MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

DIFFERENT PRODUCTS OF VECTORS AND THEIR GEOMETRICAL APPLICATIONS

Exercises

1. If $\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-a)^2 \end{vmatrix} = 0$ and vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$,
where $\vec{A} = a^2\hat{i} = a\hat{j} + \hat{k}$ etc. are non-coplanar, then prove that vectors $\vec{X}$, $\vec{Y}$ and $\vec{Z}$ where $\vec{X} = x^2\hat{i} + x\hat{j} + \hat{k}$. etc.may be coplanar.

2. OABC is a tetrahedron where O is the origin and A,B,C have position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively prove that circumcentre of tetrahedron OABC is $\dfrac{a^2\left(\vec{b}\times\vec{c}\right)+b^2\left(\vec{c}\times\vec{a}\right)+c^2\left(\vec{a}\times\vec{b}\right)}{2\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$

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3. Let k be the length of any edge of a regular tetrahedron (a tetrahedron whose edges are equal in length is called a regular tetrahedron). Show that the angel between any edge and a face not containing the edge is $\cos^{-1}(1/\sqrt{3})$.

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4. In ABC, a point P is taken on AB such that $AP/BP=1/3$ and point Q is taken on BC such that $CQ/BQ=3/1$. If R is the point of intersection of the lines $AQandCP$, ising vedctor method, find the are of ABC if the area of BRC is 1 unit

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5. Let O be an interior point of ΔABC such that $\overline{OA} + 2\overline{OB} + 3\overline{OC} = 0$. Then the ratio of a ΔABC to area of ΔAOC is

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6. The lengths of two opposite edges of a tetrahedron of $aandb;$ the shortest distane between these edgesis $d,$ and the angel between them if $\theta\cdot$ Prove using vector4s that the volume of the tetrahedron is $\frac{abdisn\theta}{6}$.

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7. Find the volume of a parallelopiped having three coterminus vectors of equal magnitude $|a|$ and equal inclination θ with each other.

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8. Let $\vec{p}$ and $\vec{q}$ any two othogonal vectors of equal magnitude 4 each. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be any three vectors of lengths $7\sqrt{15}$ and $2\sqrt{33}$, mutually perpendicular to each other. Then find the distance of the vector

$$\left(\vec{a}.\vec{p}\right)\vec{p}+\left(\vec{a}.\vec{q}\right)\vec{q}+\left(\vec{a}.\left(\vec{p}\times\vec{q}\right)\right)\left(\vec{p}\times\vec{q}\right)+\left(\vec{b}.\vec{p}\right)\vec{p}+\left(\vec{b}.\left(\vec{b}.\vec{q}\right)\right)\left(\vec{p}\times\vec{q}\right)+\left(\vec{c}.\vec{p}\right)\vec{p}+\left(\vec{c}.\vec{q}\right)\vec{q}+\left(\vec{c}.\left(\vec{p}\times\vec{q}\right)\right)$$

from the origin.

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9. Given that $\vec{A}, \vec{B}, \vec{C}$ form triangle such that $\vec{A}=\vec{B}+\vec{C}$. Find a,b,c,d such that area of the triangle is $5\sqrt{6}$ where $\vec{A}=a\vec{i}+b\vec{i}+c\vec{k}.\vec{B}=d\vec{i}+3\vec{j}+4\vec{k}$ and $\vec{C}=3\vec{i}+\vec{j}-2\vec{k}$

.

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10. A line l is passing through the point $\vec{b}$ and is parallel to vector $\vec{c}$. Determine the distance of point A$\left(\vec{a}\right)$ from the line l in from $\left|\vec{b}-\vec{a}+\frac{\left(\vec{a}-\vec{b}\right)\vec{c}}{\left|\vec{c}\right|^2}\vec{c}\right|$ or $\frac{\left|\left(\vec{b}-\vec{a}\right)\times\vec{c}\right|}{\left|\vec{c}\right|}$

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11. If $\vec{e}_1, \vec{e}_2, \vec{e}_3 and \vec{E}_1, \vec{E}_2, \vec{E}_3$ are two sets of vectors such that $\vec{e}_i \dot{\vec{E}}_j = 1$, if $i = j and \vec{e}_i \dot{\vec{E}}_j = 0 and$ if $i \ne j$, then prove that $\left[\vec{e}_1 \vec{e}_2 \vec{e}_3\right]\left[\vec{E}_1 \vec{E}_2 \vec{E}_3\right] = 1.$

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12. In a quadrilateral ABCD, it is given that $AB \mid \mid CD$ and the diagonals AC and BD are perpendicular to each other. Show that $AD.\,BC \geq AB.\,CD.$

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13. $OABC$ is regular tetrahedron in which D is the circumcentre of OAB and E is the midpoint of edge AC. Prove that DE is equal to half the edge of tetrahedron.

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14. If $A\left(\vec{a}\right), B\left(\vec{b}\right) and C\left(\vec{c}\right)$ are three non-collinear points and origin does not lie in the plane of the points $A, B and C$, then point $P\left(\vec{p}\right)$ in the plane of the ABC such that vector $\vec{O}P$ is $\perp$ to planeof ABC, show that $\vec{O}P = \frac{\left[\vec{a}\vec{b}\vec{c}\right]\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)}{4^2}, where$ is the area of the ABC.

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15. If $\vec{a}, \vec{b}, \vec{c}$ are three given non-coplanar vectors and any arbitrary vector $\vec{r}$ in space, where

$$\Delta_1 = \begin{vmatrix} \vec{r}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{r}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{r}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}, \Delta_2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{r}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{r}.\vec{b} & \vec{c}.\vec{b} \\ \vec{a}.\vec{c} & \vec{r}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{r}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{r}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{r}.\vec{c} \end{vmatrix}, \Delta = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix},$$

then prove that $\vec{r} = \frac{\Delta_1}{\Delta}\vec{a} + \frac{\Delta_2}{\Delta}\vec{b} + \frac{\Delta_3}{\Delta}\vec{c}$

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Exercises MCQ

1. Two vectors in space are equal only if they have equal component in a. a given direction b. two given directions c. three given directions d. in any arbitrary direction

A. a given direction

B. two given directions

C. three given direction

D. in any arbitrary direaction

Answer: c

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2. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the three vectors having magnitudes, 1,5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \vec{c}$. Then $\tan\theta$ is equal to

A. 0

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{3}{4}$

Answer: d

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3. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors of equal magnitude such that the angle between each pair is $\frac{\pi}{3}$. If $\left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{6}$, then $\left|\vec{a}\right| =$

A. 2

B. -1

C. 1

D. $\sqrt{6}/3$

Answer: c

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4. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors, then the vector which is equally inclined to these vectors is (A) $\vec{a} + \vec{b} + \vec{c}$ (B) $\frac{\vec{a}}{\left|\vec{a}\right|} + \frac{\vec{b}}{\left|\vec{b}\right|} + \vec{/}\left|\vec{c}\right|$ (C) $\frac{\vec{a}}{\left|\vec{a}\right|^2} + \frac{\vec{b}}{\left|\vec{b}\right|^2} + \frac{\vec{c}}{\left|\vec{c}\right|^2}$ (D) $\left|\vec{a}\right|\vec{a} - \left|\vec{b}\right|\vec{b} + \left|\vec{c}\right|\vec{c}$

A. $\vec{a} + \vec{b} + \vec{c}$

B. $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} + \frac{\vec{c}}{|\vec{c}|}$

C. $\frac{\vec{a}}{|\vec{a}|^2} + \frac{\vec{b}}{|\vec{b}|^2} + \frac{\vec{c}}{|\vec{c}|^2}$

D. $|\vec{a}|\vec{a} - |\vec{b}|\vec{b} + |\vec{c}|\vec{c}$

Answer: b

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5. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. Then the point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is (A) $(3, -1, 10$ (B) $(3, 1, -1)$ (C) $(-3, 1, 1)$ (D) $(-3, -1, -1)$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $3\hat{i} - \hat{j} + \hat{k}$

C. $3\hat{i} + \hat{j} - \hat{k}$

D. $\hat{i} - \hat{j} - \hat{k}$

Answer: c

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6. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\vec{a}.\vec{b} < 0$ and $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ then the angle between the vectors $\vec{a}$ and $\vec{b}$ is (a) π (b) $\frac{7\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

A. π

B. $7\pi/4$

C. $\pi/4$

D. $3\pi/4$

Answer: d

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7. If $\widehat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors such that $\widehat{a} + \hat{b} + \hat{c}$ is also a unit vector and θ_1, θ_2 and θ_3 are angles between the vectors $\widehat{a}, \hat{b}, \hat{b}, \hat{c}$ and $\hat{c}, \widehat{a},$ respectively m then among θ_1, θ_2 and θ_3

A. all are acute angles

B. all are right angles

C. at least one is obtuse angle

D. none of these

Answer: c

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8. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = 0 = \vec{a}.\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}\, is \frac{\pi}{3}$, then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$

A. $1/2$

B. 1

C. 2

D. none of these

Answer: b

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9. P $\left(\vec{p}\right)$ and $Q\left(\vec{q}\right)$ are the position vectors of two fixed points and $R\left(\vec{r}\right)$ is the postion vector of a variable point. If R moves such that $\left(\vec{r}-\vec{p}\right)\times\left(\vec{r}-\vec{q}\right)=\vec{0}$ then the locus of R is

A. a plane containing the origian O and parallel to two non-collinear vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$

B. the surface of a sphere described on PQ as its diameter

C. a line passing through points P and Q

D. a set of lines parallel to line PQ

Answer: c

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10. Two adjacent sides of a parallelogram ABCD are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Then the value of $\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$ is

A. $20\sqrt{5}$

B. $22\sqrt{5}$

C. $24\sqrt{5}$

D. $26\sqrt{5}$

Answer: b

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11. If $\widehat{a}, \hat{b}, and \hat{c}$ are three unit vectors inclined to each other at angle θ, then the maximum value of θ is $\frac{\pi}{3}$ b. $\frac{\pi}{4}$ c. $\frac{2\pi}{3}$ d. $\frac{5\pi}{6}$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{5}$

Answer: c

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12. Let the pair of vector $\vec{a}, \vec{b}$ and $\vec{c}, \vec{c}\,d$ each determine a plane. Then the planes are parallel if

A. $\left(\vec{a} \times \vec{c}\right) \times \left(\vec{b} \times \vec{d}\right) = \vec{0}$

B. $\left(\vec{a} \times \vec{c}\right).\left(\vec{b} \times \vec{d}\right) = \vec{0}$

C. $\left(\vec{a} \times \vec{c}\right) \times \left(\vec{c} \times \vec{d}\right) = \vec{0}$

D. $\left(\vec{a} \times \vec{c}\right).\left(\vec{c} \times \vec{d}\right) = \vec{0}$

Answer: c

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13. If $\vec{r}.\vec{a}=\vec{r}.\vec{b}=\vec{r}.\vec{c}=0$ where $\vec{a},\vec{b}$ and $\vec{c}$ are non-coplanar, then

A. $\vec{r}\perp\left(\vec{c}\times\vec{a}\right)$

B. $\vec{r}\perp\left(\vec{a}\times\vec{b}\right)$

C. $\vec{r}\perp\left(\vec{b}\times\vec{c}\right)$

D. $\vec{r}=\vec{0}$

Answer: d

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14. If $\vec{a}$ satisfies $\vec{a}\times\left(\hat{i}+2\hat{j}+\hat{k}\right)=\hat{i}-\hat{k}$ then $\vec{a}$ is equal to

A. a) $\lambda\hat{i}+(2\lambda-1)\hat{j}+\lambda\hat{k},\lambda\in R$

B. b) $\lambda\hat{i}+(1-2\lambda)\hat{j}+\lambda\hat{k},\lambda\in R$

C. c) $\lambda\hat{i} + (2\lambda + 1)\hat{j} + \lambda\hat{k}, \lambda \in R$

D. d) $\lambda\hat{i} + (1 + 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

Answer: c

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15. Vectors $3\vec{a} - 5\vec{b}$ and $2\vec{a} + \vec{b}$ are mutually perpendicular. If $\vec{a} + 4\vec{b}$ and $\vec{b} - \vec{a}$ are also mutually perpendicular, then the cosine of the angle between $\vec{a}$ and $\vec{b}$ is (a) $\frac{19}{5\sqrt{43}}$ (b) $\frac{19}{3\sqrt{43}}$ (c) $\frac{19}{\sqrt{45}}$ (d) $\frac{19}{6\sqrt{43}}$

A. $\frac{19}{5\sqrt{43}}$

B. $\frac{19}{3\sqrt{43}}$

C. $\frac{19}{\sqrt{45}}$

D. $\frac{19}{6\sqrt{43}}$

Answer: a

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16. The units vectors orthogonal to the vector $-\hat{i}+2\hat{j}+2\hat{k}$ and making equal angles with the X and Y axes islare) :

A. $\pm\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

B. $\frac{19}{5\sqrt{43}}$

C. $\pm\frac{1}{3}\left(\hat{i}+\hat{j}-\hat{k}\right)$

D. none of these

Answer: a

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17. The value of x for which the angle between $\overrightarrow{a}=2x^2\hat{i}+4x\hat{j}=\hat{k}+\hat{k}$ and $\overrightarrow{b}=7\hat{i}-2\hat{j}=x\hat{k}$, is obtuse and the angle between $\overrightarrow{b}$ and the z-axis is acute and less than $\pi/6$, are

A. $a<x<1/2$

B. $1/2 < x < 15$

C. $x < 1/2 \text{ or } x < 0$

D. none of these

Answer: d

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18. If vectors $\vec{a}$ and $\vec{b}$ are two adjacent sides of parallelograsm then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$ is (A) $\vec{b} + \frac{\vec{b} \times \vec{a}}{\left|\vec{a}\right|^2}$ (B) $\frac{\vec{a} . \vec{b}}{\vec{b}\Big|^2}$ (C) $\vec{b} - \frac{\vec{b} . \vec{a}}{\left(\left|\vec{a}\right|\right)^2}\Bigg)$

(D) $\frac{\vec{a} \times \left(\vec{b} \times \vec{a}\right)}{\vec{b}\Big|^{20}}$

A. $\vec{b} + \frac{\vec{b} \times \vec{a}}{\left|\vec{a}\right|^2}$

B. $\frac{\vec{a} . \vec{b}}{\left|\vec{b}\right|^2}$

C. $\vec{b} - \frac{\vec{b} \cdot \vec{a}}{\left|\vec{a}\right|^2}\vec{a}$

D. $\frac{\vec{a} \times \left(\vec{b} \times \vec{a}\right)}{\left|\vec{b}\right|^2}$

Answer: c

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19. A parallelogram is constructed on $3\vec{a} + \vec{b}$ and $\vec{a} - 4\vec{b}$, $where\left|\vec{a}\right| = 6$ and $\left|\vec{b}\right| = 8$ and $\vec{a}$ and $\vec{b}$ are anti parallel then the length of the longer diagonal is (A) 40 (B) 64 (C) 32 (D) 48

A. 40

B. 64

C. 32

D. 48

Answer: c

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20. Let $\vec{a}.\vec{b}=0$ where $\vec{a}$ and $\vec{b}$ are unit vectors and the vector $\vec{c}$ is inclined an anlge θ to both $\vec{a}$ and $\vec{b}$. $If \vec{c}=m\vec{a}+n\vec{b}+p\left(\vec{a}\times\vec{b}\right)$, $(m,n,p\in R)$ then

A. $\frac{\pi}{4}\leq\theta\leq\frac{\pi}{4}$

B. $\frac{\pi}{4}\leq\theta\leq\frac{3\pi}{4}$

C. $0\leq\theta\leq\frac{\pi}{4}$

D. $0\leq\theta\leq\frac{3\pi}{4}$

Answer: a

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21. $\vec{a}$ and $\vec{c}$ are unit vectors and $\left|\vec{b}\right| = 4$ the angle between $\vec{a}$ and $\vec{c}$ $is \cos^{-1}(1/4)$ and $\vec{b} - 2\vec{c} = \lambda\vec{a}$ the value of λ is

A. 3,-4

B. 1/4,3/4

C. $-3, 4$

D. $-1/4, \frac{3}{4}$

Answer: a

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22. Let the position vectors of the points $PandQ$ be $4\hat{i} + \hat{j} + \lambda\hat{k} and 2\hat{i} - \hat{j} + \lambda\hat{k},$ respectively. Vector $\hat{i} - \hat{j} + 6\hat{k}$ is perpendicular to the plane containing the origin and the points $PandQ$. Then λ equals a$-1/2$ b. $1/2$ c. 1 d. none of these

A. $-1/2$

B. $1/2$

C. 1

D. none of these

Answer: a

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23. A vector of magnitude $\sqrt{2}$ coplanar with the vectors $\overrightarrow{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = \hat{i} + \hat{j} + \hat{k}$, and perpendicular to the vector $\overrightarrow{c} = \hat{i} + \hat{j} + \hat{k}$ is

A. $-\hat{j} + \hat{k}$

B. $\hat{i}$ and $\hat{k}$

C. $\hat{i} - \hat{k}$

D. hati- hatj`

Answer: a

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24. Let P be a point interior to the acute triangle ABC. If $PA + PB + PC$ is a null vector, then w.r.t traingel ABC, point P is its

a. centroid b. orthocentre c. incentre d. circumcentre

A. centroid

B. orthocentre

C. incentre

D. circumcentre

Answer: a

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25. G is the centroid of triangle ABC and A_1 and B_1 are the midpoints of sides AB and AC, respectively. If Δ_1 is the area of quadrilateral GA_1AB_1 and Δ is the area of triangle ABC, then $\frac{\Delta}{\Delta_1}$ is equal to

