $\sqrt{px} + \sqrt{qy} = 1$ where $p, q \in R, p, q > 0$, is (a) a circle (b) a parabola (c) an ellipse (d) a hyperbola

882. Prove that the equation of the parabola whose focus is (0, 0) and tangent at the vertex is $x - y + 1 = 0$ is $x^2 + y^2 + 2xy - 4x + 4y - 4 = 0$.

883. A parabola is drawn touching the axis of x at the origin and having its vertex at a given distance k form this axis Prove that the axis of the parabola is a tangent to the parabola $x^2 = -8k(y - 2k)$.

884. The equation of the parabola whose vertex and focus lie on the axis of x at distances a and a_1 from the origin, respectively, is (a) $y^2 - 4\left(a_1 - a\right)x$ (b) $y^2 - 4\left(a_1 - a\right)(x - a)$ (c) $y^2 - 4\left(a_1 - a\right)(x - a)$ (d) *none*

885. Prove that for a suitable point P on the axis of the parabola, chord AB through the point P can be drawn such that $\left[\left(\dfrac{1}{AP^2}\right) + \left(\dfrac{1}{BP^2}\right)\right]$ is same for all positions of the chord.
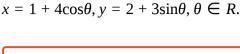
886. Two parabolas have the same focus. If their directrices are the x- and the y-axis, respectively, then the slope of their common chord is ±1 (b) $\dfrac{4}{3}$ (c) $\dfrac{3}{4}$ (d) none of these

887. The number of common chords of the parabolas $x = y^2 - 6y + 11$ and $y = x^2 - 6x + 11$ is (a) 1 (b) 2 (c) 4 (d) 6

888. Find the equation of the curve whose parametric equations are

$x = 1 + 4\cos\theta, y = 2 + 3\sin\theta, \theta \in R.$

889. Prove that any point on the ellipse whose foci are $(-1, 0)$ and $(7, 0)$ and eccentricity is $\dfrac{1}{2}$ is $\left(3 + 8\cos\theta, 4\sqrt{3}\sin\theta\right), \theta \in R$

890. Find the eccentric angle of a point on the ellipse $\dfrac{x^2}{6} + \dfrac{y^2}{2} = 1$ whose distance from the center of the ellipse is $\sqrt{5}$

891. Find the area of the greatest rectangle that can be inscribed in an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1.$

892. The auxiliary circle of a family of ellipses passes through the origin and makes intercepts of 8 units and 6 units on the x and y-axis, respectively. If the eccentricity of all such ellipses is $\dfrac{1}{2}$, then find the locus of the focus.

893. Find the number of rational points on the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1.$

894. A line passing through the origin $O(0, 0)$ intersects two concentric circles of radii $a and b$ at $P and Q,$ If the lines parallel to the X-and Y-axes through $Q and P,$ respectively, meet at point $R,$ then find the locus of R.

895. If the line $lx + my + n = 0$ cuts the ellipse $\left(\dfrac{x^2}{a^2}\right) + \left(\dfrac{y^2}{b^2}\right) = 1$ at points whose eccentric angles differ by $\dfrac{\pi}{2}$, then find the value of $\dfrac{a^2l^2 + b^2m^2}{n^2}$.

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896. Find the area of the greatest isosceles triangle that can be inscribed in the ellipse $\left(\dfrac{x^2}{a^2}\right) + \left(\dfrac{y^2}{b^2}\right) = 1$ having its vertex coincident with one extremity of the major axis.

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897. Find the eccentric angles of the extremities of the latus recta of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

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898. Find the equation of the ellipse whose axes are of length 6 and $2\sqrt{6}$ and their equations are $x - 3y + 3 = 0$ and $3x + y - 1 = 0$, respectively.

899. If the equation $(5x - 1)^2 + (5y - 2)^2 = \left(\lambda^2 - 2\lambda + 1\right)(3x + 4y - 1)^2$ represents an ellipse, then find values of λ

900. Obtain the equation of the ellipse whose focus is the point (-1, 1), and the corresponding directrix is the line $x - y + 3 = 0$, and the eccentricity is $\dfrac{1}{2}$.

901. The moon travels an elliptical path with Earth as one focus. The maximum distance from the moon to the earth is 405, 500 km and the minimum distance is 363,300 km. What is the eccentricity of the orbit?

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902. If the foci of an ellipse are $(0, \pm 1)$ and the minor axis of unit length, then find the equation of the ellipse. The axes of ellipe are the coordinate axes.

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903. Let P be a point on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ of eccentricity e. If A, A' are the vertices and S, S are the foci of the ellipse, then find the ratio area PSS'' : area APA'

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904. If C is the center of the ellipse $9x^2 + 16y^2 = 144$ and S is a focus, then find the ratio of CS to the semi-major axis.

905. Find the sum of the focal distances of any point on the ellipse $9x^2 + 16y^2 = 144$.

906. Find the lengths of the major and minor axis and the eccentricity of the ellipse $\dfrac{(3x - 4y + 2)^2}{16} + \dfrac{(4x + 3y - 5)^2}{9} = 1$

907. Find the eccentricity, one of the foci, the directrix, and the length of the latus rectum for the conic $(3x - 12)^2 + (3y + 15)^2 = \dfrac{(3x - 4y + 5)^2}{25}$.

908. An ellipse passes through the point (4, - 1) and touches the line $x + 4y - 10 = 0$. Find its equation if its axes coincide with the coordinate axes.

909. Find the point on the ellipse $16x^2 + 11y^2 = 256$ where the common tangent to it and the circle $x^2 + y^2 - 2x = 15$ touch.

910. If $\dfrac{x}{a} + \dfrac{y}{b} = \sqrt{2}$ touches the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, then find the eccentric angle θ of point of contact.

911. Find the points on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ such that the tangent at each point makes equal angles with the axes.

912. An ellipse slides between two perpendicular straight lines. Then identify the locus of its center.

913. Find the locus of the foot of the perpendicular drawn from the center upon any tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

914. Find the maximum area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ which touches the line $y = 3x + 2$.

915. A tangent is drawn to the ellipse $\dfrac{x^2}{27} + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ where $\theta\varepsilon\left(0, \dfrac{\pi}{2}\right)$. Then find the value of θ such that the sum of intercepts on the axes made by this tangent is minimum.

916. Consider an ellipse $\dfrac{x^2}{4} + y^2 = \alpha$ (α is parameter > 0) and a parabola $y^2 = 8x$. If a common tangent to the ellipse and the parabola meets the coordinate axes at A and B , respectively, then find the locus of the midpoint of AB

917. Find the angle between the pair of tangents from the point (1,2) to the ellipse $3x^2 + 2y^2 = 5$.

918. If the chord joining points $P(\alpha)$ and $Q(\beta)$ on the ellipse $\left(\dfrac{x^2}{a^2}\right) + \left(\dfrac{y^2}{b^2}\right) = 1$ subtends a right angle at the vertex $A(a, 0)$, then prove that $\tan\left(\dfrac{\alpha}{2}\right)\tan\left(\dfrac{\beta}{2}\right) = -\dfrac{b^2 \cdot}{a^2}$

919. If α and β are the eccentric angles of the extremities of a focal chord of an ellipse, then prove that the eccentricity of the ellipse is $\dfrac{\sin\alpha + \sin\beta}{\sin(\alpha + \beta)}$

920. If the area of the ellipse $\left(\dfrac{x^2}{a^2}\right) + \left(\dfrac{y^2}{b^2}\right) = 1$ is 4π , then find the maximum area of rectangle inscribed in the ellipse.

921. The center of an ellipse is C and PN is any ordinate. Point A, A' are the endpoints of the major axis. Then find the value of $\dfrac{PN^2}{AN} \cdot A' N$

922. The ratio of the area of triangle inscribed in ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ to that of triangle formed by the corresponding points on the auxiliary circle is 0.5. Then, find the eccentricity of the ellipse. (A) $\dfrac{1}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\dfrac{1}{\sqrt{2}}$ (D) $\dfrac{1}{\sqrt{3}}$

A. 1/2

B. null

C.

D. null

923. If PSQ is a focal chord of the ellipse $16x^2 + 25y^2 = 400$ such that $SP = 8$, then find the length of SQ is (a) 2 (b) 1 (c) $\dfrac{8}{9}$ (d) $\dfrac{16}{9}$

924. AOB is the positive quadrant of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ in which $OA = a, OB = b$. Then find the area between the arc AB and the chord AB of the ellipse.

925. If S and S' are two foci of ellipse $16x^2 + 25y^2 = 400$ and PSQ is a focal chord such that $SP = 16$, then find $S'Q$.

926. Find the equations of the tangents drawn from the point $(2, 3)$ to the ellipse $9x^2 + 16y^2 = 144$.

927. Prove that the area bounded by the circle $x^2 + y^2 = a^2$ and the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is equal to the area of another ellipse having semi-axis $a - b$ and b, $a > b$.

928. If the normal at $P\left(2, \dfrac{3\sqrt{3}}{2}\right)$ meets the major axis of ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ at Q, and S and S' are the foci of the given ellipse, then find the ratio $SQ : S'Q$.

929. Normal to the ellipse $\dfrac{x^2}{64} + \dfrac{y^2}{49} = 1$ intersects the major and minor axes at P and Q, respectively. Find the locus of the point dividing segment PQ in the ratio 2:1.

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930. The line $lx + my + n = 0$ is a normal to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. then

prove that $\dfrac{a^2}{l^2} + \dfrac{b^2}{m^2} = \dfrac{\left(a^2 - b^2\right)^2}{n^2}$

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931. Find the equation of the normal to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the positive end of the latus rectum.

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932. Find the points on the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ on which the normals are parallel to the line $2x - y = 1$.

933. If ω is one of the angles between the normals to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(b > a)$ at the point whose eccentric angles are θ and $\dfrac{\pi}{2} + \theta$, then prove that $\dfrac{2\cot\omega}{\sin2\theta} = \dfrac{e^2}{\sqrt{1 - e^2}}$

934. If the normal at any point P on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ meets the axes at G and g respectively, then find the ratio $PG:Pg$.

(a) $a:b$ (b) $a^2:b^2$ (c) $b:a$ (d) $b^2:a^2$

935. P is the point on the ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and Q is the corresponding point on the auxiliary circle of the ellipse. If the line joining the center C to Q meets the normal at P with respect to the given ellipse at K, then find the value of CK.

936. If the normal at one end of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through one end of the minor axis, then prove that eccentricity is constant.

937. If the normals to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are concurrent, prove that

$$\begin{vmatrix} x_1 & y_1 & x_1 y_1 \\ x_2 & y_2 & x_2 y_2 \\ x_3 & y_3 & x_3 y_3 \end{vmatrix} = 0.$$

938. Find the normal to the ellipse $\dfrac{x^2}{18} + \dfrac{y^2}{8} = 1$ at point $(3, 2)$.

939. If two points are taken on the minor axis of an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the same distance from the center as the foci, then prove that the sum of the squares of the perpendicular distances from these points on any tangent to the ellipse is $2a^2$.

940. If any tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ intercepts equal lengths l on the axes, then find l.

941. Find the slope of a common tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and a concentric circle of radius r.

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942. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

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943. If F_1 and F_2 are the feet of the perpendiculars from the foci $S_1 and S_2$ of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ on the tangent at any point P on the ellipse, then prove that $S_1F_1 + S_2F_2 \geq 8$.

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944. If the tangent at any point of the ellipse $\dfrac{x^2}{a^3} + \dfrac{y^2}{b^2} = 1$ makes an angle α with the major axis and an angle β with the focal radius of the point of contact, then show that the eccentricity of the ellipse is given by $e = \dfrac{\cos\beta}{\cos\alpha}$

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945. Two perpendicular tangents drawn to the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ intersect on the curve.

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946. A tangent having slope of $-\dfrac{4}{3}$ to the ellipse $\dfrac{x^2}{18} + \dfrac{y^2}{32} = 1$ intersects the major and minor axes at points A and $B,$ respectively. If C is the center of the ellipse, then find area of triangle ABC

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947. If the tangent to the ellipse $x^2 + 2y^2 = 1$ at point $P\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{2}\right)$ meets the auxiliary circle at point R and Q, then find the points of intersection of tangents to the circle at Q and R.

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948. Chords of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ are drawn through the positive end of the minor axis. Then prove that their midpoints lie on the ellipse.

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949. Find the locus of the middle points of all chords of $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ which are at a distance of 2 units from the vertex of parabola $y^2 = -8ax$

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950. Tangents PQ and PR are drawn at the extremities of the chord of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$, which get bisected at point $P(1, 1)$. Then find the point of intersection of the tangents.

951. If the chords of contact of tangents from two poinst $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ are at right angles, then find the value of $\dfrac{x_1 x_2}{y_1 y_2}$.

952. From the point $A(4, 3)$, tangent are drawn to the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ to touch the ellipse at B and CEF is a tangent to the ellipse parallel to line BC and towards point A. Then find the distance of A from EF.

953. An ellipse is drawn with major and minor axis of length 10 and 8 respectively. Using one focus a centre, a circle is drawn that is tangent to ellipse, with no part of the circle being outside the ellipse. The radius of the circle is

954. Find the foci of the ellipse $25(x + 1)^2 + 9(y + 2)^2 = 225$.

955. Find the equation of an ellipse whose axes are the x-and y-axis and whose one focus is at (4,0) and eccentricity is 4/5.

956. If $P(\alpha, \beta)$ is a point on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with foci $Sand S'$ and eccentricity e, then prove that the area of $\Delta SPS'$ is $be\sqrt{a^2 - \alpha^2}$

957. An arc of a bridge is semi-elliptical with the major axis horizontal. If the length of the base is 9m and the highest part of the bridge is 3m from the horizontal, then prove that the best approximation of the height of the acr 2 m from the center of the base is $\dfrac{8}{3}m$.

