MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

DIFFERENT PRODUCTS OF VECTORS AND THEIR GEOMETRICAL APPLICATIONS

Illustration

1. Find the angle between the following pairs of vectors $3\hat{i} + 2\hat{j} - 6\hat{k}, 4\hat{i} - 3\hat{j} + \hat{k}, \hat{i} - 2\hat{j} + 3\hat{k}, 3\hat{i} - 2\hat{j} + \hat{k}$

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2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non - zero vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$,then find the goemetrical relation between the vectors.

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3. if $\vec{r}.\vec{i}=\vec{r}.\vec{j}=\vec{r}.\vec{k}$ and $|\vec{r}|=3$, then find vector $\vec{r}$.

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4. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is

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5. if $\vec{a},\vec{b}$ and $\vec{c}$ are mutally perpendicular vectors of equal magnitudes, then find the angle between vectors and $\vec{a}+\vec{b}+\vec{c}$.

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6. If $|\vec{a}|+|\vec{b}|=|\vec{c}|$ and $\vec{a}+\vec{b}=\vec{c}$ then find the angle between $\vec{a}$ and $\vec{b}$.

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7. If three unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Then find the angle between $\vec{a}$ and $\vec{b}$.

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8. If θ is the angle between the unit vectors $\vec{a}$ and $\vec{b}$, then prove that

i. $\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} + \vec{b}\right|$

$ii.$ $\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$

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9. find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

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10. If the scalar projection of vector $x\hat{i} - \hat{j} + \hat{k}1$ on vector $2\hat{i} - \hat{j} + 5\hat{k}is\frac{1}{\sqrt{30}}$.

The find the value of x.

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11. If $\vec{a} = x\hat{i} + (x - 1)\hat{j} + \hat{k}$ and $\vec{b} = (x + 1)\hat{i} + \hat{j} + a\hat{k}$ make an acute angle $\forall x \in R$, then find the values of a.

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12. If $\vec{a}.\vec{i} = \vec{a}.\left(\hat{i} + \hat{j}\right) = \vec{a}.\left(\hat{i} + \hat{j} + \hat{k}\right)$. Then find the unit vector $\vec{a}$.

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13. Prove by vector method that $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

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14. In any triangle ABC, prove the projection formula $a = b\cos C + c\cos B$ using vector method.

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15. Prove that an angle inscribed in a semi-circle is a right angle using vector method.

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16. Using dot product of vectors, prove that a parallelogram, whose diagonals are equal, is a rectangle

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17. If $a + 2b + 3c = 4$, then find the least value of $a^2 + b^2 + c^2$.

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18. A unit vector a makes an angle $\frac{\pi}{4}$ with z-axis. If $a + i + j$ is a unit vector, then a can be equal to

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19. vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are of the same length and when taken pair-wise they form equal angles. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ then find vector $\vec{c}$.

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20. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a unit vector which makes equal angle with $\vec{a}, \vec{b}$ and $\vec{c}$, then find the value of $\left|\vec{a} + \vec{b} + \vec{c} + \vec{d}\right|^2$.

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21. A particle acted on by constant forces $4\vec{i} + \vec{j} - 3\vec{k}$ and $3\vec{i} + \vec{j} - \vec{k}$ is displaced from the point $\vec{i} + 2\vec{j} + 3\vec{k}$ to the point $5\vec{i} + 4\vec{j} + \vec{k}$. Find the total work done by the forces

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22. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$

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23. If $\vec{a} = 4\hat{i} + 6\hat{j}$ and $\vec{b} = 3\hat{i} + 4\hat{k}$ find the vector component of $\vec{a}$ along $\vec{b}$.

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24. If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{a} + \vec{b}\right| = 1$ then find the value of $\left|\vec{a} - \vec{b}\right|$

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25. If $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 0\hat{j} + \hat{k}$ then find vector $\vec{c}$ satisfying the following conditions, (i) that it is coplaner with $\vec{a}$ and $\vec{b}$, (ii) that it is $\perp$ to$\vec{b}$ and (iii) that $\vec{a}.\vec{c} = 7$.

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26. If $\vec{a}, \vec{b}$ and $\vec{c}$ are vectors such that $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$ and $(\vec{a} + \vec{b})$ is perpendicular to $\vec{c}$, $(\vec{b} + \vec{c})$ is perpendicular to $\vec{a}$ and $(\vec{c} + \vec{a})$ is perpendicular to $\vec{b}$ then $|\vec{a} + \vec{b} + \vec{c}| =$

(A) $4\sqrt{3}$ (B) $5\sqrt{2}$ (C) 2 (D) 12

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27. Prove that in a tetrahedron if two pairs of opposite edges are perpendicular , then the third pair is also perpendicular.

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28. In the isosceles triangle ABC, $\left|\vec{AB}\right| = \left|\vec{BC}\right| = 8$,a point E divide AB internally in the ratio $1:3$, then the cosine of the angle between $\vec{CE}$ and $\vec{CA}$ is (where $\left|\vec{CA}\right| = 12$)

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29. An arc AC of a circle subtends a right angle at then the center O. the point B divides the arc in the ratio $1:2$, If $\vec{O}A = a\&\vec{O}B = b$. then the vector $\vec{O}C$ in terms of $a\&b$, is

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30. Vector $\vec{O}A = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle passing through the positive x-axis on the way. Show that the vector in its new position is

$\frac{4\hat{i} - \hat{j} - \hat{k}\cdot}{\sqrt{2}}$

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31. The base of the pyramid $AOBC$ is an equilateral triangle OBC with each side equal to $4\sqrt{2}$, O is the origin of reference, AO is perpendicualar to the plane of OBC and $\left|\vec{AO}\right| = 2$. Then find the cosine of the angle between the skew straight lines, one passing though A and the midpoint of $OBand$ the other passing through O and the mid point of $BC^{\cdot}$

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32. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

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33. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$ then $|\vec{a} \times \vec{b}$ is a unit vector. If the angle between $\vec{a}$ and $\vec{b}$ is ?

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34. Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$ also interpret this result.

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35. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15.$

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36. If $A, BandC$ are the vetices of a triangle $ABC,$ then prove sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

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37. Using cross product of vectors, prove that $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

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38. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1) and (0, 2, 1)$.

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39. If $\vec{a}$ and $\vec{b}$ are two vectors, then prove that $\left(\vec{a}\times\vec{b}\right)^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} \end{vmatrix}$

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40. If $\left|\vec{a}\right| = 2$ then find the value of $\left|\vec{a}\times\vec{i}\right|^2 + \left|\vec{a}\times\vec{j}\right|^2 + \left|\vec{a}\times\vec{k}\right|^2$

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41. $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}, \vec{r} \times \vec{b} = \vec{a} \times \vec{b}, \vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}, \vec{a} \neq \lambda \vec{b}$ and $\vec{a}$ is not perpendicular to $\vec{b}$, then find $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$.

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42. $A, B, CandD$ are any four points in the space, then prove that $\left|\vec{A}B \times \vec{C}D + \vec{B}C \times \vec{A}D + \vec{C}A \times \vec{B}D\right| = 4$ (area of ABC .)

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43. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of the vertices A,B and C. respectively , of $\triangle ABC$. Prove that the perpendicualar distance of the vertex A from the base BC of the triangle ABC is $\dfrac{\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|}{\left|\vec{c} - \vec{b}\right|}$

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44. Using vectors, find the area of the triangle with vertices $A(1, 1, 2),\ B(2, 3, 5) and\ C(1, 5, 5)$

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45. Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$`

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46. Area of a parallelogram, whose diagonals are $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$ will be:

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47. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} \neq 0$, $|\vec{a}| = |\vec{c}| = 1$, $|\vec{b}| = 4$ and $|\vec{b} \times \vec{c}| = \sqrt{15}$. If $\vec{b} - 2\vec{c} = \lambda\vec{a}$ then find the value of λ .

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48. Find the moment about (1,-1,-1) of the force $3\hat{i} + 4\hat{j} - 5\hat{k}$ acting at (1,0,-2)

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49. A rigid body is spinning about a fixed point (3,-2,-1) with an angular velocity of 4 rad/s, the axis of rotation being in the direction of (1,2,-2). Find the velocity of the particle at point (4,1,1).

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50. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $\left(\vec{a} - \vec{d}\right)$ is parallel to $\left(\vec{b} - \vec{c}\right)$.

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51. Show by a numerical example that $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ does not imply $\vec{b} = \vec{c}$.

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52. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the vertices of a cycle quadrilateral ABCD, prove that

$$\frac{\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{d} + \vec{d} \times \vec{a}\right|}{\left(\vec{b} - \vec{a}\right).\left(\vec{d} - \vec{a}\right)} + \frac{\left|\vec{b} \times \vec{c} + \vec{c} \times \vec{d} + \vec{d} + \vec{d} \times \vec{b}\right|}{\left(\vec{b} - \vec{c}\right).\left(\vec{d} - \vec{c}\right)}$$

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53. The postion vectors of the vertrices fo aquadrilateral with A as origian are $B(\vec{b}), D(\vec{d})$ and $C(l\vec{b} + m\vec{d})$. Prove that the area of the quadrilateral is $\frac{1}{2}(l + m)\left|\vec{b} \times \vec{d}\right|$.

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54. Let $\vec{a}$ and $\vec{b}$ be unit vectors such that $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$. Then find the value of $\left(2\vec{a} + 5\vec{b}\right).\left(3\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

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55. $uandv$ are two non-collinear unit vectors such that $\left|\frac{\hat{u} + \hat{v}}{2} + \hat{u} \times \hat{v}\right| = 1.$ Prove that $\left|\hat{u} \times \hat{v}\right| = \left|\frac{\hat{u} - \hat{v}}{2}\right|$.

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56. In a $\triangle ABC$ points D,E,F are taken on the sides BC,CA and AB respectively such that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n$ prove that $\triangle DEF = \frac{n^2 - n + 1}{(n + 1)^2} \triangle ABC$

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57. Let A,B,C be points with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} + \hat{k}$ and $3\hat{i} + \hat{j} + 2\hat{k}$ respectively. Find the shortest distance between point B and plane OAC.

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58. Let $\vec{a} = x\hat{i} + 12\hat{j} - \hat{k}, \vec{b} = 2\hat{i} + 2x\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$. If the ordered set $\left[\vec{b}\vec{c}\vec{a}\right]$ is left handed, then find the value of x.

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59. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then find the value of

$$\frac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\vec{b}.\left(\vec{c}\times\vec{a}\right)}+\frac{\vec{b}.\left(\vec{c}\times\vec{a}\right)}{\vec{c}.\left(\vec{a}\times\vec{b}\right)}+\frac{\vec{c}.\left(\vec{b}\times\vec{a}\right)}{\vec{a}.\left(\vec{b}\times\vec{c}\right)}$$

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60. if the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k}$ and $3\hat{i} - \hat{k}$ from three concurrent edges of a parallelpiped, then find the volume of the parallelepied.

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61. The postion vectors of the four angular points of a tetrahedron are $A\left(\hat{j} + 2\hat{k}\right), B\left(3\hat{i} + \hat{k}\right), C\left(4\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and $D\left(2\hat{i} + 3\hat{j} + 2\hat{k}\right)$ find the volume of the tetrahedron ABCD.

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62. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors and $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$ then find the value of $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$

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63. Prove that $\left[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right]$

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64. Show that : $\left[\vec{l}\vec{m}\vec{n}\right]\left[\vec{a}\vec{b}\vec{c}\right] = \begin{vmatrix} \vec{l}.\vec{a} & \vec{l}.\vec{b} & \vec{l}.\vec{c} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}.\vec{c} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n}.\vec{c} \end{vmatrix}$

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65. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \hat{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, then find the value of

$$\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$$

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66. The value of a so thast the volume of parallelpiped formed by vectors $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k}, a\hat{i} + \hat{k}$ becomes minimum is (A) $\sqrt{93}\big)$ (B) 2 (C) $\dfrac{1}{\sqrt{3}}$ (D) 3

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67. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non coplanar vectors then $\left(\vec{u} + \vec{v} - \vec{w}\right).\left(\vec{u} - \vec{v}\right) \times \left(\vec{v} - \vec{w}\right)$ equals (A) $\vec{u}.\left(\vec{v} \times \vec{w}\right)$ (B) $\vec{u}.\vec{w} \times \vec{v}$ (C) $2\vec{u}.\left(\vec{v} \times \vec{w}\right)$ (D) 0

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68. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\left|\vec{a} \times \vec{b}\right| = 2$, then find the value of $\left[\vec{a}\vec{b}\vec{a}\right] \times \vec{b}$.

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69. Find th altitude of a parallelepiped whose three coterminous edges are vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}, \vec{B} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{C} = \hat{i} + \hat{j} + 3\hat{k} \text{with} \vec{A}$ and $\vec{B}$ as the sides of the base of the parallelepiped .

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70. If $\left[\vec{a}\vec{b}\vec{c}\right] = 2$, then find the value of $\left[\left(\vec{a} + 2\vec{b} - \vec{c}\right)\left(\vec{a} - \vec{b}\right)\left(\vec{a} - \vec{b} - \vec{c}\right)\right]$

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71. If $\vec{a}, \vec{b}$ and $\vec{c}$ are , mutually perpendicular vcetors and $\vec{a} = \alpha\left(\vec{a} \times \vec{b}\right) + \beta\left(\vec{b} \times \vec{c}\right) + \gamma\left(\vec{c} \times \vec{a}\right)$ and $\left[\vec{a}\vec{b}\vec{c}\right] = 1$, then find the value of $\alpha + \beta + \gamma$

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72. If $\vec{a}, \vec{b}a$ and $\vec{c}$ are non- coplanar vecotrs, then prove that $|\left(\vec{a}.\vec{d}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{b}.\vec{d}\right)(\vec{c} \times \vec{a}) + \left(\vec{c}.\vec{d}\right)\left(\vec{a} \times \vec{b}\right)$ is independent of $\vec{d}$ where $\vec{d}$ is a unit vector.

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73. Prove that vectors $\vec{u} = \left(al + a_1 l_1\right)\hat{i} + \left(am + a_1 m_1\right)\hat{j} + \left(an + a_1 n_1\right)\hat{k}$

$\vec{v} = \left(bl + b_1 l_1\right)\hat{i} + \left(bm + b_1 m_1\right)\hat{j} + \left(bn + b_1 n_1\right)\hat{k}$

$\vec{w} = \left(cl + c_1 l_1\right)\hat{i} + \left(cm + c_1 m_1\right)\hat{j} + \left(cn + c_1 n_1\right)\hat{k}$ are coplanar.

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74. Let G_1, G_2 and G_3 be the centroids of the trianglular faces OBC,OCA and OAB, respectively, of a tetrahedron OABC. If V_1 denotes the volume of the tetrahedron OABC and V_2 that of the parallelepiped with OG_1, OG_2 and OG_3 as three concurrent edges, then prove that $4V_1 = 9V_2$.

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75. Prove that $\hat{i} \times \left(\vec{a} \times \vec{i}\right) + \hat{j} \times \left(\vec{a} \times \vec{j}\right) + \hat{k} \times \left(\vec{a} \times \vec{k}\right) = 2\vec{a}$

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76. If $\hat{i} \times \left[\left(\vec{a} - \hat{j}\right) \times \hat{i}\right] \times \left[\left(\vec{a} - \hat{k}\right) \times \hat{j}\right] + \vec{k} \times \left[\left(\vec{a} - \vec{i}\right) \times \hat{k}\right] = 0$, then find vector $\vec{a}$.

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77. Let $vea, \vec{b}$ and $\vec{c}$ be any three vectors, then prove that $\left[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}\right] = \left[\vec{a}\vec{b}\vec{c}\right]^2$

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78. For any four vectors prove that

$\left(\vec{b}\times\vec{c}\right).\left(\vec{a}\times\vec{d}\right)+\left(\vec{c}\times\vec{a}\right).\left(\vec{b}\times\vec{d}\right)+\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=0$

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79. If $\vec{b}$ and $\vec{c}$ are two non-collinear such that $\vec{a}\ |\ \ |\left(\vec{b}\times\vec{c}\right)$. Then prove that $\left(\vec{a}\times\vec{b}\right).\left(\vec{a}\times\vec{c}\right)$ is equal to $|\vec{a}|^2\left(\vec{b}.\vec{c}\right)$`

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80. Find the vector of length 3 unit which is perpendicular to $\hat{i}+\hat{j}+\hat{k}$ and lies in the plane of $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-3\hat{j}$

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81. Let $\hat{a}, \hat{b}$,and $\hat{c}$ be the non-coplanar unit vectors. The angle between $\hat{b}$ and $\hat{c}$ is α , between $\hat{c}$ and $\hat{a}$ is β and between $\hat{a}$ and $\hat{b}$ is γ . If $A(\hat{a}\cos\alpha, 0), B(\hat{b}\cos\beta, 0)$ and $C(\hat{c}\cos\gamma, 0)$, then show that in triangle ABC, $\frac{|\hat{a} \times (\hat{b} \times \hat{c})|}{\sin A} = \frac{|\hat{b} \times (\hat{c} \times \hat{a})|}{\sin B} = \frac{|\hat{c} \times (\hat{a} \times \hat{b})|}{\sin C}$

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82. find the cosine of the angle between the vectors $\vec{a} = 3\hat{i}+2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

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83. If $\vec{b}$ is not perpendicular to $\vec{c}$. Then find the vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ and $\vec{r}.\vec{c} = 0$

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84. If $\vec{a}$ and $\vec{b}$ are two given vectors and k is any scalar,then find the vector $\vec{r}$ satisfying $\vec{r} \times \vec{a} + k\vec{r} = \vec{b}$.

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85. $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}, \vec{r} \times \vec{b} = \vec{a} \times \vec{b}, \vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}, \vec{a} \neq \lambda\vec{b}$ and $\vec{a}$ is not perpendicular to $\vec{b}$, then find $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$.

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86. if vectors $3\hat{i}$ - 2 $\hat{j}$ + m $\hat{k}$ and - $2\hat{i}$ + $\hat{j}$ + 4 hat k are perpendicular to each other , find the value of m

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87. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non- coplanar vectors and $\vec{r}$ be any arbitrary vector. Then $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{r} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{r} \times \vec{a}\right) + \left(\vec{c} \times \vec{a}\right)\left(\vec{r} \times \vec{b}\right)$ is always equal to

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88. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non coplanar and unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is (A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

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89. Prove that

$$\vec{R} + \frac{\left[\vec{R}\vec{\beta} \times \left(\vec{\beta} \times \vec{\alpha}\right)\right]\vec{\alpha}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} + \frac{\left[\vec{R}\vec{\alpha} \times \left(\vec{\alpha} \times \vec{\beta}\right)\right]\vec{\beta}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} = \frac{\left[\vec{R}\vec{\alpha}\vec{\beta}\right]\left(\vec{\alpha} \times \vec{\beta}\right)}{\left|\vec{\alpha} \times \vec{\beta}\right|^2}$$

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90. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar non-zero vectors, then prove that

$$(\vec{a}.\vec{a})\vec{b}\times\vec{c}+(\vec{a}.\vec{b})\vec{c}\times\vec{a}+(\vec{a}.\vec{c})\vec{a}\times\vec{b}=\left[\vec{b}\vec{c}\vec{a}\right]\vec{a}$$

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91. Find a set of vectors reciprocal to the set $-\hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}, \hat{i}+\hat{j}+\hat{k}$

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92. find the scalar and vector projection of $3\hat{i}-\hat{j}+4\hat{k}on2\hat{i}+3\hat{j}-6\hat{k}$

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93. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors, then prove that $\vec{a}'\times\vec{b}'\times\vec{b}, \times\vec{c}'+\vec{c}'\times\vec{a}'=\dfrac{\vec{a}+\vec{b}+\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$

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94. $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors and $\vec{r}$. Is any arbitrary vector. Prove that $\left[\vec{b}\,\vec{c}\,\vec{r}\right]\vec{a} + \left[\vec{c}\,\vec{a}\,\vec{r}\right]\vec{b} + \left[\vec{a}\,\vec{b}\,\vec{r}\right]\vec{c} = \left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{r}$.

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95. Find the angle between the following pairs of vectors $3\hat{i} + 2\hat{j} - 6\hat{k}, 4\hat{i} - 3\hat{j} + \hat{k}, \hat{i} - 2\hat{j} + 3\hat{k}, 3\hat{i} - 2\hat{j} + \hat{k}$

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96. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non - zero vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$,then find the goemetrical relation between the vectors.