A. $\frac{3}{2}$

B. 3

C. $\frac{1}{3}$

D. none of these

Answer: b

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26. Points $\vec{a}, \vec{b}\,\vec{c}$ and $\vec{d}$ are coplanar and $(\sin\alpha)\vec{a} + (2\sin 2\beta)\vec{b} + (3\sin 3\gamma)\vec{c} - \vec{d} = \vec{0}$. Then the least value of $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma$ is

A. $1/14$

B. 14

C. 6

D. $1/\sqrt{6}$

Answer: a

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27. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 1and 2. respectively, and $\left(1-3\vec{a}.\vec{b}\right)^2+\left|2\vec{a}+\vec{b}+3\left(\vec{a}\times\vec{b}\right)\right|^2=47$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\pi/3$

B. $\pi-\cos^{-1}(1/4)$

C. $\frac{2\pi}{3}$

D. $\cos^{-1}(1/4)$

Answer: c

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28. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitude 2 and 3 respectively such that $\left|2\left(\vec{a}\times\vec{b}\right)\right|+\left|3\left(\vec{a}.\vec{b}\right)\right|=k$ then the maximum value of k is (a) $\sqrt{13}$ (b) $2\sqrt{13}$ (c) $6\sqrt{13}$ (d) $10\sqrt{13}$

A. $\sqrt{13}$

B. $2\sqrt{13}$

C. $6\sqrt{13}$

D. $10\sqrt{13}$

Answer: c

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29. $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vecrtors such that $\left|\vec{a}+\vec{b}+3\vec{c}\right|=4$ Angle between $\vec{a}$ and $\vec{b}\ is\theta_1$, between $\vec{b}$ and $\vec{c}\ is\theta_2$ and between $\vec{a}$ and $\vec{b}$ varies $[\pi/6, 2\pi/3]$. Then the maximum value of $\cos\theta_1+3\cos\theta_2$ is

A. 3

B. 4

C. $2\sqrt{2}$

D. 6

Answer: b

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30. If the vector product of a constant vector $\vec{O}A$ with a variable vector $\vec{O}B$ in a fixed plane OAB be a constant vector, then the locus of B is (a).a straight line perpendicular to $\vec{O}A$ (b). a circle with centre O and radius equal to $\left|\vec{O}A\right|$ (c). a straight line parallel to $\vec{O}A$ (d). none of these

A. a straight line perpendicular to $\overrightarrow{OA}$

B. a circle with centre O and radius equal to $\left|\overrightarrow{OA}\right|$

C. a striaght line parallel to $\overrightarrow{OA}$

D. none of these

Answer: c

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31. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2$ and $|\vec{w}| = 3$ if the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v}$ and $\vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals

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32. If the two adjacent sides of two rectangles are reprresented by vectors

$\vec{p} = 5\vec{a} - 3\vec{b}, \vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}, \vec{s} = -\vec{a} +$

, respectively, then the angle between the vectors $\vec{x} = \frac{1}{3}(\vec{p} + \vec{r} + \vec{s})$ and $\vec{y} = \frac{1}{5}(\vec{r} + \vec{s})$ is

A. $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

B. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

C. $\pi \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

D. cannot of these

Answer: b

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33. If $\vec{\alpha} \mid \mid \left(\vec{b} \times \vec{\gamma}\right), then\left(\vec{\alpha} \times \vec{\beta}\right).\left(\vec{\alpha} \times \vec{\gamma}\right) =$ (A) $\left|\vec{\alpha}\right|^2\left(\vec{\beta}.\vec{\gamma}\right)$ (B) $\left|\vec{\beta}\right|^2\left(\vec{\gamma}.\vec{\alpha}\right)$ (C) $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}.\vec{\beta}\right)$ (D) $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

A. $\left|\vec{\alpha}\right|^2\left(\vec{\beta}.\vec{\gamma}\right)$

B. $\left|\vec{\beta}\right|^2\left(\vec{\gamma}.\vec{\alpha}\right)$

C. $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}.\vec{\beta}\right)$

D. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

Answer: a

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34. The position vectors of points A,B and C are $\hat{i}+\hat{j}, \hat{i}+5\hat{j}-\hat{k}$ and $2\hat{i}+3\hat{j}+5\hat{k}$, respectively the greatest angle of triangle ABC is

A. 120°

B. 90°

C. $\cos^{-1}(3/4)$

D. none of these

Answer: b

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35. Given three vectors $\vec{a}, \vec{b}, and\, \vec{c}$ two of which are non-collinear. Further if $\left(\vec{a}+\vec{b}\right)$ is collinear with $\vec{c}$, $\left(\vec{b}+\vec{c}\right)$ is collinear with $\vec{a}$, $\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{c}\right|=\sqrt{2}$. Find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

a. 3 b. -3 c. 0 d. cannot be evaluated

A. 3

B. -3

C. 0

D. cannot of these

Answer: b

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36. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left(\vec{a}+\vec{b}\right).\left(2\vec{a}+3\vec{b}\right)\times\left(3\vec{a}-2\vec{b}\right)=\vec{0}$ then angle between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. π

D. indeterminate

Answer: d

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37. If in a right-angled triangle ABC, the hypotenuse AB = p , then $\overrightarrow{AB}.\overrightarrow{AC}+\overrightarrow{BC}.\overrightarrow{BA}+\overrightarrow{CA}.\overrightarrow{CB}$is equal to

A. $2p^2$

B. $\frac{p^2}{2}$

C. p^2

D. none of these

Answer: c

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38. Resolved part of vector $\vec{a}$ and along vector $\vec{b}$ is $\vec{a}1$ and that prependicular to $\vec{b}$ is $\vec{a}2$ then $\vec{a}1\times\vec{a}2$ is equl to

A. $\dfrac{\left(\vec{a} \times \vec{b}\right) . \vec{b}}{\left|\vec{b}\right|^2}$

B. $\dfrac{\left(\vec{a} . \vec{b}\right)\vec{a}}{\left|\vec{a}\right|^2}$

C. $\dfrac{\left(\vec{a} . \vec{b}\right)\left(\vec{b} \times \vec{a}\right)}{\left|\vec{b}\right|^2}$

D. $\dfrac{\left(\vec{a} . \vec{b}\right)\left(\vec{b} \times \vec{a}\right)}{\left|\vec{b} \times \vec{a}\right|}$

Answer: c

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39. Let $\vec{a} = 2\hat{i} = \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors . A vector in the pland of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\left(\frac{2}{3}\right)}$ is (A) $2\hat{i} + 3\hat{j} + 3\hat{k}$ (B) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (C) $-2\hat{i} - \hat{j} + 5\hat{k}$ (D) $2\hat{i} + \hat{j} + 5\hat{k}$

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $-2\hat{i} - \hat{j} + 5\hat{k}$

C. $2\hat{i} + 3\hat{j} + 3\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: b

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40. If P is any arbitrary point on the circumcirlce of the equllateral trangle of side length l units, then $\left|\overrightarrow{P}A\right|^2 + \left|\overrightarrow{P}B\right|^2 + \left|\overrightarrow{P}C\right|^2$ is always equal to $2l^2$ b. $2\sqrt{3}l^2$ c. l^2 d. $3l^2$

A. $2l^2$

B. $2\sqrt{3}l^2$

C. l^2

D. $3l^2$

Answer: a

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41. If $\vec{r}$ and $\vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $\left|\vec{r}+b\vec{s}\right|$ is minimum, then the value of $\left|b\vec{s}\right|^2+\left|\vec{r}+b\vec{s}\right|^2$ is equal to

A. $2\left|\vec{r}\right|^2$

B. $\left|\vec{r}\right|^2/2$

C. $3\left|\vec{r}\right|^2$

D. $\left|\vec{r}\right|^2$

Answer: b

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42. $\vec{a}$ and $\vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that if equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{a}\times\vec{b}$ is equal to

A. $\frac{1}{\sqrt{2}}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

B. $\frac{1}{2}\left(\vec{a}\times\vec{b}+\vec{a}+\vec{b}\right)$

C. $\frac{1}{\sqrt{3}}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

D. $\frac{1}{3}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

Answer: a

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43. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a}+\vec{b}=\mu\vec{p}, \vec{b}.\vec{q}=0$ and $\left|\vec{b}\right|^2=1$ where μ is a sclar. Then $\left|\left(\vec{a}.\vec{q}\right)\vec{p}-\left(\vec{p}.\vec{q}\right)\vec{a}\right|$ is equal to

(a)$2\left|\vec{p}\,\vec{q}\right|$ (b)$(1/2)\left|\vec{p}.\vec{q}\right|$ (c)$\left|\vec{p}\times\vec{q}\right|$ (d)$\left|\vec{p}.\vec{q}\right|$

A. $2\left|\vec{p}\,\vec{q}\right|$

B. $(1/2)\left|\vec{p}\,.\,\vec{q}\right|$

C. $\left|\vec{p}\times\vec{q}\right|$

D. $\left|\vec{p}\,.\,\vec{q}\right|$

Answer: d

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44. The position vectors of the vertices A, B and C of a triangle are three unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. A vector $\vec{d}$ is such that $\vec{d}\,.\,\hat{a}=\vec{d}\,.\,\hat{b}=\vec{d}\,.\,\hat{c}$ and $\vec{d}=\lambda\left(\hat{b}+\hat{c}\right)$. Then triangle ABC is

A. acute angled

B. obtuse angled

C. right angled

D. none of these

Answer: a

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45. If a is real constant $A, BandC$ are variable angles and $\sqrt{a^2-4}\tan A+a\tan B+\sqrt{a^2+4}\tan c=6a,$ then the least vale of $\tan^2 A+\tan^2 b+\tan^2 Cis$ 6 b. 10 c. 12 d. 3

A. 6

B. 10

C. 12

D. 3

Answer: d

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46. The vertex A triangle ABC is on the line $\overrightarrow{r} = \hat{i} + \hat{j} + \lambda\hat{k}$ and the vertices $BandC$ have respective position vectors $\hat{i}and\hat{j}$. Let Delta be the area of the triangle and $\mathrm{Delta}\left[3/2, \sqrt{33}/2\right]$. Then the range of values of λ corresponding to A is $[-8, 4] \cup [4, 8]$ b. $[-4, 4]$ c. $[-2, 2]$ d. $[-4, -2] \cup [2, 4]$

A. [-8, -4]cup[4,8]`

B. $[-4, 4]$

C. [-2,2]

D. $[-4, -2] \cup [2, 4]$

Answer: c

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47. A non-zero vecto $\overrightarrow{a}$ is such tha its projections along vectors $\frac{\hat{i} + \hat{j}}{\sqrt{2}}, \frac{-\hat{i} + \hat{j}}{\sqrt{2}}$ and $\hat{k}$ are equal , then unit vector along $\overrightarrow{a}$ us

A. $\dfrac{\sqrt{2}\hat{j}-\hat{k}}{\sqrt{3}}$

B. $\dfrac{\hat{j}-\sqrt{2}\hat{k}}{\sqrt{3}}$

C. $\dfrac{\sqrt{2}}{\sqrt{3}}\hat{j}+\dfrac{\hat{k}}{\sqrt{3}}$

D. $\dfrac{\hat{j}-\hat{k}}{\sqrt{2}}$

Answer: a

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48. Position vector $\hat{k}$ is rotated about the origin by angle 135^0 in such a way that the plane made by it bisects the angel between $\hat{i} and \hat{j}$. Then its new position is $\pm\dfrac{\hat{i}}{\sqrt{2}}\pm\dfrac{\hat{j}}{\sqrt{2}}$ b. $\pm\dfrac{\hat{i}}{2}\pm\dfrac{\hat{j}}{2}-\dfrac{\hat{k}}{\sqrt{2}}$ c. $\dfrac{\hat{i}}{\sqrt{2}}-\dfrac{\hat{k}}{\sqrt{2}}$ d. none of these

A. $\pm\dfrac{\hat{i}}{\sqrt{2}}\pm\dfrac{\hat{j}}{\sqrt{2}}$

B. $\pm\dfrac{\hat{i}}{2}\pm\dfrac{\hat{j}}{2}-\dfrac{\hat{k}}{\sqrt{2}}$

C. $\dfrac{\hat{i}}{\sqrt{2}}-\dfrac{\hat{k}}{\sqrt{2}}$

D. none of these

Answer: d

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49. In a quadrilateral $ABCD$, $\vec{A}C$ is the bisector of $\vec{A}BandA\vec{A}D$, angle between $\vec{A}BandA\vec{A}D$ is $2\pi/3$, $15\left|\vec{A}C\right| = 3\left|\vec{A}B\right| = 5\left|\vec{A}D\right|$. Then the angle between $\vec{B}AandC\vec{C}D$ is $\frac{\cos^{-1}\left(\sqrt{14}\right)}{7\sqrt{2}}$ b. $\frac{\cos^{-1}\left(\sqrt{21}\right)}{7\sqrt{3}}$ c. $\frac{\cos^{-1}2}{\sqrt{7}}$ d. $\frac{\cos^{-1}\left(2\sqrt{7}\right)}{14}$

A. $\cos^{-1}\frac{\sqrt{14}}{7\sqrt{2}}$

B. $\cos^{-1}\frac{\sqrt{21}}{7\sqrt{3}}$

C. $\cos^{-1}\frac{2}{\sqrt{7}}$

D. $\cos^{-1}\frac{2\sqrt{7}}{14}$

Answer: c

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50. In AB, DE and GF are parallel to each other and AD, BG and EF ar parallel to each other . If CD: CE = CG:CB = 2:1 then the value of area $(\triangle AEG): area(\triangle ABD)$ is equal to (a) 7/2 (b)3 (c)4 (d)9/2

A. $7/2$

B. 3

C. 4

D. $9/2$

Answer: b

51. Vectors $\hat{a}$ in the plane of $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ is such that it is equally inclined to $\vec{b}$ and $\vec{d}$ where $\vec{d} = \hat{j} + 2\hat{k}$ the value of $\hat{a}$ is (a) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (b) $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ (c) $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$ (d) $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$

A. $\dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

B. $\dfrac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

C. $\dfrac{2\hat{i}+\hat{j}}{\sqrt{5}}$

D. $\dfrac{2\hat{i}+\hat{j}}{\sqrt{5}}$

Answer: b

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52. Let $ABCD$ be a tetrahedron such that the edges $AB, ACandAD$ are mutually perpendicular. Let the area of triangles $ABC, ACDandADB$ be 3, 4 and 5sq. units, respectively. Then the area of triangle BCD is a. $5\sqrt{2}$ b. 5 c. $\dfrac{\sqrt{5}}{2}$ d. $\dfrac{5}{2}$

A. $5\sqrt{2}$

B. 5

C. $\dfrac{\sqrt{5}}{2}$

D. $\dfrac{5}{2}$

Answer: a

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53. Let $\overrightarrow{f(t)} = [t]\hat{i} + (t - [t])\hat{j} + [t+1]\hat{k}, where[.]$ denotes the greatest integer function. Then the vectors $\overrightarrow{f\left(\frac{5}{4}\right)}$ and $\overrightarrow{f(t)}$,$0 < t < 1$ are (a)parallel to each other (b)perpendicular to each other (c)inclined at $\cos^{-1}\left(\frac{2}{\sqrt{7(1-t^2)}}\right)$ (d)inclined at $\cos^{-1}\left(\frac{8+t}{9\cdot\sqrt{1+t^2}}\right)$

A. parallel to each other

B. perpendicular to each other

C. inclined at $\frac{\cos^{-1}2}{\sqrt{7}(1-t^2)}$

D. inclined at $\frac{\cos^{-1}(8+t)}{9\sqrt{1+t^2}}$

Answer: d

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54. If $\vec{a}$ is parallel to $\vec{b} \times \vec{c}$, $then\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$ is equal to

(a) $\left|\vec{a}\right|^2\left(\vec{b}.\vec{c}\right)$ (b) $\left|\vec{b}\right|^2\left(\vec{a}.\vec{c}\right)$ (c) $\left|\vec{c}\right|^2\left(\vec{a}.\vec{b}\right)$ (d) none of these

A. $\left|\vec{a}\right|^2\left(\vec{b}.\vec{c}\right)$

B. $\left|\vec{b}\right|^2\left(\vec{a}.\vec{c}\right)$

C. $\left|\vec{c}\right|^2\left(\vec{a}.\vec{b}\right)$

D. none of these

Answer: a

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55. The three vectors $\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume: ______________

A. $1/3$

B. 4

C. $(3\sqrt{3})/4$

D. $4\sqrt{3}$

Answer: d

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56. If $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is a on zero vector and $\left|\left(\vec{d}.\vec{c}\right)\left(\vec{a}\times\vec{b}\right)+\left(\vec{d}.\vec{a}\right)\left(\vec{b}\times\vec{c}\right)+\left(\vec{d}.\vec{b}\right)\left(\vec{c}\times\vec{a}\right)\right|=0$ then (A) $\left|\vec{a}\right|+\left|\vec{b}\right|+\left|\vec{c}\right|=\left|\vec{d}\right|$ (B) $\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{c}\right|$ (C) $\vec{a},\vec{b},\vec{c}$ are coplanar (D) $\vec{a}+\vec{c}=2\vec{b}$

A. $\left|\vec{a}\right|=\left|\vec{b}\right|=\left|\vec{c}\right|$

B. $\left|\vec{a}\right|+\left|\vec{b}\right|+\left|\vec{c}\right|=\left|\vec{d}\right|$

C. $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar

D. none of these

Answer: c

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57. If $\left|\vec{a}\right| = 2$ and $\left|\vec{b}\right| = 3$ and $\vec{a}\,.\,\vec{b} = 0$, then $\left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times\right.\right.\right.\right.$

is equal to the given diagonal is $\vec{c} = 4\hat{k} = 8\hat{k}$ then , the volume of a parallelpiped is

A. $48\hat{b}$

B. $-48\hat{b}$

C. $48\hat{a}$

D. $-48\hat{a}$

Answer: a

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58. If two diagonals of one of its faces are $6\hat{i} + 6\hat{k}$ and $4\hat{j} + 2\hat{k}$ and of the edges not containing the given diagonals is $\vec{c} = 4\hat{j} - 8\hat{k}$, then the volume of a parallelpiped is

A. 60

B. 80

C. 100

D. 120

Answer: d

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59. The volume of a tetrahedron fomed by the coterminus edges $\vec{a}, \vec{b}$ and $\vec{c} is 3$. Then the volume of the parallelepiped formed by the coterminus edges $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ is

A. 6

B. 18

C. 36

D. 9

Answer: c

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60. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually orthogonal unit vectors , then the triple product $\left[\vec{a}+\vec{b}+\vec{c} \quad \vec{a}+\vec{b} \quad \vec{b}+\vec{c}\right]$ equals

A. 0

B. 1 or -1

C. 1

D. 3

Answer: b

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61. vector $\vec{c}$ are perpendicular to vectors $\vec{a}=(2,-3,1)$ and $\vec{b}=(1,-2,3)$ and satifies the condition $\vec{c}.\left(\hat{i}+2\hat{j}-7\hat{k}\right)=10$ then vector $\vec{c}$ is equal to

$(a)(7,5,1)$ $(b)(-7,-5,-1)$ $(c)(1,1,-1)$ (d) none of these

A. 7,5,1

B. (-7, -5, -1)

C. 1,1,-1

D. none of these

Answer: a

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62. Given $\vec{a}=x\hat{i}+y\hat{j}+2\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}, \vec{a}\perp\vec{b}, \vec{a}.\vec{c}=4$ then find the value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$.