958. An ellipse has OB as the semi-minor axis, F and F' as its foci, and $\angle FBF'$ a right angle. Then, find the eccentricity of the ellipse.

959. P is a variable on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with $\forall'$ as the major axis.

Find the maximum area of triangle APA'

960. Prove that the curve represented by

$x = 3(\cos t + \sin t), y = 4(\cos t - \sin t), t \in R,$ is an ellipse.

961. Find the center, foci, the length of the axes, and the eccentricity of

the ellipse $2x^2 + 3y^2 - 4x - 12y + 13 = 0$

962. If C is the center and A, B are two points on the conic

$4x^2 + 9y^2 - 8x - 36y + 4 = 0$ such that $\angle ACB = \dfrac{\pi}{2},$ then prove that
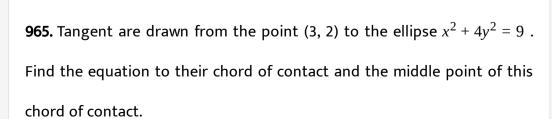
$$\frac{1}{CA^2} + \frac{1}{CB^2} = \frac{13 \cdot}{36}$$

963. Find the equation of a chord of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ joining two points $P\left(\dfrac{\pi}{4}\right)$ and $Q\left(\dfrac{5\pi}{4}\right)$.

964. Prove that the chords of contact of pairs of perpendicular tangents to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ touch another fixed ellipse.

965. Tangent are drawn from the point (3, 2) to the ellipse $x^2 + 4y^2 = 9$. Find the equation to their chord of contact and the middle point of this chord of contact.

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966. Find the locus of the point of intersection of tangents to the ellipse if the difference of the eccentric angle of the points is $\dfrac{2\pi}{3}$.

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967. Tangents are drawn from the points on the line $x - y - 5 = 0$ to $x^2 + 4y^2 = 4$. Then all the chords of contact pass through a fixed point. Find the coordinates.

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968. If from a point P, tangents PQ and PR are drawn to the ellipse $\dfrac{x^2}{2} + y^2 = 1$ so that the equation of QR is $x + 3y = 1$, then find the coordinates of P.

969. Prove that the chord of contact of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with respect to any point on the directrix is a focal chord.

970. The locus a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is (A) a parabola (B) an ellipse (C) a hyperbola (D) a circle

971. Find the locus of the point which is such that the chord of contact of tangents drawn from it to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ form a triangle of constant area with the coordinate axes.

972. A point P moves such that the chord of contact of the pair of tangents from P on the parabola $y^2 = 4ax$ touches the rectangular hyperbola $x^2 - y^2 = c^2$. Show that the locus of P is the ellipse $\dfrac{x^2}{c^2} + \dfrac{y^2}{(2a)^2} = 1$.

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973. Find the length of the chord of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$, whose middle point is $\left(\dfrac{1}{2}, \dfrac{2}{5}\right)$.

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974. Find the equation of the chord of the hyperbola $25x^2 - 16y^2 = 400$ which is bisected at the point $(5, 3)$.

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975. The locus of the point which divides the double ordinates of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ in the ratio $1:2$ internally is

976. Find the locus of the middle points of chord of an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ which are drawn through the positive end of the minor axis.

977. Find the point on the hyperbola $x^2 - 9y^2 = 9$ where the line $5x + 12y = 9$ touches it.

978. If (5, 12) and (24, 7) are the foci of an ellipse passing through the origin, then find the eccentricity of the ellipse.

979. From any point P lying in the first quadrant on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, PN is drawn perpendicular to the major axis and produced at Q so that NQ equals to PS, where S is a focus. Then the locus of Q is

$5y - 3x - 25 = 0$ $3x + 5y + 25 = 0$ $3x - 5y - 25 = 0$ (d) none of these

980. If any line perpendicular to the transverse axis cuts the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the conjugate hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ at points $P and Q$, respectively, then prove that normal at $P and Q$ meet on the x-axis.

981. If the focal distance of an end of the minor axis of an ellipse (referred to its axes as the axes of x and y, respectively) is k and the distance between its foci is $2h$, them find its equation.

982. Tangents are drawn to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, (a > b)$, and the circle $x^2 + y^2 = a^2$ at the points where a common ordinate cuts them (on the same side of the x-axis). Then the greatest acute angle between these tangents is given by (A) $\tan^{-1}\left(\dfrac{a-b}{2\sqrt{ab}}\right)$ (B) $\tan^{-1}\left(\dfrac{a+b}{2\sqrt{ab}}\right)$ (C) $\tan^{-1}\left(\dfrac{2ab}{\sqrt{a-b}}\right)$

(D) $\tan^{-1}\left(\dfrac{2ab}{\sqrt{a+b}}\right)$

983. A normal to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ meets the axes in M and N and lines MP and NP are drawn perpendicular to the axes meeting at P. Prove that the locus of P is the hyperbola $a^2x^2 - b^2y^2 = \left(a^2 + b^2\right)^2$

984. Find the eccentricity of an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ whose latus reactum is half of its major axis.

985. The slopes of the common tangents of the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$ and the circle $x^2 + y^2 = 3$ are ± 1 (b) $\pm\sqrt{2}$ (c) $\pm\sqrt{3}$ (d) none of these

986. Find the locus of the midpoints of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$.

987. The coordinates of the vertices $B and C$ of a triangle ABC are $(2, 0)$ and $(8, 0)$, respectively. Vertex A is moving in such a way that $4\dfrac{\tan B}{2}\dfrac{\tan C}{2} = 1$.

Then find the locus of A

988. If the tangents to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ make angles α and β with the major axis such that $\tan\alpha + \tan\beta = \gamma$, then the locus of their point of intersection is $x^2 + y^2 = a^2$ (b) $x^2 + y^2 = b^2$ $x^2 - a^2 = 2\lambda xy$ (d) $\lambda\left(x^2 - a^2\right) = 2xy$

989. If $P = (x, y)$, $F_1 = (3, 0)$, $F_2 = (-3, 0)$, and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equal 8 (b) 6 (c) 10 (d) 12

990. Find the condition on a and b for which two distinct chords of the hyperbola $\dfrac{x^2}{2a^2} - \dfrac{y^2}{2b^2} = 1$ passing through (a, b) are bisected by the line

$x + y = b$.

991. The point of intersection of the tangents at the point P on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and its corresponding point Q on the auxiliary circle meet on the line (a) $x = \dfrac{a}{e}$ (b) $x = 0$ (c) $y = 0$ (d) none of these

992. Find the equation of the ellipse (referred to its axes as the axes of x and y, respectively) whose foci are $(\pm 2, 0)$ and eccentricity is $\dfrac{1}{2}$

993. Tangents are drawn to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at two points whose eccentric angles are $\alpha - \beta$ and $\alpha + \beta$ The coordinates of their point of

intersection are

994. The sum of the squares of the perpendiculars on any tangents to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ from two points on the minor axis each at a distance *ae* from the center is

(a)$2a^2$ (b) $2b^2$ (c) $a^2 + b^2$ $a^2 - b^2$

995. A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3cm from the end in contact with the x-axis.

996. Tangents are drawn from the points on the line $x - y - 5 = 0$ to $x^2 + 4y^2 = 4$, then all the chords of contact pass through a fixed point, whose coordinate are

997. If $\alpha - \beta = $ constant, then the locus of the point of intersection of tangents at $P(a\cos\alpha, b\sin\alpha)$ and $Q(a\cos\beta, b\sin\beta)$ to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is: (a) a circle (b) a straight line (c) an ellipse (d) a parabola

998. Two circles are given such that one is completely lying inside the other without touching. Prove that the locus of the center of variable circle which touches the smaller circle from outside and the bigger circle from inside is an ellipse.

999. How many real tangents can be drawn from the point (4, 3) to the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$? Find the equation of these tangents and the angle between them.

1000. For an ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ with vertices A and A', drawn at the point P in the first quadrant meets the y axis in Q and the chord A'P meets the y axis in M. If 'O' is the origin then $OQ^2 - MQ^2$

1001. The first artificial satellite to orbit the earth was Sputnik I. Its highest point above earth's surface was 947 km, and its lowest point was 228 km. The center of the earth was at one focus of the elliptical orbit. The radius of the earth is 6378 km. Find the eccentricity of the orbit.

1002. Which of the following can be slope of tangent to the hyperbola $4x^2 - y^2 = 4$? (a) 1 (b) -3 (c) 2 (d) $-\dfrac{3}{2}$

1003. A tangent to the ellipes $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ at any points meet the line $x = 0$ at a point Q Let R be the image of Q in the line $y = x$, then circle whose extremities of a dameter are Q and R passes through a fixed point, the fixed point is

1004. Tangents are drawn to the hyperbola $3x^2 - 2y^2 = 25$ from the point $\left(0, \dfrac{5}{2}\right)$. Find their equations.

1005. Suppose that the foci of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$ are $\left(f_1, 0\right) and \left(f_2, 0\right)$ where $f_1 > 0 and f_2 < 0$. Let $P_1 and P_2$ be two parabolas with a common vertex at (0, 0) and with foci at $\left(f_1, 0\right) and$ (2f_2 , 0), respectively. Let T_1 be a tangent to P_1 which passes through $\left(2f_2, 0\right)$ and T_2 be a tangents to P_2 which passes through $\left(f_1, 0\right)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\dfrac{1}{m_1^2} + m_2^2\right)$ is
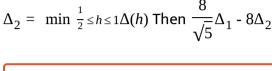
1006. From the center C of hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, perpendicular CN is drawn on any tangent to it at the point $P(a\sec\theta, b\tan\theta)$ in the first quadrant. Find the value of θ so that the area of CPN is maximum.

1007. A vertical line passing through the point $(h, 0)$ intersects the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at R. If $\Delta(h)$ Area of triangle ΔPQR, and $\Delta_1 = \max_{\frac{1}{2} \le h \le 1} \Delta(h)$ and $\Delta_2 = \min_{\frac{1}{2} \le h \le 1} \Delta(h)$ Then $\dfrac{8}{\sqrt{5}} \Delta_1 - 8\Delta_2$

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1008. A common tangent to $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$, is

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1009. Find the equation of tangents to the curve $4x^2 - 9y^2 = 1$ which are parallel to $4y = 5x + 7$.

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1010. Find the equation of the locus of the middle points of the chords of the hyperbola $2x^2 - 3y^2 = 1$, each of which makes an angle of 45^0 with the x-axis.

1011. Find the angle between the asymptotes of the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$.

1012. Match the following: List - I, List - II Let $y(x) = \cos\left(3\cos^{-1}x\right)$ $x \in [-1, 1], x \neq \pm \dfrac{\sqrt{3}}{2}$ Then $\dfrac{1}{y(x)}\left\{\left(x^2 - 1\right)\dfrac{d^2y(x)}{dx^2} + \dfrac{dy(x)}{dx}\right\}$ equals, 1 Let $A_1, A_2, A_n(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point

$A_k, k = 1, 2, n$ If $\left|\sum_{k=1}^{n-1}\left(\vec{a}_k \times \vec{a}_{k+1}\right)\right| = \left|\sum_{k=1}^{n-1}\left(\vec{a}_k \cdot \vec{a}_{k+1}\right)\right|$, then the minimum

value of n is, 2 If the normal from the point $P(h, 1)$ on the ellipse

$\dfrac{x^2}{6} + \dfrac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is, 8

Number of positive solutions satisfying the equation

$\tan^{-1}\left(\dfrac{1}{2x} + 1\right) + \tan^{-1}\left(\dfrac{1}{4x+1}\right) = \tan^{-1}\left(\dfrac{2}{x^2}\right)$ is, 9

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1013. If a hyperbola passing through the origin has $3x - 4y - 1 = 0$ and $4x - 3y - 6 = 0$ as its asymptotes, then find the equation of its transvers and conjugate axes.

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1014. Let E_1 and E_2 be two ellipse whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y - 1)^2 = 2$. The straight line $x + y = 3$ touches the curves, S, E_1 and E_2 at P,Q and R respectively. Suppose that

$PQ = PR = \dfrac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2 ,respectively, then the correct expression (s) is (are)

1015. A triangle has its vertices on a rectangular hyperbola. Prove that the orthocentre of the triangle also lies on the same hyperbola.

1016. From any point on any directrix of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a > b$, a pari of tangents is drawn to the auxiliary circle. Show that the chord of contact will pass through the correspoinding focus of the ellipse.

1017. Find the equation of the asymptotes of the hyperbola

$3x^2 + 10xy + 9y^2 + 14x + 22y + 7 = 0$

1018. A tangent is drawn to the ellipse to cut the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and

to cut the ellipse $\dfrac{x^2}{c^2} + \dfrac{y^2}{d^2} = 1$ at the points P and Q. If the tangents are at

right angles, then the value of $\left(\dfrac{a^2}{c^2}\right) + \left(\dfrac{b^2}{d^2}\right)$ is

1019. PQ and RS are two perpendicular chords of the rectangular

hyperbola $xy = c^2$ If C is the center of the rectangular hyperbola, then

find the value of product of the slopes of $CP, CQ, CR,$ and CS

1020. O is the origin & also the centre of two concentric circles having

radii of the inner & the outer circle as a&b respectively. A line OPQ is

drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y-axis & QR is drawn parallel to the x-axis. Prove that the locus of R is an ellipse touching the two circles. If the focii of this ellipse lie on the inner circle, find the ratio of inner: outer radii & find also the eccentricity of the ellipse.

1021. If the tangents to the parabola $y^2 = 4ax$ intersect the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at $A and B$, then find the locus of the point of intersection of the tangents at $A and B$

1022. The tangent at a point P on an ellipse intersects the major axis at $T, and N$ is the foot of the perpendicular from P to the same axis. Show that the circle drawn on NT as diameter intersects the auxiliary circle orthogonally.