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97. if $\vec{r}.\vec{i} = \vec{r}.\vec{j} = \vec{r}.\vec{k}$ and $\left|\vec{r}\right| = 3$, then find vector $\vec{r}$.

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98. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ is

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99. if $\vec{a}, \vec{b}$ and $\vec{c}$ are mutally perpendicular vectors of equal magnitudes, then find the angle between vectors and$\vec{a} + \vec{b} = \vec{c}$.

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100. If $|\vec{a}| + |\vec{b}| = |\vec{c}|$ and $\vec{a} + \vec{b} = \vec{c}$ then find the angle between $\vec{a}$ and $\vec{b}$.

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101. If three unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Then find the angle between $\vec{a}$ and $\vec{b}$.

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102. If θ is the angle between the unit vectors $\vec{a}$ and $\vec{b}$, then prove that

i. $\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} + \vec{b}\right|$

$ii.$ $\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$

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103. find the projection of the vector $\hat{i} + 3\hat{j} = 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

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104. If the scalar projection of vector $x\hat{i} - \hat{j} + \hat{k}1$ on vector $2\hat{i} - \hat{j} + 5\hat{k}is\frac{1}{\sqrt{30}}$.

The find the value of x.

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105. If $\vec{a} = x\hat{i} + (x - 1)\hat{j} + \hat{k}$ and $\vec{b} = (x + 1)\hat{i} + \hat{j} + a\hat{k}$ make an acute angle $\forall x \in R$, then find the values of a.

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106. If $\vec{a}.\vec{i} = \vec{a}.\left(\hat{i} + \hat{j}\right) = \vec{a}.\left(\hat{i} + \hat{j} + \hat{k}\right)$. Then find the unit vector $\vec{a}$.

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107. Prove by vector method that $\cos(A + B) = \cos A\cos B - \sin A\sin B$

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108. In any triangle ABC, prove the projection formula $a = b\cos C + c\cos B$ using vector method.

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109. Prove that an angle inscribed in a semi-circle is a right angle using vector method.

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110. Using dot product of vectors, prove that a parallelogram, whose diagonals are equal, is a rectangle

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111. If $a + 2b + 3c = 4$, then find the least value of $a^2 + b^2 + c^2$.

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112. about to only mathematics

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113. Vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are of the same length and when taken pair-wise they form equal angles. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ then find vector $\vec{c}$.

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114. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a unit vector which makes equal angle with $\vec{a}, \vec{b}$ and $\vec{c}$, then find the value of $\left|\vec{a} + \vec{b} + \vec{c} + \vec{d}\right|^2$.

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115. A particle acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the forces in SI unit is

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116. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

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117. If $\vec{a} = 4\hat{i} + 6\hat{j}$ and $\vec{b} = 3\hat{i} + 4\hat{k}$ find the vector component of $\vec{a}$ along $\vec{b}$.

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118. If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{a} + \vec{b}\right| = 1$ then find the value of $\left|\vec{a} - \vec{b}\right|$

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119. If $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 0\hat{j} + \hat{k}$ then find vector $\vec{c}$ satisfying the following conditions, (i) that it is coplaner with $\vec{a}$ and $\vec{b}$, (ii) that it is $\perp$ to $\vec{b}$ and (iii) that $\vec{a}.\vec{c} = 7$.

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120. Let $\vec{a}, \vec{b}$ and $\vec{c}$are vectors such that $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5,$ and $(\vec{a} + \vec{b})$ is perpendicular to $\vec{c}, (\vec{b} + \vec{c})$ is perpendiculatr to $\vec{a}$ and $(\vec{c} + \vec{a})$ is perpendicular to $\vec{b}$. Then find the value of $|\vec{a} + \vec{b} + \vec{c}|$.

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121. Prove that in a tetrahedron if two pairs of opposite edges are perpendicular, then the thrid pair is also perpendicular.

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122. In isosceles triangle ABC $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right| = 8$ a point E divides AB internally in the ratio 1:3, then find the angle between $\overrightarrow{CE}$ and $\overrightarrow{CA}$ (where $\left|\overrightarrow{CA}\right| = 12)$

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123. An arc AC of a circle subtends a right angle at then the center O. the point B divides the arc in the ratio $1:2$, If $\vec{O}A = a\&\vec{O}B = b$. then the vector $\vec{O}C$ in terms of $a\&b$, is

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124. Vector $\vec{O}A = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle passing through the positive x-axis on the way. Show that the vector in its new position is

$\frac{4\hat{i} - \hat{j} - \hat{k}}{\sqrt{2}}$.

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125. The base of the pyramid $AOBC$ is an equilateral triangle OBC with each side equal to $4\sqrt{2}$, O is the origin of reference, AO is perpendicualar to the plane of OBC and $\left|\vec{AO}\right| = 2$. Then find the cosine of the angle between the skew straight lines, one passing though A and the midpoint of $OBand$ the other passing through O and the mid point of BC.

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126. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = 2\hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

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127. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = \frac{\sqrt{2}}{3}$ then$|\vec{a} \times \vec{b}$ is a unit vector. If the angle between $\vec{a}$ and $\vec{b}$ is ?

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128. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$

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129. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15.$

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130. If $A, BandC$ are the vetices of a triangle $ABC,$ then prove sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

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131. Using cross product of vectors , prove that $(\sin A + B) - \sin A \cos B + \cos A \sin B$.

132. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1) and (0, 2, 1)^{\cdot}$

133. If $\vec{a}$ and $\vec{b}$ are two vectors , then prove that

$$\left(\vec{a} \times \vec{b}\right)^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} \end{vmatrix}$$

134. If $|\vec{a}| = 2$ then find the value of $|\vec{a} \times \vec{i}|^2 + |\vec{a} \times \vec{j}|^2 + |\vec{a} \times \vec{k}|^2$

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135. $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}, \vec{r} \times \vec{b} = \vec{a} \times \vec{b}, \vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}, \vec{a} \neq \lambda \vec{b}$ and $\vec{a}$ is not perpendicular to $\vec{b}$, then find $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$.

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136. $A, B, CandD$ are any four points in the space, then prove that $\left|\vec{A}B \times \vec{C}D + \vec{B}C \times \vec{A}D + \vec{C}A \times \vec{B}D\right| = 4$ (area of ABC .)

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137. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of the vertices A,B and C. respectively of $\triangle ABC$. Prove that the perpendicualar distance of the vertex A from the base BC of the triangle ABC is $\frac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|}{\left|\vec{c}-\vec{b}\right|}$

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138. Using vectors, find the area of the triangle with vertices $A(1, 1, 2),\ B(2, 3, 5) and\ C(1, 5, 5)$

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139. Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$\`

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140. find the area of a parallelogram whose diagonals are $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}-3\hat{j}+4\hat{k}.$

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141. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} \neq 0, |\vec{a}| = |\vec{c}| = 1, |\vec{b}| = 4$ and $|\vec{b} \times \vec{c}| = \sqrt{15}$. If $\vec{b} - 2\vec{c} = \lambda\vec{a}$ then find the value of λ .

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142. Find the moment about (1,-1,-1) of the force $3\hat{i} + 4\hat{j} - 5\hat{k}$ acting at (1,0,-2)

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143. A rigid body is spinning about a fixed point (3,-2,-1) with an angular velocity of 4 rad/s, the axis of rotation being in the direction of (1,2,-2). Find the velocity of the particle at point (4,1,1).

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144. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $\left(\vec{a} - \vec{d}\right)$ is parallel to $\left(\vec{b} - \vec{c}\right)$.

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145. Show by a numerical example and geometrically also that $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ does not imply $\vec{b} = \vec{\cdot}$

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146. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the vertices of a cycle quadrilateral ABCD, prove that

$$\frac{\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{d} + \vec{d} \times \vec{a}\right|}{\left(\vec{b} - \vec{a}\right).\left(\vec{d} - \vec{a}\right)} + \frac{\left|\vec{b} \times \vec{c} + \vec{c} \times \vec{d} + \vec{d} \times \vec{b}\right|}{\left(\vec{b} - \vec{c}\right).\left(\vec{d} - \vec{c}\right)} = 0$$

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147. The postion vectors of the vertices of a quadrilateral with A as origin are $B(\vec{b}), D(\vec{d})$ and $C(l\vec{b}+m\vec{d})$. Prove that the area of the quadrilateral is $\frac{1}{2}(l+m)\left|\vec{b}\times\vec{d}\right|$.

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148. Let $\vec{a}$ and $\vec{b}$ be unit vectors such that $\left|\vec{a}+\vec{b}\right|=\sqrt{3}$. Then find the value of $\left(2\vec{a}+5\vec{b}\right).\left(3\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

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149. $\hat{u}$ and $\hat{v}$ are two non-collinear unit vectors such that $\left|\frac{\hat{u}+\hat{v}}{2}+\hat{u}\times\hat{v}\right|=1$. Prove that $\left|\hat{u}\times\hat{v}\right|=\left|\frac{\hat{u}-\hat{v}}{2}\right|$

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150. In a $\triangle ABC$points D,E,F are taken on the sides BC,CA and AB respectively such that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n$ prove that $\triangle DEF = \frac{n^2 - n + 1}{(n + 1)^2} \triangle ABC$

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151. Let A,B,C be points with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} + \hat{j} + 2\hat{k}$ respectively. Find the shortest distance between point B and plane OAC.

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152. Let $\vec{a} = x\hat{i} + 12\hat{j} - \hat{k}, \vec{b} = 2\hat{i} + 2x\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$. If the ordered set $\left[\vec{b}\vec{c}\vec{a}\right]$ is left handed, then find the value of x.

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153. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then find the value of

$$\frac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\vec{b}.\left(\vec{c}\times\vec{a}\right)}+\frac{\vec{b}.\left(\vec{c}\times\vec{a}\right)}{\vec{c}.\left(\vec{a}\times\vec{b}\right)}+\frac{\vec{c}.\left(\vec{b}\times\vec{a}\right)}{\vec{a}.\left(\vec{b}\times\vec{c}\right)}$$

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154. if the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k}$ and $3\hat{i} - \hat{k}$ from three concurrent edges of a parallelpiped, then find the volume of the parallelepied.

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155. The postion vectors of the four angular points of a tetrahedron are $A\left(\hat{j} + 2\hat{k}\right), B\left(3\hat{i} + \hat{k}\right), C\left(4\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and $D\left(2\hat{i} + 3\hat{j} + 2\hat{k}\right)$ find the volume of the tetrahedron ABCD.

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156. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors and $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$ then find the value of $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$

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157. Prove that $\left[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right]$

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158. Prove that $\left[\vec{l}\vec{m}\vec{n}\right]\left[\vec{a}\vec{b}\vec{c}\right] = \begin{vmatrix} \vec{l}.\vec{a} & \vec{l}.\vec{b} & \vec{l}.\vec{c} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}.\vec{c} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n}.\vec{c} \end{vmatrix}$

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159. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \hat{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k},$ then find the value of

$$\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$$

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160. Find the value of a so that the volume of the parallelopiped formed by vectors $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum.

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161. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then prove that $(\vec{u} + \vec{v} - \vec{w}).\left[\left[(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})\right]\right] = \vec{u}.\vec{v} \times \vec{w}$

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162. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\left|\vec{a} \times \vec{b}\right| = 2$, then find the value of $\left[\vec{a}\vec{b}\vec{a}\right] \times \vec{b}$.

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163. Find the altitude of a parallelopiped whose three coterminous edges are vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}, \vec{B} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{C} = \hat{i} + \hat{j} + 3\hat{k}$ with $\vec{A}$ and $\vec{B}$ as the sides of the base of the parallelopiped .

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164. If $\left[\vec{a}\vec{b}\vec{c}\right] = 2$, then find the value of $\left[\left(\vec{a} + 2\vec{b} - \vec{c}\right)\left(\vec{a} - \vec{b}\right)\left(\vec{a} - \vec{b} - \vec{c}\right)\right]$

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165. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vector and $\vec{a} = \alpha\left(\vec{a} \times \vec{b}\right) + \beta\left(\vec{b} \times \vec{c}\right) + \gamma(\vec{c} \times \vec{a})$ and $\left[\vec{a}\vec{b}\vec{c}\right] = 1 then \vec{\alpha} + \vec{\beta} + \vec{\gamma} =$ (A) $\left|\vec{a}\right|^2$ (B) - $\left|\vec{a}\right|^2$ (C) 0 (D) none of these

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166. i. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors, prove that vectors $3\vec{a} - 7\vec{b} - 4\vec{c}, 3\vec{a} - 2\vec{b} + \vec{c}$ and $\vec{a} + \vec{b} + 2\vec{c}$ are coplanar.

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167. Prove that vectors

$$\vec{u} = \left(al + a_1 l_1\right)\hat{i} + \left(am + a_1 m_1\right)\hat{j} + \left(an + a_1 n_1\right)\hat{k}$$

$$\vec{v} = \left(bl + b_1 l_1\right)\hat{i} + \left(bm + b_1 m_1\right)\hat{j} + \left(bn + b_1 n_1\right)\hat{k}$$

$$\vec{w} = \left(wl + c_1 l_1\right)\hat{i} + \left(cm + c_1 m_1\right)\hat{j} + \left(cn + c_1 n_1\right)\hat{k}$$

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168. Let $G_1, G_2 and G_3$ be the centroids of the triangular faces $OBC, OCA and OAB,$ respectively, of a tetrahedron $OABC$. If V_1 denotes the volumes of the tetrahedron $OABC and V_2$ that of the parallelepiped with $OG_1, OG_2 and OG_3$ as three concurrent edges, then prove that $4V_1 = 9V_2$.

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169. Prove that $\hat{i} \times \left(\vec{a} \times \vec{i}\right) + \hat{j} \times \left(\vec{a} \times \vec{j}\right) + \hat{k} \times \left(\vec{a} \times \vec{k}\right) = 2\vec{a}$

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170. If $\hat{i} \times \left[\left(\vec{a} - \hat{j}\right) \times \hat{i}\right] + \hat{j} \times \left[\left(\vec{a} - \hat{k}\right) \times \hat{j}\right] + \vec{k} \times \left[\left(\vec{a} - \vec{i}\right) \times \hat{k}\right] = 0$, then find vector $\vec{a}$.

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171. Prove that: $\left[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}\right]=\left[\vec{a}\vec{b}\vec{c}\right]^2$

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172. For any four vectors prove that

$\left(\vec{b}\times\vec{c}\right).\left(\vec{a}\times\vec{d}\right)+\left(\vec{c}\times\vec{a}\right).\left(\vec{b}\times\vec{d}\right)+\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=0$

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173. If $\vec{b}$ and $\vec{c}$ are two non-collinear such that $\vec{a}$ | | $\left(\vec{b}\times\vec{c}\right)$. Then prove that $\left(\vec{a}\times\vec{b}\right).\left(\vec{a}\times\vec{c}\right)$ is equal to $|\vec{a}|^2\left(\vec{b}.\vec{c}\right)$`

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174. Find the vector of length 3 unit which is perpendicular to $\hat{i}+\hat{j}+\hat{k}$ and lies in the plane of $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-3\hat{j}$

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175. Let $\hat{a}, \hat{b}$,and $\hat{c}$ be the non-coplanar unit vectors. The angle between $\hat{b}$ and $\hat{c}$ is α , between $\hat{c}$ and $\hat{a}$ is β and between $\hat{a}$ and $\hat{b}$ is γ . If $A(\hat{a}\cos\alpha, 0), B(\hat{b}\cos\beta, 0)$ and $C(\hat{c}\cos\gamma, 0)$, then show that in triangle ABC, $\frac{\left|\hat{a} \times (\hat{b} \times \hat{c})\right|}{\sin A} = \frac{\left|\hat{b} \times (\hat{c} \times \hat{a})\right|}{\sin B} = \frac{\left|\hat{c} \times (\hat{a} \times \hat{b})\right|}{\sin C}$

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176. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplannar vectors, then prove that

$$\frac{\left|\hat{a} \times (\hat{b} \times \hat{c})\right|}{\sin A} = \frac{\left|\hat{b} \times (\hat{c} \times \hat{a})\right|}{\sin B} = \frac{\left|\hat{c} \times (\hat{a} \times \hat{b})\right|}{\sin C} = \frac{\prod \left|\hat{a} \times (\hat{b} \times \hat{c})\right|}{\left|\sum \hat{n}_1 \sin\alpha\cos\beta\cos\gamma\right|}$$

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177. If $\vec{b}$ is not perpendicular to $\vec{c}$. Then find the vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ and $\vec{r} . \vec{c} = 0$

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178. If $\vec{a}$ and $\vec{b}$ are two given vectors and k is any scalar,then find the vector $\vec{r}$ satisfying $\vec{r}\times\vec{a}+k\vec{r}=\vec{b}$.

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179. If $\vec{r}.\vec{a}=0, \vec{r}.\vec{b}=1$ and $\left[\vec{r}\vec{a}\vec{b}\right]=1, \vec{a}.\vec{b}\neq 0, \left(\vec{a}.\vec{b}\right)^2-\left|\vec{a}\right|^2\left|\vec{b}\right|^2=1,$ then find $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$.

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180. If vector $\vec{x}$ satisfying $\vec{x}\times\vec{a}+\left(\vec{x}.\vec{b}\right)\vec{c}=\vec{d}$ is given by $\vec{x}=\lambda\vec{a}+\vec{a}\times\dfrac{\vec{a}\times\left(\vec{d}\times\vec{c}\right)}{\left(\vec{a}.\vec{c}\right)\left|\vec{a}\right|^2}$, then find out the value of λ

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181. $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors and $\vec{r}$. Is any arbitrary vector. Prove that $\left[\vec{b}\vec{c}\vec{r}\right]\vec{a} + \left[\vec{c}\vec{a}\vec{r}\right]\vec{b} + \left[\vec{a}\vec{b}\vec{r}\right]\vec{c} = \left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$.

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182. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non -coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \dfrac{\vec{b} \times \vec{c}}{\sqrt{2}}, \vec{b}$ and $\vec{c}$ are non- parallel , then prove that the angle between $\vec{a}$ and $\vec{b}$is$3\pi/4$

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183. Prove that

$$\vec{R} + \frac{\left[\vec{R}\vec{\beta} \times \left(\vec{\beta} \times \vec{\alpha}\right)\right]\vec{\alpha}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} + \frac{\left[\vec{R}\vec{\alpha} \times \left(\vec{\alpha} \times \vec{\beta}\right)\right]\vec{\beta}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} = \frac{\left[\vec{R}\vec{\alpha}\vec{\beta}\right]\left(\vec{\alpha} \times \vec{\beta}\right)}{\left|\vec{\alpha} \times \vec{\beta}\right|^2}$$

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184. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar non-zero vectors, then prove that $(\vec{a}.\vec{a})\vec{b}\times\vec{c}+(\vec{a}.\vec{b})\vec{c}\times\vec{a}+(\vec{a}.\vec{c})\vec{a}\times\vec{b}=[\vec{b}\vec{c}\vec{a}]\vec{a}$

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185. Find a set of vectors reciprocal to the set $-\hat{i}+\hat{j}+\hat{k}, \hat{i}+\hat{k}, \hat{i}+\hat{j}+\hat{k}$

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186. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be a set of non- coplanar vectors and $\vec{a}'\vec{b}'$ and $\vec{c}'$ be its reciprocal set.

prove that $\vec{a}=\dfrac{\vec{b}'\times\vec{c}'}{[\vec{a}'\vec{b}'\vec{c}']}, \vec{b}=\dfrac{\vec{c}'\times\vec{a}'}{[\vec{a}'\vec{b}'\vec{c}']}$ and $\vec{c}=\dfrac{\vec{a}'\times\vec{b}'}{[\vec{a}'\vec{b}'\vec{c}']}$

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187. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors, then prove that $\vec{a}' \times \vec{b}' + \vec{b}' \times \vec{c}' + \vec{c}' \times \vec{a}' = \dfrac{\vec{a} + \vec{b} + \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$

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188. If $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar vectors and a',b' and c' constitute the reciprocal system of vectors, then prove that

$i.\ \vec{r} = \left(\vec{r}.\vec{a}'\right)\vec{a} + \left(\vec{r}.\vec{b}'\right)\vec{b} + \left(\vec{r}.\vec{c}'\right)\vec{c}$

ii. $\vec{r} = \left(\vec{r}.\vec{a}\right)\vec{a}' + \left(\vec{r}.\vec{b}\right)\vec{b}' + \left(\vec{r}.\vec{c}\right)\vec{c}'$

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Exercise 2.1

1. Find `|veca| and |vecb| if (veca+vecb).(veca-vecb)=8 and |veca|=8|vecb|.

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2. Show that $\left|\vec{a}\right|\vec{b}+\left|\vec{b}\right|\vec{a}$ is perpendicular to $\left|\vec{a}\right|\vec{b}-\left|\vec{b}\right|\vec{a}$ for any two non zero vectors `veca and vecb.