A. $\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2 = \left|\vec{a}\right|$

B. $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \left|\vec{a}\right|$

C. $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = 0$

D. $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = 0$

Answer: d

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63. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ gve three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}\ is \frac{\pi}{6}$, then prove that $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} p = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

A. 0

B. 1

C. $\frac{1}{4}\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)$

D. $\frac{3}{4}\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)$

Answer: c

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64. Let $\vec{r}, \vec{a}, \vec{b}$ and $\vec{c}$ be four non -zero vectors such that $\vec{r}.\vec{a}-0, \left|\vec{r}\times\vec{b}\right|=\left|\vec{r}\right|\left|\vec{b}\right|$ and $\left|\vec{r}\times\vec{c}\right|=\left|\vec{r}\right|\vec{c}\right|$ then [a b c] is equal to

A. |a||b||c|

B. $-|a||b||c|$

C. 0

D. none of these

Answer: c

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65. If $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 1, \vec{c} = \lambda\left(\vec{a} \times \vec{b}\right)$, angle between $\vec{c}$ and $\vec{b}$ is $2\pi/3, \left|\vec{a}\right| = \sqrt{2}, \left|\vec{b}\right| = \sqrt{3}$ and $\left|\vec{c}\right| = \frac{1}{\sqrt{3}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: b

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66. If $4\vec{a} + 5\vec{b} + 9\vec{c} = 0$ then $\left(\vec{a} \times \vec{b}\right) \times \left[\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\right]$ is equal to

A. a vector perpendicular to the plane of $\vec{a}, \vec{b}$ and $\vec{c}$

B. a scalar quantity

C. $\vec{0}$

D. none of these

Answer: c

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67. value of $\left[\vec{a} \times \vec{b}\,\vec{a} \times \vec{c}\,\vec{d}\right]$ is always equal to

A. $\left(\vec{a} . \vec{d}\right)\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

B. `(veca.vecc)[veca vecb vecd]

C. $\left(\vec{a} . \vec{b}\right)\left[\vec{a}\,\vec{b}\,\vec{d}\right]$

D. none of these

Answer: a

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68. Let $\hat{a}$ and $\hat{b}$ be mutually perpendicular unit vectors. Then for ant arbitrary $\vec{r}$.

A. $\vec{r}=\left(\vec{r}.\hat{a}\right)\hat{a}+\left(\vec{r}.\hat{b}\right)\hat{b}+\left(\vec{r}.\left(\vec{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$

B. $\vec{r}=\left(\vec{r}.\hat{a}\right)-\left(\vec{r}.\hat{b}\right)\hat{b}-\left(\vec{r}.\left(\vec{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$

C. $\vec{r}=\left(\vec{r}.\hat{a}\right)\hat{a}-\left(\vec{r}.\hat{b}\right)\hat{b}-\left(\vec{r}.\left(\vec{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$

D. none of these

Answer: a

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69. Let $\vec{a}$ and $\vec{b}$ be unit vectors that are perpendicular to each other, then $\left[\vec{a}+\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{b}\right)\right]$ is equal to

A. 1

B. 0

C. -1

D. none of these

Answer: a

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70. $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 4$ and $\vec{a}$. Vecb $= 2. If$vecc $= \left(2\vec{a} \times \vec{b}\right) - 3\vec{b}$ then find angle between $\vec{b}$ and $\vec{c}$.

A. A$\frac{\pi}{3}$

B. B$\frac{\pi}{6}$

C. C$\frac{3\pi}{4}$

D. D$\frac{5\pi}{6}$

Answer: d

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71. If $\vec{b}$ and $\vec{c}$ are unit vectors, then for any arbitary vector $\vec{a}$, $\left(\left(\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)\right)\times\left(\vec{b}\times\vec{c}\right)\right).\left(\vec{b}-\vec{c}\right)$ is always equal to

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72. If $\vec{a}.\vec{b}=\beta$ and $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}$ is

A. $\dfrac{\left(\beta\vec{a}-\vec{a}\times\vec{c}\right)}{\left|\vec{a}\right|^2}$

B. $\dfrac{\left(\beta\vec{a}+\vec{a}\times\vec{c}\right)}{\left|\vec{a}\right|^2}$

C. $\dfrac{\left(\beta\vec{c}+\vec{a}\times\vec{c}\right)}{\left|\vec{a}\right|^2}$

D. $\dfrac{\left(\beta\vec{c}+\vec{a}\times\vec{c}\right)}{\left|\vec{a}\right|^2}$

Answer: a

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73. If $a\left(\overrightarrow{\alpha}\times\overrightarrow{\beta}\right)=b\left(\overrightarrow{\beta}\times\overrightarrow{\gamma}\right)+c\left(\overrightarrow{\gamma}\times\overrightarrow{\alpha}\right)=\overrightarrow{0}$ and at least one of a,b and c is non zero then vectors $\overrightarrow{\alpha},\overrightarrow{\beta},\overrightarrow{\gamma}$ are (A) parallel (B) coplanar (C) mutually perpendicular (D) none of these

A. parallel

B. coplanar

C. mutually perpendicular

D. none of these

Answer: b

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74. if $\left(\overrightarrow{a}\times\overrightarrow{b}\right)\times\left(\overrightarrow{b}\times\overrightarrow{c}\right)=\overrightarrow{b},\ \text{where}\ \overrightarrow{a},\overrightarrow{b}\ \text{and}\ \overrightarrow{c}$ are non-zero vectors, then

A. $\overrightarrow{a},\overrightarrow{b}$ and $\overrightarrow{v}$ can be coplanar

B. $\vec{a}$, $\vec{b}$ and $\vec{c}$ must be coplanar

C. $\vec{a}$, $\vec{b}$ and $\vec{c}$ cannot be coplanar

D. none of these

Answer: c

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75. If $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = \frac{1}{2}$ for some non zero vector $\vec{r}$ and $\vec{a}, \vec{b}, \vec{c}$ are non coplanar, then the area of the triangle whose vertices are $A\left(\vec{a}\right), B\left(\vec{b}\right)$ and $C\left(\vec{c}\right)$ is

A. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$

B. $\left|\vec{r}\right|$

C. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}\right|$

D. none of these

Answer: c

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76. A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point $P(1, 0)$ can be $6\hat{i} + 8\hat{j}$ b. $-8\hat{i} + 3\hat{j}$ c. $6\hat{i} - 8\hat{j}$ d. $8\hat{i} + 6\hat{j}$

A. $6\hat{i} + 8\hat{j}$

B. $-8\hat{i} + 3\hat{j}$

C. $6\hat{i} - 8\hat{j}$

D. $8\hat{i} + 6\hat{j}$

Answer: a

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77. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\frac{\pi}{3}$ then $\left\{\vec{a} \times \left(\vec{b} + \vec{a} \times \vec{b}\right)\right\}.\ \vec{b}$ is equal to (a) $-\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

A. $\frac{-3}{4}$

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. $\frac{1}{2}$

Answer: a

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78. If $\vec{a}$ and $\vec{b}$ are othogonal unit vectors, then for a vector $\vec{r}$ non - coplanar with $\vec{a}$ and $\vec{b}$ vector $\vec{r} \times \vec{a}$ is equal to

A. $\left[\vec{r}\,\vec{a}\,\vec{b}\right]\vec{b} - \left(\vec{r}\,.\,\vec{b}\right)\left(\vec{b} \times \vec{a}\right)$

B. $\left[\vec{r}\,\vec{a}\,\vec{b}\right]\left(\vec{a} + \vec{b}\right)$

C. $\left[\vec{r}\,\vec{a}\,\vec{b}\right]\vec{a} + \left(\vec{r}\,.\,\vec{a}\right)\vec{a} \times \vec{b}$

D. none of these

Answer: a

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79. If $\vec{a}+\vec{b}, \vec{c}$ are any three non- coplanar vectors then the equation

$$\left[\vec{b}\times\vec{c}\,\vec{c}\times\vec{a}\,\vec{a}\times\vec{b}\right]x^2+\left[\vec{a}+\vec{b}\,\vec{b}+\vec{c}\,\vec{c}+\vec{a}\right]x+1+\left[\vec{b}-\vec{c}\right.$$

has roots

A. real and distinct

B. real

C. equal

D. imaginary

Answer: c

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80. Sholve the simultasneous vector equations for $\vec{x}$ and $\vec{y}:\vec{x}+\vec{c}\times\vec{y}=\vec{a}$ and $\vec{y}+\vec{c}\times\vec{x}=\vec{b}, \vec{c}\neq 0$

A. $\vec{x} = \dfrac{\vec{b} \times \vec{c} + \vec{a} + \left(\vec{c} \,.\, \vec{a}\right)\vec{c}}{1 + \vec{c} \,.\, \vec{c}}$

B. $\vec{x} = \dfrac{\vec{c} \times \vec{b} + \vec{b} + \left(\vec{c} \,.\, \vec{a}\right)\vec{c}}{1 + \vec{c} \,.\, \vec{c}}$

C. $\vec{y} = \dfrac{\vec{a} \times \vec{c} + \vec{b} + \left(\vec{c} \,.\, \vec{b}\right)\vec{c}}{1 + \vec{c} \,.\, \vec{c}}$

D. none of these

Answer: b

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81. The condition for equations $\vec{r} \times \vec{a} = \vec{b}$ and $\vec{r} \times \vec{c} = \vec{d}$ to be consistent is

A. $\vec{b} \,.\, \vec{c} = \vec{a} \,.\, \vec{d}$

B. $\vec{a} \,.\, \vec{b} = \vec{c} \,.\, \vec{d}$

C. $\vec{b} \,.\, \vec{c} + \vec{a} \,.\, \vec{d} = 0$

D. $\vec{a} \,.\, \vec{b} + \vec{c} \,.\, \vec{d} = 0$

Answer: c

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82. If $\overrightarrow{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ then $\left[\overrightarrow{a}\,\overrightarrow{b}\,\overrightarrow{i}\right]\hat{i}$ + $\left[\overrightarrow{a}\,\overrightarrow{b}\,\overrightarrow{j}\right]\hat{j}$ + $\left[\overrightarrow{a}\,\overrightarrow{b}\,\hat{k}\right]k$ is equal to

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83. If $\overrightarrow{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{b} = \hat{i} + 2\hat{j} + 2\hat{k}$, $\overrightarrow{c} = \hat{i} + \hat{j} + 2\hat{k}$ and $(1 + \alpha)\hat{i} + \beta(1$

A. $-2, \ -4, \ -\frac{2}{3}$

B. $2, \ -4, \frac{2}{3}$

C. $-2, 4, \frac{2}{3}$

D. $2, 4, \ -\frac{2}{3}$

Answer: a

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84. Let $\left(\vec{a}(x) = (\sin x)\hat{i} + (\cos x)\hat{j} \text{ and } \vec{b}(x) = (\cos 2x)\hat{i} + (\sin 2x)\hat{j}\right.$ be two variable vectors $(x \in R)$. Then $\vec{a}(x)$ and $\vec{b}(x)$ are

A. collinear for unique value of x

B. perpendicular for infinte values of x.

C. zero vectors for unique value of x

D. none of these

Answer: b

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85. For any vectors $\vec{a}$ and $\vec{b}$, $\left(\vec{a} \times \hat{i}\right) + \left(\vec{b} \times \hat{i}\right) + \left(\vec{a} \times \hat{j}\right).\left(\vec{b} \times \hat{j}\right) + \left(\vec{a} \times \hat{k}\right).\left(\vec{b}\right.$

is always equal to

A. $\vec{a} . \vec{b}$

B. $2\vec{a} . Vecb$

C. zero

D. none of these

Answer: b

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86. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non coplanar vectors and $\vec{r}$ is any vector in space, then

$$\left(\vec{a} \times \vec{b}\right) \times \left(\vec{r} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{r} \times \vec{a}\right) + \left(\vec{c} \times \vec{a}\right) \times \left(\vec{r}\right.$$

A. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

B. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

C. $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

D. none of these

Answer: b

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87. If

$$\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]} \text{ and } \vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}, \quad \text{where } \vec{a}, \vec{b} \text{ a}$$

are three non- coplanar vectors then the value of the expression $\left(\vec{a} + \vec{b} + \vec{c}\right).\left(\vec{p} + \vec{q} + \vec{r}\right)$ is $(a)3\ (b)2\ (c)1\ (d)0$

A. 3

B. 2

C. 1

D. 0

Answer: a

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88. $A\left(\vec{a}\right), B\left(\vec{b}\right) and C\left(\vec{c}\right)$ are the vertices of triangle ABC and $R\left(\vec{r}\right)$ is any point in the plane of triangle ABC, then $r\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is always equal to a. zero b. $\left[\vec{a}\vec{b}\vec{c}\right]$ c. $-\left[\vec{a}\vec{b}\vec{c}\right]$ d. none of these

A. zero

B. $\left[\vec{a}\vec{b}\vec{c}\right]$

C. $-\left[\vec{a}\vec{b}\vec{c}\right]$

D. none of these

Answer: b

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89. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar vectors and $\vec{a}\times\vec{c}$ is perpendicular to $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$, then the value of $\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right]\times\vec{c}$ is equal to

A. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{c}$

B. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{b}$

C. $\vec{0}$

D. $\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{a}$

Answer: c

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90. If V be the volume of a tetrahedron and $V\,'$ be the volume of another tetrahedran formed by the centroids of faces of the previous tetrahedron and $V = KV\,', thenK$ is equal to a. 9 b. 12 c. 27 d. 81

A. 9

B. 12

C. 27

D. 81

Answer: c

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91. $\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)\quad\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\quad\left(\vec{c}\times\vec{a}\right)\times\left(\vec{a}\times\vec{b}\right)\right]$ is equal to (where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non - zero non- colanar vectors). $(a)\left[\vec{a}\vec{b}\vec{c}\right]^2$ $(b)\left[\vec{a}\vec{b}\vec{c}\right]^3$ $(c)\left[\vec{a}\vec{b}\vec{c}\right]^4$ $(d)\left[\vec{a}\vec{b}\vec{c}\right]$

A. $\left[\vec{a}\vec{b}\vec{c}\right]^2$

B. $\left[\vec{a}\vec{b}\vec{c}\right]^3$

C. $\left[\vec{a}\vec{b}\vec{c}\right]^4$

D. $\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: c

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92. If

$\vec{r} = x_1\left(\vec{a} \times \vec{b}\right) + x_2\left(\vec{b} \times \vec{a}\right) + x_3\left(\vec{c} \times \vec{d}\right)$ and $4\left[\vec{a}\,\vec{b}\,\vec{c}\right] = 1$

is equal to

A. $\frac{1}{2}\vec{r}\cdot\left(\vec{a} + \vec{b} + \vec{c}\right)$

B. $\frac{1}{4}\vec{r}\cdot\left(\vec{a} + \vec{b} + \vec{c}\right)$

C. $2\vec{r}\cdot\left(\vec{a} + \vec{b} + \vec{c}\right)$

D. $4\vec{r}\cdot\left(\vec{a} + \vec{b} + \vec{c}\right)$

Answer: d

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93. If the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other then a vector $\vec{v}$ in terms of $\vec{a}$ and $\vec{b}$ satisfying the equations $\vec{v}\cdot\vec{a} = 0$, $\vec{v}\cdot\vec{b} = 1$ and $\left[\vec{v}\ \vec{a}\ \vec{b}\right] = 1$ is