1023. If $(a\sec\theta, b\tan\theta)$ and $(a\sec\phi, b\tan\phi)$ be two coordinate of the ends of a focal chord passing through $(ae, 0)$ of $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ then $\tan\left(\dfrac{\theta}{2}\right)\tan\left(\dfrac{\phi}{2}\right)$ equals to

1024. Find the area of the triangle formed by any tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ with its asymptotes.

1025. If a triangle is inscribed in an ellipse and two of its sides are parallel to the given straight lines, then prove that the third side touches the fixed ellipse.

1026. Normal are drawn to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at point $\theta_1 and th\eta_2$ meeting the conjugate axis at $G_1 and G_2$, respectively. If $\theta_1 + \theta_2 = \dfrac{\pi}{2}$, prove that $CG_1\dot{C}G_2 = \dfrac{a^2e^4}{e^2 - 1}$, where C is the center of the hyperbola and e is the eccentricity.

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1027. The tangent at a point $P(a\cos\varphi, b\sin\varphi)$ of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ meets its auxiliary circle at two points, the chord joining which subtends a right angle at the center. Find the eccentricity of the ellipse.

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1028. Find the product of the length of perpendiculars drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ to its asymptotes.

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1029. Tangents are drawn to the ellipse from the point

$$\left(\frac{a^2}{\sqrt{a^2 - b^2}}, \sqrt{a^2 + b^2} \right)$$. Prove that the tangents intercept on the

ordinate through the nearer focus a distance equal to the major axis.

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1030. Find the locus of the point which is such that the chord of contact

of tangents drawn from it to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ form a triangle of

constant area with the coordinate axes.

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1031. Find the point (α, β) on the ellipse $4x^2 + 3y^2 = 12$, in the first

quadrant, so that the area enclosed by the lines $y = x, y = \beta, x = \alpha$, and

the x-axis is maximum.

1032. The ellipse $E_1 : \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is $\dfrac{\sqrt{2}}{2}$ (b) $\dfrac{\sqrt{3}}{2}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{3}{4}$

1033. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are (a) $\left(\dfrac{2}{5}, \dfrac{1}{5}\right)$ (b) $\left(-\dfrac{2}{5}, \dfrac{1}{5}\right)$ $\left(-\dfrac{2}{5}, -\dfrac{1}{5}\right)$ (d) $\left(\dfrac{2}{5}, -\dfrac{1}{5}\right)$

1034. Find the eccentricity of the conic $4(2y - x - 3)^2 - 9(2x + y - 1)^2 = 80$

1035. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the midpoint of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at points. (a) $\left(\pm \dfrac{\left(3\sqrt{5}\right)}{2} \pm \dfrac{2}{7} \right)$ (b) $\left(\pm \dfrac{\left(3\sqrt{5}\right)}{2} \pm \dfrac{\sqrt{19}}{7} \right)$ (c) $\left(\pm 2\sqrt{3},\ \pm \dfrac{1}{7} \right)$ (d) $\left(\pm 2\sqrt{3} \pm \dfrac{4\sqrt{3}}{7} \right)$

1036. For all real values of m, the straight line $y = mx + \sqrt{9m^2 - 4}$ is a tangent to which of the following certain hyperbolas? (a)$9x^2 + 4y^2 = 36$ (b) $4x^2 + 9y^2 = 36$ (c)$9x^2 - 4y^2 = 36$ (d) $4x^2 - 9y^2 = 36$

1037. Let E be the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ and C be the circle $x^2 + y^2 = 9$. Let $P and Q$ be the points (1, 2) and (2, 1), respectively. Then Q lies inside C but outside E Q lies outside both $C and E$ P lies inside both C and E P lies inside C but outside E

1038. Two straight lines rotate about two fixed points $(-a, 0)$ and $(a, 0)$ in anticlockwise sense. If they start from their position of coincidence such that one rotates at a rate double the other, then find the locus of curve.

1039. Statement 1 : There can be maximum two points on the line $px + qy + r = 0$, from which perpendicular tangents can be drawn to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ Statement 2 : Circle $x^2 + y^2 = a^2 + b^2$ and the given line can intersect at maximum two distinct points.

1040. Find the vertices of the hyperbola $9x^2 - 16y^2 - 36x + 96y - 252 = 0$

1041. Statement 1 : Circles $x^2 + y^2 = 9$ and $\left(x - \sqrt{5}\right)\left(\sqrt{2}x - 3\right) + y\left(\sqrt{2}y - 2\right) = 0$ touch each other internally. Statement 2 : The circle described on the focal distance as diameter of the ellipse $4x^2 + 9y^2 = 36$ touches the auxiliary circle $x^2 + y^2 = 9$ internally.

1042. If AOB and COD are two straight lines which bisect one another at right angles, show that the locus of a point P which moves so that $PA \times PB = PC \times PD$ is a hyperbola.

Find its eccentricity.

1043. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$ is (a) $\dfrac{27}{4}$ (b) 18 (c) $\dfrac{27}{2}$ (d) 27

1044. Find the equation of hyperbola : Whose foci are (4, 2) and (8, 2) and accentricity is 2.

1045. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2,$ then the locus of the midpoint of the intercept made by the tangents between the coordinate axes is (a) $\dfrac{1}{2x^2} + \dfrac{1}{4y^2} = 1$ (b) $\dfrac{1}{4x^2} + \dfrac{1}{2y^2} = 1$ (c) $\dfrac{x^2}{2} + y^2 = 1$ (d) $\dfrac{x^2}{4} + \dfrac{y^2}{2} = 1$

1046. Two straight lines pass through the fixed points ($\pm a, 0$) and have slopes whose products is $p > 0$ Show that the locus of the points of intersection of the lines is a hyperbola.

1047. Each question has four choices: a, b, c and d, out of which only one is correct. Each question contains Statement 1 and Statement 2. Find the correct answer. Statement 1 : The locus of a moving point (x, y) satisfying $\sqrt{(x - 2)^2 + y^2} + \sqrt{(x - 2)^2 + y^2} = 4$ is an ellipse. Statement 2 : The distance between (- 2, 0) and (2, 0) is 4.

1048. Find the lengths of the transvers and the conjugate axis, eccentricity, the coordinates of foci, vertices, the lengths of latus racta,

and the equations of the directrices of the following hyperbola:

$16x^2 - 9y^2 = -144$.

1049. Statement 1 : In a triangle ABC , if base BC is fixed and the perimeter of the triangle is constant, then vertex A moves on an ellipse.

Statement 2 : If the sum of the distances of a point P from two fixed points is constant, then the locus of P is a real ellipse.

1050. In an ellipse, the sum of the distances between foci is always less than the sum of focal distances of any point on it. Statement 2 : The eccentricity of any ellipse is less than 1.

1051. Find the equation of hyperbola : whose axes are coordinate axes and the distances of one of its vertices from the foci are 3 and 1

1052. Statement 1 : The equations of the tangents drawn at the ends of the major axis of the ellipse $9x^2 + 5y^2 - 30y = 0$ is $y = 0, y = 6$. Statement 2 : The tangents drawn at the ends of the major axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are always parallel to the y-axis.

1053. Find the equation of hyperbola if centre is (1, 0), one focus is (6, 0) and transverse axis 6 .

1054. Let $E_1 and E_2$, respectively, be two ellipses

$\dfrac{x^2}{a^2} + y^2 = 1, and x^2 + \dfrac{y^2}{a^2} = 1$ (where a is a parameter). Then the locus of

the points of intersection of the ellipses $E_1 and E_2$ is a set of curves

comprising two straight lines (b) one straight line one circle (d) one

parabola

1055. Find the equation of hyperbola : Whose center is (3, 2), one focus is

(5, 2) and one vertex is (4, 2)

1056. Consider the ellipse $\dfrac{x^2}{f\left(k^2 + 2k + 5\right)} + \dfrac{y^2}{f(k + 11)} = 1$. If $f(x)$ is a

positive decr4easing function, then the set of values of k for which the

major axis is the x-axis is $(-3, 2)$ the set of values of k for which the

major axis is the y-axis is $(-\infty, 2)$ the set of values of k for which the major axis is the y-axis is $(-\infty, -3) \cup (2, \infty)$ the set of values of k for which the major axis is the y-axis is $(-3, -\infty,)$

1057. An ellipse and a hyperbola have their principal axes along the coordinate axes and have a common foci separated by distance $2\sqrt{3}$ The difference of their focal semi-axes is equal to 4. If the ratio of their eccentricities is 3/7 , find the equation of these curves.

1058. Two concentric ellipses are such that the foci of one are on the other and their major axes are equal. Let e and e' be their eccentricities. Then. the quadrilateral formed by joining the foci of the two ellipses is a parallelogram the angle θ between their axes is given by

$\theta = \cos^{-1}\sqrt{\dfrac{1}{e^2} + \dfrac{1}{e'^2}} = \dfrac{1}{e^2 e'^2}$ If $e^2 + e'^2 = 1$, then the angle between the axes of the two ellipses is 90^0 none of these

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1059. If hyperbola $\dfrac{x^2}{b^2} - \dfrac{y^2}{a^2} = 1$ passes through the focus of ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, then find the eccentricity of hyperbola.

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1060. If the tangent drawn at point $\left(t^2, 2t\right)$ on the parabola $y^2 = 4x$ is the same as the normal drawn at point $\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$ on the ellipse $4x^2 + 5y^2 = 20$, then $\theta = \cos^{-1}\left(-\dfrac{1}{\sqrt{5}}\right)$ (b) $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$ $t = -\dfrac{2}{\sqrt{5}}$ (d) $t = -\dfrac{1}{\sqrt{5}}$

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1061. If the foci of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$ coincide, then find the value of b^2

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1062. Statement 1 : Any chord of the conic $x^2 + y^2 + xy = 1$ through (0, 0) is bisected at (0, 0). Statement 2 : The center of a conic is a point through which every chord is bisected.

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1063. Find the coordinates of the foci, the eocentricity, the latus rectum, and the equations of directrices for the hyperbola $9x^2 - 16y^2 - 72x + 96y - 144 = 0$

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1064. Statement 1 : If there is exactly one point on the line $3x + 4y + 5\sqrt{5} = 0$ from which perpendicular tangents can be drawn to the ellipse $\dfrac{x^2}{a^2} + y^2 = 1, (a > 1),$ then the eccentricity of the ellipse is $\dfrac{1}{3} \cdot$

Statement 2 : For the condition given in statement 1, the given line must touch the circle $x^2 + y^2 = a^2 + 1.$

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1065. If the latus rectum of a hyperbola forms an equilateral triangle with the vertex at the center of the hyperbola ,then find the eccentricity of the hyperbola.

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1066. Statement 1 : For the ellipse $\dfrac{x^2}{5} + \dfrac{y^2}{3} = 1$, the product of the perpendiculars drawn from the foci on any tangent is 3. Statement 2 : For the ellipse $\dfrac{x^2}{5} + \dfrac{y^2}{3} = 1$, the foot of the perpendiculars drawn from the

foci on any tangent lies on the circle $x^2 + y^2 = 5$ which is an auxiliary circle of the ellipse.

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1067. If the latus rectum subtends a right angle at the center of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then find its eccentricity.

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1068. Statement 1 : The locus of the center of a variable circle touching two circle $(x - 1)^2 + (y - 2)^2 = 25$ and $(x - 2)^2 + (y - 1)^2 = 16$ is an ellipse.

Statement 2 : If a circle $S_2 = 0$ lies completely inside the circle $S_1 = 0$, then the locus of the center of a variable circle $S = 0$ that touches both the circles is an ellipse.

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1069. If PQ is a double ordinate of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the center of the hyperbola, then find the range of the eccentricity e of the hyperbola.

1070. Statement 1 : A triangle ABC right-angled at A moves so that its perpendicular sides touch the curve $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ all the time. Then the loci of the points A, $B and C$ are circle. Statement 2 : The locus of thepoint of intersection of two perpendicular tangents to the curve is a director circle.

A. Both statements 1 and 2 are true and statement 2 is the correct explanation of statement 1.

B. Both statements 1 and 2 are true but statement 2 is not the correct explanation of statement 1.

C. Statement 1 is true and statement 2 is false.

D. Both statements 1 and 2 are false.

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1071. Find the eccentricity of the hyperbola given by equations $x = \dfrac{e^t + e^{-t}}{2}$ and $y = \dfrac{e^t - e^{-t}}{3}, t \in R.$

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1072. A ray emanating from the point (5, 0) is INCIDENT on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8. Find the equation of the reflected ray after the first reflection if point P lies in the first quadrant.

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1073. Statement 1 : If the line $x + y = 3$ is a tangent to an ellipse with focie (4, 3) and (6, y) at the point (1, 2) then $y = 17.$ Statement 2 : Tangent and

normal to the ellipse at any point bisect the angle subtended by the foci at that point.

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1074. Normal is drawn at one of the extremities of the latus rectum of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ which meets the axes at points A and B. Then find the area of triangle OAB (O being the origin).

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1075. Statement 1 : The area of the ellipse $2x^2 + 3y^2 = 6$ is more than the area of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$. Statement 2 : The length f the semi-major axis of an ellipse is more that the radius of the circle.

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1076. An ellipse and a hyperbola are confocal (have the same focus) and the conjugate axis of the hyperbola is equal to the minor axis of the ellipse. If e_1 and e_2 are the eccentricities of the ellipse and the hyperbola, respectively, then prove that $\dfrac{1}{e_1^2} + \dfrac{1}{e_2^2} = 2$.

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1077. The point of intersection of the tangents at the point P on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and its corresponding point Q on the auxiliary circle meet on the line $x = \dfrac{a}{e}$ (b) $x = 0$ $y = 0$ (d) none of these

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1078. The distance between two directrices of a rectangular hyperbola is 10 units. Find the distance between its foci.