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3. If the vectors A, B, C of a triangle ABC are $(1, 2, 3), (-1, 0, 0), (0, 1, 2)$, respectively then find $\angle ABC^{\cdot}$

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4. If $\left|\vec{a}\right|=3, \left|\vec{b}\right|=4$ and the angle between $\vec{a}$ and $\vec{b} is 120^{\circ}$. Then find the value of $\left|4\vec{a}+3\vec{b}\right|$

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5. If vectors $\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other, then find the locus of th point (x,y).

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6. Let $\vec{a}\vec{b}$ and $\vec{c}$ be pairwise mutually perpendicular vectors, such that $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 2$, the find the length of $\vec{a} + \vec{b} + \vec{c}$.

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7. If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

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8. If the angle between unit vectors $\vec{a}$ and $\vec{b} is 60^\circ$. Then find the value of $|\vec{a} - \vec{b}|$.

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9. Let $\vec{u} = \hat{i} + \hat{j}, \vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$, $\left|\vec{w}.\hat{n}\right|$ is equal to (A) 0 (B) 1 (C) 2 (D) 3

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10. A, B, C, D are any four points, prove that $\vec{A}\dot{B}\vec{C}D + \vec{B}\dot{C}\vec{A}D + \vec{C}\dot{A}\vec{B}D = 0$.

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11. $P(1, 0, -1), Q(2, 0, -3), R(-1, 2, 0) and S(, -2, -1)$, then find the projection length of $\vec{P}Qon\vec{R}\dot{S}$

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12. If the vectors $3\vec{p} + \vec{q}; 5p - 3\vec{q} and 2\vec{p} + \vec{q}; 3\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors, then find the angle between vectors $\vec{p} and \vec{q}$.

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13. Let $\vec{A}$ and $\vec{B}$ be two non-parallel unit vectors in a plane. If $\left(\alpha\vec{A} + \vec{B}\right)$ bisects the internal angle between $\vec{A}$ and $\vec{B}$ then find the value of α.

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14. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{x}, \vec{a}.\vec{x} = 1, \vec{b}.\vec{x} = \frac{3}{2}, \left|\vec{x}\right| = 2$ then find theh angle between $\vec{c}$ and $\vec{x}$.

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15. If $\vec{a}$ and $\vec{b}$ are unit vectors, then find the greatest value of $\left|\vec{a} + \vec{b}\right| + \left|\vec{a} - \vec{b}\right|$.

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16. Constant forces $P_1 = \hat{i} - \hat{j} + \hat{k}, P_2 = -\hat{i} + 2\hat{j} - \hat{i}k$ and $P_3 = \hat{j} - \hat{k}$ act on a particle at a point A . Determine the work done when particle is displaced from position $A\left(4\hat{i} - 3\hat{j} - 2\hat{k}\right)$ to$B\left(6\hat{i} + \hat{j} - 3\hat{k}\right)$

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17. Find $\left|\vec{a}\right|$ and $\left|\vec{b}\right|$, if $\left(\vec{a} + \vec{b}\right).\left(\vec{a} - \vec{b}\right) = 8$ and $\left|\vec{a}\right| = 8\left|\vec{b}\right|$

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18. If A,B, C and D are four distinct points in space that $\overrightarrow{AB}$ is not perpendicular to $\overrightarrow{CD}$ and satisfies $(AB).(CD) = k\left(\left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BC}\right|^2 - \left|\overrightarrow{BD}\right|^2\right)$, then find the value of k .

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Exercise 2.2

1. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k}$ and $\vec{a} \times \vec{b} = \vec{0}$ then find (m,n)

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2. Find $\vec{a}.\vec{b}$ if $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 5,$ and $\left|\vec{a} \times \vec{b}\right| = 8$

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3. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq 0$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar vectors, then for some scalar k prove that $\vec{a} + \vec{c} = kb\vec{b}$.

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4. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}, \vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{c} = \vec{i} + \vec{j} + \vec{k}$, then find the value of $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$

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5. If the vectors $\vec{c}$, $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right handed system then $\vec{c}$ is

A. (a) $z\hat{i} - x\hat{k}$

B. (b) $\vec{0}$

C. (c) $y\hat{j}$

D. (d) $-z\hat{i} + x\hat{k}$

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6. given that $\vec{a}.\vec{b}=\vec{a}.\vec{c}, \vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}$ is not a zero vector. Show that $\vec{b}=\vec{c}$.

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7. Show that $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\vec{a}\times\vec{b}$ and give a genometrical interpretation of it.

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8. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\left|\vec{z}\right|=\dfrac{2}{\sqrt{7}}$ such that $\vec{z}+\vec{z}\times\vec{x}=\vec{y}$ then find the angle θ between $\vec{x}$ and $\vec{z}$

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9. prove that $\left(\vec{a}.\hat{i}\right)\left(\vec{a}\times\hat{i}\right)+\left(\vec{a}.\hat{j}\right)\left(\vec{a}\times\hat{j}\right)+\left(\vec{a}.\hat{k}\right)\left(\vec{a}\times\hat{k}\right)=\vec{0}$

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10. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $\lambda\vec{b}\times\vec{a}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$ then find the value of λ.

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11. A particle has an angular speed of 3 rad/s and the axis of rotation passes through the points $(1, 1, 2)and(1, 2, -2)^{\cdot}$ Find the velocity of the particle at point $P(3, 6, 4)^{\cdot}$

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12. Let $vea,\vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}.\vec{b}=0=\vec{a}.\vec{c}$. It the angle between $\vec{b}$ and $\vec{c}is\frac{\pi}{6}$ then find $\vec{a}$.

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13. If $\left|\vec{a}\times\vec{b}\right|^2+\left(\vec{a}.\vec{b}\right)^2=144$ and $\left|\vec{a}\right|=4$, then $\left|\vec{b}\right|$ is equal to ....

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14. Given $\left|\vec{a}\right|=\left|\vec{b}\right|=1$ and $\left|\vec{a}+\vec{b}\right|=\sqrt{3}\text{if}\vec{c}$ is a vector such that $\vec{c}-\vec{a}-2\vec{b}=3\left(\vec{a}\times\vec{b}\right)$ then find the value of $\vec{c}.Vecb$.

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15. Find the moment of $\vec{F}$ about point (2, -1, 3), where force $\vec{F}=3\hat{i}+2\hat{j}-4\hat{k}$ is acting on point (1, -1, 2).

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Exercise 2.3

1. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four non-coplanar unit vectors such that $\vec{d}$ makes equal angles with all the three vectors $\vec{a}, \vec{b}, \vec{c}$ then prove that $\left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right]$

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2. prove that if $\left[\vec{l}\vec{m}\vec{n}\right]$ are three non-coplanar vectors, then

$$\left[\vec{l}\vec{m}\vec{n}\right]\left(\vec{a} \times \vec{b}\right) = \begin{vmatrix} \vec{l}.\vec{a} & \vec{l}.\vec{b} & \vec{l} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n} \end{vmatrix}$$

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3. if the volume of a parallelpiped whose adjacent egges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}, \vec{c} = \vec{i} + 2\hat{j} + \alpha\hat{k} is 15$ then find of α if $(\alpha > 0)$

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4. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find the vector $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$.

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5. If $\vec{x}.\vec{a} = 0\vec{x}.\vec{b} = 0$ and $\vec{x}.\vec{c} = 0$ for some non zero vector $\vec{x}$ then show that $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

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6. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find the vector $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$.

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7. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b}$, then the value of $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is

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8. If $\vec{a} = \vec{P} + \vec{q}, \vec{P} \times \vec{b} = \vec{0}$ and $\vec{q}.\vec{b} = 0$ then prove that $\dfrac{\vec{b} \times (\vec{a} \times \vec{b})}{\vec{b}.\vec{b}} = \vec{q}$

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9. Prove that $(\vec{a}.(\vec{b} \times \hat{i}))\hat{i} + (\vec{a}.(\vec{b} \times \hat{j}))\hat{j} + (\vec{a}.(\vec{b} \times \hat{k}))\hat{k} = \vec{a} \times \vec{b}$

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10. for any four vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ prove that $\vec{d}.(\vec{a} \times (\vec{b} \times (\vec{c} \times \vec{d}))) = (\vec{b}.\vec{d})[\vec{a}\vec{c}\vec{d}]$

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11. If $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors such that $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = -\frac{1}{2}\vec{b}$,then find the angle between $\vec{a}$ and $\vec{b}$.

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12. show that $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ if and only if $\vec{a}$ and $\vec{c}$ are collinear or $\left(\vec{a} \times \vec{c}\right) \times \vec{b} = \vec{0}$

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13. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the non zero vectors such that $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. if theta is the acute angle between the vectors $\vec{b}$ and $\vec{a}$ then theta equals (A) $\frac{1}{3}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $2\frac{\sqrt{2}}{3}$

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14. If $\vec{p}, \vec{q}, \vec{r}$ denote vectors $\vec{b} \times \vec{c}, \vec{c} \times \vec{c} \times \vec{a}, \vec{a} \times \vec{b}$. Respectively, show that $\vec{a}$ is parallel to $\vec{q} \times \vec{r}, \vec{b}$ is parallel to $\vec{r} \times \vec{p}, \vec{c}$ is parallel to $\vec{p} \times \vec{q}$.\

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15. Let $\vec{a}, \vec{b}, \vec{c}$ be non -coplanar vectors and let equations $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vector $\vec{a}, \vec{b}, \vec{c}$ then prove that $\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}'$ is a null vector.

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16. Given unit vectors $\hat{m}\hat{n}$ and $\hat{p}$ such that angle between $\hat{m}$ and $\hat{n} is \alpha$ and angle between $\hat{p}$ and $\hat{m} X \hat{n} is \alpha$if [n p m] = 1/4 find alpha

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17. $\vec{a}, \vec{b}, \vec{c}$ arwe threee unit vectors and every two are two inclined to each at an angle $\cos^{-1}(3/5)$. If $\vec{a} \times \vec{b} = p\vec{a} + q\vec{b} + r\vec{c}$, where p,q,r are scalars, then find the value of q.

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18. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both vectors, $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b} is \pi/6 then \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

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Exercises

1. If $\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-a)^2 \end{vmatrix} = 0$ and vectors $\vec{A}, \vec{B}$ and $\vec{C}$, where $\vec{A} = a^2\hat{i} = a\hat{j} + \hat{k}$ etc. are non-coplanar, then prove that vectors $\vec{X}, \vec{Y}$ and $\vec{Z}$ where $\vec{X} = x^2\hat{i} + x\hat{j} + \hat{k}$. etc.may be coplanar.

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2. OABC is a tetrahedron where O is the origin and A,B,C have position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively prove that circumcentre of tetrahedron OABC is $\dfrac{a^2\left(\vec{b} \times \vec{c}\right) + b^2\left(\vec{c} \times \vec{a}\right) + c^2\left(\vec{a} \times \vec{b}\right)}{2\left[\vec{a}\vec{b}\vec{c}\right]}$

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3. Let k be the length of any edge of a regular tetrahedron (a tetrahedron whose edges are equal in length is called a regular tetrahedron). Show that the angel between any edge and a face not containing the edge is $\cos^{-1}\left(1/\sqrt{3}\right)$.

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4. In $\triangle ABC$, a point P is taken on AB such that $AP/BP = 1/3$ and point Q is taken on BC such that $CQ/BQ = 3/1$. If R is the point of intersection of the lines $AQandCP$, using vector method, find the area of ABC if the area of BRC is 1 unit

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5. Let O be an interior points of $\triangle ABC$ such that $\overrightarrow{OA} + \overrightarrow{OB} + 3\vec{OC} = \vec{0}$, then the ratio of $\triangle ABC$ to area of $\triangle AOC$ is

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6. The length of two opposite edges of a tetrahedrom are a and b, the shortest distance between these edges is d, and the angle between them is θ. Prove using vectors that the volume of the tetrahedron is $\frac{abd\sin\theta}{6}$

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7. Find the volume of a parallelopiped having three coterminus vectors of equal magnitude $|a|$ and equal inclination θ with each other.

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8. Let $\vec{p}$ and $\vec{q}$ any two othogonal vectors of equal magnitude 4 each. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be any three vectors of lengths $7\sqrt{15}$ and $2\sqrt{33}$, mutually perpendicular to each other. Then find the distance of the vector $(\vec{a}.\vec{p})\vec{p} + (\vec{a}.\vec{q})\vec{q} + (\vec{a}.(\vec{p}\times\vec{q}))(\vec{p}\times\vec{q}) + (\vec{b}.\vec{p})\vec{p} + (\vec{b}.\vec{p})\vec{q} + (\vec{b}.(\vec{b}.\vec{q}))(\vec{p}\times\vec{q}) + (\vec{c}.\vec{p})\vec{p} + (\vec{c}.\vec{q})\vec{q} + (\vec{c}.(\vec{p}\times\vec{q}))(\vec{p}\times\vec{q})$ from the origin.

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9. Given that vectors $\vec{A}, \vec{B}$ and $\vec{C}$ from a triangle such that $\vec{A} = \vec{B} + \vec{C}$. Find a, b,c and d such that the area of the triangle is $5\sqrt{16}$ where.

$\vec{A} = a\hat{i} + b\vec{j} + c\hat{k}$

$\vec{B} = d\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{C} = 3\hat{i} + \hat{j} - 2\hat{k}$

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10. A line l is passing through the point $\vec{b}$ and is parallel to vector $\vec{c}$. Determine the distance of point A$(\vec{a})$ from the line l in from

$$\left|\vec{b} - \vec{a} + \frac{\left(\vec{a} - \vec{b}\right)\vec{c}}{\left|\vec{c}\right|^2}\vec{c}\right| \text{ or } \frac{\left|\left(\vec{b} - \vec{a}\right) \times \vec{c}\right|}{\left|\vec{c}\right|}$$

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11. If $\vec{e}_1, \vec{e}_2, \vec{e}_3 and \vec{E}_1, \vec{E}_2, \vec{E}_3$ are two sets of vectors such that $\vec{e}_i . \vec{E}_j = 1$, if $i = j$ and $\vec{e}_i . \vec{E}_j = 0 and$ if $i \neq j$, then prove that $\left[\vec{e}_1 \ \vec{e}_2 \ \vec{e}_3\right]\left[\vec{E}_1 \ \vec{E}_2 \ \vec{E}_3\right] = 1.$

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12. In a quadrillateral ABCD, it is given that AB ||CD and the diagonals AC and BD are perpendiclar to each other . Show that $AD. BC \geq AB. CD.$

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13. $OABC$ is regular tetrahedron in which D is the circumcentre of OAB and E is the midpoint of edge AC. Prove that DE is equal to half the edge of tetrahedron.

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14. If A$(\vec{a})$. $B(\vec{b})$ and $C(\vec{c})$ are three non-collinear point and origin does not lie in the plane of the points A, B and C, then for any point $P(\vec{P})$ in the plane of the $\triangle ABC$ such that vector $\overrightarrow{OP}$ is $\perp$ to plane of $triangl ABC$, show that $\overrightarrow{OP} = \frac{[\vec{a}\vec{b}\vec{c}](\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})}{4\Delta^2}$

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15. If $\vec{a}, \vec{b}, \vec{c}$ are three given non-coplanar vectors and any arbitrary vector $\vec{r}$ in space, where $\Delta_1 = \begin{vmatrix} \vec{r}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{r}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{r}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{r}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{r}.\vec{b} & \vec{c}.\vec{b} \\ \vec{a}.\vec{c} & \vec{r}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}$

$\Delta_3 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{r}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{r}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{r}.\vec{c} \end{vmatrix}$, $\Delta = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}$,

then prove that $\vec{r} = \frac{\Delta_1}{\Delta}\vec{a} + \frac{\Delta_2}{\Delta}\vec{b} + \frac{\Delta_3}{\Delta}\vec{c}$

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Exercises MCQ

1. Two vectors in space are equal only if they have equal component in

A. a given direction

B. two given directions

C. three given direction

D. in any arbitrary direaction

Answer: c

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2. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the three vectors having magnitudes, 1,5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \vec{c}$. Then $\tan\theta$ is equal to

A. 0

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{3}{4}$

Answer: d

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3. $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors of equal magnitude. The angle between each pair of vectors is $\pi/3$ such that $\left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{6}$ then $\left|\vec{a}\right|$ is equal to

A. 2

B. -1

C. 1

D. $\sqrt{6}/3$

Answer: c

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4. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors, then the vector which is equally inclined to these vectors is

A. $\vec{a} + \vec{b} + \vec{c}$

B. $\dfrac{\vec{a}}{|\vec{a}|} + \dfrac{\vec{b}}{|\vec{b}|} + \dfrac{\vec{c}}{|\vec{c}|}$

C. $\dfrac{\vec{a}}{|\vec{a}|^2} + \dfrac{\vec{b}}{|\vec{b}|^2} + \dfrac{\vec{c}}{|\vec{c}|^2}$

D. $|\vec{a}|\vec{a} - |\vec{b}|\vec{b} + |\vec{c}|\vec{c}$

Answer: b

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5. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. Then the point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is (A) $(3, -1, 10$ (B) $(3, 1, -1)$ (C) $(-3, 1, 1)$ (D) $(-3, -1, -1)$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $3\hat{i} - \hat{j} + \hat{k}$

C. $3\hat{i} + \hat{j} - \hat{k}$

D. $\hat{i} - \hat{j} - \hat{k}$

Answer: c

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6. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\vec{a}.\vec{b} < 0$ and $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ then the angle between the vectors $\vec{a}$ and $\vec{b}$ is (a) π (b) $\frac{7\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

A. π

B. $7\pi/4$

C. $\pi/4$

D. $3\pi/4$

Answer: d

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7. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors such that $\hat{a}+\hat{b}+\hat{c}$ is also a unit vector and θ_1, θ_2 and θ_3 are angles between the vectors $\hat{a}, \hat{b}, \hat{b}, \hat{c}$ and $\hat{c}, \hat{a},$ respectively m then among θ_1, θ_2 and θ_3

A. all are acute angles

B. all are right angles

C. at least one is obtuse angle

D. none of these

Answer: c

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8. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b}=0=\vec{a}.\vec{c}$ and the angle between $\vec{b}$ and $\vec{c} is \pi/3$ then the value of $\left|\vec{a}\times\vec{b}-\vec{a}\times\vec{c}\right|$ is

A. 1/2

B. 1

C. 2

D. none of these

Answer: b

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9. P $(\vec{p})$ and $Q(\vec{q})$ are the position vectors of two fixed points and $R(\vec{r})$ is the postion vector of a variable point. If R moves such that $(\vec{r} - \vec{p}) \times (\vec{r} - \vec{q}) = \vec{0}$then the locus of R is

A. a plane containing the origian O and parallel to two non-collinear vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$

B. the surface of a sphere described on PQ as its diameter

C. a line passing through points P and Q

D. a set of lines parallel to line PQ

Answer: c

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10. Two adjacent sides of a parallelogram ABCD are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Then the value of $\left|\vec{AC} \times \vec{BD}\right|$ is

A. $20\sqrt{5}$

B. $22\sqrt{5}$

C. $24\sqrt{5}$

D. $26\sqrt{5}$

Answer: b

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11. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors inclined to each other at an angle θ. The maximum value of θ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{5}$

Answer: c

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12. Let the pair of vector $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ each determine a plane. Then the planes are parallel if

A. $\left(\vec{a} \times \vec{c}\right) \times \left(\vec{b} \times \vec{d}\right) = \vec{0}$

B. $\left(\vec{a} \times \vec{c}\right). \left(\vec{b} \times \vec{d}\right) = \vec{0}$

C. $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \vec{0}$

D. $\left(\vec{a} \times \vec{c}\right).\left(\vec{c} \times \vec{d}\right) = \vec{0}$

Answer: c

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13. If $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar, then

A. $\vec{r} \perp \left(\vec{c} \times \vec{a}\right)$

B. $\vec{r} \perp \left(\vec{a} \times \vec{b}\right)$

C. $\vec{r} \perp \left(\vec{b} \times \vec{c}\right)$

D. $\vec{r} = \vec{0}$

Answer: d

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14. If $\vec{a}$ satisfies $\vec{a} \times \left(\hat{i} + 2\hat{j} + \hat{k}\right) = \hat{i} - \hat{k}$ then $\vec{a}$ is equal to

