MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

DIFFERENTIATION

ILLUSTRATION

1. If $y=\sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}}, x\in\left(0,\dfrac{\pi}{2}\right)\cup\left(\dfrac{\pi}{2},\pi\right)$, then find $\dfrac{dy}{dx}\cdot$

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2. Find the derivative of $e^{\sqrt{x}}$ w.r.t. x using the first principle.

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3. If $f(x) = x\tan^{-1}x,$ find $f'\left(\sqrt{3}\right)$ using the first principle.

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4. If $f(x) = [2x]\sin 3\pi x$ then prove that $f'\left(k^{+}\right) = 6k\pi(-1)^{k},$ (where [.] denotes the greatest integer function and $k \in N$).

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5. Let $f: R \rightarrow$ satisfying $|f(x)| \leq x^{2} \forall x \in R$ be differentiable at $x = 0.$ The find $f'(0)$.

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6. A function $f: R\vec{R}$ satisfies the equation $f(x + y) = f(x)f(y)$ for all $x, y \in R and f(x) \neq 0 f$ or $all x \in R$. If $f(x)$ is differentiable at $x = 0 and f'(0) = 2,$ then prove that $f'(x) = 2f(x)$.

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7. If $y = \left(1 + x^{\frac{1}{4}}\right)\left(1 + x^{\frac{1}{2}}\right)\left(1 - x^{\frac{1}{4}}\right)$, then find $\frac{dy}{dx}\cdot$

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8. If $f(x) = x|x|$, then prove that $f'(x) = 2|x|$

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9. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \; + \frac{x^n}{n!}$, show that $\frac{dy}{dx} - y + \frac{x^n}{n!} = 0.$

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10. Find $\frac{dy}{dx}$ for $y = \sin^{-1}(\cos x)$, where $x \in (0, 2\pi)^{\cdot}$

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11. Differentiate $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to x, if `-1/(sqrt(2))

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12. `y=tan^(-1)((acosx-bsinx)/(bcosx+asinx)),w h e r e-pi/2-1`

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13. $y=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)+\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$, where $0<x<\infty$ Find $\frac{dy}{dx}$

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14. Find $\frac{dy}{dx}$ for $y=\sin\left(x^2+1\right)^{.}$

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15. If $y = \sqrt{\log\left\{\sin\left(\frac{x^2}{3} - 1\right)\right\}}, then f \in d\frac{dy}{dx}\cdot$

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16. Differentiate the function $f(x) = \sec\left(\tan\left(\sqrt{x}\right)\right)$ with respect to x

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17. Find $\frac{dy}{dx}$ for $y = \log\left(x + \sqrt{a^2 + x^2}\right)$.

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18. `y=tan^(-1)((sqrt(1+x^2)+sqrt(1-x^2))/(sqrt(1+x^2)-sqrt(1-x^2))),

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19. Find $\frac{dy}{dx}$ for $y = \tan^{-1}\sqrt{\frac{a - x}{a + x}}$, $-a<x<a$

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20. If $y = \sin^{-1}\left[x\sqrt{1 - x} - \sqrt{x}\sqrt{1 - x^2}\right)$ and \`0

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21. If $y = \frac{\tan^{-1}1}{1 + x + x^2} + \frac{\tan^{-1}1}{x^2 + 3x + 3} + \frac{\tan^{-1}1}{x^2 + 5x + 7} + +$ upto n terms, then find the value of $y'(0)$

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22. Let $f: R\vec{R}$ be a one-one onto differentiable function, such that $f(2) = 1 and f'(2) = 3.$ The find the value of $\left(\left(\frac{d}{dx}\left(f^{-1}(x)\right)\right)\right)_{x=1}$

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23. If $f(x) = \cos x \cos 2x \cos 4x \cos(8x).\cos 16x$ then find $f'\left(\frac{\pi}{4}\right)$

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24. If $\cos y = x\cos(a + y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$.

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25. Find $\frac{dy}{dx} f$ or $y = x\sin x\log x$.

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26. Evaluate $(\lim)_{h \to 0} \frac{(a + h)^2\sin^{-1}(a + h) - a^2\sin^{-1}a}{h}$

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27. Differentiate $y = \frac{e^x}{1 + \sin x}$

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28. If $f(x) = \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x$, then find $f'\left(\frac{\pi}{4}\right)$.

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29. If $\cos y = x\cos(a + y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$.

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30. If $y = \sqrt{\frac{1 - x}{1 + x}}$, prove that $\left(1 - x^2\right)\frac{dy}{dx} + y = 0$.

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31. Find the sum of the series $\left(1+2x+3x^2+(n-1)x^{n-2}\right)$ using differentiation.

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32. If $\sqrt{x}+\sqrt{y}=4$, then find $\frac{dy}{dx}$.

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33. If $xy+y^2=\tan x+y, thenf \in d\frac{dy}{dx}\cdot$

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34. If $y=x+\frac{1}{x+\frac{1}{x+\frac{1}{x+.}}}$, prove that $\frac{dy}{dx}=\frac{y}{2y-x}$.

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35. If $\sec(x + y) = xy$ find dy//dx

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36. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dot{\rightarrow} \infty}}}$, prove that $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

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37. If $\log\left(x^2 + y^2\right) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x + y}{x - y}$

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38. If $y = y(x)$ and it follows the relation $4xe^{xy} = y + 5\sin^2 x$, then $y'(0)$ is equal to_____

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39. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta) and y = a(1 - \cos\theta)$.