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1079. Statement 1 : Tangents are drawn to the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{2} = $ at the points where it is intersected by the line $2x + 3y = 1$. The point of intersection of these tangents is $(8, 6)$. Statement 2 : The equation of the chord of contact to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ from an external point is given by $\dfrac{x_1}{a^2} + \dfrac{yy_1}{b^2} - 1 = 0$

1080. Find the equation of normal to the hyperbola $3x^2 - y^2 = 1$ having slope $\dfrac{1}{3}$

1081. A triangle ABC with fixed base BC , the vertex A moves such that $\cos B + \cos C = 4\dfrac{\sin^2 A \cdot}{2}$ If $a, b and c,$ denote the length of the sides of the triangle opposite to the angles $A, B, and C$, respectively, then (a)$b + c = 4a$

(b) $b + c = 2a$ (c)the locus of point A is an ellipse (d)the locus of point A is a pair of straight lines

1082. Find the equation of normal to the hyperbola $x^2 - 9y^2 = 7$ at point $(4, 1)$.

1083. A circle has the same center as an ellipse and passes through the foci $F_1 and F_2$ of the ellipse, such that the two curves intersect at four points. Let P be any one of their point of intersection. If the major axis of the ellipse is 17 and the area of triangle PF_1F_2 is 30, then the distance between the foci is

1084. C is the center of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ The tangent at any point P on this hyperbola meet the straight lines $bx - ay = 0$ and $bx + ay = 0$ at points $Q and R$, respectively. Then prove that $C\dot{Q}C\dot{R} = a^2 + b^2$

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1085. The eccentricity of the conjugate hyperbola of the hyperbola $x^2 - 3y^2 = 1$ is (a) 2 (b) $2\sqrt{3}$ (c) 4 (d) $\dfrac{4}{5}$

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1086. The angle subtended by common tangents of two ellipses $4(x - 4)^2 + 25y^2 = 100 and 4(x + 1)^2 + y^2 {=} 4$ at the origin is $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{6}$ (d) $\dfrac{\pi}{2}$

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1087. PN is the ordinate of any point P on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $\forall'$ is its transvers axis. If Q divides AP in the ratio $a^2 : b^2$, then prove that NQ is perpendicular to $A'\overset{.}{P}$

1088. If PQR is an equilateral triangle inscribed in the auxiliary circle of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, (a > b)$, and $P'Q'R'$ is the correspoinding triangle inscribed within the ellipse, then the centroid of triangle $P'Q'R'$ lies at center of ellipse focus of ellipse between focus and center on major axis none of these

1089. Show that the equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represents a hyperbola. Find the coordinates of the centre, lengths of the axes,

eccentricity, latus-rectum, coordinates of foci and vertices, equations of the directrices of the hyperbola.

1090. Find the equation of hyperbola, whose center is $(-3, 2)$, one vertex is $(-3, 4)$ and eccentricity is $\dfrac{5}{2}$.

1091. An ellipse passing through the origin has its foci $(3, 4)$ and $(6, 8)$. The length of its semi-minor axis is b Then the value of $\dfrac{b}{\sqrt{2}}$ is____

1092. The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}t = 0 \,\&\, \sqrt{3}tx + ty - 4\sqrt{3} = 0$ (where t is a parameter) is a hyperbola whose eccentricity is:

1093. Find the eccentricity of the hyperbola with asymptotes $3x + 4y = 2$ and $4x - 3y = 2$.

1094. An ellipse having foci at (3, 3) and (- 4, 4) and passing through the origin has eccentricity equal to (a)$\dfrac{3}{7}$ (b) $\dfrac{2}{7}$ (c) $\dfrac{5}{7}$ (d) $\dfrac{3}{5}$

1095. If $S and S'$ are the foci, C is the center, and P is a point on a rectangular hyperbola, show that $SP \times S'P = (CP)^2$.

1096. $P and Q$ are the foci of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and B is an end of the minor axis. If PBQ is an equilateral triangle, then the eccentricity of the ellipse is $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{3}$ (d) $\dfrac{1}{2}$ (d) $\dfrac{\sqrt{3}}{2}$

1097. If PN is the perpendicular from a point on a rectangular hyperbola $xy = c^2$ to its asymptotes, then find the locus of the midpoint of PN

1098. A line of fixed length $a + b$ moves so that its ends are always on two fixed perpendicular straight lines. Then the locus of the point which divides this line into portions of length $a and b$ is (a) an ellipse (b) parabola (c) straight line (d) none of these

1099. The equation of the transvers and conjugate axes of a hyperbola are, respectively, $x + 2y - 3 = 0$ and $2x - y + 4 = 0$, and their respective lengths are $\sqrt{2}$ and $\dfrac{2}{\sqrt{3}}$ The equation of the hyperbola is

a) $\dfrac{2}{5}(x + 2y - 3)^2 - \dfrac{3}{5}(2x - y + 4)^2 = 1$

b) $\dfrac{2}{5}(x - y - 4)^2 - \dfrac{3}{5}(x + 2y - 3)^2 = 1$

c) $\dfrac{2}{5}(2x - y + 4)^2 - \dfrac{3}{5}(x + 2y - 3)^2 = 1$

d) $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

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1100. Show that the acute angle between the asymptotes of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1, \left(a^2 > b^2\right),$ is $2\cos^{-1}\left(\dfrac{1}{e}\right),$ where e is the eccentricity of the hyperbola.

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1101. With a given point and line as focus and directrix, a series of ellipses are described. The locus of the extremities of their minor axis is an

(a)ellipse (b)a parabola (c)a hyperbola (d)none of these

1102. If the vertex of a hyperbola bisects the distance between its center and the correspoinding focus, then the ratio of the square of its conjugate axis to the square of its transverse axis is (a) 2   (b) 4

(c) 6   (d) 3

1103. Find the equation of the hyperbola which has $3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$ as its asymptotes and which passes through the origin.

1104. If the ellipse $\dfrac{x^2}{4} + y^2 = 1$ meets the ellipse $x^2 + \dfrac{y^2}{a^2} = 1$ at four distinct points and $a = b^2 - 5b + 7$, then b does not lie in $[4, 5]$ (b) $(-\infty, 2) \cup (3, \infty)$ $(-\infty, 0)$ (d) $[2, 3]$

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1105. The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola. (a)the length of whose transvers axis is $4\sqrt{3}$ (b)the length of whose conjugate is 4 (c)whose center is $(-1, 2)$ (d)whose eccentricity is $\sqrt{\dfrac{19}{3}}$

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1106. If the base of a triangle and the ratio of tangent of half of base angles are given, then identify the locus of the opposite vertex.

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1107. S_1, S_2, are foci of an ellipse of major axis of length $10 units$ and P is any point on the ellipse such that perimeter of triangle PS_1S_2, is 15. Then eccentricity of the ellipse is:

1108. Let LL' be the latus rectum through the focus of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and A' be the farther vertex. If $A' LL'$ is equilateral, then the eccentricity of the hyperbola is (axes are coordinate axes). (a) $\sqrt{3}$ (b) $\sqrt{3} + 1$ (c) $\left(\dfrac{\sqrt{3} + 1}{\sqrt{2}}\right)$ (d) $\dfrac{\left(\sqrt{3} + 1\right)}{\sqrt{3}}$

1109. Find the equation of the common tangent in the first quadrant of the circle $x^2 + y^2 = 16$ and the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{4} = 1$. Also find the length of the intercept of the tangent between the coordinates axes.

1110. If the normal at $P(\theta)$ on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{2a^2} = 1$ meets the transvers axis at G, then prove that $AG\overset{.}{A}'G = a^2\left(e^4\sec^2\theta - 1\right)$, where $A and A'$ are the vertices of the hyperbola.

1111. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : (A) $\dfrac{4}{3}$ (B) $\dfrac{4}{\sqrt{3}}$ (C) $\dfrac{2}{\sqrt{3}}$ (D) $\sqrt{3}$

1112. Prove that in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix.

1113. Find the asymptotes of the curve $xy - 3y - 2x = 0$.

1114. With one focus of the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ as the centre, a circle is drawn which is tangent to the hyperbola with no part of the circle being outside the hyperbola. The radius of the circle is

1115. The equation of the passing through the of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 9$, and having centre at (0,3) is :
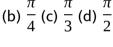
1116. Two circles are given such that they neither intersect nor touch. Then identify the locus of the center of variable circle which touches both the circles externally.

1117. If the eccentricity of the hyperbola $x^2 - y^2(\sec)^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2(\sec)^2\alpha + y^2 = 25$, then a value of α is : (a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

1118. An ellipse has OB as the semi-minor axis, $F and F'$ as its foci, and $\angle FBF'$ a right angle. Then, find the eccentricity of the ellipse.

1119. If A, B, $and C$ are three points on the hyperbola $xy = c^2$ such that AB subtends a right angle at C, then prove that AB is parallel to the normal to the hyperbola at point C

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1120. Statement 1 : If (3, 4) is a point on a hyperbola having foci (3, 0) and (λ, 0) , the length of the transverse axis being 1 unit, then λ can take the value 0 or 3. Statement 2 : $\left|S'P - SP\right| = 2a$, where $S and S'$ are the two foci, $2a$ is the length of the transverse axis, and P is any point on the hyperbola.

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1121. Find the co-ordinates of all the points P on the ellipse, $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$,for which the area of the triangle PON is maximum, where O denotes the origin and N, the foot of the perpendicular from O to tangent at P.

1122. If $\alpha + \beta = 3\pi$, then the chord joining the points α and β for the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ passes through which of the following points? Focus (b) Center One of the endpoints of the transverse exis. One of the endpoints of the conjugate exis.

1123. Statement 1 : If from any point $P\left(x_1, y_1\right)$ on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = -1$, tangents are drawn to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then the corresponding chord of contact lies on an other branch of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = -1$ Statement 2 : From any point outside the hyperbola, two tangents can be drawn to the hyperbola.

1124. Consider the family ol circles $x^2 + y^2 = r^2, 2 < r < 5$. If in the first quadrant, the common tangnet to a circle of this family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at A and B, then find the equation of the locus of the mid-point of AB.

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1125. Prove that the locus of the point of intersection of the tangents at the ends of the normal chords of the hyperbola $x^2 - y^2 = a^2$ is $a^2\left(y^2 - x^2\right) = 4x^2y^2$.

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1126. If a point $\left(x_1, y_1\right)$ lies in the shaded region $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, shown in the figure, then $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} < 0$ Statement 2 : If $P\left(x_1, y_1\right)$ lies outside the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then $\dfrac{x12}{a^2} - \dfrac{y12}{b^2} < 1$

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1127. Let P be a point on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, 0 < b < a$ and let the line parallel to y-axis passing through P meet the circle $x^2 + y^2 = a^2$ at the point Q such that P and Q are on the same side of x-axis. For two positive real numbers r and s, find the locus of the point R on PQ such that $PR:RQ = r:s$ and P varies over the ellipse.

1128. Statement 1 : If a circle $S = 0$ intersects a hyperbola $xy = 4$ at four points, three of them being (2, 2), (4, 1) and $\left(6, \dfrac{2}{3}\right)$, then the coordinates of the fourth point are $\left(\dfrac{1}{4}, 16\right)$. Statement 2 : If a circle $S = 0$ intersects a hyperbola $xy = c^2$ at $t_1, t_2, t_3,$ and t_3 then $t_1 \cdot t_2 \cdot t_3 \cdot t_4 = 1$

1129. Find the coordinates of the foci and the centre of the hyperbola

$$\left(\frac{(3x - 4y - 12)^2}{100}\right) - \left(\frac{(4x + 3y - 12)^2}{225}\right) = 1$$

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1130. show that the area of the triangle inscribed in the circle $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meet the ellipse respictively at P,Q,R so that P,Q,R lie on the same side of the major axis as A,B,C respictively. Prove that the normal to the ellipse drawn at the points P,Q and R are concurrent.

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1131. Number of points from where perpendicular tangents can be drawn to the curve $\frac{x^2}{16} - \frac{y^2}{25} = 1$ is

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1132. On which curve does the perpendicular tangents drawn to the hyperbola $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$ intersect?

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1133. The minimum area of the triangle formed by the tangent to $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and the coordinate axes is (a)ab sq. units (b) $\dfrac{a^2 + b^2}{2}$ sq units (c)$\dfrac{(a + b)^2}{2}$ sq units (d) $\dfrac{a^2 + ab + b^2}{3}$ sq. units

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1134. Statement 1 : The equations of tangents to the hyperbola $2x^2 - 3y^2 = 6$ which is parallel to the line $y = 3x + 4$ are $y = 3x - 5$ and $y = 3x + 5$. Statement 2 : For a given slope, two parallel tangents can be drawn to the hyperbola.

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1135. P is a point on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transvers axis at T. If O is the center of the hyperbola, then find the value of $OT \times ON$

1136. If $P = (x, y)$, $F_1 = (3, 0)$, $F_2 = (-3, 0)$, and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equal 8 (b) 6 (c) 10 (d) 12

1137. Statement 1 : Every line which cuts the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{16} = 1$ at two distinct points has slope lying in $(-2, 2)$. Statement 2 : The slope of the tangents of a hyperbola lies in $(-\infty, -2) \cup (2, \infty)$

1138. Find the equation of the hyperbola whose foci are $(8, 3) and (0, 3)$ and

eccentricity $= \dfrac{4 \cdot}{3}$

1139. The number of values of c such that the straight line $y = 4x + c$

touches the curve $\dfrac{x^2}{4} + \dfrac{y^2}{1} = 1$ is (a) 0 (b) 1 (c) 2 (d) infinite

1140. Find the equation of tangents to the curve $4x^2 - 9y^2 = 1$ which are

parallel to $4y = 5x + 7$.

1141. Statement 1 : The asymptotes of hyperbolas $3x + 4y = 2$ and

$4x - 3y = 5$ are the bisectors of the transvers and conjugate axes of the

hyperbolas. Statement 2 : The transverse and conjugate axes of the hyperbolas are the bisectors of the asymptotes.