A. $\frac{\vec{b}}{\left|\vec{b}\right|^2} + \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$

B. $\frac{\vec{b}}{\left|\vec{b}\right|}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|^{2}}$

C. $\frac{\vec{b}}{\left|\vec{b}\right|}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$

D. none of these

Answer: a

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94. If $\vec{a}' = \hat{i}+\hat{j}$, $\vec{b}' = \hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}' = 2\hat{i}-\hat{j}-\hat{k}$ then the altitude of the parallelepiped formed by the vectors, $\vec{a}$, $\vec{b}$ and $\vec{c}$ having base formed by $\vec{b}$ and $\vec{c}$ is (where $\vec{a}'$ is recipocal vector $\vec{a}$)

$(a)1\ (b)3\sqrt{2}/2\ (c)1/\sqrt{6}\ (d)1/\sqrt{2}$

A. 1

B. $3\sqrt{2}/2$

C. $1/\sqrt{6}$

D. $1/\sqrt{2}$

Answer: d

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95. If $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$, $\vec{c} = \hat{k} + \hat{i}$ then in the reciprocal system of vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ reciprocal $\vec{a}$ of vector $\vec{a}$ is

A. $\dfrac{\hat{i} + \hat{j} + \hat{k}}{2}$

B. $\dfrac{\hat{i} - \hat{j} + \hat{k}}{2}$

C. $\dfrac{-\hat{i} - \hat{j} + \hat{k}}{2}$

D. $\dfrac{\hat{i} + \hat{j} - \hat{k}}{2}$

Answer: d

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96. If the unit vectors $\vec{a}$ and $\vec{b}$ are inclined of an angle 2θ such that $\left|\vec{a} - \vec{b}\right| < 1$ and $0 \leq \theta \leq \pi$ then θ in the interval

A. $[0, \pi/6)$

B. $(5\pi/6, \pi]$

C. $[\pi/6, \pi/2]$

D. $(\pi/2, 5\pi/6]$

Answer: a,b

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97. $\vec{b}$ and $\vec{c}$ are non- collinear if $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a} . \vec{b}\right)\vec{b} = (4 - 2x - \sin y)\vec{b} + \left(x^2 - 1\right)\vec{c}$ and d

then

A. x =1

B. x = -1

C. $y = (4n + 1)\frac{\pi}{2}, n \in I$

D. $y(2n + 1)\frac{\pi}{2}, n \in I$

Answer: a,c

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98. Let $\vec{a}.\vec{b}=0$ where $\vec{a}$ and $\vec{b}$ are unit vectors and the vector $\vec{c}$ is inclined an anlge θ to both $\vec{a}$ and $\vec{b}$. $If \vec{c}=m\vec{a}+n\vec{b}+p\left(\vec{a}\times\vec{b}\right),(m,n,p\in R)$ then

A. $\alpha=\beta$

B. $\gamma^2=1-2\alpha^2$

C. $\gamma^2=-\cos 2\theta$

D. $\beta^2=\dfrac{1+\cos 2\theta}{2}$

Answer: a,b,c,d

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99. $\vec{a}$ and $\vec{b}$ are two given vectors. On these vectors as adjacent sides a parallelogram is constructed. The vector which is the altitude of the parallelogam and which is perpendicular to $\vec{a}$ is not equal to

A. $\dfrac{\left(\vec{a}\,.\,\vec{b}\right)}{\left|\vec{a}\right|^2}\vec{a}-\vec{b}$

B. $\dfrac{1}{\left|\vec{a}\right|^2}\left\{\left|\vec{a}\right|^2\vec{b}-\left(\vec{a}\,.\,\vec{b}\right)\vec{a}\right\}$

C. $\dfrac{\vec{a}\times\left(\vec{a}\times\vec{b}\right)}{\left|\vec{a}\right|^2}$

D. $\dfrac{\vec{a}\times\left(\vec{b}\times\vec{a}\right)}{\left|\vec{b}\right|^2}$

Answer: a,b,c

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100. If $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ is perpendicular to $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$, we may have

A. $\left(\vec{a}.\vec{c}\right)\left|\vec{b}\right|^2=\left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{c}\right)$

B. $\vec{a}.\vec{b}=0$

C. $\vec{a}.\vec{c}=0$

D. $\vec{b}.\vec{c}=0$

Answer: a,c

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101. If $\vec{p}=\frac{\vec{b}\times\vec{c}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}, \vec{q}=\frac{\vec{c}\times\vec{a}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}, \vec{r}=\frac{\vec{a}\times\vec{b}}{\left[\vec{a}\ \vec{b}\ \vec{b}\right]}$ where $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then the value of the expression $\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{p}+\vec{q}+\vec{r}\right)$ is

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102. $a_1, a_2, a_3 \in R-\{0\}$ and $a_1+a_2\cos 2x+a_3\sin^2 x=0$ " for all " x in R then (a) vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=4\hat{i}+2\hat{j}+\hat{k}$ are perpendicular to each other (b)vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ are parallel to each each other (c)if vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then on of the ordered tripplet $(a_1, a_2, a_3) = (1, -1, -2)$ (d)if $2a_1 + 3a_2 + 6a_3 = 26$, then $\left|\vec{a}\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

A. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other

B. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ are parallel to each each other

C. if vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then on of the ordered tripplet $(a_1, a_2, a_3) = (1, -1, -2)$

D. if $2a_1 + 3a_2 + 6a_3 + 6a_3 = 26$, then $\left|\vec{a}\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

Answer: a,b,c,d

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103. If $\vec{a}$ and $\vec{b}$ are two vectors and angle between them is θ, then

A. $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a} . \vec{b}\right)^2 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2$

B. $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a} . \vec{b}\right)^2$, if $\theta = \pi/4$

C. $\vec{a} \times \vec{b} = \left(\vec{a} . Vecb\right)\hat{n}$ (where $\hat{n}$ is a normal unit vector)

if $\theta f = \pi/4$

D. $\left(\vec{a} \times \vec{b}\right) . \left(\vec{a} + \vec{b}\right) = 0$

Answer: a,b,c,d

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104. Let $\vec{a}$ and $\vec{b}$ be two non- zero perpendicular vectors. A vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$ can be

A. $\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

B. $2\vec{b}-\frac{\vec{a}\times\vec{b}}{\left|\vec{b}\right|^2}$

C. $\left|\vec{a}\right|\vec{b}-\frac{\vec{a}\times\vec{b}}{\left|\vec{b}\right|^2}$

D. $\left|\vec{b}\right|\vec{b}-\frac{\vec{a}\times\vec{b}}{\left|\vec{b}\right|^2}$

Answer: a,b,cd,

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105. If vector $\vec{b}=\left(tan\alpha,\ -1,2\sqrt{\sin\alpha/2}\right)and\vec{c}=\left(tan\alpha,tan\alpha,\ -\frac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and vector $\vec{a}=(1,3,\sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is $a.\ \alpha=(4n+1)\pi+\tan^{-1}2$ $b.\ \alpha=(4n+1)\pi-\tan^{-1}2$ $c.\ \alpha=(4n+2)\pi+\tan^{-1}2$ $d.\ \alpha=(4n+2)\pi-\tan^{-1}2$

A. $\alpha=(4n+1)\pi+\tan^{-1}2$

B. $\alpha = (4n+1)\pi - \tan^{-1} 2$

C. $\alpha = (4n+2)\pi + \tan^{-1} 2$

D. $\alpha = (4n+2)\pi - \tan^{-1} 2$

Answer: b,d

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106. Let $\vec{r}$ be a unit vector satisfying $\vec{r} \times \vec{a} = \vec{b}$, where $\left|\vec{a}\right| = \sqrt{3}$ and $\left|\vec{b}\right| = \sqrt{2}$, then (a) $\vec{r} = \frac{2}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$ (b) $\vec{r} = \frac{1}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$ (c) $\vec{r} = \frac{2}{3}\left(\vec{a} - \vec{a} \times \vec{b}\right)$ (d) $\vec{r} = \frac{1}{3}\left(-\vec{a} + \vec{a} \times \vec{b}\right)$

A. $\vec{r} = \frac{2}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

B. $\vec{r} = \frac{1}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

C. $\vec{r} = \frac{2}{3}\left(\vec{a} - \vec{a} \times \vec{b}\right)$

D. $\vec{r} = \frac{1}{3}\left(-\vec{a} + \vec{a} \times \vec{b}\right)$

Answer: b,d

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107. If $\vec{a}$ and $\vec{b}$ are unequal unit vectors such that $\left(\vec{a}-\vec{b}\right)\times\left[\left(\vec{b}+\vec{a}\right)\times\left(2\vec{a}+\vec{b}\right)\right]=\vec{a}+\vec{b}$ then angle θ between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. $\pi/4$

D. π

Answer: b,d

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108. If $\vec{a}$ and $\vec{b}$ are two unit vectors perpenicualar to each other and $\vec{c} = \lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \left(\vec{a} \times \vec{b}\right)$, then which of the following is (are) true ?

A. $\lambda_1 = \vec{a} . \vec{c}$

B. $\lambda_2 = \left|\vec{b} \times \vec{c}\right|$

C. $\lambda_3 = \left|\vec{a} \times \vec{b}\right| \times \vec{c}\Big|$

D. $\lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \left(\vec{a} \times \vec{b}\right)$

Answer: a,d

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109. If vectors $\vec{a}$ and $\vec{b}$ are non collinear then $\frac{\vec{a}}{\left|\vec{a}\right|} + \frac{\vec{b}}{\left|\vec{b}\right|}$ is (A) a unit vector (B) in the plane of $\vec{a}$ and $\vec{b}$ (C) equally inclined to $\vec{a}$ and $\vec{b}$ (D) perpendicular to $\vec{a} \times \vec{b}$

A. a unit vector

B. in the plane of $\vec{a}$ and $\vec{b}$

C. equally inclined to $\vec{a}$ and $\vec{b}$

D. perpendicular to $\vec{a} \times \vec{b}$

Answer: b,c,d

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110. If $\vec{a}$ and $\vec{b}$ are non - zero vectors such that $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-2\vec{b}\right|$ then

A. $2\vec{a}.\vec{b}=\left|\vec{b}\right|^2$

B. $\vec{a}.\vec{b}=\left|\vec{b}\right|^2$

C. least value of $\vec{a}.Vecb+\frac{1}{\left|\vec{b}\right|^2+2}$ is $\sqrt{2}$

D. least value of $\vec{a}.\vec{b}+\frac{1}{\left|\vec{b}\right|^2+2}$ is $\sqrt{2}-1$

Answer: a,d

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111. Let $\vec{a}\vec{b}$ and $\vec{c}$ be non- zero vectors aned $\vec{V}_1 = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ and $\vec{V}_2 = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$.vectors $\vec{V}_1$ and $\vec{V}_2$ are equal . Then

A. $\vec{a}$ and $\vec{b}$ ar orthogonal

B. $\vec{a}$ and $\vec{c}$ are collinear

C. $\vec{b}$ and $\vec{c}$ ar orthogonal

D. $\vec{b} = \lambda\left(\vec{a} \times \vec{c}\right)$ when λ is a scalar

Answer: b,d

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112. Vectors $\vec{A}$ and $\vec{B}$ satisfying the vector equation $\vec{A}+\vec{B}=\vec{a}, \vec{A}\times\vec{B}=\vec{b}$ and $\vec{A}.\vec{a}=1$. where veca and $\vec{b}$ are given vectosrs, are

A. $\vec{A}=\dfrac{\left(\vec{a}\times\vec{b}\right)-\vec{a}}{a^2}$

B. $\vec{B}=\dfrac{\left(\vec{b}\times\vec{a}\right)+\vec{a}(a^2-1)}{a^2}$

C. $\vec{A}=\dfrac{\left(\vec{a}\times\vec{b}\right)+\vec{a}}{a^2}$

D. $\vec{B}=\dfrac{\left(\vec{b}\times\vec{a}\right)-\vec{a}(a^2-1)}{a^2}$

Answer: b,c,

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113. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}, \vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=3\hat{j}-2\hat{k}$. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{;} \vec{c}, \vec{a},$ respectively. Then

A. $\vec{x} \cdot \vec{d} = -1$

B. $\vec{y} \cdot \vec{d} = 1$

C. $\vec{z} \cdot \vec{d} = 0$

D. $\vec{r} \cdot \vec{d} = 0$, where $\vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

Answer: c.d

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114. Vectors perpendicular to $\hat{i} - \hat{j} - \hat{k}$ and in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ are (A) $\hat{i} + \hat{k}$ (B) $2\hat{i} + \hat{j} + \hat{k}$ (C) $3\hat{i} + 2\hat{j} + \hat{k}$ (D) $-4\hat{i} - 2\hat{j} - 2\hat{k}$

A. $\hat{i} + \hat{k}$

B. $2\hat{i} + \hat{j} + \hat{k}$

C. $3\hat{i} + 2\hat{j} + \hat{k}$

D. $-4\hat{i} - 2\hat{j} - 2\hat{k}$

Answer: b,d

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115. If the sides $\overrightarrow{AB}$ of an equilateral triangle ABC lying in the xy-plane is $3\hat{i}$ then the side $\overrightarrow{CB}$ can be (A) $-\frac{3}{2}\left(\hat{i}-\sqrt{3}\right)$ (B) $\frac{3}{2}\left(\hat{i}-\sqrt{3}\right)$ (C) $-\frac{3}{2}\left(\hat{i}+\sqrt{3}\right)$ (D) $\frac{3}{2}\left(\hat{i}+\sqrt{3}\right)$

A. $-\frac{3}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$

B. $-\frac{3}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$

C. $-\frac{3}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$

D. $\frac{3}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$

Answer: b,d

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116. Let $\widehat{a}$ be a unit vector and $\hat{b}$ a non zero vector non parallel to $\vec{a}$. Find the angles of the triangle tow sides of which are represented by the vectors. $\sqrt{3}\left(\widehat{\times}\ \vec{b}\right)$ and $\vec{b}-\left(\widehat{a}.\ \vec{b}\right)\widehat{a}$

A. $\tan^{-1}(\sqrt{3})$

B. $\tan^{-1}(1/\sqrt{3})$

C. $\cot^{-1}(0)$

D. tant^(-1)(1)`

Answer: a,b,c

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117. $\vec{a}, \vec{b}$ and $\vec{c}$ are unimodular and coplanar. A unit vector $\vec{d}$ is perpendicualt to them , $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\frac{1}{6}\hat{i}-\frac{1}{3}\hat{j}+\frac{1}{3}\hat{k}$, and the angle between $\vec{a}$ and $\vec{b}\, is 30^{\circ}$ then $\vec{c}$ is

A. $\left(\hat{i}-2\hat{j}+2\hat{k}\right)/3$

B. $\left(-\hat{i}+2\hat{j}-2\hat{k}\right)/3$

C. $\left(-\hat{i}+2\hat{j}-\hat{k}\right)/3$

D. $\left(-2\hat{i}-2\hat{j}+\hat{k}\right)/3$

Answer: a,b

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118. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=$

A. $2\left(\vec{a}\times\vec{b}\right)$

B. $6\left(\vec{b}\times\vec{c}\right)$

C. $3\left(\vec{c}\times\vec{a}\right)$

D. $\vec{0}$

Answer: c,d

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119. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - \left(\vec{a} . \vec{b}\right)\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $\left|\vec{v}\right|$ is

A. $\left|\vec{u}\right|$

B. $\left|\vec{u}\right| + \left|\vec{u} . \vec{b}\right|$

C. $\left|\vec{u}\right| + \left|\vec{u} . \vec{a}\right|$

D. none of these

Answer: b,d

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120. if $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}$, where $\vec{c} \neq \vec{0}$ then (a) $\left|\vec{a}\right| = \left|\vec{c}\right|$ (b) $\left|\vec{a}\right| = \left|\vec{b}\right|$ (c) $\left|\vec{b}\right| = 1$ (d) $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = 1$

A. $\left|\vec{a}\right| = \left|\vec{c}\right|$

B. $\left|\vec{a}\right| = \left|\vec{b}\right|$

C. $\left|\vec{b}\right| = 1$

D. $\left|\vec{a}\right| = \vec{b}| = |\vec{c}\ | \ = 1$

Answer: a,c

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121. Let $\vec{a}, \vec{b},$ and $\vec{c}$ be three non- coplanar vectors and $\vec{d}$ be a non - zero , which is perpendicular to $\left(\vec{a} + \vec{b} + \vec{c}\right). Now \vec{d} = \left(\vec{a} \times \vec{b}\right)\sin x + \left(\vec{b} \times \vec{c}\right)\cos y + 2\left(\vec{c} \times\right.$