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40. If $x = a\ sec^3\theta$ and $y = a\ tan^3\theta$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

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41. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, then move that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$.

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42. Let $y = x^3 - 8x + 7 and x = f(t) I \cdot \frac{f(dy)}{dx} = 2 and x = 3a = 0,$ then find the value of $\frac{dx}{dt} a = 0.$

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43. Find the derivative of $\dfrac{\sqrt{x}(x+4)^{\frac{3}{2}}}{(4x-3)^{\frac{4}{3}}}$

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44. If $x^m y^n = (x+y)^{m+n}, provethat \dfrac{dy}{dx} = \dfrac{y}{x}\cdot$

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45. Differentiate $(\log x)^{\cos x}$ with respect to x.

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46. If $f(x) = |x|^{|\sin x|}$, then find $f'\left(-\dfrac{\pi}{4}\right)$

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47. If $y = x^{x} \wedge x \wedge ((((\infty))))$, find $\frac{dy}{dx}\cdot$

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48. If $f(x) = \lim_{h \to 0} \frac{(\sin(x+h))^{\log_e(x+h)} - (\sin x)^{\log_e x}}{h}$ then find $f(\pi/2)$.

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49. If $x < 1, provethat \frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \infty = \frac{1}{1-x}$

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50. Differentiate log sin x w.r.t. $\sqrt{x}$.

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51. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$,

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52. Find the derivative of $f(\tan x) wrt g(\sec x) at x = \frac{\pi}{4}$, where $f'(1) = 2 and g'\left(\sqrt{2}\right) = 4.$

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53. Let $= \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then find the values of f(0) and f' $(\pi/2)$.

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54. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$. $Thenf \in dthevlueof$ $\lim_{x \to 0} \frac{f'(x)}{x}$.

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55. If $y = \cos^{-1}x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

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56. about to only mathematics

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57. If $y = x\log\{\frac{x}{(a + bx)}]$, then show that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^{2\cdot}$

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58. If $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$, prove that $\dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

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59. If $y = e^a \cos^{(-1)x}$, $-1 \le x < 1$, *showthat* $\left(1 - x^2\right)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$

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60. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$,find(d^2y)/(dx^2)dot

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61. If g is inverse of f then prove that $f''(g(x)) = -g''(x)(f'(g(x)))^3$.

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62. Let f(x) and g(x) be real valued functions such that f(x)g(x)=1, $\forall x \in R$. If $f''(x)$ and $g''(x)$ exists $\forall x \in R$ and $f'(x)$ and $g'(x)$ are never zero, then prove that $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)} = \frac{2f'(x)}{f(x)}$

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63. If $f(x), g(x) and h(x)$ are three polynomial of degree 2, then prove that

$\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ is a constant polynomial.

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64. If $f\left(\frac{x+y}{3}\right) = \frac{2+f(x)+f(y)}{3}$ for all real x and y and f'(2)=2, then determine y=f(x).

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65.

If $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$ for all $x, y \in R, (xy \neq 1)$, and $\lim_{x \to 0}\frac{f(x)}{x}=2$ then f

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66. Let $f:R-\{0\} \rightarrow R$ be a function which is differentiable in its domain and satisfying the equation $f(x+y)=f(x)+f(y)+\frac{x+y}{xy}-\frac{1}{x+y}$, also f'(1)=2. Then find the function.

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67. Find function f(x) which is differentiable and satisfy the relation $f(x+y)=f(x)+f(y)+\left(e^{x}-1\right)\left(e^{y}-1\right) \forall x, y \in R$, and $f'(0)=2$.

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68. If $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}, \forall y, f(y) \neq 0$ and $f'(1) = 2$, find f(x) .

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69. Let $f: R\vec{R}$ be a function satisfying condition $f\left(x + y^3\right) = f(x) + [f(y)]^3 f\ or\ all x, y \in \dot{R}$ If $f'(0) \geq 0$, find $\dot{f(10)}$

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70. Let $f(x + y) = f(x) + f(y) + 2xy - 1$ for all real $xandy$ and $f(x)$ be a differentiable function. If $f'(0) = \cos\alpha$, the prove that $f(x) > 0 \forall x \in \dot{R}$

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71. If $y = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}, x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$, then find $\frac{dy}{dx}\cdot$

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72. Find the derivative of $e^{\sqrt{x}}$ w.r.t. x using the first principle.

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73. If $f(x) = x\tan^{-1}x$, find $f'(\sqrt{3})$ using the first principle.

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74. If $f(x) = [2x]\sin 3\pi x$ then prove that $f'(k^+) = 6k\pi(-1)^k$, (where [.] denotes the greatest integer function and $k \in N$).

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75. Let $f: R \to R$ satisfying $|f(x)| \le x^2, \forall x \in R,$ differentiable at x = 0 then find f'(0)

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76. A function $f: R \rightarrow R$ satisfies the equation f(x+y)=f(x)f(y) for all $x, y \in R$ and $f(x) \neq 0$ for all $x \in R$. If f(x) is differenitable at x=0 and f'(0)=2, then prove that $f'(x) = 2f(x)$.

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77. If $y = \left(1 + x^{\frac{1}{4}}\right)\left(1 + x^{\frac{1}{2}}\right)\left(1 - x^{\frac{1}{4}}\right)$, then find $\frac{dy}{dx} \cdot$

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78. If $f(x) = x|x|$, then prove that $f'(x) = 2|x|$

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79. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + + \frac{x^n}{n!}$, show that $\frac{dy}{dx} - y + \frac{x^n}{n!} = 0$.