1142. The line passing through the extremity A of the major exis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets is auxiliary circle at the point M. Then the area of the triangle with vertices at $A, M,$ and O (the origin) is $(a)\dfrac{31}{10}$ $(b)\dfrac{29}{10}$ $(c)\dfrac{21}{10}$ $(d)\dfrac{27}{10}$

1143. Find the value of m for which $y = mx + 6$ is tangent to the hyperbola $\dfrac{x^2}{100} - \dfrac{y^2}{49} = 1$

1144. If a hyperbola passes through the foci of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$.

Its transverse and conjugate axes coincide respectively with the major and minor axes of the ellipse and if the product of eccentricities of hyperbola and ellipse is 1 then the equation of a. hyperbola is

$\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ b. the equation of hyperbola is $\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$ c. focus of hyperbola is (5, 0) d. focus of hyperbola is $\left(5\sqrt{3}, 0\right)$

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1145. One the x - y plane, the eccentricity of an ellipse is fixed (in size and position) by 1) both foci 2) both directrices 3)one focus and the corresponding directrix 4)the length of major axis.

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1146. Find the equation of tangent to the conic $x^2 - y^2 - 8x + 2y + 11 = 0$ at (2, 1)

1147. Statement 1 : A bullet is fired and it hits a target. An observer in the same plane heard two sounds: the crack of the rifle and the thud of the bullet striking the target at the same instant. Then the locus of the observer is a hyperbola where the velocity of sound is smaller than the velocity of the bullet. Statement 2 : If the difference of distances of a point P from two fixed points is constant and less than the distance between the fixed points, then the locus of P is a hyperbola.

1148. The equation of one directrix of a hyperbola is $2x + y = 1$, the corresponding focus is (1, 2) and eccentricity is $\sqrt{3}$. Find the equation of hyperbola and coordinates of the centre and second focus.

1149. The distance of a point on the ellipse $\dfrac{x^2}{6} + \dfrac{y^2}{2} = 1$ from the center is 2. Then the eccentric angle of the point is $\dfrac{\pi}{4}$ (b) $\dfrac{3\pi}{4}$ (c) $\dfrac{5\pi}{6}$ (d) $\dfrac{\pi}{6}$

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1150. A hyperbola having the transverse axis of length $2\sin\theta$ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

$x^2\cosec^2\theta - y^2\sec^2\theta = 1$ 　 $x^2\sec^2\theta - y^2\cosec^2\theta = 1$ 　 $x^2\sin^2\theta - y^2\cos^2\theta = 1$

$x^2\cos^2\theta - y^2\sin^2\theta = 1$

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1151. If it is posssible to draw the tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ having slope 2, then find its range of eccentricity.

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1152. The set of values of m for which it is possible to draw the chord $y = \sqrt{m}x + 1$ to the curve $x^2 + 2xy + \left(2 + \sin^2\alpha\right)^y \wedge 2 = 1$, which subtends a right angle at the origin for some value of α, is [2, 3] (b) [0, 1] [1, 3] (d) none of these

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1153. Consider a branch of the hypebola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is (A) $1 - \sqrt{\dfrac{2}{3}}$ (B) $\sqrt{\dfrac{3}{2}} - 1$ (C) $1 + \sqrt{\dfrac{2}{3}}$ (D) $\sqrt{\dfrac{3}{2}} + 1$

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1154. Find the equations of the tangents to the hyperbola $x^2 - 9y^2 = 9$ that are drawn from (3, 2).

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1155. Let a and b be nonzero real numbers. Then the equation $\left(ax^2 + by^2 + c\right)\left(x^2 - 5xy + 6y^2\right) = 0$ represents. four straight lines, when $c = 0$ and a, b are of the same sign. two straight lines and a circle, when $a = b$ and c is of sign opposite to that a two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

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1156. $\dfrac{x^2}{r^2 - r - 6} + \dfrac{y^2}{r^2 - 6r + 5} = 1$ will represent ellipse if r lies in the interval (a).(-∞,2) (b). (3,∞) (c). (5,∞) (d).(1,∞)

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1157. Find the equations to the common tangents to the two hyperbolas $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$

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1158. A parabola is drawn with focus at one of the foci of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. If the latus rectum of the ellipse and that of the parabola are same, then the eccentricity of the ellipse is (a) $1 - \dfrac{1}{\sqrt{2}}$ (b) $2\sqrt{2} - 2$ (c) $\sqrt{2} - 1$ (d) none of these

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1159. Let P(6,3) be a point on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ If the normal at the point intersects the x-axis at (9,0), then the eccentricity of the hyperbola is

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1160. Find the equation of the common tangent to the curves $y^2 = 8x$ and xy=-1.

1161. If the maximum distance of any point on the ellipse $x^2 + 2y^2 + 2xy = 1$ from its center is r, then r is equal to

1162. If a hyperbola passes through the foci of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$. Its transverse and conjugate axes coincide respectively with the major and minor axes of the ellipse and if the product of eccentricities of hyperbola and ellipse is 1 then the equation of a. hyperbola is $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ b. the equation of hyperbola is $\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$ c. focus of hyperbola is $(5, 0)$ d. focus of hyperbola is $\left(5\sqrt{3}, 0\right)$

1163. Let P_i and P_i' be the feet of the perpendiculars drawn from the foci S and S' on a tangent T_i to an ellipse whose length of semi-major axis is 20. If $\displaystyle\sum_{i=0}^{10} \left(SP_i\right)\left(S'\Pi'\right) = 2560$, then the value of eccentricity is (a) $\dfrac{1}{5}$ (b) $\dfrac{2}{5}$ (c) $\dfrac{3}{5}$ (d) $\dfrac{4}{5}$

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1164. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal to that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then (a) the foci of ellipse are $(\pm 1, 0)$ (b) equation of ellipse is $x^2 + 2y^2 = 2$ (c) the foci of ellipse are $\left(\pm \sqrt{2}, 0\right)$ (d) equation of ellipse is $\left(x^2 + y^2 = 4\right)$

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1165. The number of points on the ellipse $\dfrac{x^2}{50} + \dfrac{y^2}{20} = 1$ from which a pair of perpendicular tangents is drawn to the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ is 0 (b) 2 (c) 1 (d) 4

1166. let the eccentricity of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. if the hyperbola passes through a focus of the ellipse then: (a) the equation of the hyperbola is $\dfrac{x^2}{3} - \dfrac{y^2}{2} = 1$ (b) a focus of the hyperbola is $(2, 0)$ (c) the eccentricity of the hyperbola is $\sqrt{\dfrac{5}{3}}$ (d) the equation of the hyperbola is $x^2 - 3y^2 = 3$
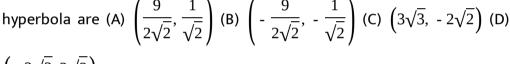
1167. The equation of the ellipse whose axes are coincident with the coordinates axes and which touches the straight lines $3x - 2y - 20 = 0$ and

$x + 6y - 20 = 0$ is (b) $\dfrac{x^2}{40} + \dfrac{y^2}{10} = 1$ (b) $\dfrac{x^2}{5} + \dfrac{y^2}{8} = 1$ (c) $\dfrac{x^2}{10} + \dfrac{y^2}{40} = 1$ (d) $\dfrac{x^2}{40} + \dfrac{y^2}{30} = 1$

1168. Tangents are drawn to the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ parallel to the sraight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are (A) $\left(\dfrac{9}{2\sqrt{2}}, \dfrac{1}{\sqrt{2}} \right)$ (B) $\left(-\dfrac{9}{2\sqrt{2}}, -\dfrac{1}{\sqrt{2}} \right)$ (C) $\left(3\sqrt{3}, -2\sqrt{2} \right)$ (D) $\left(-3\sqrt{3}, 2\sqrt{2} \right)$

1169. An ellipse with major and minor axes lengths $2a$ and $2b$, respectively, touches the coordinate axes in the first quadrant. If the foci are $\left(x_1, y_1 \right) and \left(x_2, y_2 \right)$, then the value of $x_1 x_2$ and $y_1 y_2$ is 9a) a^2 (b) b^2 (c) $a^2 b^2$ (d) $a^2 + b^2$

1170. Consider the hyperbola $H, x^2 - y^2 = 1$ and a circle S with centre $N(x, 0)$. Suppose that H and S touch each other at point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle PMN, then the correct expression(s) is/are.

1171. Let d be the perpendicular distance from the centre of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ to the tangent drawn at a point P on the ellipse. If F_1 & F_2 are the two foci of the ellipse, then show the $(PF_1 - PF_2)^2 = 4a^2 \left[1 - \dfrac{b^2}{d^2} \right]$.

.

1172. From a point P(1, 2), pair of tangents are drawn to hyperbola, one tangent ot each arm of hyperbola. Equations of asymptotes of hyperbola are $\sqrt{3}x - y + 5 = 0$ and $\sqrt{3}x + y - 1 = 0$. Find the eccentricity of hyperbola.

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1173. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. The angle between the tangents at P and Q of the ellipse $x^2 + 2y^2 = 6$ is

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1174. The combined equation of the asymptotes of the hyperbola $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ is (a) $2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$ (b) $2x^2 + 5xy + 2y^2 + 4x + 5y - 2 = 0$ (c) $2x^2 + 5xy + 2y^2 = 0$ (d) none of these

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1175. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0, y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

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1176. Let any double ordinate PNP' of the hyperbola $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$ be produced on both sides to meet the asymptotes in Q and Q' . Then $PQ. P'Q$ is equal to

(a) 25 (b) 16 (c) 41 (d) none of these

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1177. Tangents drawn from the point $P(2, 3)$ to the circle $x^2 + y^2 - 8x + 6y + 1 = 0$ points A and B. The circumcircle of the $\triangle PAB$ cuts the director circle of ellipse $\dfrac{(x - 5)^2}{9} + \dfrac{(y - 3)^2}{b^2} = 1$ orthogonally. Find the value of b^2.

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1178. For hyperbola whose center is at (1, 2) and the asymptotes are parallel to lines $2x + 3y = 0$ and $x + 2y = 1$, the equation of the hyperbola passing through (2, 4) is $(2x + 3y - 5)(x + 2y - 8) = 40$ $(2x + 3y - 8)(x + 2y - 8) = 40$ $(2x + 3y - 8)(x + 2y - 5) = 30$ none of these

1179. If from a point $P(0, \alpha)$, two normals other than the axes are drawn to the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ such that |alpha| < k then the value of $4k$ is

1180. The chord of contact of a point P w.r.t a hyperbola and its auxiliary circle are at right angle. Then the point P lies on conjugate hyperbola one of the directrix one of the asymptotes (d) none of these

1181. If the mid-point of a chord of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ (0, 3), then length of the chord is (1) $\frac{32}{5}$ (2) 16 (3) $\frac{4}{5}$ 12 (4) 32

1182. If the intercepts made by tangent, normal to a rectangular hyperbola $x^2 - y^2 = a^2$ with x-axis are a_1, a_2 and with y-axis are b_1, b_2 then $a_1, a_2 + b_1 b_2 =$

1183. Let the distance between a focus and the corresponding directrix of an ellipse be 8 and the eccentricity be $\frac{1}{2}$. If the length of the minor axis is k, then $\frac{\sqrt{3k}}{2}$ is _____

1184. If $S = 0$ is the equation of the hyperbola $x^2 + 4xy + 3y^2 - 4x + 2y + 1 = 0$, then the value of K for which $S + K = 0$ represents its asymptotes is (a)20 (b) -16 (c) -22 (d) 18

1185. Consider an ellipse E, $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, centered at point O andhaving AB and CD as its major and minor axes, respectively. If S_1 is one of the focus of the ellipse, the radius of the incircle of triangle OCS_1 is 1 unit, and $OS_1 = 6$ units, then the value of $\dfrac{a - b}{2}$ is_____

1186. If two distinct tangents can be drawn from the Point $(\alpha, 2)$ on different branches of the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ then (1) $|\alpha| < \dfrac{3}{2}$ (2) $|\alpha| > \dfrac{2}{3}$ (3)$|\alpha| > 3$ (4) $\alpha = 1$

1187. Suppose x and y are real numbers and that $x^2 + 9y^2 - 4x + 6y + 4 = 0$. Then the maximum value of $\dfrac{(4x - 9y)}{2}$ is_____

1188. A hyperbola passes through (2,3) and has asymptotes $3x - 4y + 5 = 0$ and $12x + 5y - 40 = 0$. Then, the equation of its transverse axis is

(a) $77x - 21y - 265 = 0$ (b) $21x - 77y + 265 = 0$ (c) $21x - 77y - 265 = 0$ (d) $21x + 77y - 265 = 0$

1189. Rectangle ABCD has area 200. An ellipse with area 200π passes through A and C and has foci at B and D. Find the perimeter of the rectangle.

1190. From any point on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, tangents are drawn to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 2.$ The area cut-off by the chord of contact on the asymptotes is equal to $\dfrac{a}{2}$ (b) ab (c) $2ab$ (d) $4ab$

1191. The locus of the image of the focus of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1, (a > b),$ with respect to any of the tangents to the ellipse is:

(a) $(x + 4)^2 + y^2 = 100$ (b) $(x + 2)^2 + y^2 = 50$ (c) $(x - 4)^2 + y^2 = 100$ (d) $(x + 2)^2 + y^2 = 50$

1192. about to only mathematics

1193. A point on the ellipse $x^2 + 3y^2 = 37$ where the normal is parallel to the line $6x - 5y = 2$ is (5, -2) (b) (5, 2) (c) (-5, 2) (d) (-5, -2)

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1194. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec c\phi, b\tan\phi)$ (where $\theta + \phi = \dfrac{\pi}{2}$ be two points on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ If (h, k) is the point of intersection of the normals at P and Q then k is equal to (A) $\dfrac{a^2 + b^2}{a}$ (B) $-\left(\dfrac{a^2 + b^2}{a}\right)$ (C)

$\dfrac{a^2 + b^2}{b}$ (D) $-\left(\dfrac{a^2 + b^2}{b}\right)$

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1195. If a pair of variable straight lines $x^2 + 4y^2 + \alpha xy = 0$ (where α is a real parameter) cut the ellipse $x^2 + 4y^2 = 4$ at two points A and B, then the locus of the point of intersection of tangents at A and B is

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1196. The line $2x + y = 1$ is tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

1197. The equation $3x^2 + 4y^2 - 18 + 16y + 43 = k$ represents an empty set, if $k < 0$ represents an ellipse, if $k > 0$ represents a point, if $k = 0$ cannot represent a real pair of straight lines for any value of k

1198. Which of the following is/are true about the ellipse $x^2 + 4y^2 - 2x - 16y + 13 = 0$? (a)the latus rectum of the ellipse is 1. (b)The distance between the foci of the ellipse is $4\sqrt{3}$ (c)The sum of the focal distances of a point $P(x, y)$ on the ellipse is 4. (d)Line $y = 3$ meets the

tangents drawn at the vertices of the ellipse at points P and Q. Then PQ subtends a right angle at any of its foci.