A. $\lambda\hat{i} + (2\lambda - 1)\hat{j} + \lambda\hat{k}, \lambda \in R$

B. $\lambda\hat{i} + (1 - 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

C. $\lambda\hat{i} + (2\lambda + 1)\hat{j} + \lambda\hat{k}, \lambda \in R$

D. $\lambda\hat{i} + (1 + 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

Answer: c

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15. Vectors $3\vec{a} - 5\vec{b}$ and $2\vec{a} + \vec{b}$ are mutually perpendicular. If $\vec{a} + 4\vec{b}$ and $\vec{b} - \vec{a}$ are also mutually perpendicular, then the cosine of the angle between $\vec{a}$ and $\vec{b}$ is (a) $\frac{19}{5\sqrt{43}}$ (b) $\frac{19}{3\sqrt{43}}$ (c) $\frac{19}{\sqrt{45}}$ (d) $\frac{19}{6\sqrt{43}}$

A. $\frac{19}{5\sqrt{43}}$

B. $\frac{19}{3\sqrt{43}}$

C. $\frac{19}{\sqrt{45}}$

D. $\frac{19}{6\sqrt{43}}$

Answer: a

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16. the unit vector orthogonal to vector $-\hat{i}+2\hat{j}+2\hat{k}$ and making equal angles with the x- and y-axes is

A. $\pm\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

B. $\frac{19}{5\sqrt{43}}$

C. $\pm\frac{1}{3}\left(\hat{i}+\hat{j}-\hat{k}\right)$

D. none of these

Answer: a

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17. The value of x for which the angle between $\vec{a}=2x^2\hat{i}+4x\hat{j}+\hat{k}$ and $\vec{b}=7\hat{i}-2\hat{j}+\hat{k}$ is obtuse and the angle between $\vec{b}$

and the z-axis is acute and less then $\pi/6$

A. $a < x < 1/2$

B. $1/2 < x < 15$

C. $x < 1/2 \text{ or } x < 0$

D. none of these

Answer: b

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18. If vectors $\vec{a}$ and $\vec{b}$ are two adjecent sides of a paralleogram, then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$is

A. $\vec{b} + \dfrac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$

B. $\dfrac{\vec{a}.\vec{b}}{\left|\vec{b}\right|^2}$

C. $\vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$

D. $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{b}|^2}$

Answer: a

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19. A parallelogram is constructed on $3\vec{a} + \vec{b}$ and $\vec{a} - 4\vec{b}$, *where* $|\vec{a}| = 6$ and $|\vec{b}| = 8$ and $\vec{a}$ and $\vec{b}$ are anti parallel then the length of the longer diagonal is (A) 40 (B) 64 (C) 32 (D) 48

A. 40

B. 64

C. 32

D. 48

Answer: c

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20. Let $\vec{a}.\vec{b} = 0$ where $\vec{a}$ and $\vec{b}$ are unit vectors and the vector $\vec{c}$ is inclined an anlge θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c} = m\vec{a} + n\vec{b} + p\left(\vec{a} \times \vec{b}\right), (m, n, p \in R)$ then

A. $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$

B. $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

C. $0 \leq \theta \leq \frac{\pi}{4}$

D. $0 \leq \theta \leq \frac{3\pi}{4}$

Answer: a

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21. $\vec{a}$ and $\vec{c}$ are unit vectors and $\left|\vec{b}\right| = 4$ the angle between $\vec{a}$ and $\vec{b}$ is $\cos^{-1}(1/4)$ and $\vec{b} - 2\vec{c} = \lambda\vec{a}$ the value of λ is

A. 3,-4

B. 1/4,3/4

C. $-3, 4$

D. $-1/4, \frac{3}{4}$

Answer: a

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22. Let the position vectors of the points $PandQ$ be $4\hat{i} + \hat{j} + \lambda\hat{k} and 2\hat{i} - \hat{j} + \lambda\hat{k},$ respectively. Vector $\hat{i} - \hat{j} + 6\hat{k}$ is perpendicular to the plane containing the origin and the points $PandQ$. Then λ equals $1/2$ b. $1/2$ c. 1 d. none of these

A. $-1/2$

B. $1/2$

C. 1

D. none of these

Answer: a

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23. A vector of magnitude $\sqrt{2}$ coplanar with the vectors $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, and perpendicular to the vector $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ is

A. $-\hat{j} + \hat{k}$

B. $\hat{i}$ and $\hat{k}$

C. $\hat{i}$ - $\hat{k}$

D. hati- hatj`

Answer: a

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24. Let P be a point interior to the acute triangle ABC. If $PA + PB + PC$ is a null vector, then w.r.t traingel ABC, point P is its a. centroid b. orthocentre c. incentre d. circumcentre

A. centroid

B. orthocentre

C. incentre

D. circumcentre

Answer: a

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25. G is the centroid of triangle ABC and A_1 and B_1 are the midpoints of sides AB and AC, respectively. If Δ_1 is the area of quadrilateral GA_1AB_1 and Δ is the area of triangle ABC, then $\frac{\Delta}{\Delta_1}$ is equal to

A. $\frac{3}{2}$

B. 3

C. $\frac{1}{3}$

D. none of these

Answer: b

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26. Points $\vec{a}, \vec{b}\vec{c}$ and $\vec{d}$ are coplanar and $(\sin\alpha)\vec{a} + (2\sin 2\beta)\vec{b} + (3\sin 3\gamma)\vec{c} - \vec{d} = \vec{0}$. Then the least value of $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma$ is

A. $1/14$

B. 14

C. 6

D. $1/\sqrt{6}$

Answer: a

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27. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 1and 2. respectively, and $\left(1 - 3\vec{a}.\vec{b}\right)^2 + \left|2\vec{a} + \vec{b} + 3\left(\vec{a} \times \vec{b}\right)\right|^2 = 47$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\pi/3$

B. $\pi - \cos^{-1}(1/4)$

C. $\frac{2\pi}{3}$

D. $\cos^{-1}(1/4)$

Answer: c

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28. If $\vec{a}$ and $\vec{b}$ are any two vectors of $magnitude$ 2 and 3 respectively such that $\left|2\left(\vec{a} \times \vec{b}\right)\right| + \left|3\left(\vec{a}.\vec{b}\right)\right| = k$ then the maximum value of k is

A. $\sqrt{13}$

B. $2\sqrt{13}$

C. $6\sqrt{13}$

D. $10\sqrt{13}$

Answer: c

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29. $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vecrtors such that $\left|\vec{a}+\vec{b}+3\vec{c}\right|=4$ Angle between $\vec{a}$ and $\vec{b} is \theta_1$, between $\vec{b}$ and $\vec{c} is \theta_2$ and between $\vec{a}$ and $\vec{b}$ varies $[\pi/6, 2\pi/3]$. Then the maximum value of $\cos\theta_1+3\cos\theta_2$ is

A. 3

B. 4

C. $2\sqrt{2}$

D. 6

Answer: b

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30. If the vector product of a constant vector $\vec{OA}$ with a variable vector $\vec{OB}$ in a fixed plane OAB be a constant vector, then the locus of B is (a).a straight line perpendicular to $\vec{OA}$ (b). a circle with centre O and radius equal to $\left|\vec{OA}\right|$ (c). a straight line parallel to $\vec{OA}$ (d). none of these

A. a straight line perpendicular to $\overrightarrow{OA}$

B. a circle with centre O and radius equal to $\left|\overrightarrow{OA}\right|$

C. a striaght line parallel to $\overrightarrow{OA}$

D. none of these

Answer: c

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31. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}|3$. If the projection of $\vec{v} along \vec{u}$ is equal to that of $\vec{w} along \vec{v}, \vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals (A) 2 (B) $\sqrt{7}$ (C) $\sqrt{14}$ (D) 14

A. 2

B. $\sqrt{7}$

C. $\sqrt{14}$

D. 14

Answer: c

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32. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $\vec{u}$ and $\vec{v}$ are any two vectors.

Prove that $\vec{u} \times \vec{v} = \frac{1}{\left[\vec{a}\vec{b}\vec{c}\right]}\begin{vmatrix} \vec{u}.\vec{a} & \vec{v}.\vec{a} & \vec{a} \\ \vec{u}.\vec{b} & \vec{v}.\vec{b} & \vec{b} \\ \vec{u}.\vec{c} & \vec{v}.\vec{c} & \vec{c} \end{vmatrix}$

A. $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

B. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

C. $\pi\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

D. cannot of these

Answer: b

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33. if $\vec{\alpha}\ |\ |\ \left(\vec{\beta}\times\vec{\gamma}\right),\ \text{ then }\ \left(\vec{\alpha}\times\vec{\gamma}\right)$ equal to

A. $\left|\vec{\alpha}\right|^2\left(\vec{\beta}.\vec{\gamma}\right)$

B. $\left|\vec{\beta}\right|^2\left(\vec{\gamma}.\vec{\alpha}\right)$

C. $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}.\vec{\beta}\right)$

D. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

Answer: a

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34. The position vectors of points A,B and C are $\hat{i}+\hat{j}, \hat{i}+5\hat{j}-\hat{k}$ and $2\hat{i}+3\hat{j}+5\hat{k}$, respectively the greatest angle of triangle ABC is

A. 120°

B. 90°

C. $\cos^{-1}(3/4)$

D. none of these

Answer: b

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35. Given three vectors e$\vec{a}, \vec{b}$ and $\vec{c}$ two of which are non-collinear. Futrther if $\left(\vec{a}+\vec{b}\right)$ is collinear with $\vec{c}, \left(\vec{b}+\vec{c}\right)$ is collinear with $\vec{a}, |\vec{a}| = \left|\vec{b}\right| = |\vec{c}| = \sqrt{2}$ find the value of $\vec{a}. Vecb + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

A. 3

B. -3

C. 0

D. cannot of these

Answer: b

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36. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left(\vec{a}+\vec{b}\right).\left(2\vec{a}+3\vec{b}\right)\times\left(3\vec{a}-2\vec{b}\right)=\vec{0}$ then angle between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. π

D. indeterminate

Answer: d

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37. If in a right-angled triangle ABC, the hypotenuse $AB = p, then \vec{A}B\dot{A}C + \vec{B}C\dot{B}A + \vec{C}A\dot{C}B$ is equal to $2p^2$ b. $\frac{p^2}{2}$ c. p^2 d. none of these

A. $2p^2$

B. $\frac{p^2}{2}$

C. p^2

D. none of these

Answer: c

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38. Resolved part of vector $\vec{a}$ and along vector $\vec{b}$ is $\vec{a}1$ and that prependicular to $\vec{b}$ is $\vec{a}2$ then $\vec{a}1 \times \vec{a}2$ is equl to

A. $\dfrac{\left(\vec{a}\times\vec{b}\right).\vec{b}}{\left|\vec{b}\right|^2}$

B. $\dfrac{\left(\vec{a}.\vec{b}\right)\vec{a}}{\left|\vec{a}\right|^2}$

C. $\dfrac{\left(\vec{a}.\vec{b}\right)\left(\vec{b}\times\vec{a}\right)}{\left|\vec{b}\right|^2}$

D. $\dfrac{\left(\vec{a}.\vec{b}\right)\left(\vec{b}\times\vec{a}\right)}{\left|\vec{b}\times\vec{a}\right|}$

Answer: c

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39. $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{j} + 2\hat{j} - \hat{k}, \vec{c} = \hat{i} + \hat{j} - 2\hat{k}$. A vector coplanar with $\vec{b}$ and $\vec{c}$. Whose projection on $\vec{a}$ is magnitude $\sqrt{\dfrac{2}{3}}$ is

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $-2\hat{i} - \hat{j} + 5\hat{k}$

C. $2\hat{i} + 3\hat{j} + 3\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: b

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40. If P is any arbitary point on the circumcurcle of the equilateral triangle of side length l units, then $\left|\overrightarrow{PA}\right|^2 + \left|\overrightarrow{PB}\right|^2 + \left|\overrightarrow{PC}\right|^2$ is always equal to

A. $2l^2$

B. $2\sqrt{3}l^2$

C. l^2

D. $3l^2$

Answer: a

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41. If $\vec{r}$ and $\vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $|\vec{r} + b\vec{s}|$ is minimum, then the value of $|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2$ is equal to

A. $2|\vec{r}|^2$

B. $|\vec{r}|^2/2$

C. $3|\vec{r}|^2$

D. $|\vec{r}|^2$

Answer: d

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42. $\vec{a}$ and $\vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that if equally inclined to $\vec{a}, \vec{b}$ and $\vec{a} \times \vec{b}$ is equal to

A. $\frac{1}{\sqrt{2}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

B. $\frac{1}{2}\left(\vec{a} \times \vec{b} + \vec{a} + \vec{b}\right)$

C. $\frac{1}{\sqrt{3}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

D. $\frac{1}{3}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

Answer: a

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43. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a} + \vec{b} = \mu\vec{p}, \vec{b}. \vec{q} = 0$ and $\left|\vec{b}\right|^2 = 1$ where μ is a sclar. Then $\left|\left(\vec{a}. \vec{q}\right)\vec{p} - \left(\vec{p}. \vec{q}\right)\vec{a}\right|$ is equal to

(a)$2\left|\vec{p}\vec{q}\right|$ (b)$(1/2)\left|\vec{p}. \vec{q}\right|$ (c)$\left|\vec{p} \times \vec{q}\right|$ (d)$\left|\vec{p}. \vec{q}\right|$

A. $2\left|\vec{p}\vec{q}\right|$

B. $(1/2)\left|\vec{p}. \vec{q}\right|$

C. $\left|\vec{p} \times \vec{q}\right|$

D. $\left|\vec{p}. \vec{q}\right|$

Answer: d

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44. The position vectors of the vertices A, B and C of a triangle are three unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. A vector $\vec{d}$ is such that $\vec{d}.\hat{a} = \vec{d}.Hatb = \vec{d}.\hat{c}$ and $\vec{d} = \lambda\left(\hat{b} + \hat{c}\right)$. Then triangle ABC is

A. acute angled

B. obtuse angled

C. right angled

D. none of these

Answer: a

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45. If a is real constant $A, BandC$ are variable angles and $\sqrt{a^2 - 4}\tan A + a\tan B + \sqrt{a^2 + 4}\tan c = 6a$, then the least vale of $\tan^2 A + \tan^2 b + \tan^2 C is\ 6$ b. 10 c. 12 d. 3

A. 6

B. 10

C. 12

D. 3

Answer: d

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46. The vertex A triangle ABC is on the line $\vec{r} = \hat{i} + \hat{j} + \lambda\hat{k}$ and the vertices $BandC$ have respective position vectors $\hat{i} and \hat{j}$ Let Delta be the area of the triangle and $\mathrm{Delta}\left[3/2, \sqrt{33}/2\right]$. Then the range of values of λ corresponding to a is $[-8, 4] \cup [4, 8]$ b. $[-4, 4]$ c. $[-2, 2]$ d. $[-4, -2] \cup [2, 4]$

A. [-8, -4]cup[4,8]`

B. $[-4, 4]$

C. [-2,2]

D. $[-4, -2] \cup [2, 4]$

Answer: c

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47. A non-zero vecto $\vec{a}$ is such tha its projections along vectors $\frac{\hat{i}+\hat{j}}{\sqrt{2}}, \frac{-\hat{i}+\hat{j}}{\sqrt{2}}$ and $\hat{k}$ are equal , then unit vector along $\vec{a}$ us

A. $\frac{\sqrt{2}\hat{j}-\hat{k}}{\sqrt{3}}$

B. $\frac{\hat{j}-\sqrt{2}\hat{k}}{\sqrt{3}}$

C. $\frac{\sqrt{2}}{\sqrt{3}}\hat{j}+\frac{\hat{k}}{\sqrt{3}}$

D. $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$

Answer: a

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48. Position vector $\hat{k}$ is rotated about the origin by angle 135° in such a way that the plane made bt it bisects the angle between $\hat{i}$ and $\hat{j}$. Then its new position is

A. $\pm\frac{\hat{i}}{\sqrt{2}} \pm \frac{\hat{j}}{\sqrt{2}}$

B. $\pm\frac{\hat{i}}{2} \pm \frac{\hat{j}}{2} - \frac{\hat{k}}{\sqrt{2}}$

C. $\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{k}}{\sqrt{2}}$

D. none of these

Answer: d

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49. In a quadrilateral $ABCD, \vec{A}C$ is the bisector of $\vec{A}Band\vec{A}D$, angle between $\vec{A}Band\vec{A}D$ is $2\pi/3$, $15\left|\vec{A}C\right| = 3\left|\vec{A}B\right| = 5\left|\vec{A}D\right|^{\cdot}$ Then the angle between $\vec{B}Aand\vec{C}D$ is $\frac{\cos^{-1}\left(\sqrt{14}\right)}{7\sqrt{2}}$ b. $\frac{\cos^{-1}\left(\sqrt{21}\right)}{7\sqrt{3}}$ c. $\frac{\cos^{-1}2}{\sqrt{7}}$ d. $\frac{\cos^{-1}\left(2\sqrt{7}\right)}{14}$

A. $\cos^{-1}\frac{\sqrt{14}}{7\sqrt{2}}$

B. $\cos^{-1}\frac{\sqrt{21}}{7\sqrt{3}}$

C. $\cos^{-1}\frac{2}{\sqrt{7}}$

D. $\cos^{-1}\frac{2\sqrt{7}}{14}$

Answer: c

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50. In AB, DE and GF are parallel to each other and AD, BG and EF ar parallel to each other . If CD: CE = CG:CB = 2:1 then the value of area

$(\triangle AEG) : area(\triangle ABD)$ is equal to (a) 7/2 (b)3 (c)4 (d)9/2

A. $7/2$

B. 3

C. 4

D. $9/2$

Answer: b

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51. Vectors $\hat{a}$ in the plane of $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ is such that it is equally inclined to $\vec{b}$ and $\vec{d}$ where $\vec{d} = \hat{j} + 2\hat{k}$ the value of $\hat{a}$ is (a) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (b) $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ (c) $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$ (d) $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$

A. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

B. $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

C. $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$

D. $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$

Answer: b

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52. Let $ABCD$ be a tetrahedron such that the edges $AB, ACandAD$ are mutually perpendicular. Let the area of triangles $ABC, ACDandADB$ be 3, 4 and 5sq. units, respectively. Then the area of triangle BCD is a. $5\sqrt{2}$ b. 5 c. $\frac{\sqrt{5}}{2}$ d. $\frac{5}{2}$

A. $5\sqrt{2}$

B. 5

C. $\frac{\sqrt{5}}{2}$

D. $\frac{5}{2}$

Answer: a

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53. Let $\overrightarrow{f}(t)=[t]\hat{i}+(t-[t])\hat{j}+[t+1]\hat{k}, where[.]$ denotes the greatest integer function. Then the vectors $\overrightarrow{f}\left(\frac{5}{4}\right) and f(t), 0<t<1$ are(a) parallel to each other(b) perpendicular(c) inclined at $\cos^{-1}2\left(\sqrt{7\left(1-t^2\right)}\right)$ (d)inclined at $\cos^{-1}\left(\frac{8+t}{\sqrt{1+t^2}}\right)$;

A. parallel to each other

B. perpendicular to each other

C. inclined at $\frac{\cos^{-1}2}{\sqrt{7}\left(1-t^2\right)}$

D. inclined at $\frac{\cos^{-1}(8+t)}{9\sqrt{1+t^2}}$

Answer: d

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54. If $\vec{a}$ is parallel to $\vec{b} \times \vec{c}$, $then\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$ is equal to (a) $|\vec{a}|^2\left(\vec{b}.\vec{c}\right)$ (b) $\left|\vec{b}\right|^2\left(\vec{a}.\vec{c}\right)$ (c) $|\vec{c}|^2\left(\vec{a}.\vec{b}\right)$ (d) none of these

A. $|\vec{a}|^2\left(\vec{b}.\vec{c}\right)$

B. $\left|\vec{b}\right|^2\left(\vec{a}.\vec{c}\right)$

C. $|\vec{c}|^2\left(\vec{a}.\vec{b}\right)$

D. none of these

Answer: a

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55. The three vectors $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume: ____________

A. $1/3$

B. 4

C. $\left(3\sqrt{3}\right)/4$

D. $4\sqrt{3}$

Answer: d

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56. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} + \vec{c} \times \vec{a}$ is a non- zero vector and $|\left(\vec{d}.\vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d}.\vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d}.\vec{b}\right)\left(\vec{c} \times \vec{a}\right) = 0$ then

A. $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$

B. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = \left|\vec{d}\right|$

C. $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

D. none of these

Answer: c

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57. If $\left|\vec{a}\right| = 2$ and $\left|\vec{b}\right| = 3$ and $\vec{a}.\vec{b} = 0$, then $\left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)\right)$ is equal to the given diagonal is $\vec{c} = 4\hat{k} = 8\hat{k}$ then , the volume of a parallelpiped is

A. $48\hat{b}$

B. $-48\hat{b}$

C. $48\hat{a}$

D. $-48\hat{a}$

Answer: a

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58. If two diagonals of one of its faces are $6\hat{i} + 6\hat{k}$ and $4\hat{j} + 2\hat{k}$ and of the edges not containing the given diagonals is $\vec{c} = 4\hat{j} - 8\hat{k}$, then the volume of a parallelpiped is

A. 60

B. 80

C. 100

D. 120

Answer: d

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59. The volume of a tetrahedron fomed by the coterminus edges $\vec{a}, \vec{b}$ and $\vec{c} is 3$. Then the volume of the parallelepiped formed by the coterminus edges $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ is

A. 6

B. 18

C. 36

D. 9

Answer: c

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60. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually orthogonal unit vectors , then the triple product $\left[\vec{a}+\vec{b}+\vec{c} \quad \vec{a}+\vec{b} \quad \vec{b}+\vec{c}\right]$ equals

A. 0

B. 1 or -1

C. 1

D. 3

Answer: b

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61. vector $\vec{c}$ are perpendicular to vectors $\vec{a}=(2,\ -3,1)$ and $\vec{b}=(1,\ -2,3)$ and satifies the condition $\vec{c}.\left(\hat{i}+2\hat{j}-7\hat{k}\right)=10$ then vector $\vec{c}$ is equal to $(a)(7,5,1)\ (b)(-7,\ -5,\ -1)\ (c)(1,1,\ -1)\ (d)$ none of these

A. 7,5,1

B. (-7, -5, -1)

C. 1,1,-1

D. none of these

Answer: a

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62. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j}, \vec{a} \perp \vec{b}, \vec{a}.\vec{c} = 4$ then find the value of $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$.