. Then

A. $\dfrac{\vec{d}.\left(\vec{a} + \vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]} = 2$

B. $\dfrac{\vec{d}.\left(\vec{a} + \vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]} = -2$

C. minimum value of $x^2 + y^2 is \pi^2/4$

D. minimum value of $x^2 + y^2 is 5\pi^2/4$

Answer: b,d

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122. If $\vec{a}, \vec{b}, and \leftrightarrow c$ are three unit vecrtors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{1}{1}\vec{b}, then\left(\vec{b} and \vec{c}\right.$ being non-parallel) angle between $\vec{a} and \vec{b}$ is$\pi/3$ b.a n g l eb e t w e e n $\vec{a} and \vec{c}$ i s$\pi/3$ c. a. angle between $\vec{a} and \vec{b}$ is$\pi/2$ d. a. angle between $\vec{a} and \vec{c}$ is$\pi/2$

A. angle between $\vec{a}$ and $\vec{b}\ is\pi/3$

B. angle between $\vec{a}$ and $\vec{c}\ is\pi/3$

C. angle between $\vec{a}$ and $\vec{b}\ is\pi/2$

D. angle between $\vec{a}$ and $\vec{c}\ is\pi/2$

Answer: b,c

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123. If in triangle ABC, $\overrightarrow{AB} = \frac{\overrightarrow{u}}{\left|\overrightarrow{u}\right|} - \frac{\overrightarrow{v}}{\left|\overrightarrow{v}\right|}$ and $\overrightarrow{AC} = \frac{2\overrightarrow{u}}{\left|\overrightarrow{u}\right|}$, where $\left|\overrightarrow{u}\right| \neq \left|\overrightarrow{v}\right|$, then $(a) 1 + \cos 2A + \cos 2B + \cos 2C = 0$ (b)$\sin A = \cos C$ (c)projection of AC on BC is equal to BC (d) projection of AB on BC is equal to AB

A. $1 + \cos 2A + \cos 2B + \cos 2C = 0$

B. $\sin A = \cos C$

C. projection of AC on BC is equal to BC

D. projection of AB on BC is equal to AB

Answer: a,b,c

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124. $\left[\overrightarrow{a} \times \overrightarrow{b} \quad \overrightarrow{c} \times \overrightarrow{d} \quad \overrightarrow{e} \times \overrightarrow{f}\right]$ is equal to

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125. The scalars l and m such that $l\vec{a}+m\vec{b}=\vec{c}$, where $\vec{a},\vec{b}$ and $\vec{c}$ are given vectors, are equal to

A. $l=\dfrac{\left(\vec{c}\times\vec{b}\right).\left(\vec{a}\times\vec{b}\right)}{\left(\vec{a}\times\vec{b}\right)^2}$

B. $l=\dfrac{\left(\vec{c}\times\vec{a}\right).\left(\vec{b}\times\vec{a}\right)}{\left(\vec{b}\times\vec{a}\right)}$

C. $m=\dfrac{\left(\vec{c}\times\vec{a}\right).\left(\vec{b}\times\vec{a}\right)}{\left(\vec{b}\times\vec{a}\right)^2}$

D. $m=\dfrac{\left(\vec{c}\times\vec{a}\right).\left(\vec{b}\times\vec{a}\right)}{\left(\vec{b}\times\vec{a}\right)}$

Answer: a,c

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126. If $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right).\left(\vec{a}\times\vec{d}\right)=0$ then which of the following may be true ?

A. $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are nenessarily coplanar

B. $\vec{a}$ lies in the plane of $\vec{c}$ and $\vec{d}$

C. $\vec{b}$ lies in the plane of $\vec{a}$ and $\vec{d}$

D. $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{d}$

Answer: b,c,d

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127. $A, B, CandD$ are four points such that $\vec{A}B = m\left(2\hat{i} - 6\hat{j} + 2\hat{k}\right), \vec{B}C = \left(\hat{i} - 2\hat{j}\right)and\vec{C}D = n\left(-6\hat{i} + 15\hat{j} - 3\hat{k}\right)$ If CD intersects AB at some point E, then a. $m \geq 1/2$ b.$n \geq 1/3$ c. $m = n$ d. $m < n$

A. (a) $m \geq 1/2$

B. (b) $n \geq 1/3$

C. (c) m= n

D. (d) $m < n$

Answer: a,b

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128. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non -coplanar and l, m, n are distinct scalars such that

$\left[l\vec{a}+m\vec{b}+n\vec{c} \quad l\vec{b}+m\vec{c}+n\vec{a} \quad l\vec{c}+m\vec{a}+n\vec{b}\right]=0$ then

A. $a)l+m+n=0$

B. b) roots of the equation $lx^2+mx+n=0$ are equal

C. c)$l^2+m^2+n^2=0$

D. d)$l^3+m^2+n^3=3lmn$

Answer: a,b,d

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129. Let $\overrightarrow{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \overrightarrow{\beta} = b\hat{i} + c\hat{j} + a\hat{k}$ and $\overrightarrow{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplnar vectors with $a \neq b$, and $\overrightarrow{v} = \hat{i} + \hat{j} + \hat{k}$. Then $\overrightarrow{v}$ is perpendicular to

A. $\overrightarrow{\alpha}$

B. $\overrightarrow{\beta}$

C. $\overrightarrow{\gamma}$

D. none of these

Answer: a,b,c

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130. if vectors $\overrightarrow{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \overrightarrow{B} = \hat{i} + \hat{j} + 5\hat{k}$ and $\overrightarrow{C}$ from a left - handed system, then $\overrightarrow{C}$ is

A. a)$11\hat{i} - 6\hat{j} - \hat{k}$

B. b)$-11\hat{i} - 6\hat{j} - \hat{k}$

C. c)$-11\hat{i} - 6\hat{j} + \hat{k}$

D. d)$-11\hat{i} + 6\hat{j} - \hat{k}$

Answer: b,d

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131. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$, $\vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k}$, $\text{then } \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is

(a)parallel to $(y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k}$ (b)orthogonal to $\hat{i} + \hat{j} + \hat{k}$ (c)orthogonal to $(y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ (d)orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

A. parallel to $(y-z)\hat{i} + (z-x)\hat{j} + (x-y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$

D. orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

Answer: a,b,c,d

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132. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$ then

A. $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{0}$

B. $\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$

C. $\vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

D. $\vec{c} \times \vec{a} \times \vec{b} = \vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

Answer: a,c,d

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133. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{;} \vec{c}, \vec{a},$ respectively. Then

A. (a) $\overrightarrow{z}.\overrightarrow{d}=0$

B. (b) $\overrightarrow{x}.\overrightarrow{d}=1$

C. (c) $\overrightarrow{y}.\overrightarrow{d}=32$

D. (d) $\overrightarrow{r}.\overrightarrow{d}=0, \quad \text{where } \overrightarrow{r}=\lambda\overrightarrow{x}+\mu\overrightarrow{y}+\gamma\overrightarrow{z}$

Answer: a,d

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134. A parallelogram is constructed on the vectors $\overrightarrow{a}=3\overrightarrow{\alpha}-\overrightarrow{\beta}, \overrightarrow{b}=\overrightarrow{\alpha}+3\overrightarrow{\beta}. If\left|\overrightarrow{\alpha}\right|=\left|\overrightarrow{\beta}\right|=2$ and angle between $\overrightarrow{\alpha}$ and $\overrightarrow{\beta}\, is\frac{\pi}{3}$ then the length of a diagonal of the parallelogram is

A. $4\sqrt{5}$

B. $4\sqrt{3}$

C. $4\sqrt{7}$

D. none of these

Answer: b,c

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Reasoning type

1. (a)Statement 1: Vector $\overrightarrow{c} = -5\hat{i} + 7\hat{j} + 2\hat{k}$ is along the bisector of angle between $\overrightarrow{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\overrightarrow{b} = 8\hat{i} + \hat{j} - 4\hat{k}$.

Statement 2 : $\overrightarrow{c}$ is equally inclined to $\overrightarrow{a}$ and $\overrightarrow{b}$.

A. (a) Both the statements are true and statement 2 is the correct explanation for statement 1.

B. (b) Both statements are true but statement 2 is not the correct explanation for statement 1.

C. (c) Statement 1 is true and Statement 2 is false

D. (d) Statement 1 is false and Statement 2 is true.

Answer: b

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2. Statement1: A component of vector $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in the direction perpendicular to the direction of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k} is \hat{i} - \hat{j}$

Statement 2: A component of vector in the direction of $\vec{a} = \hat{i} + \hat{j} + \hat{k} is 2\hat{i} + 2\hat{j} + 2\hat{k}$

A. (a) Both the statements are true and statement 2 is the correct explanation for statement 1.

B. (b) Both statements are true but statement 2 is not the correct explanation for statement 1.

C. (c) Statement 1 is true and Statement 2 is false

D. (d)Statement 1 is false and Statement 2 is true.

Answer: c

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3. Statement 1: Distance of point D(1,0,-1) from the plane of points A(1,-2,0) , B (3, 1,2) and C(-1,1,-1) is $\frac{8}{\sqrt{229}}$

Statement 2: volume of tetrahedron formed by the points A,B, C and D is $\frac{\sqrt{229}}{2}$

A. (a) Both the statements are true and statement 2 is the correct explanation for statement 1.

B. (b) Both statements are true but statement 2 is not the correct explanation for statement 1.

C. (c) Statement 1 is true and Statement 2 is false

D. (d) Statement 1 is false and Statement 2 is true.

Answer: d

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4. Let $\vec{r}$ be a non - zero vector satisfying $\vec{r}.\vec{a}=\vec{r}.\vec{b}=\vec{r}.\vec{c}=0$ for given non- zero vectors $\vec{a}\,\vec{b}$ and $\vec{c}$

Statement 1: $\left[\vec{a}-\vec{b}\,\vec{b}-\vec{c}\,\vec{c}-\vec{a}\right]=0$

Statement 2: $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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5. Statement 1: If $a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}\,\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ are three mutually perpendicular unit vectors then $a_1\hat{i}+b_1\hat{j}+c_1\hat{k}$, $a_2\hat{i}+b_2\hat{j}+c_2\hat{k}$ and $a_3\hat{i}+b_3\hat{j}+c_3\hat{k}$ may be mutually perpendicular unit vectors.

Statement 2 : value of determinant and its transpose are the same.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: a

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6. Statement 1: $\overrightarrow{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}, \overrightarrow{B} = \hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{C} = \hat{i} + 2\hat{j} + \hat{k}$ then $\left|\overrightarrow{A} \times \left(\overrightarrow{A} \times \left(\overrightarrow{A} \times \overrightarrow{B}\right)\right).\overrightarrow{C}\right| = 243$ Statement 2: $\left|\overrightarrow{A} \times \left(\overrightarrow{A} \times \left(\overrightarrow{A} \times \overrightarrow{B}\right)\right).\overrightarrow{C}\right| = \left|\overrightarrow{A}\right|^2 \left|\left[\overrightarrow{A}\,\overrightarrow{B}\,\overrightarrow{C}\right]\right|$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: d

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7. Statement 1: $\vec{a}, \vec{b}$ and $\vec{c}$ arwe three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are non- coplanar. If $\left[\vec{d}\,\vec{b}\,\vec{c}\right] = \left[\vec{d}\,\vec{a}\,\vec{b}\right] = \left[\vec{d}\,\vec{c}\,\vec{a}\right] = 1, \text{ then } \vec{d} = \vec{a} + \vec{b} + \vec{c}$

Statement 2: $\left[\vec{d}\,\vec{b}\,\vec{c}\right] = \left[\vec{d}\,\vec{a}\,\vec{b}\right] = \left[\vec{d}\,\vec{c}\,\vec{a}\right] \Rightarrow \vec{d}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

A. (a) Both the statements are true and statement 2 is the correct explanation for statement 1.

B. (b) Both statements are true but statement 2 is not the correct explanation for statement 1.

C. (c) Statement 1 is true and Statement 2 is false

D. (d) Statement 1 is false and Statement 2 is true.

Answer: b

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8. Consider three vectors $\vec{a}, \vec{b}$ and $\vec{c}$

Statement 1:

$\vec{a} \times \vec{b} = \left(\left(\hat{i} \times \vec{a}\right). \vec{b}\right)\hat{i} + \left(\left(\hat{j} \times \vec{a}\right). \vec{b}\right)\hat{j} + \left(\hat{k} \times \vec{a}\right). \vec{b}\right)\hat{k}$

Statement 2: $\vec{c} = \left(\hat{i}. \vec{c}\right)\hat{i} + \left(\hat{j}. \vec{c}\right)\hat{j} + \left(\hat{k}. \vec{c}\right)\hat{k}$

A. (a) Both the statements are true and statement 2 is the correct explanation for statement 1.

B. (b) Both statements are true but statement 2 is not the correct explanation for statement 1.

C. (c) Statement 1 is true and Statement 2 is false

D. (d) Statement 1 is false and Statement 2 is true.

Answer: a

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Comprehension type

1. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} =$

Vector $\vec{u}$ is

A. $\vec{a} - \frac{2}{3}\vec{b} + \vec{c}$

B. $\vec{a} + \frac{4}{3}\vec{b} + \frac{8}{3}\vec{c}$

C. $2\vec{a} - \vec{b} + \frac{1}{3}\vec{c}$

D. $\frac{4}{3}\vec{a} - \vec{b} + \frac{2}{3}\vec{c}$

Answer: b

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2. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} =$

Vector $\vec{u}$ is

A. $2\vec{a} - 3\vec{c}$

B. $3\vec{b} - 4c$

C. $-4\vec{c}$

D. $\vec{a} + \vec{b} + 2\vec{c}$

Answer: c

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3. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} =$

Vector $\vec{u}$ is

A. $\frac{2}{3}\left(2\vec{c} - \vec{b}\right)$

B. $\frac{1}{3}\left(\vec{a} - \vec{b} - \vec{c}\right)$

C. $\frac{1}{3}\vec{a} - \frac{2}{3}\vec{b} - 2\vec{c}$

D. $\frac{4}{3}\left(\vec{c} - \vec{b}\right)$

Answer: d

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4. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$. Find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.

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5. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$. Find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.

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6. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}$ $, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\overrightarrow{\times} x \vec{y} = \vec{c}$, find vecx, vecy, vecz $\in termsof$veca,vecb and vecc`.

A. $\frac{1}{2}\left[\left(\vec{a} - \vec{c}\right) \times \vec{c} - \vec{b} + \vec{a}\right]$

B. $\frac{1}{2}\left[\left(\vec{a} - \vec{b}\right) \times \vec{c} + \vec{b} - \vec{a}\right]$

C. $\frac{1}{2}\left[\vec{c} \times \left(\vec{a} - \vec{b}\right) + \vec{b} + \vec{a}\right]$

D. none of these

Answer: b

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7. If

$\vec{x} \cdot x\vec{y} = \vec{a}, \vec{y} \times \vec{z} = \vec{b}, \vec{x}.\vec{b} = \gamma, \vec{x}.\vec{y} = 1$ and $\vec{y}.\vec{z} = 1$

then find x,y,z in terms of `veca,vecb and gamma.

A. A. $\frac{1}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

B. B. $\frac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \vec{b} - \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

C. C . $\frac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \vec{b} + \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

D. D. none of these

Answer: b

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8. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x} \times \left(\vec{y} \times \vec{z}\right) = \vec{a}, \vec{y} \times \left(\vec{z} \times \vec{x}\right) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$. Find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.

A. $\dfrac{\vec{a} \times \vec{b}}{\gamma}$

B. $\vec{a} + \dfrac{\vec{a} \times \vec{b}}{\gamma}$

C. $\vec{a} + \vec{b} + \dfrac{\vec{a} \times \vec{b}}{\gamma}$

D. none of these

Answer: a

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9. If

$\vec{x} \cdot x\vec{y} = \vec{a}, \vec{y} \times \vec{z} = \vec{b}, \vec{x}.\vec{b} = \gamma, \vec{x}.\vec{y} = 1 \text{ and } \vec{y}.\vec{z} = 1$

then find x,y,z in terms of `veca,vecb and gamma.

A. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} \times \left(\vec{a} \times \vec{b}\right)\right]$

B. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} - \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

C. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} + \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

D. none of these

Answer: c

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10. Given two orthogonal vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ each of length unity. Let $\overrightarrow{P}$ be the vector satisfying the equation $\overrightarrow{P} \times \overrightarrow{B} = \overrightarrow{A} - \overrightarrow{P}$. then $\overrightarrow{P}$ is equal to

A. $\overrightarrow{P}$

B. $-\overrightarrow{P}$

C. $2\overrightarrow{B}$

D. $\overrightarrow{A}$

Answer: b

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11. Given two orthogonal vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ each of length unity. Let $\overrightarrow{P}$ be the vector satisfying the equation $\overrightarrow{P} \times \overrightarrow{B} = \overrightarrow{A} - \overrightarrow{P}$. then $\overrightarrow{P}$ is equal to

A. $\frac{\overrightarrow{A}}{2} + \frac{\overrightarrow{A} \times \overrightarrow{B}}{2}$

B. $\frac{\overrightarrow{A}}{2} + \frac{\overrightarrow{B} \times \overrightarrow{A}}{2}$

C. $\frac{\overrightarrow{A} \times \overrightarrow{B}}{2} - \frac{\overrightarrow{A}}{2}$

D. $\overrightarrow{A} \times \overrightarrow{B}$

Answer: B

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12. Given two orthogonal vectors $\overrightarrow{A}$ and $VecB$ each of length unity. Let $\overrightarrow{P}$ be the vector satisfying the equation $\overrightarrow{P} \times \overrightarrow{B} = \overrightarrow{A} - \overrightarrow{P}$. then which of the following statements is false ?