1199. If a ray of light incident along the line $3x + \left(5 - 4\sqrt{2}\right)y = 15$ gets reflected from the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$, then its reflected ray goes along the line. (a)$x\sqrt{2} - y + 5 = 0$ (b) $\sqrt{2}y - x + 5 = 0$ (c)$\sqrt{2}y - x - 5 = 0$ (d) none of these

1200. If the sum of the slopes of the normal from a point P to the hyperbola $xy = c^2$ is equal to $\lambda\left(\lambda \in R^+\right)$, then the locus of point P is (a) $x^2 = \lambda c^2$ (b) $y^2 = \lambda c^2$ (c)$xy = \lambda c^2$ (d) none of these

1201. If the tangent at the point $P(\theta)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x = 15$, then $\theta = $ (a) $\dfrac{2\pi}{3}$ (b) $\dfrac{4\pi}{3}$ (c) $\dfrac{5\pi}{3}$ (d) $\dfrac{\pi}{3}$

1202. If the normal to the given hyperbola at the point $\left(ct, \dfrac{c}{t}\right)$ meets the curve again at $\left(ct', \dfrac{c}{t'}\right)$, then (A) $t^3t' = 1$ (B) $t^3t' = -1$ (C) $tt' = 1$ (D) $tt' = -1$

1203. If the equation of the ellipse is $3x^2 + 2y^2 + 6x - 8y + 5 = 0$, then which of the following is/are true? a. $e = \dfrac{1}{\sqrt{3}}$ b.Center is $(-1, 2)$ c.Foci are $(-1, 1) and (-1, 3)$ d.Directrices are $y = 2 \pm \sqrt{3}$

1204. A normal to the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{1} = 1$ has equal intercepts on the positive x- and y-axis. If this normal touches the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, then $a^2 + b^2$ is equal to 5 (b) 25 (c) 16 (d) none of these

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1205. If the chord through the points whose eccentric angles are θ and φ on the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ passes through a focus, then the value of $\tan\left(\dfrac{\theta}{2}\right)\tan\left(\dfrac{\varphi}{2}\right)$ is $\dfrac{1}{9}$ (b) -9 (c) -$\dfrac{1}{9}$ (d) 9

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1206. The number of points on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 3$ from which mutually perpendicular tangents can be drawn to the circle $x^2 + y^2 = a^2$ is/are (a) 0 (b) 2 (c) 3 (d) 4

1207. The coordinates (2, 3) and (1, 5) are the foci of an ellipse which passes through the origin. Then the equation of the (a)tangent at the origin is $\left(3\sqrt{2}-5\right)x + \left(1-2\sqrt{2}\right)y = 0$ (b)tangent at the origin is $\left(3\sqrt{2}+5\right)x + \left(1+2\sqrt{2}y\right) = 0$ (c)tangent at the origin is $\left(3\sqrt{2}+5\right)x - \left(2\sqrt{2}+1\right)y = 0$ (d)tangent at the origin is $\left(3\sqrt{2}-5\right) - y\left(1-2\sqrt{2}\right) = 0$

1208. If tangents $PQ and PR$ are drawn from a variable point P to thehyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1, (a > b)$, so that the fourth vertex S of parallelogram $PQSR$ lies on the circumcircle of triangle PQR , then the locus of P is $x^2 + y^2 = b^2$ (b) $x^2 + y^2 = a^2$ $x^2 + y^2 = a^2 - b^2$ (d) none of these

1209. If the variable line $y = kx + 2h$ is tangent to an ellipse $2x^2 + 3y^2 = 6$, then the locus of $P(h, k)$ is a conic C whose eccentricity is 3. Then the value of $3e^2$ is_____
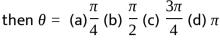
1210. The locus of a point, from where the tangents to the rectangular hyperbola $x^2 - y^2 = a^2$ contain an angle of 45^0, is (a) $\left(x^2 + y^2\right)^2 + a^2\left(x^2 - y^2\right) = 4a^2$ (b)$2\left(x^2 + y^2\right)^2 + 4a^2\left(x^2 - y^2\right) = 4a^2$ (c) $\left(x^2 + y^2\right)^2 + 4a^2\left(x^2 - y^2\right) = 4a^2$ (d)$\left(x^2 + y^2\right) + a^2\left(x^2 - y^2\right) = a^4$

1211. The value of a for the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, (a > b)$, if the extremities of the latus rectum of the ellipse having positive ordinates lie on the parabola $x^2 = 2(y - 2)$ is ___

1212. The tangent at a point P on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ meets one of the directrix at F If PF subtends an angle θ at the corresponding focus, then $\theta =$ (a)$\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{3\pi}{4}$ (d) π

1213. If $x, y \in R$, satisfies the equation $\dfrac{(x-4)^2}{4} + \dfrac{y^2}{9} = 1$, then the difference between the largest and the smallest valus of the expression $\dfrac{x^2}{4} + \dfrac{y^2}{9}$ is_____

1214. The locus of the foot of the perpendicular from the center of the hyperbola $xy = 1$ on a variable tangent is $\left(x^2 + y^2\right)^2 = 4xy$ (b) $\left(x^2 - y^2\right) = \dfrac{1}{9}\left(x^2 - y^2\right) = \dfrac{7}{144}$ (d) $\left(x^2 - y^2\right) = \dfrac{1}{16}$

1215. Column I, Column II An ellipse passing through the origin has its foci (3, 4) and (6,8). Then the length of its minor axis is, p. 8 If PQ is a focal chord of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ which passes through $S \equiv (3, 0)$ and $PS = 2$, then the length of chord PQ is, q. $10\sqrt{2}$ If the line $y = x + K$ touches the ellipse $9x^2 + 16y^2 = 144,$ then the difference of values of K is, r. 10 The sum of distances of a point on the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ from the foci is, s. 12

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1216. Find the range of parameter a for which a unique circle will pass through the points of intersection of the hyperbola $x^2 - y^2 = a^2$ and the parabola $y = x^2$ Also, find the equation of the circle.

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1217. Column I, Column II stick of length 10 units rests against the floor and a wall of a room. If the stick begins to slide on the floor then the locus of its middle point is (Assume the origin to be the intersection of the floor and the wall and the coordinate axes along the floor and the wall), p. $\sqrt{6}$ ∀' is the major axis of the ellipse $3x^2 + 2y^2 + 6x - 4y - 1 = 0$ and P is a variable point on it. Then the greatest area of triangle APA' is, q. $2\sqrt{7}$ The distance between the foci of the curve represented by the equation $x = 1 + 4\cos\theta, y = 2 + 3\sin\theta$ is, r. $\dfrac{128}{3}$ Tangents are drawn to the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{7} = 1$ at the endpoints of the latus rectum. The area of the quadrilateral so formed is, s. $\sqrt{5}$

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1218. Show that the midpoints of focal chords of a hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ lie on another similar hyperbola.

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1219. If the normal at the point $P(\theta)$ to the ellipse $\dfrac{x^2}{14} + \dfrac{y^2}{5} = 1$ intersects it again at the point $Q(2\theta)$, then $\cos\theta$ is equal to (A) $\dfrac{2}{3}$ (B) $\dfrac{-2}{3}$ (C) $\dfrac{3}{4}$ (D) non of these

1220. A tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ cuts the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at P and Q. Show that the locus of the midpoint of PQ is

$$\left(\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}\right)^2 = \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}.$$

1221. Prove that the part of the tangent at any point of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ intercepted between the point of contact and the transvers axis is a harmonic mean between the lengths of the perpendiculars drawn from the foci on the normal at the same point.

1222. A variable line $y = mx - 1$ cuts the lines $x = 2y$ and $y = -2x$ at points A and B. Prove that the locus of the centroid of triangle OAB(O being the origin) is a hyperbola passing through the origin.

1223. Statement 1 : If a and b are real numbers and $c > 0$, then the locus represented by the equation $|ay - bx| = c\sqrt{(x - a)^2 + (y - b)^2}$ is an ellipse.

Statement 2 : An ellipse is the locus of a point which moves in a plane such that the ratio of its distances from a fixed point (i.e., focus) to that from the fixed line (i.e., directrix) is constant and less than 1.

1224. Two tangents to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ having m_1 and m_2 cut the axes at four concyclic points. Fid the value of $m_1 m_2$

1225. A tangent having slope of $-\dfrac{4}{3}$ to the ellipse $\dfrac{x^2}{18} + \dfrac{y^2}{32} = 1$ intersects the major and minor axes at points A and B, respectively. If C is the center of the ellipse, then find area of triangle ABC.

1226. Let P be a point on the hyperbola $x^2 - y^2 = a^2$, where a is a parameter, such that P is nearest to the line $y = 2x$. Find the locus of P.

1227. Let P be any point on a directrix of an ellipse of eccentricity e, S be the corresponding focus, and C the center of the ellipse. The line PC meets the ellipse at A. The angle between PS and tangent a A is α. Then α is equal to $\tan^{-1}e$ (b) $\dfrac{\pi}{2}$ $\tan^{-1}\left(1 - e^2\right)$ (d) none of these

1228. If one of varying central conic (hyperbola) is fixed in magnitude and position, prove that the locus of the point of contact of a tangent drawn to it from a fixed point on the other axis is a parabole.

1229. If a tangent of slope is 2 of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is normal to the circle $x^2 + y^2 + 4x + 1 = 0$, then the maximum value of ab is 4 (b) 2 (c) 1 (d) none of these

1230. A transvers axis cuts the same branch of a hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at $P and P'$ and the asymptotes at Q and Q' . Prove that $PQ = P'Q'$ and $PQ' = P'Q$.

1231. If $\left(\sqrt{3}\right)bx + ay = 2ab$ touches the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, then the eccentric angle of the point of contact is $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

1232. The eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0$ is 1 (b) $\sqrt{2}$ (c) 2 (d) $\dfrac{1}{2}$

1233. If the ellipse $\dfrac{x^2}{a^2 - 7} + \dfrac{y^2}{13 = 5a} = 1$ is inscribed in a square of side length $\sqrt{2}a$, then a is equal to $\dfrac{6}{5}$ $\left(-\infty, -\sqrt{7}\right) \cup \left(\sqrt{7}, \dfrac{13}{5}\right)$ $\left(-\infty, -\sqrt{7}\right) \cup \left(\dfrac{13}{5}, \sqrt{7}, \right)$ no such a exists

1234. The curve for which the length of the normal is equal to the length of the radius vector is/are (a) circles (b) rectangular hyperbola (c) ellipses (d) straight lines

1235. The locus of the point of intersection of the tangent at the endpoints of the focal chord of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1(b < a)$(a) is a an circle (b) ellipse (c) hyperbola (d) pair of straight lines

1236. A tangent drawn to hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at $P\left(\dfrac{\pi}{6}\right)$ froms a triangle of area $3a^2$ square units, with the coordinate axes, then the square of its eccentricity is (A) 15 (B) 24 (C) 17 (D) 14

1237. The normal at a variable point P on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ of eccentricity e meets the axes of the ellipse at Q and R. Then the locus of the midpoint of QR is a conic with eccentricity e' such that (a)e' is independent of e (b) $e' = 1$ (c)$e' = e$ (d) $e' = \dfrac{1}{e}$

1238. If the distance between the foci and the distance between the two directricies of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ are in the ratio 3:2, then $b : a$ is (a)$1 : \sqrt{2}$ (b) $\sqrt{3} : \sqrt{2}$ (c)$1 : 2$ (d) $2 : 1$

1239. Any ordinate MP of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ meets the auxiliary circle at Q. Then locus of the point of intersection of normals at P and Q to the

respective curves is $x^2 + y^2 = 8$ (b) $x^2 + y^2 = 34$ $x^2 + y^2 = 64$ (d)

$x^2 + y^2 = 15$

1240. 1. If the distance between two parallel tangents drawn to the hyperbola 1 is 2, then their slope is equal 49 b. t d. none of these

1241. The number of distinct normal lines that can be drawn to the ellipse $\dfrac{x^2}{169} + \dfrac{y^2}{25} = 1$ from the point $P(0, 6)$ is (A) one (B) two (C) three (D) four

1242. A normal is drawn to the hyperbolas $(y - xm)(my + x) = a^2$ and $\left(m^2 - 1\right)\left(y^2 - x^2\right) + 4mxy = b^2$ find the angle between the hyperbolas.