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80. Find $\frac{dy}{dx}$ for $y = \sin^{-1}(\cos x), x \in (0, \pi) \cup (\pi, 2\pi)$.

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81. Differentiate $\sin^{-1}\left(2x\sqrt{1 - x^2}\right)$ with respect to x if $\frac{1}{\sqrt{2}} < x < 1$

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82.

If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and $\frac{a}{b}\tan x > -1$, then find $\frac{d}{d}$

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83. $y = \sin^{-1}\left(\frac{x}{\sqrt{1 + x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right)$, Find dy/dx

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84. Find $\frac{dy}{dx}$ for $y = \sin\left(x^2 + 1\right)$.

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85. If $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$, then find $\frac{dy}{dx}$.

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86. Differentiate the function f(x) = sec $\left(\tan\sqrt{x}\right)$ with respect to x,

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87. Find $\frac{dy}{dx}$ for $y = \log\left(x + \sqrt{a^2 + x^2}\right)$.

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88. $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, *where* -1

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89. Find $\frac{dy}{dx}$ for $y = \tan^{-1}\left\{\sqrt{\frac{a-x}{a+x}}\right\}$, where $-a < x < a$

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90. if $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ and $0 < x < 1$, then find $\frac{dy}{dx}$.

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91. If $y = \tan^{-1}\frac{1}{1+x+x^2} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \ldots$ upto n terms, then find the value of y'(0).

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100. If $y = \sqrt{\frac{1 - x}{1 + x}}, provethat\left(1 - x^2\right)\frac{dy}{dx} + y = 0$

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101. Find the sum of the series $1 + 2x + 3x^2 + (n - 1)x^{n-2}$ using differentiation.

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102. If $\sqrt{x} + \sqrt{y} = 4$, then find $\frac{dy}{dx}$.

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103. If $xy + y^2 = \tan x + y$, then find $\frac{dy}{dx}$.

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104. If $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \ldots}}}$, prove that $\frac{dy}{dx} = \frac{y}{2y - x}$.

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105. If sec (x+y) = xy, then find $\frac{dy}{dx}$

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106. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \to \infty}}}$, prove that $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

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107. If $\log\left(x^2 + y^2\right) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x + y}{x - y}$

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108. If $y = y(x)$ and it follows the relation $4xe^{xy} = y + 5\sin^2 x$, then $y'(0)$ is equal to_____

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109. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$.

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118. If $f(x) = \lim_{h \to 0} \frac{(\sin(x+h))^{\log_e(x+h)} - (\sin x)^{\log_e x}}{h}$ then find $f(\pi/2)$.

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119. If $0 < x < 1$, prove that $\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \ldots\infty = \frac{1}{1-x}$

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122. Let $= \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then find the values of f(0) and f' $(\pi/2)$.

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123. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$ then find the value of

$\lim_{x \to 0} \frac{f(x)}{x}$

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124. If $y = \cos^{-1}x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

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125. if $y = \left(x^2 - 1\right)^m$, then the $(2m)th$ differential coefficient of y is

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126. If $y = x\log\{\frac{x}{(a + bx)}]$, then show that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^{2\cdot}$

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127. If $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

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128. If $y = e^{a\cos^{-1}x}, \ -1 \le x \le 1,$ show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$

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129. If x=a (cos t + t sin t) and y=a (sin t- t cos t), find $\frac{d^2y}{dx^2}$

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130. If g is inverse of f then prove that $f''(g(x)) = -g''(x)(f'(g(x)))^3$.

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131. Let f(x) and g(x) be real valued functions such that f(x)g(x)=1, $\forall x \in R$. If $f''(x)$ and $g''(x)$ exists $\forall x \in R$ and $f'(x)$ and $g'(x)$ are never zero, then prove that $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)} = \frac{2f'(x)}{f(x)}$

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132. if f(x) g(x) and h(x) are three polynomials of degree 2,

then prove that $\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ is a constant polynomial.

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133. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2} f \text{ or } allrealxandy$ If $f'(0)$ exists and equals $-1 andf(0) = 1, thenf \in df(2)^{\cdot}$

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134. $f(x) + f\left(y_{=} f\left(\frac{x+y}{1-xy}\right)\right)$ for all$x, y \in R^{\cdot}$ $(xy \neq 1), and(\lim)_{x\vec{0}} \frac{f(x)}{x} = 2. F \in df\left(\frac{1}{\sqrt{3}}\right) andf'(1)^{\cdot}$

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135. Find function f(x) which is differentiable and satisfy the relation $f(x+y) = f(x) + f(y) + \left(e^x - 1\right)\left(e^y - 1\right) \forall x, y \in R$, and $f'(0) = 2$.

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136. If $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}, \forall y, f(y) \neq 0$ and $f'(1) = 2$, find f(x) .