A. vectors $\overrightarrow{P}$, $\overrightarrow{A}$ and $\overrightarrow{P} \times \overrightarrow{B}$ ar linearly dependent.

B. vectors $\vec{P}$, $\vec{B}$ and $\vec{P} \times \vec{B}$ ar linearly independent

C. $\vec{P}$ is orthogonal to $\vec{B}$ and has length $\frac{1}{\sqrt{2}}$.

D. none of these

Answer: d

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13. Let

$\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then

$\vec{a}_2$ is equal to

A. (a) $\frac{943}{49}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

B. (b) $\frac{943}{49^2}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

C. (c) $\frac{943}{49}\left(-2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

D. (d) $\frac{943}{49^2}\left(-2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

Answer: b

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14. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then

$\vec{a}_1 . \vec{b}$ is equal to

A. (a) -41

B. (b) $-41/7$

C. (c) 41

D. (d) 287

Answer: a

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15. Let $\overrightarrow{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \overrightarrow{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\overrightarrow{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\overrightarrow{a}_1$ be the projection of $\overrightarrow{a}$ on $\overrightarrow{b}$ and $\overrightarrow{a}_2$ be the projection of $\overrightarrow{a}_1$ on $\overrightarrow{c}$

. Then

$\overrightarrow{a}_2$ is equal to

A. $\overrightarrow{a}$ and $vcea_2$ are collinear

B. $\overrightarrow{a}_1$ and $\overrightarrow{c}$ are collinear

C. $\overrightarrow{a} m \overrightarrow{a}_1$ and $\overrightarrow{b}$ are coplanar

D. $\overrightarrow{a}, \overrightarrow{a}_1$ and a_2 are coplanar

Answer: c

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16. Consider a triangular pyramid ABCD the position vectors of whose angular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 2, 3)$ and $D(0, -5, 4)$

Let G be the point of intersection of the medians of the triangle BCD. The length of the vec AG is

A. $\sqrt{17}$

B. $\sqrt{51}/3$

C. $3/\sqrt{6}$

D. $\sqrt{59}/4$

Answer: b

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17. Consider a triangular pyramid ABCD the position vectors of whone agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCT. The length of the perpendicular from the vertex D on the opposite face

A. (a) 24

B. (b) $8\sqrt{6}$

C. (c) $4\sqrt{6}$

D. (d) none of these

Answer: c

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18. Consider a triangular pyramid ABCD the position vectors of whose agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCD. The length of the vector $\overline{AG}$ is

A. $14/\sqrt{6}$

B. $2/\sqrt{6}$

C. $3/\sqrt{6}$

D. $\sqrt{5}$

Answer: a

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19. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D (x,y,z) The distance between the parallel lines AB and CD is

A. (a) $\sqrt{6}$

B. (b) $3\sqrt{6/5}$

C. (c) $2\sqrt{2}$

D. (d) 3

Answer: c

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20. Vertices of a parallelogram taken in order are A(2,-1,4)B(1,0,-1)C(1,2,3) and D.

Distance of the point P (8, 2,-12) from the plane of the parallelogram is

A. $\frac{4\sqrt{6}}{9}$

B. $\frac{32\sqrt{6}}{9}$

C. $\frac{16\sqrt{6}}{9}$

D. none

Answer: b

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21. Vertices of a parallelogram taken in order are A(2,-1,4)B(1,0,-1)C(1,2,3) and D.

Distance of the point P (8, 2,-12) from the plane of the parallelogram is

A. 14, 4,2

B. 2,4,14

C. 4,2,14

D. 2,14,4

Answer: d

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22. Let $\overrightarrow{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\overrightarrow{r}.\left(10\hat{j}-8\hat{i}-\overrightarrow{r}\right)=40$ and $p_1=\max\left\{\left|\overrightarrow{r}+2\hat{i}-3\hat{j}\right|^2\right\}, p_2=\min\left\{\left|\overrightarrow{r}+2\hat{i}-3\hat{j}\right|^2\right\}$. A tangent line is drawn to the curve $y=\frac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B

A. (a) 9

B. (b) $2\sqrt{2}-1$

C. (c) $6\sqrt{6}+3$

D. (d) $9-4\sqrt{2}$

Answer: d

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23. Let $\overrightarrow{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\overrightarrow{r}.\left(10\hat{j}-8\hat{i}-\overrightarrow{r}\right)=40$ and $p_1=\max\left\{\left|\overrightarrow{r}+2\hat{i}-3\hat{j}\right|^2\right\}, p_2=\min\left\{\left|\overrightarrow{r}+2\hat{i}-3\hat{j}\right|^2\right\}$. Then p_1+p_2 is equal to

A. 2

B. 10

C. 18

D. 5

Answer: c

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24. Let $\overrightarrow{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\overrightarrow{r}.\left(10\hat{j}-8\hat{i}-\overrightarrow{r}\right)=40$ and $p_1=\max\left\{\left|\overrightarrow{r}+2\hat{i}-3\hat{j}\right|^2\right\}, p_2=\min\left\{\left|\overrightarrow{r}+2\hat{i}-3\hat{j}\right|^2\right\}$. Then p_1+p_2 is equal to

A. 1

B. 2

C. 3

D. 4

Answer: c

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25. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC} = \vec{b}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$

vector $\overrightarrow{AB}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{\left|\vec{a}\right|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{\left|\vec{a}\right|^2} + \frac{3\left(\vec{b} \times \vec{a}\right)}{\left|\vec{a}\right|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{\left|\vec{a}\right|^2} - \frac{3\left(\vec{b} \times \vec{a}\right)}{\left|\vec{a}\right|^2}$

D. none of these

Answer: a

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26. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a}$ is $\frac{\left|\vec{a}\right|}{3}$ vector $\overrightarrow{AD}$ is

A. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times\left(\vec{b}-\vec{c}\right)}{\left|\vec{a}\right|^2}$

B. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times\left(\vec{b}-\vec{c}\right)}{\left|\vec{a}\right|^2}+\frac{3\left(\vec{b}\times\vec{a}\right)}{\left|\vec{a}\right|^2}$

C. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times\left(\vec{b}-\vec{c}\right)}{\left|\vec{a}\right|^2}-\frac{3\left(\vec{b}\times\vec{a}\right)}{\left|\vec{a}\right|^2}$

D. none of these

Answer: C

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27. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB}\times\overrightarrow{AC}=\vec{b}$ and $\overrightarrow{AD}\times\overrightarrow{AB}=\vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a}$ is $\frac{|\vec{a}|}{3}$

vector $\overrightarrow{AB}$ is

A. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times(\vec{b}-\vec{c})}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times(\vec{b}-\vec{c})}{|\vec{a}|^2}+\frac{3(\vec{b}\times\vec{a})}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times(\vec{b}-\vec{c})}{|\vec{a}|^2}-\frac{3(\vec{b}\times\vec{a})}{|\vec{a}|^2}$

D. none of these

Answer: A

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Martrix - match type

1.

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2.

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3.

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4. Given two vectors $\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{j} - \hat{k}$

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5. Given two vectors $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$

find $\left|\vec{a} \times \vec{b}\right|$

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6.

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7. Volume of parallelpiped formed by vectors $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ is 36 sq. units.

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8.

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9.

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10.

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Integer type

1. If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then find the greatest postive integer in the range of $\frac{3\left|\vec{a}+\vec{b}\right|}{2}+2\left|\vec{a}-\vec{b}\right|$

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2. Let $\vec{u}$ be a vector on rectangular coodinate system with sloping angle 60° suppose that $\left|\vec{u}-\hat{i}\right|$ is geomtric mean of $\left|\vec{u}\right|$ and $\left|\vec{u}-2\hat{i}\right|$, where $\hat{i}$ is the unit vector along the x-axis . Then find the value of $\frac{\sqrt{2}-1}{\left|\vec{u}\right|}$

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3. Find the absolute value of parameter t for which the area of the triangle whose vertices the $A(-1, 1, 2); B(1, 2, 3) and C(t, 1, 1)$ is minimum.

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4. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k},\ \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k},\ \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ and

$$\left[3\vec{a} + \vec{b}\ \ 3\vec{b} + \vec{c}\ \ 3\vec{c} + \vec{a}\right] = \lambda\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \text{ then find the value of }$$

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5. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - 3\hat{k},\ \vec{b} = \hat{i} + 2\alpha\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \alpha\hat{j} + \hat{k}$. Find the value of 6α. Such that

$$\left\{\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\right\} \times \left(\vec{c} \times \vec{a}\right) = 0$$

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6. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying $\left[(a-2)\alpha^2+(b-3)\alpha+c\right]\vec{x}+\left[(a-2)\beta^2+(b-3)\beta+c\right]\vec{y}+\left[(a-2)\right.$ are three distinct real numbers, then find the value of $\left(a^2+b^2+c^2-4\right)$.

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7. Let $\vec{u}$ and $\vec{v}$ be unit vectors such that $\vec{u}\times\vec{v}+\vec{u}=\vec{w}$ and $\vec{w}\times\vec{u}=\vec{v}$. Find the value of $\left[\vec{u}\,\vec{v}\,\vec{w}\right]$

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8. The volume of the tetrahedron whose vertices are the points with positon vectors $\hat{i}-6\hat{j}+10\hat{k},\ -\hat{i}-3\hat{j}+7\hat{k}, 5\hat{i}-\hat{j}+\lambda\hat{k}$ and $7\hat{i}-4\hat{j}+7\hat{k}$ is 11 cubic units if the value of λ is

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9. Given that $\vec{u} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{v} = 2\hat{i} + \hat{k} + 4\hat{k}$, $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$ and $\left(\vec{u} . \vec{R} - 15\right.$

. Then find the greatest integer less than or equal to $\left|\vec{R}\right|$.

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10. Let a three- dimensional vector $\vec{V}$ satisfy the condition , $2\vec{V} + \vec{V} \times \left(\hat{i} + 2\hat{j}\right) = 2\hat{i} + \hat{k}$. If $3\left|\vec{V}\right| = \sqrt{m}$. Then find the value of m.

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11. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} . \vec{b} = 0 = \vec{a} . \vec{c}$ and the angle between $\vec{b}$ and $\vec{c}\, is \frac{\pi}{3}$, then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$

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12. Let $\overrightarrow{O}A = \overrightarrow{a}, \overrightarrow{O}B = 10\overrightarrow{a} + 2\overrightarrow{b} and \overrightarrow{O}C = \overrightarrow{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB$, and let q denote the area of parallelogram with $OA and OC$ as adjacent sides. If $p = kq,$ then find$k$.

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13. Find the work done by the force $F = 3\hat{i} - \hat{j} - 2\hat{k}$ acting on a particle such that the particle is displaced from point $A(-3, -4, 1) to point B(-1, -1, -2)$.

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14. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are vectors in space given by $\overrightarrow{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\overrightarrow{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$ then find the value of $\left(2\overrightarrow{a} + \overrightarrow{b}\right).\left[\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \left(\overrightarrow{a} - 2\overrightarrow{b}\right)\right]$

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15. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = i + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$ then find the value of $\vec{r}.\vec{b}$.

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16. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9$ then find the value of $\left|2\vec{a} + 5\vec{b}\right.$

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17. Let $\vec{a}, \vec{b},$ and $\vec{c}$ be three non coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$ where p,q,r are scalars then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

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Subjective type

1. From a point O inside a triangle ABC, perpendiculars $OD, OEandOf$ are drawn to rthe sides $BC, CAandAB$, respecrtively. Prove that the perpendiculars from $A, B, andC$ to the sides $EF, FDandDE$ are concurrent.

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2. $A_1, A_2, \ldots A_n$ are the vertices of a regular plane polygon with n sides and O ars its centre. Show that $\sum_{i=1}^{n-1}\left(\overrightarrow{OA}_i \times \overrightarrow{OA}_{i+1}\right) = (n-1)\left(\overrightarrow{OA}_1 \times \overrightarrow{OA}_2\right)$

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3. If c is a given non - zero scalar, and $\vec{A}$ and $\vec{B}$ are given non- zero , vectors such that $\vec{A} \perp \vec{B}$. Then find vector, $\vec{X}$ which satisfies the equations $\vec{A} . \vec{X} = c$ and $\vec{A} \times \vec{X} = \vec{B}$.

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4. $A, B, C and D$ are any four points in the space, then prove that $\left|\vec{A}B \times \vec{C}D + \vec{B}C \times \vec{A}D + \vec{C}A \times \vec{B}D\right| = 4$ (area of ABC .)

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5. If the vectors $\vec{a}, \vec{b},$ and $\vec{c}$ are coplanar show that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a} . \vec{a} & \vec{a} . \vec{b} & \vec{a} . \vec{c} \\ \vec{b} . \vec{a} & \vec{b} . \vec{b} & \vec{b} . \vec{c} \end{vmatrix} = 0$$

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6.

$\overrightarrow{A} = \left(2\overrightarrow{i} + \overrightarrow{k}\right), \overrightarrow{B} = \left(\overrightarrow{i} + \overrightarrow{j} + \overrightarrow{k}\right)$ and $\overrightarrow{C} = 4\overrightarrow{i} - \overrightarrow{3}j + 7\overrightarrow{k}$

determine a $\overrightarrow{R}$ satisfying $\overrightarrow{R} \times \overrightarrow{B} = \overrightarrow{C} \times \overrightarrow{B}$ and $\overrightarrow{R}.\overrightarrow{A} = 0$

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7. Determine the value of c so that for the real x, vectors cx $\hat{i} - 6\hat{j} - 3\hat{k}$ and $x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other .

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8. If vectors, $\overrightarrow{b}, \overrightarrow{c}$ and $\overrightarrow{d}$ are not coplanar, the prove that vector $\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \left(\overrightarrow{c} \times \overrightarrow{d}\right) + \left(\overrightarrow{a} \times \overrightarrow{c}\right) \times \left(\overrightarrow{d} \times \overrightarrow{b}\right) + \left(\overrightarrow{a} \times \overrightarrow{d}\right) \times \left(\overrightarrow{b}\right.$

is parallel to $\overrightarrow{a}$.

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9. The position vectors of the vertices A, B and C of a tetrahedron ABCD are $\hat{i}+\hat{j}+\hat{k}$, $\hat{k}$, $\hat{i}$ and $\hat{3}i$,respectively. The altitude from vertex D to the opposite face ABC meets the median line through Aof triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is2√2/3, find the position vectors of the point E for all its possible positions

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10. Let a , b and c be non-coplanar unit vectors equally inclined to one another at an acute angle θ then [a b c] in terms of θ is equal to :

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11. If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are vectors such that $\left|\vec{B}\right| = \left|\vec{C}\right|$ prove that $\left[\left(\vec{A}+\vec{B}\right)\times\left(\vec{A}+\vec{C}\right)\right]\times\left(\vec{B}+\vec{C}\right).\left(\vec{B}+\vec{C}\right)=0$

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12. For any two vectors $\vec{u}$ and $\vec{v}$ prove that $\left(1+\left|\vec{u}\right|^2\right)\left(1+\left|\vec{v}\right|^2\right)=\left(1-\vec{u}.\vec{v}\right)^2+\left|\vec{u}+\vec{v}+\left(\vec{u}\times\vec{v}\right)\right|^2$

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13. Let $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w}+\vec{w}\times\vec{u}=\vec{v}$, then prove that $\left|\left(\vec{u}\times\vec{v}\right).\vec{w}\right|\leq\frac{1}{2}$ and that the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

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14. Find 3-dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying $\vec{v}_1\cdot\vec{v}_1=4, \vec{v}_1\cdot\vec{v}_2=-2, \vec{v}_1\cdot\vec{v}_3=6,$
$\vec{v}_2\cdot\vec{v}_2=2, \vec{v}_2\cdot\vec{v}_3=-5, \vec{v}_3\cdot\vec{v}_3=29$

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15. Let V be the volume of the parallelepied formed by the vectors, $\vec{a} = a_1\hat{i} = a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. are non- negative real numbers and $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$ show that $V \leq L^3$

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16. $\vec{u}, \vec{v}$ and $\vec{w}$ are three nono-coplanar unit vectors and α, β and γ are the angles between $\vec{u}$ and $\vec{u}, \vec{v}$ and $\vec{w}$ and $\vec{w}$ and $\vec{u}$, respectively and $\vec{x}, \vec{y}$ and $\vec{z}$ are unit vectors along the bisectors of the angles α, β and γ. respectively, prove that $\left[\vec{x} \times \vec{y}\,\vec{y} \times \vec{z}\,\vec{z} \times \vec{x}\right) = \frac{1}{16}\left[\vec{u}\,\vec{v}\,\vec{w}\right]^2 \frac{\sec^2\alpha}{2}\frac{\sec^2\beta}{2}\frac{\sec^2\gamma}{2}$.

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17. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ ar distinct vectors such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$. Prove that

$\left(\vec{a}-\vec{d}\right).\left(\vec{c}-\vec{b}\right)\neq 0, i.e., \vec{a}.\vec{b}+\vec{d}.\vec{c}\neq\vec{d}.\vec{b}+\vec{a}.\vec{c}.$

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18. $P_1 nd P_2$ are planes passing through origin $L_1 and L_2$ are two lines on $P_1 and P_2,$ respectively, such that their intersection is the origin. Show that there exist points $A, Band C,$ whose permutation $A', B' and C',$ respectively, can be chosen such that A is on $L_1, Bon P_1$ but not on $L_1 and C$ not on $P_1;$ A' is on $L_2, B' on P_2$ but not on $L_2 and C'$ not on $P_2.$

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19. about to only mathematics

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fill in the blanks

1. Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ be vectors of legth , 3,4and 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}$, $\vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to$\vec{A}+\vec{B}$ then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is ________.