1243. An ellipse has point $(1, -1) and (2, -1)$ as its foci and $x + y - 5 = 0$ as one of its tangents. Then the point where this line touches the ellipse is

(a) $\left(\dfrac{32}{9}, \dfrac{22}{9}\right)$ (b) $\left(\dfrac{23}{9}, \dfrac{2}{9}\right)$ (c) $\left(\dfrac{34}{9}, \dfrac{11}{9}\right)$ (d) none of these

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1244. The equation of the transvers axis of the hyperbola $(x - 3)^2 + (y + 1)^2 = (4x + 3y)^2$ is $x + 3y = 0$ (b) $4x + 3y = 9$ $3x - 4y = 13$ (d) $4x + 3y = 0$
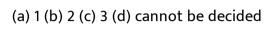
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1245. Find the values of a for which three distinct chords drawn from $(a, 0)$ to the ellipse $x^2 + 2y^2 = 1$ are bisected by the parabola $y^2 = 4x$.

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1246. If a variable line has its intercepts on the coordinate axes e and e', where $\dfrac{e}{2}$ and $\dfrac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the line always touches the circle $x^2 + y^2 = r^2$, where $r =$

(a) 1 (b) 2 (c) 3 (d) cannot be decided

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1247. Prove that if any tangent to the ellipse is cut by the tangents at the endpoints of the major axis at T and T', then the circle whose diameter is TT' will pass through the foci of the ellipse.

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1248. A straight line has its extremities on two fixed straight lines and cuts off from them a triangle of constant area c^2 Then the locus of the middle point of the line is $2xy = c^2$ (b) $xy + c^2 = 0$ $4x^2y^2 = c$ (d) none of these

1249. A circle has the same center as an ellipse and passes through the foci $F_1 and F_2$ of the ellipse, such that the two curves intersect at four points. Let P be any one of their point of intersection. If the major axis of the ellipse is 17 and the area of triangle PF_1F_2 is 30, then the distance between the foci is

1250. The length of the transverse axis of the rectangular hyperbola $xy = 18$ is (a) 6 (b) 12 (c) 18 (d) 9

1251. If P is any point on ellipse with foci $S_1 \& S_2$ and eccentricity is $\dfrac{1}{2}$ such that $\angle PS_1S_2 = \alpha$, $\angle PS_2S_1 = \beta$, $\angle S_1PS_2 = \gamma$, then

$\cot\left(\dfrac{\alpha}{2}\right)$, $\cot\left(\dfrac{\gamma}{2}\right)$, $\cot\left(\dfrac{\beta}{2}\right)$ are in

1252. The locus of the point which is such that the chord of contact of tangents drawn from it to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ forms a triangle of constant area with the coordinate axes is a straight line (b) a hyperbola an ellipse (d) a circle

1253. Find the range of eccentricity of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, (where a > b) such that the line segment joining the foci does not subtend a right angle at any point on the ellipse.

1254. The angle between the lines joining origin to the points of intersection of the line $\sqrt{3}x + y = 2$ and the curve $y^2 - x^2 = 4$ is (A) $\tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$ (B) $\dfrac{\pi}{6}$ (C) $\tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ (D) $\dfrac{\pi}{2}$

1255. the equation of the chord of contact of the pair of tangents drawn to the ellipse $4x^2 + 9y^2 = 36$ from the point (m, n) where $mn = m + n$, m, n being nonzero positive integers, is $2x + 9y = 18$ (b) $2x + 2y = 1$ $4x + 9y = 18$ (d) none of these

1256. The equation to the chord joining two points $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ on the rectangular hyperbola $xy = c^2$ is: (A) $\dfrac{x}{x_1 + x_2} + \dfrac{y}{y_1 + y_2} = 1$ (B) $\dfrac{x}{x_1 - x_2} + \dfrac{y}{y_1 - y_2} = 1$ (C) $\dfrac{x}{y_1 + y_2} + \dfrac{y}{x_1 + x_2} = 1$ (D) $\dfrac{x}{y_1 - y_2} + \dfrac{y}{x_1 - x_2} = 1$

1257. The equation of the line passing through the center and bisecting the chord $7x + y - 1 = 0$ of the ellipse $\dfrac{x^2}{1} + \dfrac{y^2}{7} = 1$ is (a)$x = y$ (b) $2x = y$ (c) $x = 2y$ (d) $x + y = 0$

1258. If $P\left(x_1, y_1\right), Q\left(x_2, y_2\right), R\left(x_3, y_3\right)$ and $S\left(x_4, y_4\right)$ are four concyclic points on the rectangular hyperbola) and $xy = c^2$, then coordinates of the orthocentre ofthe triangle PQR is

1259. Let P be any point on any directrix of an ellipse. Then the chords of contact of point P with respect to the ellipse and its auxiliary circle intersect at (a)some point on the major axis depending upon the position

of point P (b)the midpoint of the line segment joining the center to the corresponding focus (c)the corresponding focus (d)none of these

1260. Suppose the circle having equation $x^2 + y^2 = 3$ intersects the rectangular hyperbola $xy = 1$ at points $A, B, C, and D$ The equation $x^2 + y^2 - 3 + \lambda(xy - 1) = 0, \lambda \in R$, represents. a pair of lines through the origin for $\lambda = -3$ an ellipse through $A, B, C, and D$ for $\lambda = -3$ a parabola through $A, B, C, and D$ for $\lambda = -3$ a circle for any $\lambda \in R$

1261. The length of the sides of the square which can be made by four perpendicular tangents $PQ and PR$ are drawn to the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$.

Then the angle subtended by QR at the origin is $\dfrac{\tan^{-1}\left(\sqrt{6}\right)}{65}$ (b)

$\dfrac{\tan^{-1}\left(4\sqrt{6}\right)}{65}$ $\dfrac{\tan^{-1}\left(8\sqrt{6}\right)}{65}$ (d) $\dfrac{\tan^{-1}\left(48\sqrt{6}\right)}{455}$

1262. If two points $P\&Q$ on the hyperbola $,\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ whose centre is C

be such that CP is perpendicularal to CQ and $a < b1$,then prove that

$\dfrac{1}{CP^2} + \dfrac{1}{CQ^2} = \dfrac{1}{a^2} - \dfrac{1}{b^2}.$

1263. The line $y = mx - \dfrac{\left(a^2 - b^2\right)m}{\sqrt{a^2 + b^2 m^2}}$ is normal to the ellise $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ for

all values of m belonging to (0, 1) (b) (0, ∞) (c) R (d) none of these

1264. Let C be a curve which is the locus of the point of intersection of

lines $x = 2 + m$ and $my = 4 - m$. A circle $s \equiv (x - 2)^2 + (y + 1)^2 = 25$

intersects the curve C at four points: $P, Q, R, and S$. If O is center of the curve C, then $OP^2 + OP^2 + OR^2 + OS^2$ is

1265. If the normals at $P(\theta)$ and $Q\left(\dfrac{\pi}{2} + \theta\right)$ to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ meet the major axis at $G and g$, respectively, then $PG^2 + Qg^2 =$

$$b^2\left(1 - e^2\right)(2 - e)^2 \ a^2\left(e^4 - e^2 + 2\right) a^2\left(1 + e^2\right)\left(2 + e^2\right) b^2\left(1 + e^2\right)\left(2 + e^2\right)$$

1266. The ellipse $4x^2 + 9y^2 = 36$ and the hyperbola $a^2x^2 - y^2 = 4$ intersect at right angles. Then the equation of the circle through the points of intersection of two conics is

1267. If the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is inscribed in a rectangle whose length to breadth ratio is $2:1$, then the area of the rectangle is 4. $\dfrac{a^2 + b^2}{7}$ (b) 4. $\dfrac{a^2 + b^2}{3}$ 12. $\dfrac{a^2 + b^2}{5}$ (d) 8. $\dfrac{a^2 + b^2}{5}$

1268. The chord PQ of the rectangular hyperbola $xy = a^2$ meets the axis of x at A; C is the midpoint of PQ; and O is the origin. Then $\triangle ACO$ is equilateral (b) isosceles right-angled (d) right isosceles

1269. If tangents PQ and PR are drawn from a point on the circle $x^2 + y^2 = 25$ to the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1, (b < 4)$, so that the fourth vertex S of parallelogram $PQSR$ lies on the circumcircle of triangle PQR, then the eccentricity of the ellipse is

(a) $\dfrac{\sqrt{5}}{4}$

(b) $\dfrac{\sqrt{7}}{4}$

(c) $\dfrac{\sqrt{7}}{2}$

(d) $\dfrac{\sqrt{5}}{3}$

1270. The curve $xy = c, (c > 0)$, and the circle $x^2 + y^2 = 1$ touch at two points. Then the distance between the point of contacts is 1 (b) 2 (c) $2\sqrt{2}$ (d) none of these

1271. An ellipse is sliding along the coordinate axes. If the foci of the ellipse are (1, 1) and (3, 3), then the area of the director circle of the ellipse (in square units) is 2π (b) 4π (c) 6π (d) 8π

1272. If $S_1 and S_2$ are the foci of the hyperbola whose length of the transverse axis is 4 and that of the conjugate axis is 6, and $S_3 and S_4$ are the foci of the conjugate hyperbola, then the area of quadrilateral $S_1 S_3 S_2 S_4$ is 24 (b) 26 (c) 22 (d) none of these

1273. The locus of the point which is such that the chord of contact of tangents drawn from it to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ forms a triangle of constant area with the coordinate axes is a straight line (b) a hyperbola an ellipse (d) a circle

1274. The equation of conjugate axis of the hyperbola $xy - 3y - 4x + 7 = 0$ is $y + x = 3$ (b) $y + x = 7$ $y - x = 3$ (d) none of these

1275. If S and S'' are the foci of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$, and P is any point

on it, then the range of values of SPS' P is (a)$9 \le f(\theta) \le 16$ (b) $9 \le f(\theta) \le 25$

(c)$16 \le f(\theta) \le 25$ (d) $1 \le f(\theta) \le 16$

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1276. The asymptote of the hyperbola $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ form with ans tangen

to the hyperbola triangle whose area is $a^2\tan\lambda$ in magnitude then its

eccentricity is: (a) $\sec\lambda$ (b) $\csc\lambda$ (c) $\sec^2\lambda$ (d) $\csc^2\lambda$

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1277. Let d_1 and d_2 be the length of the perpendiculars drawn from the foci

S and S' of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ to the tangent at any point P on the

ellipse. Then, $SP : S'P = \ d_1 : d_2$ (b) $d_2 : d_1 \ d12 : d22$ (d) $\sqrt{d_1} : \sqrt{d_2}$

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1278. The asymptotes of the hyperbola $xy = hx + ky$ are $x - k = 0$ and $y - h = 0$ $x + h = 0$ and $y + k = 0$ $x - k = 0$ and $y + h = 0$ $x + k = 0$ and $y - h = 0$

1279. The line $x = t^2$ meets the ellipse $x^2 + \dfrac{y^2}{9} = 1$ at real and distinct points if and only if. $|t| < 2$ (b) $|t| < 1$ $|t| > 1$ (d) none of these

1280. The equation of a rectangular hyperbola whose asymptotes are $x = 3$ and $y = 5$ and passing through (7,8) is $xy - 3y + 5x + 3 = 0$ $xy + 3y + 4x + 3 = 0$ $xy - 3y + 5x - 3 = 0$ $xy - 3y + 5x + 3 = 0$

1281. The eccentric angle of a point on the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$ at a distance of 5/4 units from the focus on the positive x-axis is $\cos^{-1}\left(\dfrac{3}{4}\right)$ (b)

$\pi - \cos^{-1}\left(\dfrac{3}{4}\right)$ $\pi + \cos^{-1}\left(\dfrac{3}{4}\right)$ (d) none of these

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1282. The center of a rectangular hyperbola lies on the line $y = 2x$ If one of the asymptotes is $x + y + c = 0$, then the other asymptote is

$6x + 3y - 4c = 0$ (b) $3x + 6y - 5c = 0$ $3x - 6y - c = 0$ (d) none of these

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1283. $(x - 1)(y - 2) = 5$ and $(x - 1)^2 + (y + 2)^2 = r^2$ intersect at four points A, B, C, D and if centroid of $\triangle ABC$ lies on line $y = 3x - 4$, then locus of D is

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1284. The eccentricity of the locus of point $(2h + 2, k)$, where (h, k) lies on the circle $x^2 + y^2 = 1$, is $\frac{1}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{1}{\sqrt{3}}$

1285. If the foci of a hyperbola lie on $y = x$ and one of the asymptotes is $y = 2x$, then the equation of the hyperbola, given that it passes through $(3, \quad 4)$, is (a)$x^2 - y^2 - \frac{5}{2}xy + 5 = 0$ (b) $2x^2 - 2y^2 + 5xy + 5 = 0$ (c) $2x^2 + 2y^2 - 5xy + 10 = 0$ (d) none of these

1286. The auxiliary circle of a family of ellipses passes through the origin and makes intercepts of 8 units and 6 units on the x and y-axis, respectively. If the eccentricity of all such ellipses is $\frac{1}{2}$, then find the locus of the focus.