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137. Let $f: R\vec{R}$ be a function satisfying condition $f\left(x + y^3\right) = f(x) + [f(y)]^3 f \text{ or } all x, y \in R^{\cdot}$ If $f'(0) \geq 0$, find $f(10)^{\cdot}$

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138. Let $f(x + y) = f(x) + f(y) + 2xy - 1$ for all real $xandy$ and $f(x)$ be a differentiable function. If $f'(0) = \cos\alpha$, the prove that $f(x) > 0 \forall x \in R^{\cdot}$

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Solved Examples

1. If $f(x) = (\log)_{x^2}(\log x)$, then $f'(x)$ at $x = e$ is 0 (b) 1 (c) $\frac{1}{e}$ (d) $\frac{1}{2}e$

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2. Given that $\cos\left(\frac{x}{2}\right).\cos\left(\frac{x}{4}\right).\cos\left(\frac{x}{8}\right)\ldots. = \frac{\sin x}{x}$ Prove that $\left(\frac{1}{2^2}\right)\sec^2\left(\frac{x}{2}\right) + \left(\frac{1}{2^4}\right)\sec^2\left(\frac{x}{4}\right) + \ldots. = \operatorname{cosec}^2 x - \frac{1}{x^2}$

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3. If $y = f\left(a^x\right) and f'(\sin x) = (\log)_e x, then f \in d\frac{dy}{dx},$ if it exists, where `pi//2

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4. If P_n is the sum of a $\ddot{GP}$ upto n terms $(n \geq 3)$, then prove that $(1 - r)\frac{dP_n}{dr} = (1 - n)P_n + nP_{n-1},$ where r is the common ratio of $\ddot{GP}$

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5. If $g(x) = \frac{f(x)}{(x - a)(x - b)(x - c)}, where f(x)$ is a polynomial of degree < 3, then

$\int g(x)dx = |1af(a)\log|x - a|1bf(b)\log|x - b|1cf(c)\log|x - c|| \div |1aa^2 1 \wedge 21 \wedge 2| + k$

$\frac{dg(x)}{dx} = |1af(a)\log(x - a)^2 1bf(b)\log(x - b)^2 1cf(c)\log(x - b)^2| \div |a^2 a1b^2 b1c^2 c1|$

$\frac{dg(x)}{dx} = |1af(a)\log(x - a)^{-2} 1bf(b)\log(x - b)^{-2} 1cf(c)\log(x - b)^{-2}| \div |1aa^2 1 \wedge 21 \wedge 2$

$\int g(x)dx = |1af(a)\log|x - a|1bf(b)\log|x - b|1cf(c)\log|x - c|| \div |a^2 a1b^2 b1c^2 c1| + k$

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6. If $x = \operatorname{cosec}\theta - \sin\theta and y = \operatorname{cosec}^n\theta - \sin^n\theta$, then show that $\left(x^2 + 4\right)\left(\frac{dy}{dx}\right)^2 = n^2\left(y^2 + 4\right)$.

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7. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$, then prove that $\frac{y'}{y} = \frac{1}{x}\left[\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right]$

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8. Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

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9. If $y = \left(\frac{1}{2}\right)^{n-1}\cos\left(n\cos^{-1}x\right)$, then prove that y satisfies the differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0$

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10. Let $f(x) and g(x)$ be two function having finite nonzero third-order derivatives $f''' and g'''$ for all $x \in R$. If $f(x)g(x) = 1$ for all $x \in R$, then prove that $f'''(/)f' - g'''/g' = 3\left(f''/f - g''/g\right)$.

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11. If a curve is represented parametrically by the equation $x = f(t)$ and $y = g(t)$ then prove that $\frac{d^2y}{dx^2} = -\left[\frac{g'(t)}{f'(t)}\right]^3\left(\frac{d^2x}{dy^2}\right)$

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12. If $f\left(\frac{x+y}{3}\right) = \frac{2 + f(x) + f(y)}{3}$ for all real $x and y$ and $f'(2) = 2$, then determine $y = f(x)$.

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13. If $f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x}$ holds for all real x and y greater than 0 and $f(x)$ is a differentiable function for all $x > 0$ such that $f(e) = \frac{1}{e}$, then find $f(x)$

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14. If $\left|a_1 \sin x + a_2 \sin 2x + \quad + a_n \sin nx\right| \leq |\sin x|$ for $x \in R$, then prove that $\left|a_1 + 2a_2 + 3a_3 + na_n\right| \leq 1$

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15. Suppose $p(x) = a_0 + a_1x + a_2x^2 + \quad + a_nx^n$. If $|p(x)| \leq e^{x-1} - 1|$ for all $x \geq 0$, prove that $\left|a_1 + 2a_2 + \quad + na_n\right| \leq 1.$

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16. If $f(x) = \log_x(\log x)$, then find $f'(x) at x = e$

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17. Given that $\frac{\cos x}{2} \cdot \frac{\cos x}{4} \cdot \frac{\cos x}{8} .... = \frac{\sin x}{x}$ Then find the sum $\frac{1}{2^2} \frac{\sec^2 x}{2} + \frac{1}{2^4} \frac{\sec^2 x}{4} + ...$

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18. If $y = f\left(a^x\right) and f'(\sin x) = (\log)_e x, then f \in d\frac{dy}{dx},$ if it exists, where `pi/2

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19. If P_n is the sum of a $\ddot{GP}$ upto n terms $(n \geq 3)$, then prove that $(1 - r)\frac{dP_n}{dr} = (1 - n)P_n + nP_{n-1},$ where r is the common ratio of $\ddot{GP}$

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20. If $x = \text{cosec}\theta - \sin\theta and y = \text{cosec}^n\theta - \sin^n\theta$, then show that $\left(x^2 + 4\right)\left(\frac{dy}{dx}\right)^2 = n^2\left(y^2 + 4\right)$.

Watch Video Solution

21. If $y = \frac{ax^2}{(x - a)(x - b)(x - c)} + \frac{bx}{(x - b)(x - c)} + \frac{c}{x - c} + 1$, then prove that $\frac{y'}{y} = \frac{1}{x}\left[\frac{a}{a - x} + \frac{b}{b - x} + \frac{c}{c - x}\right]$

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22.