A. $\left[\vec{a}\vec{b}\vec{c}\right]^2 = |\vec{a}|$

B. $\left[\vec{a}\vec{b}\vec{c}\right] = |\vec{a}|$

C. $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

D. $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

Answer: d

63. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ then the value of

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{is}$$

A. 0

B. 1

C. $\frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

D. $\frac{3}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

Answer: c

64. Let $\vec{r}, \vec{a}, \vec{b}$ and $\vec{c}$ be four non-zero vectors such that $\vec{r}.\vec{a} = 0, \left|\vec{r} \times \vec{b}\right| = \left|\vec{r}\right|\left|\vec{b}\right|, \left|\vec{r} \times \vec{c}\right| = \left|\vec{r}\right|\left|\vec{c}\right|$ then $\left[\vec{a} \ \vec{b} \ \vec{c}\right] =$

A. |a||b||c|

B. - $|a||b||c|$

C. 0

D. none of these

Answer: c

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65. If $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\left[\vec{a} \ \vec{b} \ \vec{c}\right] = 1, \vec{c} = \lambda\left(\vec{a} \times \vec{b}\right)$, angle between $\vec{c}$ and $\vec{b}$ is $2\pi/3$, $\left|\vec{a}\right| = \sqrt{2}, \left|\vec{b}\right| = \sqrt{3}$ and $\left|\vec{c}\right| = \frac{1}{\sqrt{3}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. (a) $\frac{\pi}{6}$

B. (b) $\frac{\pi}{4}$

C. (c) $\frac{\pi}{3}$

D. (d) $\frac{\pi}{2}$

Answer: b

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66. If $4\vec{a} + 5\vec{b} + 9\vec{c} = 0$ then $\left(\vec{a} \times \vec{b}\right) \times \left[\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\right]$ is equal to

A. a vector perpendicular to the plane of $\vec{a}, \vec{b}$ and $\vec{c}$

B. a scalar quantity

C. $\vec{0}$

D. none of these

Answer: c

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67. Value of $\left[\vec{a}\times\vec{b},\vec{a}\times\vec{c},\vec{d}\right]$ is always equal to (a) (⃗ a .⃗ d) [⃗ a⃗ b⃗ c] (b) (⃗ a .⃗ c) [⃗ a⃗ b⃗ d] (c) (⃗ a .⃗ b) [⃗ a⃗ b⃗ d] (d) none of these

A. $\left(\vec{a}.\vec{d}\right)\left[\vec{a}\vec{b}\vec{c}\right]$

B. `(veca.vecc)[veca vecb vecd]

C. $\left(\vec{a}.\vec{b}\right)\left[\vec{a}\vec{b}\vec{d}\right]$

D. none of these

Answer: a

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68. Let $\hat{a}$ and $\hat{b}$ be mutually perpendicular unit vectors. Then for ant arbitrary $\vec{r}$.

A. $\vec{r}=\left(\vec{r}.\hat{a}\right)\hat{a}+\left(\vec{r}.\hat{b}\right)\hat{b}+\left(\vec{r}.\left(\vec{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$

B. $\vec{r}=\left(\vec{r}.\hat{a}\right)-\left(\vec{r}.\hat{b}\right)\hat{b}-\left(\vec{r}.\left(\vec{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$

C. $\vec{r}=\left(\vec{r}.\hat{a}\right)\hat{a}-\left(\vec{r}.\hat{b}\right)\hat{b}-\left(\vec{r}.\left(\vec{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$

D. none of these

Answer: a

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69. Let $\vec{a}$ and $\vec{b}$ be unit vectors that are perpendicular to each other, then $\left[\vec{a} + \left(\vec{a} \times \vec{b}\right) + \left(\vec{a} \times \vec{b}\right)\right]$ is equal to

A. 1

B. 0

C. -1

D. none of these

Answer: a

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70. $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 4$ and $\vec{a}$. Vecb $= 2. If$ vecc $= \left(2\vec{a} \times \vec{b}\right) - 3\vec{b}$ then find angle between $\vec{b}$ and $\vec{c}$.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{6}$

Answer: d

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71. If $\vec{b}$ and $\vec{c}$ are unit vectors, then for any arbitary vector $\vec{a}, \left(\left(\left(\vec{a} \times \vec{b}\right) + \left(\vec{a} \times \vec{c}\right)\right) \times \left(\vec{b} \times \vec{c}\right)\right). \left(\vec{b} - \vec{c}\right)$ is always equal to

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72. If $\vec{a}.\vec{b} = \beta$ and $\vec{a} \times \vec{b} = \vec{c}$, then $\vec{b}$ is

A. $\dfrac{\left(\beta\vec{a} - \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

B. $\dfrac{\left(\beta\vec{a} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

C. $\dfrac{\left(\beta\vec{c} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

D. $\dfrac{\left(\beta\vec{c} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

Answer: a

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73. If $a\left(\vec{\alpha} \times \vec{\beta}\right) \times \left(\vec{\beta} \times \vec{\gamma}\right) + c\left(\vec{\gamma} \times \vec{\alpha}\right) = 0$ and at leasy one of a,b and c is non-zerp , then vector $\vec{\alpha}, \vec{\beta}$ and γ are

A. parallel

B. coplanar

C. mutually perpendicular

D. none of these

Answer: b

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74. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right) = \vec{b}, where \vec{a}, \vec{b}$ and $\vec{c}$ are non zero vectors then (A) $\vec{a}, \vec{b}$ and $\vec{c} canbecoplanar(B)$veca,vecb and vecc$\mu stbecoplanar(C)$ veca,vecb and vecc cannot be coplanar (D) none of these

A. $\vec{a}, \vec{b}$ and $\vec{v}$ can be coplanar

B. $\vec{a}, \vec{b}$ and $\vec{c}$ must be coplanar

C. $\vec{a}, \vec{b}$ and $\vec{c}$ cannot be coplanar

D. none of these

Answer: c

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75. If $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = \frac{1}{2}$ for some non zero vector $\vec{r}$ and $\vec{a}, \vec{b}, \vec{c}$ are non coplanar, then the area of the triangle whose vertices are $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ is

A. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$

B. $\left|\vec{r}\right|$

C. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}\right|$

D. none of these

Answer: c

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76. A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point $P(1, 0)$ can be (a).$6\hat{i} + 8\hat{j}$ (b). $-8\hat{i} + 3\hat{j}$ (c). $6\hat{i} - 8\hat{j}$ (d). $8\hat{i} + 6\hat{j}$

A. $6\hat{i} + 8\hat{j}$

B. $-8\hat{i} + 3\hat{j}$

C. $6\hat{i} - 8\hat{j}$

D. $8\hat{i} + 6\hat{j}$

Answer: a

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77. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\frac{\pi}{3}$ then $\left\{\vec{a} \times \left(\vec{b} + \vec{a} \times \vec{b}\right)\right\}.\vec{b}$ is equal to (a) $-\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

A. $\frac{-3}{4}$

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. $\frac{1}{2}$

Answer: a

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78. If $\vec{a}$ and $\vec{b}$ are othogonal unit vectors, then for a vector $\vec{r}$ non - coplanar with $\vec{a}$ and $\vec{b}$ vector $\vec{r}\times\vec{a}$ is equal to

A. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{b}-\left(\vec{r}.\vec{b}\right)\left(\vec{b}\times\vec{a}\right)$

B. $\left[\vec{r}\vec{a}\vec{b}\right]\left(\vec{a}+\vec{b}\right)$

C. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{a}+\left(\vec{r}.\vec{a}\right)\vec{a}\times\vec{b}$

D. none of these

Answer: a

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79. If $\vec{a}+\vec{b},\vec{c}$ are any three non- coplanar vectors then the equation $\left[\vec{b}\times\vec{c}\vec{c}\times\vec{a}\vec{a}\times\vec{b}\right]x^2+\left[\vec{a}+\vec{b}\vec{b}+\vec{c}\vec{c}+\vec{a}\right]x+1+\left[\vec{b}-\vec{c}\vec{c}-\vec{c}-\vec{a}\vec{a}-\vec{b}\right]=0$ has roots

A. real and distinct

B. real

C. equal

D. imaginary

Answer: c

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80. Sholve the simultasneous vector equations for $\vec{x}$ and $\vec{y} : \vec{x} + \vec{c} \times \vec{y} = \vec{a}$ and $\vec{y} + \vec{c} \times \vec{x} = \vec{b}, \vec{c} \neq 0$

A. $\vec{x} = \dfrac{\vec{b} \times \vec{c} + \vec{a} + (\vec{c}.\vec{a})\vec{c}}{1 + \vec{c}.\vec{c}}$

B. $\vec{x} = \dfrac{\vec{c} \times \vec{b} + \vec{b} + (\vec{c}.\vec{a})\vec{c}}{1 + \vec{c}.\vec{c}}$

C. $\vec{y} = \dfrac{\vec{a} \times \vec{c} + \vec{b} + (\vec{c}.\vec{b})\vec{c}}{1 + \vec{c}.\vec{c}}$

D. none of these

Answer: b

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81. The condition for equations $\vec{r} \times \vec{a} = \vec{b}$ and $\vec{r} \times \vec{c} = \vec{d}$ to be consistent is

A. $\vec{b}.\vec{c} = \vec{a}.\vec{d}$

B. $\vec{a}.\vec{b} = \vec{c}.\vec{d}$

C. $\vec{b}.\vec{c} + \vec{a}.\vec{d} = 0$

D. $\vec{a}.\vec{b} + \vec{c}.\vec{d} = 0$

Answer: c

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82. If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, then $\left[\vec{a}\vec{b}\vec{c}\right] =$

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83. If

$\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + \hat{j} + 2\hat{k}$ and $(1 + \alpha)\hat{i} + \beta(1 + \alpha)\hat{j} + \gamma(1 + \alpha)($

A. $-2, \ -4, \ -\frac{2}{3}$

B. $2, \ -4, \frac{2}{3}$

C. $-2, 4, \frac{2}{3}$

D. $2, 4, \ -\frac{2}{3}$

Answer: a

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84. Let $\left(\vec{a}(x) = (\sin x)\hat{i} + (\cos x)\hat{j}\right.$ and $\vec{b}(x) = (\cos 2x)\hat{i} + (\sin 2x)\hat{j}$ be two variable vectors $(x \in R)$. Then $\vec{a}(x)$ and $\vec{b}(x)$ are

A. collinear for unique value of x

B. perpendicular for infinte values of x.

C. zero vectors for unique value of x

D. none of these

Answer: b

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85. For any vectors $\vec{a}$ and $\vec{b}$, $\left(\vec{a}\times\hat{i}\right)+\left(\vec{b}\times\hat{i}\right)+\left(\vec{a}\times\hat{j}\right).\left(\vec{b}\times\hat{j}\right)+\left(\vec{a}\times\hat{k}\right).\left(\vec{b}\times\hat{k}\right)$ is always equal to

A. $\vec{a}.\vec{b}$

B. $2\vec{a}.Vecb$

C. zero

D. none of these

Answer: b

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86. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non coplanar vectors and $\vec{r}$ is any vector in space, then $\left(\times \vec{b}\right), \left(\vec{r} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right) \times \left(\vec{r} \times \vec{a}\right) + \left(\vec{c} \times \vec{a}\right) \times \left(\vec{r} \times \vec{b}\right) =$

(A) $\left[\vec{a}\vec{b}\vec{c}\right]$ (B) $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ (C) $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$ (D) $4\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

A. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

B. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

C. $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

D. none of these

Answer: b

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87. If $\vec{P} = \dfrac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]} . \vec{q} = \dfrac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}$ and $\vec{r} = \dfrac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are three non- coplanar vectors then the value of the expression $\left(\vec{a} + \vec{b} + \vec{c}\right). \left(\vec{q} + \vec{q} + \vec{r}\right)$ is

A. 3

B. 2

C. 1

D. 0

Answer: a

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88. $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ are the vertices of triangle ABC and $R(\vec{r})$ is any point in the plane of triangle ABC, then $\vec{r}, (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$ is always equal to

A. zero

B. $[\vec{a}\vec{b}\vec{c}]$

C. - $[\vec{a}\vec{b}\vec{c}]$

D. none of these

Answer: b

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89. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar vectors and $\vec{a} \times \vec{c}$ is perpendicular to $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$, then the value of $\left[\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right] \times \vec{c}$ is equal to

A. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{c}$

B. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{b}$

C. $\vec{0}$

D. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{a}$

Answer: c

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90. If V be the volume of a tetrahedron and V' be the volume of another tetrahedran formed by the centroids of faces of the previous tetrahedron and $V = KV'$, $then K$ is equal to a. 9 b. 12 c. 27 d. 81

A. 9

B. 12

C. 27

D. 81

Answer: c

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91. $\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right) \left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right) \left(\vec{c} \times \vec{a}\right) \times \left(\vec{a} \times \vec{b}\right)\right]$ is equal to (where $\vec{a}, \vec{b}$ and $\vec{c}$ are non - zero non- colanar vectors). $(a)\left[\vec{a}\vec{b}\vec{c}\right]^2$ $(b)\left[\vec{a}\vec{b}\vec{c}\right]^3$ $(c)\left[\vec{a}\vec{b}\vec{c}\right]^4$ $(d)\left[\vec{a}\vec{b}\vec{c}\right]$

A. $\left[\vec{a}\vec{b}\vec{c}\right]^2$

B. $\left[\vec{a}\vec{b}\vec{c}\right]^3$

C. $\left[\vec{a}\vec{b}\vec{c}\right]^4$

D. $\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: c

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92. If

$\vec{r} = x_1\left(\vec{a} \times \vec{b}\right) + x_2\left(\vec{b} \times \vec{a}\right) + x_3\left(\vec{c} \times \vec{d}\right)$ and $4\left[\vec{a}\vec{b}\vec{c}\right] = 1$ then $x_1 + x_2 + x_3$ is equal to

A. $\frac{1}{2}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

B. $\frac{1}{4}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

C. $2\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

D. $4\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

Answer: d

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93. If $\vec{a} \perp \vec{b}$ then vector $\vec{v}$ in terms of $\vec{a}$ and $\vec{b}$ satisfying the equations $\vec{v}. Veca = 0nad\vec{v}. Vecb = 1$ and $\left[\vec{a}\vec{a}\vec{b}\right] = 1$ is

A. $\dfrac{\vec{b}}{\left|\vec{b}\right|^2} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$

B. $\dfrac{\vec{b}}{\left|\vec{b}\right|} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$

C. $\dfrac{\vec{b}}{\left|\vec{b}\right|} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|}$

D. none of these

Answer: a

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94. If $\vec{a}' = \hat{i} + \hat{j}, \vec{b}' = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c}' = 2\hat{i} - \hat{j} - \hat{k}$ then the altitude of the parallelepiped formed by the vectors, $\vec{a}, \vec{b}$ and $\vec{c}$ having base formed by $\vec{b}$ and $\vec{c}$ is (where $\vec{a}'$ is recipocal vector $\vec{a}$) $(a)1\ \ (b)3\sqrt{2}/2\ \ (c)1/\sqrt{6}$ $(d)1/\sqrt{2}$

A. 1

B. $3\sqrt{2}/2$

C. $1/\sqrt{6}$

D. $1/\sqrt{2}$

Answer: d

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95. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}, \vec{c} = \hat{k} + \hat{i}$ then in the reciprocal system of vectors $\vec{a}, \vec{b}, \vec{c}$ reciprocal $\vec{a}$ of vector $\vec{a}$ is

A. $\dfrac{\hat{i} + \hat{j} + \hat{k}}{2}$

B. $\dfrac{\hat{i} - \hat{j} + \hat{k}}{2}$

C. $\dfrac{-\hat{i} - \hat{j} + \hat{k}}{2}$

D. $\dfrac{\hat{i} + \hat{j} - \hat{k}}{2}$

Answer: d

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96. If unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $\left|\vec{a} - \vec{b}\right| < 1$ and $0 \leq \theta \leq \pi$, then θ lies in the interval

A. $[0, \pi/6)$

B. $(5\pi/6, \pi]$

C. $[\pi/6, \pi/2]$

D. $(\pi/2, 5\pi/6]$

Answer: a,b

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97. $\vec{b}$ and $\vec{c}$ are non- collinear if $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a}.\vec{b}\right)\vec{b} = (4 - 2x - \sin y)\vec{b} + \left(x^2 - 1\right)\vec{c}nad\left(\vec{a}.\vec{c}\right)\vec{a} = \vec{a}$ then

a. x =1 b. x = -1 c. $y = (4n + 1)\frac{\pi}{2}, n \in I$ d. $y(2n + 1)\frac{\pi}{2}, n \in I$

A. x =1

B. x = -1

C. $y = (4n + 1)\frac{\pi}{2}, n \in I$

D. $y(2n + 1)\frac{\pi}{2}, n \in I$

Answer: a,c

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98. Unit vectors $\vec{a}$ and $\vec{b}$ ar perpendicular , and unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. $If \alpha\vec{a} + \beta\vec{b} + \gamma\left(\vec{a} \times \vec{b}\right)$ then.