1287. Four points are such that the line joining any two points is perpendicular to the line joining other two points. If three point out of these lie on a rectangular hyperbola, then the fourth point will lie on

1288. A man running around a race course notes that the sum of the distances of two flagposts from him a always 10m and the distance between the flag posts is 8m. Then the area of the path he encloses in square meters is 15π (b) 20π (c) 27π (d) 30π

1289. If tangents OQ and OR are dawn to variable circles having radius r and the centre lying on the rectangular hyperbola xy = 1, then the locus of the circumcentre of triangle OQR is (O being the origin)

1290. Let S and S' be two foci of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. If a circle described on SS' as diameter intersects the ellipse at real and distinct points, then the eccentricity e of the ellipse satisfies (a) $e = \dfrac{1}{\sqrt{2}}$ (b)

$e \in \left(\dfrac{1}{\sqrt{2}}, 1 \right)$ (c) $e \in \left(0, \dfrac{1}{\sqrt{2}} \right)$ (d) none of these

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1291. The equation, $2x^2 + 3y^2 - 8x - 18y + 35 = K$ represents

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1292. If the curves $\dfrac{x^2}{4} + y^2 = 1$ and $\dfrac{x^2}{a^2} + y^2 = 1$ for a suitable value of a cut on four concyclic points, the equation of the circle passing through these four points is $x^2 + y^2 = 2$ (b) $x^2 + y^2 = 1$ $x^2 + y^2 = 4$ (d) none of these

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1293. If the normal at P to the rectangular hyperbola $x^2 - y^2 = 4$ meets the axes at G and g and C is the center of the hyperbola, then (a)$PG = PC$ (b) $Pg = PC$ (c)$PG = Pg$ (d) $Gg = 2PC$

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1294. Each question has four choice: a, b, c and d, out of which only one is correct. Each question contains Statement 1 and Statement 2. Find the correct answer. Both the Statements are true but Statement 2 is the correct explanation of Statement 1. Both the Statement are True but Statement 2 is not the correct explanation of Statement 1. Statement 1 is True and Statement 2 is False. Statement 1 is False and Statement 2 is True Statement 1: The lines $(a + b)x + (a - 2b)y = a$ are con-current at the point $\left(\dfrac{2}{3}, \dfrac{1}{3}\right)$. Statement 2: The lines $x + y - 1 = 0$ and $x - 2y = 0$ intersect at the point $\left(\dfrac{2}{3}, \dfrac{1}{3}\right)$.

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1295. Each of the four inequalities given below defines a region in the xy plane. One of these four regions does nothave the following property. For any two points (x_1, y_2) and (y_1, y_2) in the region the piont $\left(\dfrac{x_1 + x_2}{2} \cdot \dfrac{y_1 + y_2}{2} \right)$ is also in the region. The inequality defining this region is

A. $x^2 + 2y^2 \leq 1$

B. Max $\{|x|, |y|\} \leq 1$

C. $x^2 - y^2 \leq 1$

D. $y^2 - x \leq 0$

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1296. The lines parallel to the normal to the curve $xy = 1$ is/are

$3x + 4y + 5 = 0$ (b) $3x - 4y + 5 = 0$ $4x + 3y + 5 = 0$ (d) $3y - 4x + 5 = 0$

1297. From the point (2, 2) tangent are drawn to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$. Then the point of contact lies in the first quadrant (b) second quadrant third quadrant (d) fourth quadrant

1298. If the two intersecting lines intersect the hyperbola and neither of them is a tangent to it, then the number of intersecting points are

(a)1   (b) 2   (c) 3   (d) 4

1299. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, let n be the number of points on the plane through which perpendicular tangents are drawn. (a) If $n = 1$, then

$e = \sqrt{2}$ (b) If $n > 1$, then $1 < e < \sqrt{2}$ (c) If $n = 0$, then $e > \sqrt{2}$ (d) None of these

1300. The differential equation $\dfrac{dy}{dx} = \dfrac{2x}{3y}$ represents a family of hyperbolas

(except when it represents a pair of lines) with eccentricity. (a) $\sqrt{\dfrac{3}{5}}$ (b) $\sqrt{\dfrac{5}{3}}$

(c) $\sqrt{\dfrac{2}{5}}$ (d) $\sqrt{\dfrac{5}{2}}$

1301. Circle are drawn on the chords of the rectangular hyperbola $xy = 4$ parallel to the line $y = x$ as diameters. All such circles pass through two fixed points whose coordinates are $(2, 2)$ (b) $(2, -2)$ (c) $(-2, 2)$ (d) $(-2, -2)$

1302. The equation $(x - \alpha)^2 + (y - \beta)^2 = k(lx + my + n)^2$ represents (a) a parabola for $k = \left(l^2 + m^2\right)^{-1}$ (b) an ellipse for $0 < k < \left(l^2 + m^2\right)^{-1}$ (c)a hyperbola for $k > \left(1^2 + m^2\right)^{-1}$ (d) a point circle for $k = 0$

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1303. If $x, y \in R$, then the equation $3x^4 - 2(19y + 8)x^2 + \left(361y^2 + 2\left(100 + y^4\right)\right) + 64, = 2\left(190y + 2y^2\right)$ represents in rectangular Cartesian system a/an (a)parabola (b) hyperbola (c)circle (d) ellipse

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1304. The equation $\left|\sqrt{x^2 + (y - 1)^2} - \sqrt{x^2 + (y + 1)^2}\right| = K$ will represent a hyperbola for $(a)K \in (0, 2)$ (b) $K \in (-2, 1)$ $K \in (1, \infty)$ (d) $K \in (0, \infty)$

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1305. The exhaustive set value of α^2 such that there exists a tangent to the ellipse $x^2 + \alpha^2 y^2 = \alpha^2$ and the portion of the tangent intercepted by the hyperbola $\alpha^2 x^2 - y^2$ subtends a right angle at the center of the curves is (a) $\left[\dfrac{\sqrt{5}+1}{2}, 2\right]$ (b) $(1, 2]$ (c) $\left[\dfrac{\sqrt{5}-1}{2}, 1\right]$ (d) $\left[\dfrac{\sqrt{5}-1}{2}, 1\right] \cup \left[1, \dfrac{\sqrt{5}+1}{2}\right]$

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1306. A variable chord of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1, (b > a)$, subtends a right angle at the center of the hyperbola if this chord touches. a fixed circle concentric with the hyperbola a fixed ellipse concentric with the hyperbola a fixed hyperbola concentric with the hyperbola a fixed parabola having vertex at $(0, 0)$.

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1307. Show that the equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represents a hyperbola. Find the coordinates of the centre, lengths of the axes,

eccentricity, latus-rectum, coordinates of foci and vertices, equations of the directrices of the hyperbola.

1308. For which of the hyperbolas, can we have more than one pair of perpendicular tangents? (a)$\frac{x^2}{4} - \frac{y^2}{9} = 1$ (b) $\frac{x^2}{4} - \frac{y^2}{9} = -1$ (c)$x^2 - y^2 = 4$ (d) $xy = 44$

1309. If (5, 12) and (24, 7) are the foci of an ellipse passing through the origin, then find the eccentricity of the ellipse.

1310. If $(5, 12) and (24, 7)$ are the foci of a hyperbola passing through the origin, then $e = \frac{\sqrt{386}}{12}$ (b) $e = \frac{\sqrt{386}}{13}$ $LR = \frac{121}{6}$ (d) $LR = \frac{121}{3}$

1311. Tangents are drawn from any point on the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ to the circle $x^2 + y^2 = 9$. Find the locus of the midpoint of the chord of contact.

1312. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ at four points $P\left(x_1, y_1\right), Q\left(x_2, y_2\right), R\left(x_3, y_3\right),$ and $S\left(x_4, y_4\right),$ then $x_1 + x_2 + x_3 + x_4 = 0\ y_1 + y_2 + y_3 + y_4 = 0\ x_1 x_2 x_3 x_4 = C^4\ y_1 y_2 y_3 y_4 = C^4$

1313. If the foci of $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ coincide with the foci of $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ and the eccentricity of the hyperbola is 2, then $a^2 + b^2 = 16$ there is no

director circle to the hyperbola the center of the director circle is (0, 0). the length of latus rectum of the hyperbola is 12

1314. The locus of a point whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola $xy = 1$ is a/an ellipse (b) circle hyperbola (d) parabola

1315. The equation $\dfrac{x^2}{1-r} - \dfrac{y^2}{1+r} = 1, r > 1,$ represents (a)an ellipse (b) a hyperbola (c)a circle (d) none of these

1316. An ellipse has eccentricity $\frac{1}{2}$ and one focus at the point $P\left(\frac{1}{2}, 1\right)$. Its one directrix is the common tangent nearer to the point the P to the hyperbola $x^2 - y^2 = 1$ and the circle $x^2 + y^2 = 1$. The equation of the ellipse , in the standard form is-

A. $9\left(x - \frac{1}{3}\right)^2 + 12(y - 1)^2 = 1$

B. $8\left(x - \frac{1}{3}\right)^2 + 12(y - 1)^2 = 1$

C. $9\left(x - \frac{1}{3}\right)^2 + 8(y - 1)^2 = 1$

D. none of these

Answer: A

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1317. A tangent drawn to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P\left(\frac{\pi}{6}\right)$ froms a triangle of area $3a^2$ square units, with the coordinate axes, then the square of its

eccentricity is (A) 15 (B) 24 (C) 17 (D) 14

1318. If the eccentricity of the hyperbola $x^2 - y^2(\sec)^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2(\sec)^2\alpha + y^2 = 25,$ then a value of α is : (a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

1319. If L is the length of the latus rectum of the hyperbola for which $x = 3 and y = 2$ are the equations of asymptotes and which passes through the point $(4, 6)$, then the value of $\dfrac{L}{\sqrt{2}}$ is_____

1320. If the chord $x\cos\alpha + y\sin\alpha = p$ of the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{18} = 1$ subtends a right angle at the center, and the diameter of the circle, concentric with the hyperbola, to which the given chord is a tangent is d, then the value of $\dfrac{d}{4}$ is_____

1321. If the vertex of a hyperbola bisects the distance between its center and the correspoinding focus, then the ratio of the square of its conjugate axis to the square of its transverse axis is (a) 2   (b) 4   (c) 6   (d) 3

1322. If the distance between two parallel tangents having slope m drawn to the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{49} = 1$ is 2, then the value of $2|m|$ is_____

1323. The area of triangle formed by the tangents from the point (3, 2) to the hyperbola $x^2 - 9y^2 = 9$ and the chord of contact w.r.t. the point (3, 2) is _____.

1324. If a variable line has its intercepts on the coordinate axes e and e', where $\dfrac{e}{2}$ and $\dfrac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the line always touches the circle $x^2 + y^2 = r^2$, where $r =$

(a)1 (b) 2 (c) 3 (d) cannot be decided

1325. If tangents drawn from the point $(a, 2)$ to the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$ are perpendicular, then the value of a^2 is ____

1326. If the hyperbola $x^2 - y^2 = 4$ is rotated by 45^0 in the anticlockwise direction about its center keeping the axis intact, then the equation of the hyperbola is $xy = a^2$, where a^2 is equal to_____

1327. Find the point on the hyperbola $\dfrac{x^2}{24} - \dfrac{y^2}{18} = 1$ which is nearest to the line $3x + 2y + 1 = 0$ and compute the distance between the point and the line.

1328. The number of possible tangents which can be drawn to the curve $4x^2 - 9y^2 = 36,$ which are perpendicular to the straight line $5x + 2y - 10 = 0$, is (A) 0 (B) 1 (C) 2 (D) 4

1329. If values of a, for which the line $y = ax + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of the equation $x^2 - \left(a_1 + b_1\right)x - 4 = 0$, then the values of $a_1 + b_1$ is

1330. If the angle between the asymptotes of hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is 120^0 and the product of perpendiculars drawn from the foci upon its any tangent is 9, then the locus of the point of intersection of perpendicular tangents of the hyperbola can be (a)$x^2 + y^2 = 6$ (b) $x^2 + y^2 = 9$ (c) $x^2 + y^2 = 3$ (d) $x^2 + y^2 = 18$

1331. The sides $ACandAB$ of a ABC touch the conjugate hyperbola of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. If the vertex A lies on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, then the side BC must touch parabola (b) circle hyperbola (d) ellipse

1332. The tangent at a point P on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ passes through the point $(0, -b)$ and the normal at P passes through the point $\left(2a\sqrt{2}, 0\right)$. Then the eccentricity of the hyperbola is

1333. If $ax + by = 1$ is tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then $a^2 - b^2$ is equal to (A) $\dfrac{1}{a^2 e^2}$ (B) $a^2 e^2$ (C) $b^2 e^2$ (D) none of these

1334. The locus of a point whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola $xy = 1$ is a/an (a)ellipse (b) circle (c)hyperbola (d) parabola

1335. Locus of the feet of the perpendiculars drawn from either foci on a variable tangent to the hyperbola $16y^2 - 9x^2 = 1$ is

1336. The locus of the foot of the perpendicular from the center of the hyperbola $xy = 1$ on a variable tangent is (a) $\left(x^2 + y^2\right)^2 = 4xy$ (b) $\left(x^2 - y^2\right) = \frac{1}{9}$ $\left(x^2 - y^2\right) = \frac{7}{144}$ (d) $\left(x^2 - y^2\right) = \frac{1}{16}$

1337. If the line $2x + \sqrt{6}y = 2$ touches the hyperbola $x^2 - 2y^2 = 4$, then the point of contact is

1338. Which of the following is independent of α in the hyperbola

$$\left(0 < \alpha < \frac{\pi}{2}\right). \frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$$

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1339. Consider the graphs of $y = Ax^2$ and $y^2 + 3 = x^2 + 4y$, where A is a positive constant and $x, y \in R$.Number of points in which the two graphs intersect, is

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1340. Tangents are drawn from the point (α, β) to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined at angle θ and ϕ to the x-axis. If $\tan\theta \cdot \tan\phi = 2$, then the value $2\alpha^2 - \beta^2$ is _____.

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1341. The eccentricity of the hyperbola

$$\left| \sqrt{(x-3)^2 + (y-2)^2} - \sqrt{(x+1)^2 + (y+1)^2} \right| = 1 \text{ is } \_\_\_\_\_$$

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1342. If $y = mx + c$ is tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, having eccentricity 5, then the least positive integral value of m is _____

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1343. $A(-2, 0)$ and $B(2, 0)$ are two fixed points and P 1s a point such that $PA - PB = 2$ Let S be the circle $x^2 + y^2 = r^2$, then match the following. If $r = 2$, then the number of points P satisfying $PA - PB = 2$ and lying on $x^2 + y^2 = r^2$ is

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1344. Statement 1 : Given the base BC of the triangle and the radius ratio of the excircle opposite to the angles $B and C$. Then the locus of the vertex A is a hyperbola. Statement 2 : $|SP - SP| = 2a$, where $S and S'$ are the two foci $2a$ is the length of the transvers axis, and P is any point on the hyperbola.