If $y = \frac{2}{\sqrt{a^2 - b^2}}\left\{\tan^{-1}\left(\sqrt{\frac{a - b}{a + b}}\tan\frac{x}{2}\right)\right\}$, then show that $\frac{d^2y}{dx^2} = \frac{b\sin x}{(a + b\cos x)^2}$.

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23. If $y = \left(\frac{1}{2}\right)^{n-1}\cos\left(n\cos^{-1}x\right)$, then prove that y satisfies the differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0$

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24. If a curve is represented parametrically by the equation $x = f(t)$ and $y = g(t)$ then prove that $\frac{d^2y}{dx^2} = -\left[\frac{g'(t)}{f'(t)}\right]^3\left(\frac{d^2x}{dy^2}\right)$

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25. If $f\left(\frac{x+y}{3}\right) = \frac{2+f(x)+f(y)}{3}$ for all real $xandy$ and $f'(2) = 2$, then determine $y = f(x)$.

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26. If $f(x) = \frac{f(x)}{y} + \frac{f(y)}{x}$ holds for all real x and y greater than $0 and f(x)$ is a differentiable function for all $x > 0$ such that $f(e) = \frac{1}{e}, then f \in df(x)^{\cdot}$

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27. If $\left|a_1 \sin x + a_2 \sin 2x + \quad + a_n \sin nx\right| \leq |\sin x|$ for $x \in R$, then prove that $\left|a_1 + 2a_2 + 3a_3 + na_n\right| \leq 1$

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28. Suppose $p(x) = a_0 + a_1 x + a_2 x^2 + \quad + a_n x^{n^{\cdot}}$ If $|p(x)| \leq e^{x-1} - 1|$ for all $x \geq 0$, prove that $\left|a_1 + 2a_2 + \quad + na_n\right| \leq 1.$

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Concept Application 3.1

1. Using the definition of derivative find the derivative of $\sqrt{\sin x}$

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Concept Application 3.2

1. Find the derivative of $\sqrt{4 - x}$ w.r.t. x using the first principle.

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2. If $y = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$, then find $\frac{dy}{dx}$

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3. $y = \tan^{-1}\frac{3x - x^3}{2x^2 - 1}, \ -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

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4. If `y=sec^(-1)(1/(2x^2-1));0

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5. Find $\frac{dy}{dx}$ if $y = \frac{\tan^{-1}(4x)}{1 + 5x^2} + \frac{\tan^{-1}(2 + 3x)}{3 - 2x}$

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6. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$, where $x \neq 0$

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7. $y = \tan^{-1}\left(\frac{x}{1 + \sqrt{1 - x^2}}\right)$

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8. Find $\frac{dy}{dx}$ for the function: $y = \sin^{-1}\sqrt{(1-x)} + \cos^{-1}\sqrt{x}$

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9. $y = \sqrt{\sin\sqrt{x}}$

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10. $y = e^{\sin x^3} find \frac{dy}{dx}$

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11. $y = \log\sqrt{\sin\sqrt{e^x}}$

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12. Find $\frac{dy}{dx}$ for the function: $y = a^{\sin^{-1}x} \wedge (2)$

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13. Find $\frac{dy}{dx}$ if $y = \log\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

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14. $y = \sin^{-1}\left[\sqrt{x - ax} - \sqrt{a - ax}\right]$

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15. Find $\frac{dy}{dx}$ for the functions: $y = x^3 e^x \sin x$

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16. Find $\frac{dy}{dx}$ for the function: $y = (\log)_e\sqrt{\frac{1 + \sin x}{1 - \sin x}}, where x = \frac{\pi}{3}$

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17. Find $\frac{dy}{dx}$ for the functions: $y = \frac{x + \sin x}{x + \cos x}$

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18. If $y = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right)\ldots\left(1 + x^{2^n}\right)$ then $\frac{dy}{dx}$ at $x = 0$ is

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19. $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$ then $\frac{dy}{dx} =$

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20. If g is the inverse function of and f'(x) = sin x then prove that g'(x) = cosec (g(x))

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21. Find the derivative of $\sqrt{4 - x}$ w.r.t. x using the first principle.

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22. $Find\frac{dy}{dx}, y = \sin^{-1}\frac{2x}{1 + x^2}, \ -1 \le x \le 1$

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23. $Find\frac{dy}{dx}, y = \tan^{-1}\left[\frac{3x - x^3}{1 - 3x^2}\right], \ -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

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24. $y = \sec^{-1}\frac{1}{2x^2 - 1}, 0 < x < \frac{1}{\sqrt{2}}$

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25. Find $\frac{dy}{dx}$ if $y = \frac{\tan^{-1}(4x)}{1 + 5x^2} + \frac{\tan^{-1}(2 + 3x)}{3 - 2x}$

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26. $\text{F i n d}\frac{dy}{dx}\text{if} y = \tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$, where $x \neq 0$

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27. $y = \tan^{-1}\left(\frac{x}{1 + \sqrt{1 - x^2}}\right)$

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28. Find $\frac{dy}{dx}$ for the function: $y = \sin^{-1}\sqrt{(1 - x)} + \cos^{-1}\sqrt{x}$

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29. $y = \sqrt{\sin\sqrt{x}}$

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30. $y = e^{\sin x^3} find \frac{dy}{dx}$

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31. $y = \log\sqrt{\sin\sqrt{e^x}}$