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2. The unit vector perendicular to the plane determined by $P(1,\ -1,2), Q(2,0,\ -1)$ and $R(0,2,1)$.

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3. The area of the triangle whose vertices are

$A(1,\ -1,2), B(2,1-1)C(3,\ -1,2)$ is ....

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4. If $\vec{A}, \vec{B}, \vec{C}$ are non-coplanar vectors then $\frac{\vec{A}.\vec{B}\times\vec{C}}{\vec{C}\times\vec{A}.\vec{B}}+\frac{\vec{B}.\vec{A}\times\vec{C}}{\vec{C}.\vec{A}\times\vec{B}}=$

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5. If $\vec{A}=(1,1,1)$ and $\vec{C}=(0,1,-1)$ are given vectors then find a vector $\vec{B}$ satisfying equations $\vec{A}\times\vec{B}=\vec{C}$ and $\vec{A}.\vec{B}=3$

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6. Let $\vec{b}=4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. Find all vetors in te same plane having projection 1 and 2 along $\vec{b}$ and $\vec{c}$ respectively.

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7. The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are _______ and _________, respectively.

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8. A unit vector coplanar with $\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$ and perpendicular to $\vec{i} + \vec{j} + \vec{k}$ is ______

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9. A non vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\vec{i}, \vec{i} + \vec{j}$ and thepane determined by the vectors $\vec{i} - \vec{j}, \vec{i} + \vec{k}$ then angle between $\vec{a}$ and $\vec{i} - 2\vec{j} + 2\vec{k}$ is = (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

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10. If $\vec{b}$ and $\vec{c}$ are any two mutually perpendicular unit vectors and $\vec{a}$ is any vector, then $\left(\vec{a}.\vec{b}\right)\vec{b}+\left(\vec{a}.\vec{c}\right)\vec{c}+\frac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left|\vec{b}\times\vec{c}\right|^2}\left(\vec{b}\times\vec{c}\right)=$ (A) 0 (B) $\vec{a}\,(C)$veca /2(D)2veca`

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11. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1,1 and 2 resectively. If $\vec{a}\times\left(\vec{a}\times\vec{c}\right)+\vec{b}=\vec{0}$ then the acute angel between $\vec{a}$ and $\vec{c}$ is

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12. A, B C and D are four points in a plane with position vectors, $\vec{a}, \vec{b}\,\vec{c}$ and $\vec{d}$ respectively, such that

$\left(\vec{a}-\vec{d}\right).\left(\vec{b}-\vec{c}\right)=\left(\vec{b}-\vec{d}\right).\left(\vec{c}-\vec{a}\right)=0$ then point D is the _____ of triangle ABC.

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13. If $\vec{A}=\lambda\left(\vec{u}\times\vec{v}\right)+\mu\left(\vec{v}\times\vec{w}\right)+v\left(\vec{w}\times\vec{u}\right)$ and $\left[\vec{u}\,\vec{v}\,\vec{w}\right]=\frac{1}{5}then\lambda$

(A) 5 (B) 10 (C) 15 (D) none of these

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14. If $\vec{a}=\hat{j}+\sqrt{3}\hat{k}, \vec{b}=-\hat{j}+\sqrt{3}\hat{k}$ and $\vec{c}=2\sqrt{3}\hat{k}$ form a triangle , then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is

____

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True and false

1. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be unit vectors such that $\vec{A}.\vec{B} = \vec{A}.\vec{C} = 0$ and the angle between $\vec{B}$ and $\vec{C}$ be$\pi/3$. Then $\vec{A} = \pm 2\left(\vec{B} \times \vec{C}\right)$.

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2. If $\vec{x}.\vec{a} = 0\vec{x}.\vec{b} = 0$ and $\vec{x}.\vec{c} = 0$ for some non zero vector $\vec{x}$ then show that $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

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3. for any three vectors, $\vec{a}, \vec{b}$ and $\vec{c}$, $\left(\vec{a} - \vec{b}\right).\left(\vec{b} - \vec{c}\right) \times \left(\vec{c} - \vec{a}\right)$ =

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Exercise 2.1

1. Find $\left|\vec{a}\right|$ and $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=8$ and $\left|\vec{a}\right|=8\left|\vec{b}\right|$

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2. Show that $\left|\vec{a}\right|\vec{b}+\left|\vec{b}\right|\vec{a}$ is perpendicular to $\left|\vec{a}\right|\vec{b}-\left|\vec{b}\right|\vec{a}$ for any two non zero vectors `veca and vecb.

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3. If the vertices A,B,C of a triangle ABC are (1,2,3),(-1,0,0) ,(0,1,2) , respectively, then find $\angle ABC$.

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4. If $\left|\vec{a}\right|=3, \left|\vec{b}\right|=4$ and the angle between $\vec{a}$ and $\vec{b} is 120^{\circ}$. Then find the value of $\left|4\vec{a}+3\vec{b}\right|$

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5. If vectors $\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other, then find the locus of th point (x,y).

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6. Let $\vec{a}\vec{b}$ and $\vec{c}$ be pairwise mutually perpendicular vectors, such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 2, \left|\vec{c}\right| = 2$, the find the length of $\vec{a} + \vec{b} + \vec{c}$.

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7. If $\vec{a} + \vec{b} + \vec{c} = 0, \left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5, \left|\vec{c}\right| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

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8. If the angle between unit vectors $\overrightarrow{a}$ and $\overrightarrow{b} is 60^{\circ}$. Then find the value of $\left|\overrightarrow{a}-\overrightarrow{b}\right|$.

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9. Let $\overrightarrow{u}=\hat{i}+\hat{j}, \overrightarrow{v}=\hat{i}-\hat{j}$ and $\overrightarrow{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\widehat{n}$ is a unit vector such that $\overrightarrow{u}.\widehat{n}=0$ and $\overrightarrow{v}.\widehat{n}=0$, $\left|\overrightarrow{w}.\widehat{n}\right|$ is equal to (A) 0 (B) 1 (C) 2 (D) 3

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10. A,B,C and d are any four points prove that $\overrightarrow{AB}.\overrightarrow{CD}+\overrightarrow{BC}.\overrightarrow{AD}+\overrightarrow{CA}.\overrightarrow{BD}=0$

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11. $P(1, 0, -1), Q(2, 0, -3), R(-1, 2, 0) and S(3, -2, -1)$, then find the projection length of $\overrightarrow{P}Q$ and $\overrightarrow{R}S$.

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12. If the vectors $3\overrightarrow{P} + \overrightarrow{q}, 5\overrightarrow{P} - 3\overrightarrow{q}$ and $2\overrightarrow{p} + \overrightarrow{q}, 4\overrightarrow{p} - 2\overrightarrow{q}$ are pairs of mutually perpendicular vectors, the find the angle between vectors $\overrightarrow{p}$ and $\overrightarrow{q}$.

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13. Let $\overrightarrow{A}$ and $\overrightarrow{B}$ be two non-parallel unit vectors in a plane. If $\left(\alpha\overrightarrow{A} + \overrightarrow{B}\right)$ bisets the internal angle between $\overrightarrow{A}$ and $\overrightarrow{B}$ then find the value of α.

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14. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{x}, \vec{a} . \vec{x} = 1, \vec{b} . \vec{x} = \frac{3}{2}, \left|\vec{x}\right| = 2$ then find theh angle between $\vec{c}$ and $\vec{x}$.

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15. If $\vec{a}$ and $\vec{b}$ are unit vectors, then find the greatest value of $\left|\vec{a} + \vec{b}\right| + \left|\vec{a} - \vec{b}\right|$.

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16. Constant forces $P_1 = \hat{i} - \hat{j} + \hat{k}, P_2 = -\hat{i} + 2\hat{j} - \hat{i}k$ and $P_3 = \hat{j} - \hat{k}$ act on a particle at a point A . Determine the work done when particle is displaced from position $A\left(4\hat{i} - 3\hat{j} - 2\hat{k}\right)$ $\text{to} B\left(6\hat{i} + \hat{j} - 3\hat{k}\right)$

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17. If $\left|\vec{a}\right| = 5, \left|\vec{a} - \vec{b}\right| = 8$ and $\left|\vec{a} + \vec{b}\right| = 10$ then find $\left|\vec{b}\right|$

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18. If A, B, C, D are four distinct point in space such that AB is not perpendicular to CD and satisfies $\vec{A}B\dot{\vec{C}}D = k\left(\left|\vec{A}D\right|^2 + \left|\vec{B}C\right|^2 - \left|\vec{A}C\right|^2 = \left|\vec{B}D\right|^2\right)$, then find the value of $k\cdot$

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Exercise 2.2

1. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k}$ and $\vec{a} \times \vec{b} = \vec{0}$ then find (m,n)

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2. If $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 5$ and $\left|\vec{a} \times \vec{b}\right| = 8$ then find the value of $\vec{a} \cdot \vec{b}$

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3. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq 0$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar vectors, then for some scalar k prove that $\vec{a} + \vec{c} = k\vec{b}$.

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4. If $\vec{a} = 2\vec{j} + 3\vec{j} - \vec{k}, \vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{c} = \vec{i} + \vec{j} + \vec{k}$, then find the value of $\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{a} \times \vec{c}\right)$

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5. find the vector $\vec{c}, \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right -handed system, then find $\vec{c}$.

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6. given that $\vec{a}.\vec{b}=\vec{a}.\vec{c}, \vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}$ is not a zero vector. Show that $\vec{b}=\vec{c}$.

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7. Show that $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\vec{a}\times\vec{b}$ and give a geometrical interpretation of it.

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8. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\left|\vec{z}\right|=\frac{2}{\sqrt{7}}$ such that $\vec{z}+\vec{z}\times\vec{x}=\vec{y}$ then find the angle θ between $\vec{x}$ and $\vec{z}$

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9. Prove that $\left(\vec{a}.\hat{i}\right)\left(\vec{a}\times\hat{i}\right)+\left(\vec{a}.\hat{j}\right)\left(\vec{a}\times\hat{j}\right)+\left(\vec{a}.\hat{k}\right)\left(\vec{a}\times\hat{k}\right)=\vec{0}$

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10. Let a,b,c be three non-zero vectors such that $a+b+c=0$, then $\lambda b\times a+b\times c+c\times a=0, where\lambda$ is

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11. A particle has an angular speed of 3 rad/s and the axis of rotation passes through the points $(1, 1, 2) and (1, 2, -2)$. Find the velocity of the particle at point $P(3, 6, 4)$.

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12. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = 0 = \vec{a}.\vec{c}$. It the angle between $\vec{b}$ and $\vec{c}$ $is \frac{\pi}{6}$ then find $\vec{a}$.

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13. if $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a}.\vec{b}\right)^2 = 144$ and $\left|\vec{a}\right| = 4$ the find the value of $\left|\vec{b}\right|$

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14. Given $\left|\vec{a}\right| = \left|\vec{b}\right| = 1$ and $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$ if $\vec{c}$ is a vector such that $\vec{c} - \vec{a} - 2\vec{b} = 3\left(\vec{a} \times \vec{b}\right)$ then find the value of $\vec{c}.\vec{b}$.

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15. Find the moment of $\vec{F}$ about point (2, -1, 3), where force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is acting on point (1, -1, 2).

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Exercise 2.3

1. If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are four non-coplanar unit vectors such that $\vec{d}$ makes equal angles with all the three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ then prove that $\left[\vec{d}\ \vec{a}\ \vec{b}\right] = \left[\vec{d}\ \vec{c}\ \vec{b}\right] = \left[\vec{d}\ \vec{c}\ \vec{a}\right]$

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2. If $\vec{l}$, $\vec{m}$, $\vec{n}$ are three non coplanar vectors prove that $\left[\vec{}\right.$ vecm vecn] (vecaxxvecb) =|(vec1.veca, vec1.vecb, vec1),(vecm.veca, vecm.vecb, vecm), (vecn.veca, vecn.vecb, vecn)|`

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3. if the volume of a parallelpiped whose adjacent egges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}, \vec{c} = \vec{i} + 2\hat{j} + \alpha\hat{k}$ is 15 then find of α if $(\alpha > 0)$

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4. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find the vector $\vec{c}$ such that $\vec{a} . \vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$.

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5. If $\vec{x} . Veca = 0, \vec{x} . Vecb = 0$ and $\vec{x} . \vec{c} = 0$ for some non-zero vector $\vec{x}$. Then prove that $\left[\vec{a} \vec{b} \vec{c}\right] = 0$

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6. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find the vector $\vec{c}$ such that $\vec{a} . \vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$.

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7. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b}$ then prove that $|\vec{a}| = |\vec{b}| = |\vec{c}|$

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8. If $\vec{a} = \vec{P} + \vec{q}, \vec{P} \times \vec{b} = \vec{0}$ and $\vec{q} . \vec{b} = 0$ then prove that

$$\frac{\vec{b} \times \left(\vec{a} \times \vec{b}\right)}{\vec{b} . \vec{b}} = \vec{q}$$

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9. Prove that

$$\left(\vec{a}.\left(\vec{b}\times\hat{i}\right)\right)\hat{i}+\left(\vec{a}.\left(\vec{b}\times\hat{j}\right)\right)\hat{j}+\left(\vec{a}.\left(\vec{b}\times\hat{k}\right)\right)\hat{k}=\vec{a}\times\vec{b}$$

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10. for any four vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ prove that

$$\vec{d}.\left(\vec{a}\times\left(\vec{b}\times\left(\vec{c}\times\vec{d}\right)\right)\right)=\left(\vec{b}.\vec{d}\right)\left[\vec{a}\,\vec{c}\,\vec{d}\right]$$

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11. If $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors such that $\vec{a}\times\left(\vec{a}\times\vec{b}\right)=\frac{1}{2}\vec{b}$,then find the angle between $\vec{a}$ and $\vec{b}$.

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12. show that $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ if and only if $\vec{a}$ and $\vec{c}$ are collinear or $\left(\vec{a}\times\vec{c}\right)\times\vec{b}=\vec{0}$

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13. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that no two are collinear and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$ if θ is the acute angle between vectors $\vec{b}$ and $\vec{c}$ then find value of $\sin\theta$.

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14. If $\vec{p}, \vec{q}, \vec{r}$ denote vectors $\vec{b} \times \vec{c}, \vec{c} \times \vec{c} \times \vec{a}, \vec{a} \times \vec{b}$. Respectively, show that $\vec{a}$ is parallel to $\vec{q} \times \vec{r}$, $\vec{b}$ is parallel to $\vec{r} \times \vec{p}$, $\vec{c}$ is parallel to $\vec{p} \times \vec{q}$.\

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15. Let $\vec{a}, \vec{b}, \vec{c}$ be non -coplanar vectors and let equations $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vector $\vec{a}, \vec{b}, \vec{c}$ then prove that $\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}'$ is a null vector.

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16. Given unit vectors $\widehat{m}\widehat{n}$ and $\hat{p}$ such that angle between $\widehat{m}$ and $\widehat{n} is \alpha$ and angle between $\hat{p}$ and $\widehat{m} X \widehat{n} is \alpha$if [n p m] = 1/4 find alpha

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17. $\vec{a}, \vec{b}, and \vec{c}$ are three unit vectors and every two are inclined to each other at an angel $\cos^{-1}(3/5)$. If $\vec{a} \times \vec{b} = p\vec{a} + q\vec{b} + r\vec{c}, wherep, q, r$ are scalars, then find the value of q.