A. $\alpha = \beta$

B. $\gamma^2 = 1 - 2\alpha^2$

C. $\gamma^2 = -\cos 2\theta$

D. $\beta^2 = \frac{1 + \cos 2\theta}{2}$

Answer: a,b,c,d

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99. If vectors $\vec{a}$ and $\vec{b}$ are two adjecent sides of a paralleogram, then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$is

A. $\dfrac{\left(\vec{a}.\vec{b}\right)}{|\vec{a}|^2}\vec{a} - \vec{b}$

B. $\dfrac{1}{|\vec{a}|^2}\left\{|\vec{a}|^2\vec{b} - \left(\vec{a}.\vec{b}\right)\vec{a}\right\}$

C. $\dfrac{\vec{a} \times \left(\vec{a} \times \vec{b}\right)}{|\vec{a}|^2}$

D. $\dfrac{\vec{a} \times \left(\vec{b} \times \vec{a}\right)}{|\vec{b}|^2}$

Answer: a,b,c

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100. If $\vec{a} \times (\vec{b} \times \vec{c})$ is perpendicular to $(\vec{a} \times \vec{b}) \times \vec{c}$, we may have

A. $(\vec{a}.\vec{b})|\vec{b}|^2 = (\vec{a}.\vec{b})(\vec{b}.\vec{c})$

B. $\vec{a}.\vec{b} = 0$

C. $\vec{a}.\vec{c} = 0$

D. $\vec{b}.\vec{c} = 0$

Answer: a,c

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101. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be vectors forming right- hand triad . Let

$\vec{P} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q}\frac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]}$ and $\vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]} If x \cup R^+$ then

A. $x[\vec{a}\vec{b}\vec{c}] + \frac{[\vec{p}\vec{q}\vec{r}]}{x}$ has least value 2

B. $x^2[\vec{a}\vec{b}\vec{c}]^2 + \frac{[\vec{p}\vec{q}\vec{r}]}{x^2}$ has least value $(3/2^{2/3})$

C. $[\vec{p}\vec{q}\vec{r}] > 0$

D. none of these

Answer: a,c

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102. $a_1, a_2, a_3 \in R - \{0\}$ and $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ " for all " x in R then (a) vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other (b)vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ are parallel to each each other (c)if vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then on of the ordered trippplet $(a_1, a_2, a_3) = (1, -1, -2)$ (d)if $2a_1 + 3a_2 + 6a_3 = 26,$ then $\left|\vec{a}\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

A. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other

B. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ are parallel to each each other

C. if vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then on of the

ordered trippplet $(a_1, a_2, a_3) = (1, -1, -2)$

D. if $2a_1 + 3a_2 + 6a_3 + 6a_3 = 26$, then $\left|\vec{a}\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

Answer: a,b,c,d

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103. If $\vec{a}$ and $\vec{b}$ are two vectors and angle between them is θ, then

A. $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2 = |\vec{a}|^2\left|\vec{b}\right|^2$

B. $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2$, if $\theta = \pi/4$

C. $\vec{a} \times \vec{b} = \left(\vec{a}. Vecb\right)\hat{n}$ (where $\hat{n}$ is a normal unit vector) if $\theta f = \pi/4$

D. $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} + \vec{b}\right) = 0$

Answer: a,b,c,d

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104. Let $\vec{a}$ and $\vec{b}$ be two non- zero perpendicular vectors. A vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$ can be

A. $\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

B. $2\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

C. $\left|\vec{a}\right|\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

D. $\left|\vec{b}\right|\vec{b} - \dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

Answer: a,b,cd,

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105. If vector $\vec{b} = \left(tan\alpha, -1, 2\sqrt{\sin\alpha/2}\right) and \vec{c} = \left(tan\alpha, tan\alpha, -\dfrac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and vector $\vec{a} = (1, 3, \sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is

A. $\alpha = (4n + 1)\pi + \tan^{-1}2$

B. $\alpha = (4n + 1)\pi - \tan^{-1}2$

C. $\alpha = (4n + 2)\pi + \tan^{-1}2$

D. $\alpha = (4n + 2)\pi - \tan^{-1}2$

Answer: b,d

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106. Let $\vec{r}$ be a unit vector satisfying $\vec{r} \times \vec{a} = \vec{b}$, where $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = \sqrt{2}$, then $(a)\vec{r} = \frac{2}{3}(\vec{a} + \vec{a} \times \vec{b})$ (b) $\vec{r} = \frac{1}{3}(\vec{a} + \vec{a} \times \vec{b})$ (c) $\vec{r} = \frac{2}{3}(\vec{a} - \vec{a} \times \vec{b})$ (d) $\vec{r} = \frac{1}{3}(-\vec{a} + \vec{a} \times \vec{b})$

A. $\vec{r} = \frac{2}{3}(\vec{a} + \vec{a} \times \vec{b})$

B. $\vec{r} = \frac{1}{3}(\vec{a} + \vec{a} \times \vec{b})$

C. $\vec{r} = \frac{2}{3}(\vec{a} - \vec{a} \times \vec{b})$

D. $\vec{r} = \frac{1}{3}(-\vec{a} + \vec{a} \times \vec{b})$

Answer: b,d

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107. If $\vec{a}$ and $\vec{b}$ are unequal unit vectors such that $\left(\vec{a} - \vec{b}\right) \times \left[\left(\vec{b} + \vec{a}\right) \times \left(2\vec{a} + \vec{b}\right)\right] = \vec{a} + \vec{b}$ then angle θ between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. $\pi/4$

D. π

Answer: b,d

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108. If $\vec{a}$ and $\vec{b}$ are two unit vectors perpenicualar to each other and $\vec{c} = \lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\left(\vec{a} \times \vec{b}\right)$, then which of the following is (are) true ?

A. $\lambda_1 = \vec{a}.\vec{c}$

B. $\lambda_2 = \left|\vec{b} \times \vec{c}\right|$

C. $\lambda_3 = \ | \ \left(\vec{a} \times \vec{b}\right| \times \vec{c}\Big|$

D. $\lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\left(\vec{a} \times \vec{b}\right)$

Answer: a,d

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109. If vectors $\vec{a}$ and $\vec{b}$ are non collinear then $\dfrac{\vec{a}}{|\vec{a}|} + \dfrac{\vec{b}}{|\vec{b}|}$ is (A) a unit vector (B) in the plane of $\vec{a}$ and $\vec{b}$ (C) equally inclined to $\vec{a}$ and $\vec{b}$ (D) perpendicular to $\vec{a} \times \vec{b}$

A. a unit vector

B. in the plane of $\vec{a}$ and $\vec{b}$

C. equally inclined to $\vec{a}$ and $\vec{b}$

D. perpendicular to $\vec{a} \times \vec{b}$

Answer: b,c,d

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110. If $\vec{a}$ and $\vec{b}$ are non - zero vectors such that $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-2\vec{b}\right|$ then

A. $2\vec{a}.\vec{b}=\left|\vec{b}\right|^2$

B. $\vec{a}.\vec{b}=\left|\vec{b}\right|^2$

C. least value of $\vec{a}.Vecb+\dfrac{1}{\left|\vec{b}\right|^2+2}$ is $\sqrt{2}$

D. least value of $\vec{a}.\vec{b}+\dfrac{1}{\left|\vec{b}\right|^2+2}$ is $\sqrt{2}-1$

Answer: a,d

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111. Let $\vec{a}\vec{b}$ and $\vec{c}$ be non- zero vectors aned $\vec{V}_1=\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ and $\vec{V}_2=\left(\vec{a}\times\vec{b}\right)\times\vec{c}$.vectors $\vec{V}_1$ and $\vec{V}_2$ are equal .

Then

A. $\vec{a}$ and $\vec{b}$ ar orthogonal

B. $\vec{a}$ and $\vec{c}$ are collinear

C. $\vec{b}$ and $\vec{c}$ ar orthogonal

D. $\vec{b} = \lambda(\vec{a} \times \vec{c})$ when λ is a scalar

Answer: b,d

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112. Vectors $\vec{A}$ and $\vec{B}$ satisfying the vector equation $\vec{A} + \vec{B} = \vec{a}, \vec{A} \times \vec{B} = \vec{b}$ and $\vec{A}. \vec{a} = 1$. where veca and $\vec{b}$ are given vectosrs, are

A. $\vec{A} = \dfrac{(\vec{a} \times \vec{b}) - \vec{a}}{a^2}$

B. $\vec{B} = \dfrac{(\vec{b} \times \vec{a}) + \vec{a}(a^2 - 1)}{a^2}$

C. $\vec{A} = \dfrac{\left(\vec{a} \times \vec{b}\right) + \vec{a}}{a^2}$

D. $\vec{B} = \dfrac{\left(\vec{b} \times \vec{a}\right) - \vec{a}\left(a^2 - 1\right)}{a^2}$

Answer: b,c,

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113. A vector $\left(\vec{d}\right)$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$ let $\vec{x}, \vec{y}, \vec{z}$ be three in the plane of $\vec{a}, \vec{b}, \vec{b}, \vec{c}, \vec{c}, \vec{a}$ respectively, then

A. $\vec{x}. \vec{d} = -1$

B. $\vec{y}. \vec{d} = 1$

C. vecz.vecd=0`

D. vecr.vecd=0, " where " vecr=lambda vecx + mu vecy +deltavecz`

Answer: c.d

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114. Vectors Perpendicular to $\hat{i} - \hat{j} - \hat{k}$ and in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ are

A. $\hat{i} + \hat{k}$

B. $2\hat{i} + \hat{j} + \hat{k}$

C. $3\hat{i} + 2\hat{j} + \hat{k}$

D. $-4\hat{i} - 2\hat{j} - 2\hat{k}$

Answer: b,d

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115. if side $\overrightarrow{AB}$ of an equilateral triangle ABC lying in the x-y plane is $3\hat{i}$.

Then side $\overrightarrow{CB}$ can be

A. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$

B. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$

C. $-\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

D. $\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

Answer: b,d

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116. The angles of a triangle , two of whose sides are respresented by vectors $\sqrt{3}\left(\hat{a} \times \vec{b}\right)$ and $\hat{b} - \left(\hat{a}.\, Vecb\right)\hat{a}$ where $\vec{b}$ is a non - zero vector and $\vec{a}$ is a unit vector in the direction of $\vec{a}$. Are

A. $\tan^{-1}\left(\sqrt{3}\right)$

B. $\tan^{-1}\left(1/\sqrt{3}\right)$

C. $\cot^{-1}(0)$

D. tant^(-1)(1)`

Answer: a,b,c

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117. $\vec{a}, \vec{b}$ and $\vec{c}$ are unimodular and coplanar. A unit vector $\vec{d}$ is perpendicualt to them , $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$, and the angle between $\vec{a}$ and $\vec{b} is 30^\circ$ then $\vec{c}$ is

A. $\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)/3$

B. $\left(- \hat{i} + 2\hat{j} - 2\hat{k}\right)/3$

C. $\left(- \hat{i} + 2\hat{j} - \hat{k}\right)/3$

D. $\left(- 2\hat{i} - 2\hat{j} + \hat{k}\right)/3$

Answer: a,b

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118. If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$ then $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} =$

A. $2\left(\vec{a} \times \vec{b}\right)$

B. $6\left(\vec{b} \times \vec{c}\right)$

C. $3\left(\vec{c} \times \vec{a}\right)$

D. $\vec{0}$

Answer: c,d

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119. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - \left(\vec{a}.\vec{b}\right)\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $\left|\vec{v}\right|$ is

A. $\left|\vec{u}\right|$

B. $\left|\vec{u}\right| + \left|\vec{u}.\vec{b}\right|$

C. $\left|\vec{u}\right| + \left|\vec{u}.\vec{a}\right|$

D. none of these

Answer: b,d

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120. if $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}$, where $\vec{c} \neq \vec{0}$ then (a) $|\vec{a}| = |\vec{c}|$ (b) $|\vec{a}| = |\vec{b}|$ (c) $|\vec{b}| = 1$ (d) $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

A. $|\vec{a}| = |\vec{c}|$

B. $|\vec{a}| = |\vec{b}|$

C. $|\vec{b}| = 1$

D. $|\vec{a}| = \vec{b}| = |\vec{c}\ | = 1$

Answer: a,c

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121. Let $\vec{a}, \vec{b}$, and $\vec{c}$ be three non- coplanar vectors and $\vec{d}$ be a non -zero , which is perpendicular to $(\vec{a} + \vec{b} + \vec{c})$. $Now \vec{d} = (\vec{a} \times \vec{b})\sin x + (\vec{b} \times \vec{c})\cos y + 2(\vec{c} \times \vec{a})$. Then

A. $\dfrac{\vec{d}.\left(\vec{a}+\vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]}=2$

B. $\dfrac{\vec{d}.\left(\vec{a}+\vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]}=-2$

C. minimum value of $x^2+y^2 is \pi^2/4$

D. minimum value of $x^2+y^2 is 5\pi^2/4$

Answer: b,d

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122. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{1}{2}\vec{b}$, then $\left(\vec{b}\right.$ and $\vec{c}$ being non parallel)

A. angle between $\vec{a}$ and $\vec{b} is \pi/3$

B. angle between $\vec{a}$ and $\vec{c} is \pi/3$

C. angle between $\vec{a}$ and $\vec{b} is \pi/2$

D. angle between $\vec{a}$ and $\vec{c} is \pi/2$

Answer: b,c

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123. If in triangle ABC, $\overrightarrow{AB} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|}$ and $\overrightarrow{AC} = \frac{2\vec{u}}{|\vec{u}|}$, where $|\vec{u}| \neq |\vec{v}|$, then $(a) 1 + \cos 2A + \cos 2B + \cos 2C = 0$ (b)$\sin A = \cos C$ (c)projection of AC on BC is equal to BC (d) projection of AB on BC is equal to AB

A. $1 + \cos 2A + \cos 2B + \cos 2C = 0$

B. $\sin A = \cos C$

C. projection of AC on BC is equal to BC

D. projection of AB on BC is equal to AB

Answer: a,b,c

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124. $\left[\vec{a}\times\vec{b} \quad \vec{c}\times\vec{d} \quad \vec{e}\times\vec{f}\right]$ is equal to

(a) $\left[\vec{a}\vec{b}\vec{d}\right]\left[\vec{c}\vec{e}\vec{f}\right]-\left[\vec{a}\vec{b}\vec{c}\right]\left[\vec{d}\vec{e}\vec{f}\right]$

(b) $\left[\vec{a}\vec{b}\vec{e}\right]\left[\vec{f}\vec{c}\vec{d}\right]-\left[\vec{a}\vec{b}\vec{f}\right]\left[\vec{e}\vec{c}\vec{d}\right]$

(c) $\left[\vec{c}\vec{d}\vec{a}\right]\left[\vec{b}\vec{e}\vec{f}\right]-\left[\vec{a}\vec{d}\vec{b}\right]\left[\vec{a}\vec{e}\vec{f}\right]$

(d) $\left[\vec{a}\vec{c}\vec{e}\right]\left[\vec{b}\vec{d}\vec{f}\right]$

A. $\left[\vec{a}\vec{b}\vec{d}\right]\left[\vec{c}\vec{e}\vec{f}\right]-\left[\vec{a}\vec{b}\vec{c}\right]\left[\vec{d}\vec{e}\vec{f}\right]$

B. $\left[\vec{a}\vec{b}\vec{e}\right]\left[\vec{f}\vec{c}\vec{d}\right]-\left[\vec{a}\vec{b}\vec{f}\right]\left[\vec{e}\vec{c}\vec{d}\right]$

C. $\left[\vec{c}\vec{d}\vec{a}\right]\left[\vec{b}\vec{e}\vec{f}\right]-\left[\vec{a}\vec{d}\vec{b}\right]\left[\vec{a}\vec{e}\vec{f}\right]$

D. $\left[\vec{a}\vec{c}\vec{e}\right]\left[\vec{b}\vec{d}\vec{f}\right]$

Answer: a,b,c

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125. The scalars l and m such that $l\vec{a}+m\vec{b}=\vec{c}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are given vectors, are equal to

A. $l = \frac{(\vec{c} \times \vec{b}).(\vec{a} \times \vec{b})}{(\vec{a} \times \vec{b})^2}$

B. $l = \frac{(\vec{c} \times \vec{a}).(\vec{b} \times \vec{a})}{(\vec{b} \times \vec{a})}$

C. $m = \frac{(\vec{c} \times \vec{a}).(\vec{b} \times \vec{a})}{(\vec{b} \times \vec{a})^2}$

D. $m = \frac{(\vec{c} \times \vec{a}).(\vec{b} \times \vec{a})}{(\vec{b} \times \vec{a})}$

Answer: a,c

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126. If $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}).(\vec{a} \times \vec{d}) = 0$ then which of the following may be true ?

A. $\vec{a}, \vec{b}$ and $\vec{d}$ are nenessarily coplanar

B. $\vec{a}$ lies iin the plane of $\vec{c}$ and $\vec{d}$

C. $\vec{v}b$ lies in the plane of $\vec{a}$ and $\vec{d}$

D. $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{d}$

Answer: b,c,d

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127. A,B C and dD are four points such that $\overrightarrow{AB} = m\left(2\hat{i} - 6\hat{j} + 2\hat{k}\right)\overrightarrow{BC} = \left(\hat{i} - 2\hat{j}\right)$ and $\overrightarrow{CD} = n\left(-6\hat{i} + 15\hat{j} - 3\hat{k}\right)$. If CD intersects AB at some points E, then

A. $m \geq 1/2$

B. $n \geq 1/3$

C. m= n

D. $m < n$

Answer: a,b

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128. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non -coplanar and l, m, n are distinct scalars such that

$\left[l\vec{a} + m\vec{b} + n\vec{c} \quad l\vec{b} + m\vec{c} + n\vec{a} \quad l\vec{c} + m\vec{a} + n\vec{b}\right] = 0$ then

A. $l + m + n = 0$

B. roots of the equation $lx^2 + mx + n = 0$ are equal

C. $l^2 + m^2 + n^2 = 0$

D. $l^3 + m^2 + n^3 = 3lmn$

Answer: a,b,d

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129. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \vec{\beta} = b\hat{i} + c\hat{j} + a\hat{k}$ and $\vec{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplnar vectors with $a \neq b$, and $\vec{v} = \hat{i} + \hat{j} + \hat{k}$. Then $\vec{v}$ is perpendicular to

A. $\vec{\alpha}$

B. $\vec{\beta}$

C. $\vec{\gamma}$

D. none of these

Answer: a,b,c

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130. If vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{B} = \hat{i} + \hat{j} + 5\hat{k}$ and $\vec{C}$ form a left handed system then $\vec{C}$ is (A) $11\hat{i} - 6\hat{j} - \hat{k}$ (B) $-11\hat{i} + 6\hat{j} + \hat{k}$ (C) $-11\hat{i} + 6\hat{j} - \hat{k}$ (D) $-11\hat{i} + 6\hat{j} - \hat{k}$

A. $11\hat{i} - 6\hat{j} - \hat{k}$

B. $-11\hat{i} - 6\hat{j} - \hat{k}$

C. $-11\hat{i} - 6\hat{j} + \hat{k}$

D. $-11\hat{i} + 6\hat{j} - \hat{k}$

Answer: b,d

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131. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k}$, then $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$is

(a)parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$ (b)orthogonal to $\hat{i} + \hat{j} + \hat{k}$

(c)orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$ (d)orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

A. parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$

D. orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

Answer: a,b,c,d

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132. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$ then

A. $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{0}$

B. $\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$

C. $\vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

D. $\vec{c} \times \vec{a} \times \vec{b} = \vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

Answer: a,c,d

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133. A vector $\left(\vec{d}\right)$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$ let $\vec{x}, \vec{y}, \vec{z}$ be three in the plane of $\vec{a}, \vec{b}, \vec{b}, \vec{c}, \vec{c}, \vec{a}$ respectively, then

A. $\vec{z}.\vec{d} = 0$

B. $\vec{x}.\vec{d} = 1$

C. $\vec{y}.\vec{d} = 32$

D. $\vec{r}.\vec{d} = 0, \text{ where } \vec{r} = \lambda\vec{x} + \mu\vec{y} + \gamma\vec{z}$

Answer: a,d

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134. A parallelogram is constructed on the vectors $\vec{a} = 3\vec{\alpha} - \vec{\beta}, \vec{b} = \vec{\alpha} + 3\vec{\beta}. If \left|\vec{\alpha}\right| = \left|\vec{\beta}\right| = 2$ and angle between $\vec{\alpha}$ and $\vec{\beta} is \frac{\pi}{3}$ then the length of a diagonal of the parallelogram is

A. $4\sqrt{5}$

B. $4\sqrt{3}$

C. $4\sqrt{7}$

D. none of these

Answer: b,c

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Reasoning type

1. (a)Statement 1: Vector $\vec{c} = -5\hat{i} + 7\hat{j} + 2\hat{k}$ is along the bisector of angle between $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = 8\hat{i} + \hat{j} - 4\hat{k}$.

Statement 2 : $\vec{c}$ is equally inclined to $\vec{a}$ and $\vec{b}$.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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2. Statement1: A component of vector $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in the direction perpendicular to the direction of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k} is \hat{i} - \hat{j}$

Statement 2: A component of vector in the direction of $\vec{a} = \hat{i} + \hat{j} + \hat{k} is 2\hat{i} + 2\hat{j} + 2\hat{k}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: c

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3. Statement 1: Distance of point D(1,0,-1) from the plane of points A(1,-2,0) , B (3, 1,2) and C(-1,1,-1) is $\frac{8}{\sqrt{229}}$

Statement 2: volume of tetrahedron formed by the points A,B, C and D is $\frac{\sqrt{229}}{2}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: d

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4. Let $\vec{r}$ be a non - zero vector satisfying $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ for given non- zero vectors $\vec{a}\vec{b}$ and $\vec{c}$

Statement 1: $\left[\vec{a} - \vec{b}\vec{b} - \vec{c}\vec{c} - \vec{a}\right] = 0$

Statement 2: $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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5. Statement 1: If $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b}\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are three mutually perpendicular unit vectors then $a_1\hat{i} + b_1\hat{j} + c_1\hat{k}, a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ and $a_3\hat{i} + b_3\hat{j} + c_3\hat{k}$ may be mutually perpendicular unit vectors.