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32. Find $\frac{dy}{dx}$ for the function: $y = a^{\left(\sin^{-1}x\right)^2}$

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33. Find $\frac{dy}{dx}$ if $y = \log\left\{e^x\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

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34. $y = \sin^{-1}\left[\sqrt{x - ax} - \sqrt{a - ax}\right] provethat \frac{dy}{dx} is \frac{1}{2\sqrt{x(1-x)}}$

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35. Find $\frac{dy}{dx}$ for the functions: $y = x^3 e^x \sin x$

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36. Find $\frac{dy}{dx}$ for the function: $y = (\log)_e \sqrt{\frac{1 + \sin x}{1 - \sin x}}, wherex = \frac{\pi}{3}$

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37. $y = \frac{x + \sin x}{x + \cos x}, \ find \frac{dy}{dx}$

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38. If $y = (1 + x)\left(1 + x^2\right)\left(1 + x^4\right)...\left(1 + x^{2^n}\right)$ then $\frac{dy}{dx}$ at $x = 0$ is

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39. If $x\sqrt{1 + y} + y\sqrt{1 + x} = 0, \ \text{ prove that } \frac{dy}{dx} = -\frac{1}{(x + 1)^2}.$

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40. If g is the inverse function of and f'(x) = sin x then prove that g'(x) = cosec (g(x))

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Concept Application 3.3

1. Let $f(x) = \frac{\left(2^x + 2^{-x}\right)\sin x\sqrt{\tan^{-1}\left(x^2 - x + 1\right)}}{\left(7x^2 + 3x + 1\right)^3}$. Then find the value of f'(0).

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2. If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$

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3. If `y=btan^(-1)(x/a+tan^(-1)y/x),find (dy)/(dx)'

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4. If $\log_e\left(\log_e x - \log_e y\right) = e^{x^2 y}\left(1 - \log_e x\right)$, then find the value of $y'(e)$.

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5. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots\infty}}}}$, then prove that $\frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$

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6. Let $f(x) = \frac{\left(2^x + 2^{-x}\right)\sin x\sqrt{\tan^{-1}\left(x^2 - x + 1\right)}}{\left(7x^2 + 3x + 1\right)^3}$. Then find the value of f'(0).

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7. If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$.

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8. If $y = b\tan^{-1}\left(\frac{x}{a} + \tan^{-1}y/x\right)$, $find\frac{dy}{dx}\cdot$

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9. If $\log_e\left(\log_e x - \log_e y\right) = e^{x^2 y}\left(1 - \log_e x\right)$, then find the value of $y'(e)$.

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10. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots\infty}}}}$ then prove that $\frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$

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Concept Application 3.4

1. Statement 1: Let $f: R\vec{R}$ be a real-valued function $\forall x, y \in R$ such that $|f(x) - f(y)| \leq |x - y|^3$. Then $f(x)$ is a constant function. Statement 2: If the derivative of the function w.r.t. x is zero, then function is constant.

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2. If $x = \frac{2t}{1 + t^2}, y = \frac{1 - t^2}{1 + t^2}, thenf \in d\frac{dy}{dx}a = 2.$

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3. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$ then show that, $\frac{dy}{dx} = -\frac{y}{x}.$

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4. Find $\frac{dy}{dx}$ if x= 3 cos theta - cos 2theta and y= sin theta - sin 2theta.`

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5. If $x = 3\cos\theta - 2\cos^3\theta, y = 3\sin\theta - 2\sin^3\theta$, then $\frac{dy}{dx}$ is

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6. If $x = a\left(\cos t + \frac{1}{2}\log\tan' \frac{t}{2}\right.$ and y=asint$thenf \in d$(dy)/(dx)at=pi/4`

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7. Statement 1: Let $f: R\vec{R}$ be a real-valued function $\forall x, y \in R$ such that $|f(x) - f(y)| \leq |x - y|^3$. Then $f(x)$ is a constant function. Statement 2: If the derivative of the function w.r.t. x is zero, then function is constant.

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8. If $x = \frac{2t}{1 + t^2}, y = \frac{1 - t^2}{1 + t^2}$, then find $\frac{dy}{dx}$ at $t = 2$.

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9. If $x = \sqrt{a^{\sin} \wedge (- 1t), y = \sqrt{a^{\cos} \wedge ((- 1)t)}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

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10. Find $\frac{dy}{dx}$ if $x = \cos\theta - \cos2\theta$

and $y = \sin\theta - \sin2\theta$

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11. Find $\frac{dy}{dx}$ if $x = 3\cos\theta - 2\cos^3\theta, y = 3\sin\theta - 2\sin^3\theta$.

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Concept Application 3.5

1. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x

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2. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

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3. If $xy = e^{(x-y)}$, then find $\frac{dy}{dx}$

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4. If $y^x = x^y, thenf \in d\frac{dy}{dx}\cdot$

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5. If $x = e^y + e^{(y + \to \infty)}$, where $x > 0$, $then \dfrac{dy}{dx}$

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6. Find $\dfrac{dy}{dx} f$ or $y = x^{x^{\cdot}}$

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7. Differentiate $(x\cos x)^x$ with respect to $\dot{x}$

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8. If $y = (\tan x)^{(\tan x)^{\tan x}}$, then find $\dfrac{dy}{dx}$.

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9. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x

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10. If $y\log x = x - y$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

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11. If $xy = e^{(x-y)}$, then find $\frac{dy}{dx}$.