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18. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ gve three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both

$\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}\, is \frac{\pi}{6}$, then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} p = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$$

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single correct answer type

1. The scalar $\vec{A} . \left(\vec{B} + \vec{C}\right) \times \left(\vec{A} + \vec{B} + \vec{C}\right)$ equals (A) 0 (B) $\left[\vec{A}\,\vec{B}\,\vec{C}\right] + \left[\vec{B}\,\vec{C}\,\vec{A}\right]$ (C) $\left[\vec{A}\,\vec{B}\,\vec{C}\right]$ (D) none of these

A. 0

B. $\left[\vec{A}\,\vec{B}\,\vec{C}\right] + \left[\vec{B}\,\vec{C}\,\vec{A}\right]$

C. $\left[\vec{A}\,\vec{B}\,\vec{C}\right]$

D. none of these

Answer: a

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2. For non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $\left|\left(\vec{a} \times \vec{b}\right). \vec{c}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$ holds if and only if

A. $\vec{a}. \vec{b} = 0, \vec{b}. \vec{c} = 0$

B. $\vec{b}. \vec{c} = 0, \vec{c}, \vec{a} = 0$

C. $\vec{c}. \vec{a} = 0, \vec{a}, \vec{b} = 0$

D. $\vec{a}. \vec{b} = \vec{b}. \vec{c} = \vec{c}. \vec{a} = 0$

Answer: d

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3. The volume of he parallelepiped whose sides are given by $\vec{O}A = 2i - 2, j, \vec{O}B = i + j - kand\vec{O}C = 3i - k$ is a. $4/13$ b. 4 c. $2/7$ d. 2

A. $4/13$

B. 4

C. $2/7$

D. 2

Answer: d

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4. Let $\vec{a}, \vec{b}, \vec{c}$ be three noncolanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations $\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$ then the value of the expression $\left(\vec{a} + \vec{b}\right). \vec{p} + \left(\vec{b} + \vec{c}\right). \vec{q} + \left(\vec{c} + \vec{a}\right). \vec{r}.$ is equal to (A) 0 (B) 1 (C) 2 (D) 3

A. 0

B. 1

C. 2

D. 3

Answer: d

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5. Let $\overrightarrow{a} = \hat{i} - \hat{j}, \overrightarrow{b} = \hat{j} - \hat{k}, \overrightarrow{c} = \hat{k} - \hat{i}. If \hat{d}$ is a unit vector such that $\overrightarrow{a}.\hat{d} = 0 = \left[\overrightarrow{b}\overrightarrow{c}\overrightarrow{d}\right]$ then $\hat{d}$ equals

A. $\pm\frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$

B. $\pm\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

C. $\pm\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

D. $\pm\hat{k}$

Answer: a

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6. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non coplanar and unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}\Bigg)$ then the angle between vea and $\vec{b}$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

A. $3\pi / 4$

B. $\pi / 4$

C. $\pi / 2$

D. π

Answer: a

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7. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$ if $\left|\vec{u}\right| = 3, \left|\vec{v}\right| = 4$ and $\left|\vec{w}\right| = 5$ then $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$ is (a) 47 (b) -25 (c) 0 (d) 25

A. 47

B. -25

C. 0

D. 25

Answer: b

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8. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then $\left(\vec{a}+\vec{b}+\vec{c}\right).\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}+\vec{c}\right)\right]$ equals

A. 0

B. $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

C. $2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

D. $-\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

Answer: d

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9. Let $\vec{p}, \vec{q}, \vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation

$$\vec{p} \times \left\{\left(\vec{x} - \vec{q}\right) \times \vec{p}\right\} + \vec{q} \times \left\{\left(\vec{x} - \vec{r}\right) \times \vec{q}\right\} + \vec{r} \times \left\{\left(\vec{x} - \vec{p}\right) \times \vec{r}$$

,

then $\vec{x}$ is given by

A. (a) $\frac{1}{2}\left(\vec{p} + \vec{q} - 2\vec{r}\right)$

B. (b) $\frac{1}{2}\left(\vec{p} + \vec{q} + \vec{r}\right)$

C. (c) $\frac{1}{3}\left(\vec{p} + \vec{q} + \vec{r}\right)$

D. (d) $\frac{1}{3}\left(2\vec{p} + \vec{q} - \vec{r}\right)$

Answer: b

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10. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\vec{b} = \hat{i} + \hat{j}$ if c is a vector such that $\vec{a} . \vec{c} = \left|\vec{c}\right|, \left|\vec{c} - \vec{a}\right| = 2\sqrt{2}$ and the angle between

$\vec{a} \times \vec{b}$ and $\vec{i} s30^{\circ}$, then $\left|\left(\vec{a} \times \vec{b}\right)\right| \times \vec{c}\Big|$ is equal to

A. $2/3$

B. $3/2$

C. 2

D. 3

Answer: b

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11. Let $\vec{a} = 2i + j + k$, $\vec{b} = i + 2j - k$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is pependicular to $\vec{a}$. Then $\vec{c}$ is

A. $\dfrac{1}{\sqrt{2}}(-j+k)$

B. $\dfrac{1}{\sqrt{3}}(i-j-k)$

C. $\dfrac{1}{\sqrt{5}}(i-2j)$

D. $\dfrac{1}{\sqrt{3}}(i-j-k)$

Answer: a

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12. If the vectors $\vec{a}, \vec{b}, \vec{c}$ form the sides BC,CA and AB respectively of a triangle ABC then (A) $\vec{a}.\left(\vec{b} \times \vec{c}\right) = \vec{0}$ (B) $\vec{a} \times \left(\vec{b} x \vec{c}\right) = \vec{0}$ (C) $\vec{a}.\vec{b} = \vec{c} = \vec{c} = \vec{a}.a \ne 0$ (D) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\vec{0}$

A. $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$

B. $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

C. $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$

D. $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

Answer: b

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13. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be such that $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \vec{0}$. Let P_1 and P_2 be planes determined by pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ $respectively. Thenthe\angle between$P_1 and P_2$is(A)0(B)$pi/4$(C)$pi/3$(D)$pi/2`

A. 0

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

Answer: a

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14. If $\vec{a}, \vec{b}, \vec{c}$ are unit coplanar vectors then the scalar triple product $\left[2\vec{a} - \vec{b}, 2\vec{b} - c, 2c - \vec{a}\right]$ is equal to (A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$

A. 0

B. 1

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: a

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15. if $\widehat{a}, \hat{b}$ and $\hat{c}$ are unit vectors. Then $\left|\widehat{a}-\hat{b}\right|^2+\left|\hat{b}-\hat{c}\right|^2+\left|\overrightarrow{c}-\overrightarrow{a}\right|^2$ does not exceed

A. 4

B. 9

C. 8

D. 6

Answer: b

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16. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is (A) 45^0 (B) 60^0 (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

A. 45°

B. 60°

C. $\cos^{-1}(1/3)$

D. $\cos^{-1}(2/7)$

Answer: b

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17. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{W} = \hat{i} + 3\hat{k}$. if $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $\left[\vec{U}\,\vec{V}\,\vec{W}\right]$ is

A. -1

B. $\sqrt{10}+\sqrt{6}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: c

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18. Find the value of a so that the volume of the parallelopiped formed by vectors $\hat{i}+a\hat{j}+\hat{k}, \hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum.

A. -3

B. 3

C. $1/\sqrt{3}$

D. $\sqrt{3}$

Answer: c

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19. If $\overrightarrow{a} = \left(\hat{i} + \hat{j} + \hat{k}\right), \overrightarrow{a} . \overrightarrow{b} = 1 \text{ and } \overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}, \text{ then } \overrightarrow{b}$ is

(a) $\hat{i} - \hat{j} + \hat{k}$ (b) $2\hat{i} - \hat{k}$ (c) $\hat{i}$ (d) $2\hat{i}$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{i} - \hat{k}$

C. $\hat{i}$

D. $2\hat{i}$

Answer: c

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20. The unit vector which is orthogonal to the vector $5\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is (a) $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$ (b) $\frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$ (c) $\frac{3\hat{j} - \hat{k}}{\sqrt{10}}$ (d) $\frac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

A. $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$

B. $\dfrac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

C. $\dfrac{3\hat{j}-\hat{k}}{\sqrt{10}}$

D. $\dfrac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

Answer: c

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21. if $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero, non- coplanar vectors and

$$\vec{b}_1=\vec{b}-\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}, \vec{b}_2=\vec{b}+\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}, \vec{c}_1=\vec{c}-\frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a}+\frac{\vec{b}.}{|\vec{c}|}$$

$$-\frac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a}=\frac{\vec{b}.\vec{c}}{|\vec{b}|^2}\vec{b}_1$$

, then the set of orthogonal vectors is

A. $\left(\vec{a}, \vec{b}_1, \vec{c}_3\right)$

B. $\left(\vec{c}a, \vec{b}_1, \vec{c}_2\right)$

C. $\left(\vec{a}, \vec{b}_1, \vec{c}_1\right)$

D. $\left(\vec{a}, \vec{b}_2, \vec{c}_2\right)$

Answer: c

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22. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \overrightarrow{=} \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c} is \frac{1}{\sqrt{3}}$ is (A) $4\hat{i} - \hat{j} + 4\hat{k}$ (B) $\hat{i} + \hat{j} - 3\hat{k}$ (C) $2\hat{i} + \hat{j} - 2\hat{k}$ (D) $4\hat{i} + \hat{j} - 4\hat{k}$

A. $4\hat{i} - \hat{j} + 4\hat{k}$

B. $3\hat{i} + \hat{j} - 3\hat{k}$

C. $2\hat{i} + \hat{j} - 2\hat{k}$

D. $4\hat{i} + \hat{j} - 4\hat{k}$

Answer: a

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23. Lelt two non collinear unit vectors $\widehat{a}$ and $\hat{b}$ form and acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\widehat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\widehat{u}$ be the unit vector along $\overrightarrow{OP}$ Then (A)

$$\widehat{u} = \frac{\widehat{a} + \hat{b}}{\left|\widehat{a} + \hat{b}\right|} \text{ and } M = \left(1 + \widehat{a}.\,\hat{b}\right)^{\frac{1}{2}}$$ (B)

$$\widehat{u} = \frac{\widehat{a} - \hat{b}}{\left|\widehat{a} - \hat{b}\right|} \text{ and } M = \left(1 + \widehat{a}.\,\hat{b}\right)^{\frac{1}{2}}$$ (C)

$$\widehat{u} = \frac{\widehat{a} + \hat{b}}{\left|\widehat{a} + \hat{b}\right|} \text{ and } M = \left(1 + 2\widehat{a}.\,\hat{b}\right)^{\frac{1}{2}}$$ (D)

$$\widehat{u} = \frac{\widehat{a} - \hat{b}}{\left|\widehat{a} - \hat{b}\right|} \text{ and } M = \left(1 + 2\widehat{a}.\,\hat{b}\right)^{\frac{1}{2}}$$

A. , $\widehat{u} = \dfrac{\widehat{a} + \hat{b}}{\left|\widehat{a} + \hat{b}\right|}$ and $M = \left(1 + \widehat{a}.\,\hat{b}\right)^{1/2}$

B. , $\widehat{u} = \dfrac{\widehat{a} - \hat{b}}{\left|\widehat{a} - \hat{b}\right|}$ and $M = \left(1 + \widehat{a}.\,\hat{b}\right)^{1/2}$

C. $\widehat{u} = \dfrac{\widehat{a} + \hat{b}}{\left|\widehat{a} + \hat{b}\right|}$ and $M = \left(1 + 2\widehat{a}.\,\hat{b}\right)^{1/2}$

D. , $\widehat{u} = \dfrac{\widehat{a} - \hat{b}}{\left|\widehat{a} - \hat{b}\right|}$ and $M = \left(1 + 2\widehat{a}.\,\hat{b}\right)^{1/2}$

Answer: a

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24. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a} \times \vec{b}\right).\left(\vec{c} \times \vec{d}\right) = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$ then (A) $\vec{a}, \vec{b}, \vec{c}$ are non coplanar (B) $\vec{b}, \vec{c}, \vec{d}$ are non coplanar (C) $\vec{b}, \vec{d}$ are non paralel (D) $\vec{a}, \vec{d}$ are paralel and $\vec{b}, \vec{c}$ are parallel

A. $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar

B. $\vec{b}, \vec{c}$ and $\vec{d}$ are non-coplanar

C. $\vec{b}$ and $\vec{d}$ are non- parallel

D. $\vec{a}$ and $\vec{d}$ are parallel and $\vec{b}$ and $\vec{c}$ are parallel

Answer: c

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25. Two adjacent sides of a parallelogram $ABCD$ are given by $\vec{A}B = 2\hat{i} + 10\hat{j} + 11\hat{k} and \vec{A}D = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD

becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angel α is given by a. $\frac{8}{9}$ b. $\frac{\sqrt{17}}{9}$ c. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: b

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26. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a

A. Parallelogram, which is neither a rhombus nor a rectangle

B. square

C. rectangle, but not a square

D. rhombus, but not a square.

Answer: a

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27. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vectors $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}\, is \frac{1}{\sqrt{3}}$ is given by

A. $\hat{i} - 3\hat{j} + 3\hat{k}$

B. $-3\hat{i} - 3\hat{j} + \hat{k}$

C. $3\hat{i} - \hat{j} + 3\hat{k}$

D. $\hat{i} + 3\hat{j} - 3\hat{k}$

Answer: c

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28. Let $\overline{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overline{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overline{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overline{PT}, \overline{PQ}$ and $\overline{PS}$ is

A. 5

B. 20

C. 10

D. 30

Answer: c

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Multiple correct answers type

1. Let

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non- zero vectors such that $\vec{c}$ is a unit vectors perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$ then

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

A. (a) 0

B. (b) 1

C. (c) $\frac{1}{4}\left(a_1^2 + a_2^2 + a_2^2\right)\left(b_1^2 + b_2^2 + b_2^2\right)$

D. (d) $\frac{3}{4}\left(a_1^2 + a_2^2 + a_2^2\right)\left(b_1^2 + b_2^2 + b_2^2\right)\left(c_1^2 + c_2^2 + c_2^2\right)$

Answer: c

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2. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0) and \vec{b} = (0, 1, 1)$ is a. one b. two c. three d. infinite

A. one

B. two

C. three

D. infinite

Answer: b

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3. Let $\overrightarrow{a} = 2\hat{i} + \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors . A vector in the plane of $\overrightarrow{b}$ and $\overrightarrow{c}$ whose projection on $\overrightarrow{a}$ is of magnitude $\sqrt{\left(\frac{2}{3}\right)}$ is (A) $2\hat{i} + 3\hat{j} + 3\hat{k}$ (B) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (C) $-2\hat{i} - \hat{j} + 5\hat{k}$ (D) $2\hat{i} + \hat{j} + 5\hat{k}$

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $2\hat{i} + 3\hat{j} + 3\hat{k}$

C. $-2\hat{i} - \hat{j} + 5\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: a,c

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4. For three vectors, $\vec{u}$, $\vec{v}$ and $\vec{w}$ which of the following expressions is not equal to any of the remaining three ?

A. (a) $\vec{u} . (\vec{v} \times \vec{w})$

B. (b) $(\vec{v} \times \vec{w}) . \vec{u}$

C. (c) $\vec{v} . (\vec{u} \times \vec{w})$

D. (d) $(\vec{u} \times \vec{v}) . \vec{w}$

Answer: c

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5. Which of the following expressions are meaningful? $\vec{u}.\left(\vec{v}\times\vec{w}\right)$ b. $\left(\vec{u}.\vec{v}\right).\vec{w}$ c. $\left(\vec{u}.\vec{v}\right).\vec{w}$ d. $\vec{u}\times\left(\vec{v}.\vec{w}\right)$

A. $\vec{u}.\left(\vec{v}\times\vec{w}\right)$

B. $\left(\vec{u}.\vec{v}\right).\vec{w}$

C. $\left(\vec{u}.\vec{v}\right)\vec{w}$

D. $\vec{u}\times\left(\vec{v}.Vecw\right)$

Answer: a,c

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6. If $\vec{a}$ and $\vec{b}$ are two non collinear vectors and $\vec{u}=\vec{a}-\left(\vec{a}.\vec{b}\right).\vec{b}$ and $\vec{v}=\vec{a}x\vec{b}$ then $\vec{v}$ is

A. $\left|\vec{u}\right|$

B. $\left|\vec{u}\right|+\left|\vec{u}.Veca\right|$

C. $\left|\vec{u}\right|+\left|\vec{u}.\vec{b}\right|$

D. $\left|\vec{u}\right| + \vec{u} . \left(\vec{a} + \vec{b}\right)$

Answer: a,c

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7. $\vec{P} = \left(2\hat{i} - 2\hat{j} + \hat{k}\right)$, then find $\left|\vec{P}\right|$

A. a unit vector

B. makes an angle $\pi / 3$ with vector $\left(2\hat{i} - 4\hat{j} + 3\hat{k}\right)$

C. parallel to vector $\left(- \hat{i} + \hat{j} - \frac{1}{2}\hat{k}\right)$

D. perpendicular to vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

Answer: a,c,d

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8. Let $\overrightarrow{a}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. If $P_1$is parallel to the vectors $2\bar{j}+3\bar{k}$ and $4\bar{j}-3\bar{k}$ and P_2 is parallel to $\bar{j}-\bar{k}$ and $3\bar{I}+3\bar{j}$, then the angle between $\overrightarrow{a}$ and $2\bar{i}+\bar{j}-2\bar{k}$ is :

A. $\pi/2$

B. $\pi/4$

C. $\pi/6$

D. $3\pi/4$

Answer: b,d

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9. The vectors which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is /are (A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

A. $\hat{j}-\hat{k}$

B. $-\hat{i}+\hat{j}$

C. $\hat{i}-\hat{j}$

D. $-\hat{j}+\hat{k}$

Answer: a,d

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10. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$ if $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then

A. (a) $\vec{b}=\left(\vec{b}.\vec{z}\right)\left(\vec{z}-\vec{x}\right)$

B. (b) $\vec{a}=\left(\vec{a}.\vec{y}\right)\left(\vec{y}-\vec{z}\right)$

C. (c) $\vec{a}.\vec{b}=-\left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right)$

D. (d) $\vec{a}=\left(\vec{a}.\vec{y}\right)\left(\vec{z}-\vec{y}\right)$

Answer: a,b,c

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11. Let PQR be a triangle . Let $\vec{a} = \overline{QR}, \vec{b} = \overline{RP}$ and $\vec{c} = \overline{PQ}$. if $\left|\vec{a}\right| = 12, \left|\vec{b}\right| = 4\sqrt{3}$ and $\vec{b} . \vec{c}$

then which of the following is (are) true ?

A. (a) $\dfrac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = 12$

B. (b) $\dfrac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = 30$

C. (c) $\left|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}\right| = 48\sqrt{3}$

D. (d) $\vec{a} . \vec{b} = -72$

Answer: a,c,d

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