Statement 2 : value of determinant and its transpose are the same.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: a

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6. Statement 1: $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}, \vec{B} = \hat{u} + \hat{j} - 2\hat{k}$ and $\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$ then

$\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = 243$

Statement 2: $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = \left|\vec{A}\right|^2\left|\left[\vec{A}\vec{B}\vec{C}\right]\right|$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: d

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7. Statement 1: $\vec{a}, \vec{b}$ and $\vec{c}$ arwe three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are non- coplanar. If $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] = 1,$ then $\vec{d} = \vec{a} + \vec{b} + \vec{c}$

Statement 2: $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] \Rightarrow \vec{d}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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8. Consider three vectors $\vec{a}, \vec{b}$ and $\vec{c}$

Statement 1: $\vec{a} \times \vec{b} = \left(\left(\hat{i} \times \vec{a}\right).\vec{b}\right)\hat{i} + \left(\left(\hat{j} \times \vec{a}\right).\vec{b}\right)\hat{j} + \left(\hat{k} \times \vec{a}\right).\vec{b}\right)\hat{k}$

Statement 2: $\vec{c} = \left(\hat{i}.\vec{c}\right)\hat{i} + \left(\hat{j}.\vec{c}\right)\hat{j} + \left(\hat{k}.\vec{c}\right)\hat{k}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: a

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Comprehension type

1. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u}+\vec{v}+\vec{w}=\vec{a}, \vec{u}\times(\vec{v}\times\vec{w})=\vec{b}, (\vec{u}\times\vec{v})\times\vec{w}=\vec{c}, \vec{a}.\vec{u}=3/2, \vec{a}.\vec{v}=7/4$ and

Vector $\vec{w}$is

A. $\vec{a}-\frac{2}{3}\vec{b}+\vec{c}$

B. $\vec{a}+\frac{4}{3}\vec{b}+\frac{8}{3}\vec{c}$

C. $2\vec{a}-\vec{b}+\frac{1}{3}\vec{c}$

D. $\frac{4}{3}\vec{a}-\vec{b}+\frac{2}{3}\vec{c}$

Answer: b

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2. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} = 3/2, \vec{a}.\vec{v} = 7/4$ and

Vector $\vec{w}$is

A. (a) $\vec{a} - \frac{2}{3}\vec{b} + \vec{c}$

B. (b) $\vec{a} + \frac{4}{3}\vec{b} + \frac{8}{3}\vec{c}$

C. (c) $2\vec{a} - \vec{b} + \frac{1}{3}\vec{c}$

D. (d) $\frac{4}{3}\vec{a} - \vec{b} + \frac{2}{3}\vec{c}$

Answer: c

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3. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} = 3/2, \vec{a}.\vec{v} = 7/4$ and Vector $\vec{u}$ is

A. (a) $\vec{a} - \frac{2}{3}\vec{b} + \vec{c}$

B. (b) $\vec{a} + \frac{4}{3}\vec{b} + \frac{8}{3}\vec{c}$

C. (c) $2\vec{a} - \vec{b} + \frac{1}{3}\vec{c}$

D. (a) $\frac{4}{3}\vec{a} - \vec{b} + \frac{2}{3}\vec{c}$

Answer: d

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4. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$, find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}$ and $\vec{c}$.

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5. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$, find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}$ and $\vec{c}$.

A. $\frac{1}{2}\left[(\vec{a} + \vec{c}) \times \vec{b} - \vec{b} - \vec{a}\right]$

B. $\frac{1}{2}\left[(\vec{a} - \vec{c}) \times \vec{b} + \vec{b} + \vec{a}\right]$

C. $\frac{1}{2}\left[(\vec{a} - \vec{b}) \times \vec{c} + \vec{b} + \vec{a}\right]$

D. $\frac{1}{2}\left[(\vec{a} - \vec{c}) \times \vec{a} + \vec{b} - \vec{a}\right]$

Answer: c

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6. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$. Find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.

A. $\frac{1}{2}\left[(\vec{a} - \vec{c}) \times \vec{c} - \vec{b} + \vec{a}\right]$

B. $\frac{1}{2}\left[\left(\vec{a}-\vec{b}\right)\times\vec{c}+\vec{b}-\vec{a}\right]$

C. $\frac{1}{2}\left[\vec{c}\times\left(\vec{a}-\vec{b}\right)+\vec{b}+\vec{a}\right]$

D. none of these

Answer: b

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7. If $\vec{x}\times\vec{y}=\vec{a}, \vec{y}\times\vec{z}=\vec{b}, \vec{x}.\vec{b}=\gamma, \vec{x}.\vec{y}=1$ and $\vec{y}.\vec{z}=1$ then find x,y,z in terms of $\vec{a}, \vec{b}$ and γ.

A. $\frac{1}{\left|\vec{a}\times\vec{b}\right|^2}\left[\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right]$

B. $\frac{\gamma}{\left|\vec{a}\times\vec{b}\right|^2}\left[\vec{a}\times\vec{b}-\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right]$

C. $\frac{\gamma}{\left|\vec{a}\times\vec{b}\right|^2}\left[\vec{a}\times\vec{b}+\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right]$

D. none of these

Answer: b

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8. If $\vec{x}\times\vec{y}=\vec{a}, \vec{y}\times\vec{z}=\vec{b}, \vec{x}.\vec{b}=\gamma, \vec{x}.\vec{y}=1$ and $\vec{y}.\vec{z}=1$ then find x,y,z in terms of $\vec{a}, \vec{b}$ and γ.

A. $\dfrac{\vec{a}\times\vec{b}}{\gamma}$

B. $\vec{a}+\dfrac{\vec{a}\times\vec{b}}{\gamma}$

C. $\vec{a}+\vec{b}+\dfrac{\vec{a}\times\vec{b}}{\gamma}$

D. none of these

Answer: a

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9. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^{0} with each other. If $\vec{x}\times\left(\vec{y}\times\vec{z}\right)=\vec{a}, \vec{y}\times\left(\vec{z}\times\vec{x}\right)=\vec{b}$ and $\vec{x}\times\vec{y}=\vec{c}$. Find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.

A. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} \times \left(\vec{a} \times \vec{b}\right)\right]$

B. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} - \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

C. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} + \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

D. none of these

Answer: c

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10. Given two orthogonal vectors $\vec{A}$ and $\vec{B}$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then $\left(\vec{P} \times \vec{B}\right) \times \vec{B}$ is equal to

A. $\vec{P}$

B. $-\vec{P}$

C. $2\vec{B}$

D. $\vec{A}$

Answer: b

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11. Given two orthogonal vectors $\vec{A}$ and $\vec{B}$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then $\vec{P}$ is equal to

A. $\frac{\vec{A}}{2} + \frac{\vec{A} \times \vec{B}}{2}$

B. $\frac{\vec{A}}{2} + \frac{\vec{B} \times \vec{A}}{2}$

C. $\frac{\vec{A} \times \vec{B}}{2} - \frac{\vec{A}}{2}$

D. $\vec{A} \times \vec{B}$

Answer: b

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12. Given two orthogonal vectors $\vec{A}$ and $VecB$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then which of the following statements is false ?

A. vectors $\vec{P}, \vec{A}$ and $\vec{P} \times \vec{B}$ ar linearly dependent.

B. vectors $\vec{P}, \vec{B}$ and $\vec{P} \times \vec{B}$ ar linearly independent

C. $\vec{P}$ is orthogonal to $\vec{B}$ and has length $\frac{1}{\sqrt{2}}$.

D. none of these

Answer: d

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13. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then $\vec{a}_2$ is equal to

A. $\frac{943}{49}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

B. $\frac{943}{49^2}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

C. $\frac{943}{49}\left(- 2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

D. $\frac{943}{49^2}\left(- 2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

Answer: b

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14. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = - 2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then $\vec{a}_1 . \vec{b}$ is equal to

A. -41

B. $-41/7$

C. 41

D. 287

Answer: a

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15. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then $\vec{a}_2$ is equal to

A. $\vec{a}$ and $vcea_2$ are collinear

B. $\vec{a}_1$ and $\vec{c}$ are collinear

C. $\vec{a}m\vec{a}_1$ and $\vec{b}$ are coplanar

D. $\vec{a}, \vec{a}_1$ and a_2 are coplanar

Answer: c

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16. Consider a triangular pyramid ABCD the position vectors of whose agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCD. The length of the vector $\overline{AG}$ is

A. $\sqrt{17}$

B. $\sqrt{51}/3$

C. $3/\sqrt{6}$

D. $\sqrt{59}/4$

Answer: b

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17. Consider a triangular pyramid ABCD the position vectors of whose agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCD. The length of the vector $\overline{AG}$ is

A. 24

B. $8\sqrt{6}$

C. $4\sqrt{6}$

D. none of these

Answer: c

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18. Consider a triangular pyramid ABCD the position vectors of whose agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCD. The length of the vector $\overline{AG}$ is

A. $14/\sqrt{6}$

B. $2/\sqrt{6}$

C. $3/\sqrt{6}$

D. none of these

Answer: a

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19. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D (x,y,z) The distance between the parallel lines AB and CD is

A. $\sqrt{6}$

B. $3\sqrt{6/5}$

C. $2\sqrt{2}$

D. 3

Answer: c

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20. Vertices of a parallelogram taken in order are A(2,-1,4)B(1,0,-1)C(1,2,3) and D.

Distance of the point P (8, 2,-12) from the plane of the parallelogram is

A. $\frac{4\sqrt{6}}{9}$

B. $\frac{32\sqrt{6}}{9}$

C. $\frac{16\sqrt{6}}{9}$

D. none

Answer: b

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21. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D (x,y,z) The distance between the parallel lines AB and CD is

A. 14, 4,2

B. 2,4,14

C. 4,2,14

D. 2,14,4

Answer: d

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22. Let $\vec{r}$ be a position vector of a variable point in Cartesian OXY plane such that $\vec{r}.\left(10\hat{j}-8\hat{i}-\vec{r}\right)=40$ and $P_1=\max\left\{\left|\vec{r}+2\hat{i}-3\hat{j}\right|^2\right\}, P_2=\min\left\{\left|\vec{r}+2\hat{i}-3\hat{j}\right|^2\right\}$. A tangenty line is drawn to the curve $y=8/x^2$ at point .A with abscissa 2. the drawn line cuts the x-axis at a point B.

p_2 is equal to

A. 9

B. $2\sqrt{2}-1$

C. $6\sqrt{6}+3$

D. $9-4\sqrt{2}$

Answer: d

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23. Let $\vec{r}$ be a position vector of a variable point in Cartesian OXY plane such that $\vec{r}.\left(10\hat{j} - 8\hat{i} - \vec{r}\right) = 40$ and $P_1 = \max\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}, P_2 = \min\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}$. A tangenty line is drawn to the curve $y = 8/x^2$ at point .A with abscissa 2. the drawn line cuts the x-axis at a point B.

p_2 is equal to

A. 2

B. 10

C. 18

D. 5

Answer: c

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24. Let $\vec{r}$ be a position vector of a variable point in Cartesian OXY plane such that $\vec{r}.\left(10\hat{j} - 8\hat{i} - \vec{r}\right) = 40$ and $P_1 = \max\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}, P_2 = \min\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}$. A tangenty line is drawn to the curve $y = 8/x^2$ at point .A with abscissa 2. the drawn line cuts the x-axis at a point B.

p_2 is equal to

A. 1

B. 2

C. 3

D. 4

Answer: c

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25. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away

from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a}$ is $\frac{|\vec{a}|}{3}$ vector $\overrightarrow{AD}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} - \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

D. none of these

Answer: a

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26. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away

from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC} = \vec{b}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$ vector $\overrightarrow{AB}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} - \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

D. none of these

Answer: b

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27. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away

from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC} = \vec{b}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a}$ is $\frac{|\vec{a}|}{3}$ vector $\overrightarrow{AB}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} - \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

D. none of these

Answer: c

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Martrix - match type

1.

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2.

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3.

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4. Given two vectors $\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{j} - \hat{k}$

Find

a. $\vec{a} \times \vec{b}$ then use this to find the area of the triangle.

b. The area of the parallelogram

c. The area of a paralleogram whose diagonals are

2 veca and 4vecb

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5. Given two vectors $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$

find $\left|\vec{a} \times \vec{b}\right|$

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6.

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7. find $\left|\vec{x}\right|$, if for a unit vector $\vec{a}$, $\left(\vec{x} - \vec{a}\right)\left(\vec{x} + \vec{a}\right)$=12

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8.

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9.

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10.

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Integer type

1. If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then find the greatest postive integer in the range of $\dfrac{3\left|\vec{a}+\vec{b}\right|}{2}+2\left|\vec{a}-\vec{b}\right|$

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2. Let $\vec{u}$ be a vector on rectangular coodinate system with sloping angle 60° suppose that $\left|\vec{u} - \hat{i}\right|$ is geomtric mean of $\left|\vec{u}\right|$ and $\left|\vec{u} - 2\hat{i}\right|$, where $\hat{i}$ is the unit vector along the x-axis . Then find the value of $\left(\sqrt{2} - 1\right)\left|\vec{u}\right|$

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3. Find the absolute value of parameter t for which the area of the triangle whose vertices the $A(-1, 1, 2); B(1, 2, 3) and C(5, 1, 1)$ is minimum.

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4. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ and

$$\left[3\vec{a} + \vec{b} \;\; 3\vec{b} + \vec{c} \;\; 3\vec{c} + \vec{a}\right] = \lambda \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \text{ then find the value of } \frac{\lambda}{4}$$

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5. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\alpha\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \alpha\hat{j} + \hat{k}$. Find the value of 6α. Such that $\left\{\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\right\} \times \left(\vec{c} \times \vec{a}\right) = 0$

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6. If , $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying $\left[(a - 2)\alpha^2 + (b - 3)\alpha + c\right]\vec{x} + [(a - 2)\beta + c]\vec{y} + \left[(a - 2)\gamma^2 + (b - 3)\gamma + c\right]\left(\vec{x} \times \vec{y}\right) =$ are three distinct distinct real numbers, then find the value of $\left(a^2 + b^2 + c^2 - 4\right)$

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7. Let $\vec{u}$ and $\vec{v}$ be unit vectors such that $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \vec{v}$. Find the value of $\left[\vec{u}\vec{v}\vec{w}\right]$

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8. The volume of the tetrahedron whose vertices are the points with positon vectors $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 7\hat{k}$, $5\hat{i} - \hat{j} + \lambda\hat{k}$ and $7\hat{i} - 4\hat{j} + 7\hat{k}$ is 11 cubic units if the value of λ is

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9. Given that $\vec{u} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{v} = 2\hat{i} + \hat{k} + 4\hat{k}$, $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$ and $\left(\vec{u}.\vec{R} - 15\right)\hat{i} + \left(\vec{c}.\vec{R} - 30\right)\hat{j}$. Then find the greatest integer less than or equal to $\left|\vec{R}\right|$.

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10. Let a three- dimensional vector $\vec{V}$ satisfy the condition , $2\vec{V} + \vec{V} \times \left(\hat{i} + 2\hat{j}\right) = 2\hat{i} + \hat{k}$. If $3\left|\vec{V}\right| = \sqrt{m}$. Then find the value of m.

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11. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}. \vec{b} = 0 = \vec{a}. \vec{c}$ and the angle between $\vec{b}$ and $\vec{c} is \frac{\pi}{3}$, then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$

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12. Let $\vec{O}A = \vec{a}, \vec{O}B = 10\vec{a} + 2\vec{b} and \vec{O}C = \vec{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB$, and let q denote the area of parallelogram with $OA and OC$ as adjacent sides. If $p = kq$, then find$k$.

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13. Find the work done by the force $F = 3\hat{i} - \hat{j} - 2\hat{k}$ acrting on a particle such that the particle is displaced from point $A(-3, -4, 1) \top o \in tB(-1, -1, -2)$.

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14. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$ then find the value of $\left(2\vec{a} + \vec{b}\right). \left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$

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15. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = i + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$ then find the value of $\vec{r}.\vec{b}$.

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16. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9$ then find the value of $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right|$.

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17. $Let \vec{a}, \vec{b}$, and $\vec{c}$ be three non-coplanar ubit vectors such the angle between every pair of them is $\frac{\pi}{3}$. if $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p,q and r are scalars , then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

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Subjective type

1. from a point O inside a triangle ABC, perpendiculars, OD, OE and OF are drawn to the sides, BC, CA and AB respectively , prove that the perpendiculars from A, B and C to the sides EF, FD and DE are concurrent.

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2. $A_1, A_2, \ldots . A_n$ are the vertices of a regular plane polygon with n sides and O ars its centre. Show that $\sum_{i=1}^{n-1} \left(\overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}}\right) = (n - 1)\left(\overrightarrow{OA_1} \times \overrightarrow{OA_2}\right)$

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3. If c is a given non - zero scalar, and $\vec{A}$ and $\vec{B}$ are given non- zero , vectors such that $\vec{A} \perp \vec{B}$. Then find vector, $\vec{X}$ which satisfies the equations $\vec{A}.\vec{X} = c$ and $\vec{A} \times \vec{X} = \vec{B}$.

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4. $A, B, CandD$ are any four points in the space, then prove that $\left|\vec{AB} \times \vec{CD} + \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}\right|$ = 4 (area of ABC .)

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5. If vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar, show that $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = \vec{0}$

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6. $\vec{A} = \left(2\vec{i} + \vec{k}\right), \vec{B} = \left(\vec{i} + \vec{j} + \vec{k}\right)$ and $\vec{C} = 4\vec{i} - 3\vec{j} + 7\vec{k}$ determine a $\vec{R}$ satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}.\vec{A} = 0$

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7. Determine the value of c so that for the real x, vectors $cx\hat{i} - 6\hat{j} - 3\hat{k}$ and $x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other .

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8. Prove that:

$$\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) + \left(\vec{a} \times \vec{c}\right) \times \left(\vec{d} \times \vec{b}\right) + \left(\vec{a} \times \vec{d}\right) \times \left(\vec{b} \times \vec{c}\right) = -2\left[\vec{b}\vec{c}\vec{d}\right]\vec{a}$$

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9. The position vectors of the vertices A, B and C of a tetrahedron ABCD are $\hat{i} + \hat{j} + \hat{k}$, $\hat{k}$, $\hat{i}$ and $\hat{3}i$,respectively. The altitude from vertex D to the

opposite face ABC meets the median line through Aof triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is2√2/3, find the position vectors of the point E for all its possible positions

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10. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non - coplanar unit vectors, equally inclined to one another at an angle θ. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, find scalars p, q and r in terms of θ.

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11. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\left|\vec{b}\right| = \left|\vec{c}\right|$ then $\left\{\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} + \vec{c}\right)\right\} \times \left(\vec{b} \times \vec{c}\right).\left(\vec{b} + \vec{c}\right) =$

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12. For any two vectors $\vec{u}$ and $\vec{v}$ prove that $\left(1+|\vec{u}|^2\right)\left(1+|\vec{v}|^2\right)=(1-\vec{u}.\vec{v})^2+|\vec{u}+\vec{v}+(\vec{u}\times\vec{v})|^2$

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13. Let $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w}+\vec{w}\times\vec{u}=\vec{v}$, then prove that $\left|(\vec{u}\times\vec{v}).\vec{w}\right|\leq\frac{1}{2}$ and that the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

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14. Find 3-dimensional vectors $\vec{v}_1,\vec{v}_2,\vec{v}_3$ satisfying

$\vec{v}_1\cdot\vec{v}_1=4,\vec{v}_1\cdot\vec{v}_2=-2,\vec{v}_1\cdot\vec{v}_3=6,$

$\vec{v}_2\cdot\vec{v}_2=2,\vec{v}_2\cdot\vec{v}_3=-5,\vec{v}_3\cdot\vec{v}_3=29$

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15. Let V be the volume of the parallelepied formed by the vectors, $\vec{a} = a_1\hat{i} = a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. if $a_r b_r nadc_r$ are non- negative real numbers and $\sum_{r=1}^{3}\left(a_r + b_r + c_r\right) = 3L$ show that $V \leq L^3$

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16. $\vec{u}, \vec{v}$ and $\vec{w}$ are three nono-coplanar unit vectors and α, β and γ are the angles between $\vec{u}$ and $\vec{u}, \vec{v}$ and $\vec{w}$ and $\vec{w}$ and $\vec{u}$, respectively and $\vec{x}, \vec{y}$ and $\vec{z}$ are unit vectors along the bisectors of the angles α, β and γ. respectively, prove that $\left[\vec{x} \times \vec{y}\vec{y} \times \vec{z}\vec{z} \times \vec{x}\right) = \frac{1}{16}[\vec{u}\vec{v}\vec{w}]^2 \frac{\sec^2\alpha}{2}\frac{\sec^2\beta}{2}\frac{\sec^2\gamma}{2}$.