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12. If $y^x = x^y, thenf \in d\frac{dy}{dx}$.

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13. If $x = e^{y} + e^{(y + \to \infty)}$, where $x > 0$, $thenf \in d\frac{dy}{dx}$

Watch Video Solution

14. Find $\frac{dy}{dx}f$ or $y = x^{x^{\cdot}}$

Watch Video Solution

15. Differentiate $(x\cos x)^{x}$ with respect to $x^{\cdot}$

Watch Video Solution

16. If $y = (\tan x)^{(\tan x)^{\tan x}}$, then find $\frac{dy}{dx}$.

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Concept Application 3.6

1. Find the derivative of $\frac{\tan^{-1}(2x)}{1-x^2} \cdot\cdot wrt \frac{\sin^{-1}(2x)}{1+x^2}$

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2. The differential cofficient of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t $\sqrt{1-x^2}$ is-

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3. Differentiate $\frac{x}{\sin x}$ w.r.t . sinx.

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4.

If $y = f\left(x^3\right), z = g\left(x^5\right), f'(x) = \tan x$, and $g'(x) = \sec x$, then find the value of of

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5. Find the derivative of $\tan^{-1}\frac{2x}{1-x^2}$ w.r.t. $\sin^{-1}\frac{2x}{1+x^2}, |x| < 1.$

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6. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x = \frac{1}{2}.$

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7. Differentiate $\frac{x}{\sin x}$ w.r.t . sinx.

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8.

If $y = f\left(x^3\right), z = g\left(x^5\right), f'(x) = \tan x,$ and $g'(x) = \sec x,$ then find the value of of

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Concept Application 3.7

1. If $f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$, then prove that

$f'(x) = 3x^2 + 2x\left(a^2+b^2+c^2\right).$

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2. Let $f(x) = \begin{vmatrix} \cos\left(x+x^2\right) & \sin\left(x+x^2\right) & -\cos\left(x+x^2\right) \\ \sin\left(x-x^2\right) & \cos\left(x-x^2\right) & \sin\left(x-x^2\right) \\ \sin 2x & 0 & \sin\left(2x^2\right) \end{vmatrix}.$

Find the value of f'(0).

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3. Let $g(x)=\begin{vmatrix} f(x+c) & f(x+2c) & f(x+3c) \\ f(c) & f(2c) & f(3c) \\ f'(c) & f'(2c) & f'(3c) \end{vmatrix}$,

where c is constant, then find $\lim_{x\to 0}\frac{g(x)}{x}$.

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4. If $f(x)=\begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$, then prove that

$f'(x)=3x^2+2x\left(a^2+b^2+c^2\right)$.

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5. Let $f(x)=\begin{vmatrix} \cos\left(x+x^2\right) & \sin\left(x+x^2\right) & -\cos\left(x+x^2\right) \\ \sin\left(x-x^2\right) & \cos\left(x-x^2\right) & \sin\left(x-x^2\right) \\ \sin 2x & 0 & \sin\left(2x^2\right) \end{vmatrix}$.

Find the value of f'(0).

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6. Let $g(x) = \begin{vmatrix} f(x+c) & f(x+2c) & f(x+3c) \\ f(c) & f(2c) & f(3c) \\ f'(c) & f'(2c) & f'(3c) \end{vmatrix}$,

where c is constant, then find $\lim_{x \to 0} \frac{g(x)}{x}$.

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Concept Application 3.8

1. If $f(x) = (1+x)^2$, then the value of $f(x0) + f'(0) + \frac{f^0}{2!} + \frac{f^0}{3!} + \frac{f^n(0)}{n!}$.

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2. If $e^y(x+1) = 1, showthat \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{2\cdot}$

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3. Prove that $\frac{d^n}{dx^n}\left(e^{2x}+e^{-2x}\right)=2^n\left[e^{2x}+(-1)^n e^{-2x}\right]$

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4. If y =sin (sin x) and $\frac{d^2y}{dx^2}+\frac{dy}{dx}$ tan x + f(x) = 0, then find f(x).

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5. If y=log (1+ sin x), prove that $y_4+y_3y_1+y_2^2=0.$

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6. If $f(x)=\left|x^n \quad n! \quad 2; \cos x \quad \cos\left(\frac{n\pi}{2}\right) \quad 4; \sin x \quad \sin\left(\frac{n\pi}{2}\right) \quad 8\right|$ then find the value of $\frac{d^n}{dx^n}([f(x)])_{x=0}n \in z$

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7. If $x = a\cos\theta, y = b\sin\theta$, then prove that $\frac{d^3y}{dx^3} = \frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta^{\cdot}$

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8. If $x = a\cos^3\theta, y = b\sin^3\theta, f \in d\frac{d^3y}{dx^3}$ at $\theta = 0$.

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9. If $f(x) = (1 + x)^2$, then the value of $f(x0) + f'(0) + \frac{f^0}{2!} + \frac{f^0}{3!} + \frac{f^n(0)}{n!}^{\cdot}$

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10. If $e^y(x + 1) = 1, showthat\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{2\cdot}$

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11. Prove that $\frac{d^n}{dx^n}\left(e^{2x}+e^{-2x}\right)=2^n\left[e^{2x}+(-1)^n e^{-2x}\right]$

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12. If y sin (sin x) and $\frac{d^2y}{dx^2}+\frac{dy}{dx}$ tan x + f(x) = 0, then find f(x).