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17. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ ar distinct vectors such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$. Prove that $\left(\vec{a} \times \vec{d}\right).\left(\vec{b}.\vec{c}\right) \neq 0, i.e., \vec{a}.\vec{b} + \vec{d}.\vec{c} \neq \vec{d}.\vec{b} + \vec{a}.\vec{c}$.

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18. P_1ndP_2 are planes passing through origin L_1andL_2 are two lines on P_1andP_2, respectively, such that their intersection is the origin. Show that there exist points $A, BandC$, whose permutation $A^{'}, B^{'} andC^{'}$, respectively, can be chosen such that A is on $L_1, BonP_1$ but not on L_1andC not on P_1; A' is on $L_2, B' onP_2$ but not on L_2andC' not on P_2.

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19. about to only mathematics

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fill in the blanks

1. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be vectors of legth , 3,4and 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}, \vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to$\vec{A}+\vec{B}$ then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is ________.

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2. The unit vector perendicular to the plane determined by P (1,-1,2) ,C(3,-1,2) is _______.

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3. The area of the triangle whose vertices are A (1,-1,2) , B (1,2, -1) ,C (3, -1, 2) is _______.

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4. If $\vec{A}, \vec{B}$ and $\vec{C}$ are three non - coplanar vectors, then

$\dfrac{\vec{A}.\vec{B}\times\vec{C}}{\vec{C}\times\vec{A}.\vec{B}} + \dfrac{\vec{B}.\vec{A}\times\vec{C}}{\vec{C}.\vec{A}\times\vec{B}}$=________

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5. If $\vec{A} = (1, 1, 1)$ and $\vec{C} = (0, 1, -1)$ are given vectors the vector $\vec{B}$ satisfying the equations $\vec{A}\times\vec{B} = \vec{C}$ and $\vec{A}.\vec{B} = 3$ is _______.

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6. Let $\vec{b} = 4\hat{i} + 3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy- plane. All vectors in the sme plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$., respectively, are given by _______

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7. The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are _______ and _________, respectively.

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8. A unit vector coplanar with $\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$ and perpendicular to $\vec{i} + \vec{j} + \vec{k}$ is ______

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9. A non vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\vec{i}, \vec{i} + \vec{j}$ and thepane determined by the vectors $\vec{i} - \vec{j}, \vec{i} + \vec{k}$ then angle between $\vec{a}$ and $\vec{i} - 2\vec{j} + 2\vec{k}$ is = (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

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10. Find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$

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11. let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2, respectively, if $\vec{a}\times(\vec{a}\times\vec{c})+\vec{b}=\vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is ______

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12. A, B C and D are four points in a plane with position vectors, $\vec{a}, \vec{b}\vec{c}$ and $\vec{d}$ respectively, such that $(\vec{a}-\vec{d}).(\vec{b}-\vec{c})=(\vec{b}-\vec{d}).(\vec{c}-\vec{a})=0$ then point D is the _____ of triangle ABC.

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13. Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = 10\vec{a} + 2\vec{b}$ and $\overrightarrow{OC} = \vec{b}$ where , O, A and C are non-collinear points. Let p denote that area of the quadrilateral OABC. And let q denote the area of the parallelogram with OA and OC as adjacent sides. If p=kq, then k= ________

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14. If $\vec{a} = \hat{j} + \sqrt{3}\hat{k} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle , then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is ____

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True and false

1. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be unit vectors such that $\vec{A}.\vec{B} = \vec{A}.\vec{C} = 0$ and the angle between $\vec{B}$ and $\vec{C}$ be$\pi/3$. Then $\vec{A} = \pm 2\left(\vec{B} \times \vec{C}\right)$.

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2. If $\vec{X}.\vec{A}=0, \vec{X}.\vec{B}=0$ and $\vec{X}.\vec{C}=0$ for some non-zero vector $\vec{x}1, then$[vecA vecB vecC] =0`

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3. for any three vectors, $\vec{a}, \vec{b}$ and $\vec{c}, \left(\vec{a}-\vec{b}\right).\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)=$

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single correct answer type

1. The scalar $\vec{A}.\left(\vec{B}.\vec{C}\right)\times\left(\vec{A}+\vec{B}+\vec{C}\right)$ equals

A. 0

B. $\left[\vec{A}\vec{B}\vec{C}\right]+\left[\vec{B}\vec{C}\vec{A}\right]$

C. $\left[\vec{A}\vec{B}\vec{C}\right]$

D. none of these

Answer: a

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2. For non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $\left|\left(\vec{a} \times \vec{b}\right). \vec{c}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$ holds if and only if

A. $\vec{a}. \vec{b} = 0, \vec{b}. \vec{c} = 0$

B. $\vec{b}. \vec{c} = 0, \vec{c}, \vec{a} = 0$

C. $\vec{c}. \vec{a} = 0, \vec{a}, \vec{b} = 0$

D. $\vec{a}. \vec{b} = \vec{b}. \vec{c} = \vec{c}. \vec{a} = 0$

Answer: d

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3. The volume of he parallelepiped whose sides are given by $\vec{O}A = 2i - 2, j, \vec{O}B = i + j - kand\vec{O}C = 3i - k$ is a. $4/13$ b. 4 c. $2/7$ d. 2

A. $4/13$

B. 4

C. $2/7$

D. 2

Answer: d

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4. Let $\vec{a}, \vec{b}, \vec{c}$ be three noncolanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations $\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$ then the value of the expression $\left(\vec{a} + \vec{b}\right).\vec{p} + \left(\vec{b} + \vec{c}\right).\vec{q} + \left(\vec{c} + \vec{a}\right).\vec{r}$. is equal to (A) 0 (B) 1 (C) 2 (D) 3

A. 0

B. 1

C. 2

D. 3

Answer: d

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5. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k}, \vec{c} = \hat{k} - \hat{i}. If \hat{d}$ is a unit vector such that $\vec{a}. \hat{d} = 0 = \left[\vec{b}\vec{c}\vec{d}\right]$ then $\hat{d}$ equals

A. $\pm\dfrac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$

B. $\pm\dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

C. $\pm\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

D. $\pm\hat{k}$

Answer: a

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6. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non coplanar and unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}\Bigg)$ then the angle between vea and $\vec{b}$ is (A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

A. $3\pi/4$

B. $\pi/4$

C. $\pi/2$

D. π

Answer: a

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7. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w}=0$ if $|\vec{u}|=3, |\vec{v}|=4$ and $|\vec{w}|=5$ then $\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}$ is

A. 47

B. -25

C. 0

D. 25

Answer: b

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8. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then $(\vec{a}+\vec{b}+\vec{c}).\left[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})\right]$ equals

A. 0

B. $\left[\vec{a}\vec{b}\vec{c}\right]$

C. $2\left[\vec{a}\vec{b}\vec{c}\right]$

D. - $\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: d

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9. $\vec{p}, \vec{q}$ and $\vec{r}$ are three mutually prependicular vectors of the same magnitude . If vector $\vec{x}$ satisfies the equation $\vec{p}s \times \left(\left(\vec{x} - \vec{q}\right) \times \vec{p}\right) + \vec{q} \times \left(\left(\vec{x} - \vec{r}\right) \times \vec{q}\right) + \vec{r} \times \left(\left(\vec{x} - \vec{p}\right) \times \vec{r}\right) = \vec{0}$ then $\vec{x}$ is given by

A. $\frac{1}{2}\left(\vec{p} + \vec{q} - 2\vec{r}\right)$

B. $\frac{1}{2}\left(\vec{p} + \vec{q} + \vec{r}\right)$

C. $\frac{1}{3}\left(\vec{p} + \vec{q} + \vec{r}\right)$

D. $\frac{1}{3}\left(2\vec{p} + \vec{q} - \vec{r}\right)$

Answer: b

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10. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\vec{b} = \hat{i} + \hat{j}$ if c is a vector such that $\vec{a}.\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{i} s 30°$, then $|(\vec{a} \times \vec{b})| \times \vec{c}|$ is equal to

A. $2/3$

B. $3/2$

C. 2

D. 3

Answer: b

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11. Let $\vec{a} = 2i + j + k, \vec{b} = i + 2j - k$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is pependicular to $\vec{a}$. Then $\vec{c}$ is

A. $\frac{1}{\sqrt{2}}(-j + k)$

B. $\frac{1}{\sqrt{3}}(i - j - k)$

C. $\frac{1}{\sqrt{5}}(i - 2j)$

D. $\frac{1}{\sqrt{3}}(i - j - k)$

Answer: a

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12. If the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ form the sides, BC , CA and AB, respectively, of triangle ABC, then

A. $\vec{a}. \vec{b} + \vec{b}. \vec{c} + \vec{c}. \vec{a} = 0$

B. $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

C. $\vec{a}. \vec{b} = \vec{b}. \vec{c} = \vec{c}. \vec{a}$

D. $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

Answer: b

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13. Let vectors $\vec{a}, \vec{b}\vec{a}$ and $\vec{d}$ be such that $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \vec{0}$. Let P_1 and P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$, respectively. Then the angle between P_1 and P_2 is

A. 0

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

Answer: a

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14. If $\vec{a}, \vec{b}, \vec{c}$ are unit coplanar vectors then the scalar triple product $\left[2\vec{a} - \vec{b}, 2\vec{b} - c, \vec{2}c - \vec{a}\right]$ is equal to (A) 0 (B) 1 (C) $-\sqrt{3}$ (D) $\sqrt{3}$

A. 0

B. 1

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: a

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15. if $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors. Then $\left|\hat{a} - \hat{b}\right|^2 + \left|\hat{b} - \hat{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2$ does not exceed

A. 4

B. 9

C. 8

D. 6

Answer: b

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16. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is

A. 45°

B. 60°

C. $\cos^{-1}(1/3)$

D. $\cos^{-1}(2/7)$

Answer: b

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17. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{W} = \hat{i} + 3\hat{k}$. if $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $\left[\vec{U}\vec{V}\vec{W}\right]$ is

A. -1

B. $\sqrt{10} + \sqrt{6}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: c

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18. Find the value of a so that the volume of the parallelopiped formed by vectors $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum.

A. -3

B. 3

C. $1/\sqrt{3}$

D. $\sqrt{3}$

Answer: c

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19. If $\vec{a} = \left(\hat{i} + \hat{j} + \hat{k}\right)$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ is (a)$\hat{i} - \hat{j} + \hat{k}$ (b) $2\hat{i} - \hat{k}$ (c) $\hat{i}$ (d) $2\hat{i}$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{i} - \hat{k}$

C. $\hat{i}$

D. $2\hat{i}$

Answer: c

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20. The unit vector which is orthogonal to the vector $5\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is (a) $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$ (b) $\frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$ (c) $\frac{3\hat{j} - \hat{k}}{\sqrt{10}}$ (d) $\frac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

A. $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$

B. $\frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$

C. $\frac{3\hat{i} - \hat{k}}{\sqrt{10}}$

D. $\frac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

Answer: c

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21. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero, non- coplanar vectors and

$\vec{b}_1 = \vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{b}_2 = \vec{b} + \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{c}_1 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1$,

$\vec{c}_2 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}\vec{c}}{|\vec{b}_1|^2}\vec{b}_1$, $\vec{c}_3 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1$,

$\vec{c}_4 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a} = \frac{\vec{b}.\vec{c}}{|\vec{b}|^2}\vec{b}_1$, then the set of mutually orthogonal vectors is

A. (a) $\left(\vec{a}, \vec{b}_1, \vec{c}_3\right)$

B. (b) $\left(\vec{a}, \vec{b}_1, \vec{c}_2\right)$

C. (c) $\left(\vec{a}, \vec{b}_1, \vec{c}_1\right)$

D. (d) $\left(\vec{a}, \vec{b}_2, \vec{c}_2\right)$

Answer: c

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22. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$ A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projections on $\vec{c} is 1/\sqrt{3}$ is

A. $4\hat{i} - \hat{j} + 4\hat{k}$

B. $3\hat{i} + \hat{j} - 3\hat{k}$

C. $2\hat{i} + \hat{j} - 2\hat{k}$

D. $4\hat{i} + \hat{j} - 4\hat{k}$

Answer: a

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23. Let two non-collinear unit vectors $\vec{a}$ and $\vec{b}$ form an acute angle. A point P moves so that at any time t, time position vector, $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cot t + \hat{b}\sin t$. When p is farthest fro origing o, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$ .then

A. , $\hat{u} = \dfrac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|}$ and $M = \left(1 + \hat{a}.\hat{b}\right)^{1/2}$

B. , $\hat{u} = \dfrac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|}$ and $M = \left(1 + \hat{a}.\hat{b}\right)^{1/2}$

C. $\hat{u} = \dfrac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|}$ and $M = \left(1 + 2\hat{a}.\hat{b}\right)^{1/2}$

D. , $\hat{u} = \dfrac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|}$ and $M = \left(1 + 2\hat{a}.\hat{b}\right)^{1/2}$

Answer: a

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24. If $\vec{a}, \vec{c}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a} \times \vec{b}\right).\left(\vec{c} \times \vec{d}\right) = 1$ and $\vec{a}.\vec{b} = \dfrac{1}{2}$then

A. $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar

B. $\vec{b}, \vec{c}$ and $\vec{d}$ are non-coplanar

C. $\vec{b}$ and $\vec{d}$ are non- parallel

D. $\vec{a}$ and $\vec{d}$ are parallel and $\vec{b}$ and $\vec{c}$ are parallel

Answer: c

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25. Two adjacent sides of a parallelogram $ABCD$ are given by $\vec{A}B = 2\hat{i} + 10\hat{j} + 11\hat{k} and \vec{A}D = -\hat{i} + 2\hat{j} + 2\hat{k}$ The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angel α is given by $\frac{8}{9}$ b. $\frac{\sqrt{17}}{9}$ c. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: b

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26. Let P,Q, R and S be the points on the plane with postion vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{j}+2\hat{j}$, respectively, the quadrilateral PQRS must be a

A. Parallelogram, which is neither a rhombus nor a rectangle

B. square

C. rectangle, but not a square

D. rhombus, but not a square.

Answer: a

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27. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vectors $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c} is \frac{1}{\sqrt{3}}$ is given by

A. $\hat{i} - 3\hat{j} + 3\hat{k}$

B. $-3\hat{i} - 3\hat{j} + \hat{k}$

C. $3\hat{i} - \hat{j} + 3\hat{k}$

D. $\hat{i} + 3\hat{j} - 3\hat{k}$

Answer: c

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28. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS. And $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be onther vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is

A. 5

B. 20

C. 10

D. 30

Answer: c

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Multiple correct answers type

1. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b} is \pi/6$ then the value of

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{is}$$

A. 0

B. 1

C. $\frac{1}{4}\left(a_1^2 + a_2^2 + a_2^2\right)\left(b_1^2 + b_2^2 + b_2^2\right)$

D. $\frac{3}{4}\left(a_1^2 + a_2^2 + a_2^2\right)\left(b_1^2 + b_2^2 + b_2^2\right)\left(c_1^2 + c_2^2 + c_2^2\right)$

Answer: c

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2. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0) and \vec{b} = (0, 1, 1)$ is a. one b. two c. three d. infinite

A. one

B. two

C. three

D. infinite

Answer: b

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3. $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{j} + 2\hat{j} - \hat{k}, \vec{c} = \hat{i} + \hat{j} - 2\hat{k}$. A vector coplanar with $\vec{b}$ and $\vec{c}$. Whose projection on $\vec{a}$ is magnitude $\sqrt{\frac{2}{3}}$ is

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $2\hat{i} + 3\hat{j} + 3\hat{k}$

C. $-2\hat{i} - \hat{j} + 5\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: a,c

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4. For three vectors, $\vec{u}, \vec{v}$ and $\vec{w}$ which of the following expressions is not equal to any of the remaining three ?

A. $\vec{u}.\left(\vec{v} \times \vec{w}\right)$

B. $\left(\vec{v} \times \vec{w}\right). \vec{u}$

C. $\vec{v}. \left(\vec{u} \times \vec{w}\right)$

D. $\left(\vec{u} \times \vec{v}\right). \vec{w}$

Answer: c

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5. Which of the following expressions are meaningful? $\vec{u}. \left(\vec{v} \times \vec{w}\right)$ b. $\left(\vec{u}. \vec{v}\right). \vec{w}$ c. $\left(\vec{u}. \vec{v}\right). \vec{w}$ d. $\vec{u} \times \left(\vec{v}. \vec{w}\right)$

A. $\vec{u}. \left(\vec{v} \times \vec{w}\right)$

B. $\left(\vec{u}. \vec{v}\right). \vec{w}$

C. $\left(\vec{u}. \vec{v}\right)\vec{w}$

D. $\vec{u} \times \left(\vec{v}. Vecw\right)$

Answer: a,c

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6. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $\vec{u}$ and $\vec{v}$ are any two vectors.

Prove that $\vec{u} \times \vec{v} = \dfrac{1}{\left[\vec{a}\vec{b}\vec{c}\right]} \begin{vmatrix} \vec{u}.\vec{a} & \vec{v}.\vec{a} & \vec{a} \\ \vec{u}.\vec{b} & \vec{v}.\vec{b} & \vec{b} \\ \vec{u}.\vec{c} & \vec{v}.\vec{c} & \vec{c} \end{vmatrix}$

A. $\left|\vec{u}\right| + \vec{u}.\left(\vec{a}x\vec{b}\right)$

B. $\left|\vec{u}\right| + \left|\vec{u}.\vec{a}\right|$

C. $\left|\vec{u}\right| + \left|\vec{u}.\vec{b}\right|$

D. $\left|\vec{u}\right| + \vec{u}.\left(\vec{a} + \vec{b}\right)$

Answer: a,c

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7. Vector $\frac{1}{3}\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ is

A. a unit vector

B. makes an angle $\pi/3$ with vector $\left(2\hat{i} - 4\hat{j} + 3\hat{k}\right)$

C. parallel to vector $\left(- \hat{i} + \hat{j} - \frac{1}{2}\hat{k}\right)$

D. perpendicular to vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

Answer: a,c,d

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8. Let $\vec{A}$ be a vector parallel to the line of intersection of planes P_1 and P_2. Plane P_1 is parallel to vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k} nad P_2$ is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$. Then the angle between vector $\vec{A}$ and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is

A. $\pi/2$

B. $\pi/4$

C. $\pi/6$

D. $3\pi/4$

Answer: b,d

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9. The vector(s) which is /are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ and perpendicular to vector $\hat{i} + \hat{j} + \hat{k}$, is /are

A. $\hat{j} - \hat{k}$

B. $-\hat{i} + \hat{j}$

C. $\hat{i} - \hat{j}$

D. $-\hat{j} + \hat{k}$

Answer: a,d

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10. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$ if $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

A. $\vec{b} = \left(\vec{b}.\vec{z}\right)\left(\vec{z} - \vec{x}\right)$

B. $\vec{a} = \left(\vec{a}.\vec{y}\right)\left(\vec{y} - \vec{z}\right)$

C. $\vec{a}.\vec{b} = -\left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right)$

D. $\vec{a} = \left(\vec{a}.\vec{y}\right)\left(\vec{z} - \vec{y}\right)$

Answer: a,b,c

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11. Let PQR be a triangle . Let $\vec{a} = \overline{QR}, \vec{b} = \overline{RP}$ and $\vec{c} = \overline{PQ}$. if $\left|\vec{a}\right| = 12, \left|\vec{b}\right| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$ then which of the following is (are) true ?

A. $\dfrac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = 12$

B. $\dfrac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = 30$

C. $\left|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}\right| = 48\sqrt{3}$

D. $\vec{a}.\vec{b} = -72$

Answer: a,c,d

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