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13. If y=log (1+ sin x), prove that $y_4+y_3y_1+y_2^2=0.$

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14. If $x=a\cos\theta, y=b\sin\theta,$ then prove that $\frac{d^3y}{dx^3}=\frac{3b}{a^3}\text{cosec}^4\theta\cot\theta$.

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15. If $x=a\cos^3\theta, y=b\sin^3\theta, f\in d\frac{d^3y}{dx^3}$ at $\theta=0.$

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Concept Application 3.9

1. Let $f(x+y)=f(x)f(y)$ for all $x and y$. Suppose $f(5)=2 and f'(0)=3$. Find $f'(5)$.

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2. Let $f(xy)=f(x)f(y)\,\forall x, y \in R and f$ is differentiable at $x=1$ such that $f'(1)=1$. Also, $f(1)\neq 0, f(2)=3$. Then find $f'(2)$.

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3. Let f be a function such that $f(x+y)=f(x)+f(y)$ for all x and y and $f(x)=\left(2x^2+3x\right)g(x)$ for all x, where g is continuous and g(0) = 3. Then find f'(x)

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4. Let $g : R \rightarrow R$ be a differentiable function satisfying $g(x) = g(y)g(x - y) \forall x, y \in R$ and $g'(0) = a$ and $g'(3) = b$, Then find the value c

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5. Let $f\left(x^m y^n\right) = mf(x) + nf(y)$ for all $x, y \in R^+$ and for all $m, n \in R$. If $f'(x)$ exists and has the value $\frac{e}{x}$, then find $(\lim)_{x \to 0} \frac{f(1 + x)}{x}$

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6. If $f\left(\frac{x + 2y}{3}\right) = \frac{f(x) + 2f(y)}{3} \forall x, y \in R and f'(0) = 1, f(0) = 2$, then find $f(x)$.

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7.

Prove that $\lim_{h \to 0} \dfrac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x)$ (without using L' Hospital's

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8. Let $f(x+y) = f(x)f(y)$ for all $xandy$. Suppose $f(5) = 2andf'(0) = 3$. Find $f'(5)$.

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9. Let $f(xy) = f(x)f(y) \forall x, y \in Randf$ is differentiable at $x = 1$ such that $f'(1) = 1$. Also, $f(1) \neq 0, f(2) = 3$. Then find $f'(2)$.

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10. Let f be a function such that $f(x+y) = f(x) + f(y)$ for all $xandyandf(x) = \left(2x^2 + 3x\right)g(x)$ for all$x$, where $g(x)$ is continuous and

$g(0) = 3$. Then find $f'(x)$.

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11. Let $g: R \vec{R}$ be a differentiable function satisfying $g(x) = g(y)g(x - y) \forall x, y \in R$ and $g'(0) = a and g'(3) = b$. Then find the value of $g'(-3)$.

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12. Let $f\left(x^m y^n\right) = mf(x) + nf(y)$ for all $x, y \in R^+$ and for all $m, n \in R$. If $f'(x)$ exists and has the value $\frac{e}{x}$, then find $(\lim)_{x \vec{0}} \frac{f(1 + x)}{x}$

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13. If $f\left(\frac{x + 2y}{3}\right) = \frac{f(x) + 2f(y)}{3} \forall x, y \in R and f'(0) = 1, f(0) = 2$, then find $f(x)$.

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14. Prove that

$$\lim_{h \to 0} \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x) \text{ (without using L' Hospital's rule).}$$

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Execrises

1. If $y = a\sin x + b\cos x$, then $\left(\frac{dy}{dx}\right)^2 + y^2$ is

A. function of x

B. function of y

C. function of x and y

D. constant

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2. `d/(dx)sqrt((1-sin2x)/(1+sin2x))i se q u a lto,(0

A. $\sec^2 x$

B. $-\sec^2\left(\frac{\pi}{4} - x\right)$

C. $\sec^2\left(\frac{\pi}{4} + x\right)$

D. $\sec^2\left(\frac{\pi}{4} - x\right)$

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3. If $f(x) = |\cos x| + |\sin x|$, then $\frac{dy}{dx}$ at $x = \frac{2\pi}{3}$ is equal to

A. $\frac{1 - \sqrt{3}}{2}$

B. 0

C. $\frac{1}{2}\left(\sqrt{3} - 1\right)$

D. none of these

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4. If f(x) $= \left|\log_e |x|\right|$, then f'(x) equals

A. $\frac{1}{|x|}$, where $x \neq 0$

B. $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

C. $-\frac{1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

D. $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

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5. If $f(x) = \sqrt{1 - \sin 2x}$, then $f'(x)$ is equal to

(a) $-(\cos x+\sin x)$, for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(b) $\cos x+\sin x$, for $x \in \left(0, \frac{\pi}{4}\right)$

(c) $-(\cos x+\sin x)$, for $x \in \left(0, \frac{\pi}{4}\right)$

(d) $\cos x-\sin x$, for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

A. $-(\cos x+\sin x)$, for x $\in(\pi/4, \pi/2)$

B. cosx + sin x for x $\in(0, \pi/4)$

C. $-(\cos x+\sin x)$, for $x \in(0, \pi/4)$

D. $\cos x-\sin x$, for $x \in(\pi/4, \pi/2)$

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6. Instead of the usual definition of derivative $Df(x)$, if we define a new kind of derivative $D^{\cdot}F(x)$ by the formula

$D \cdot (x) = (\lim)_{h \vec{0}} \frac{f^2(x+h) - f^2(x)}{h}$, $where f^2(x)$ mean

$[f(x)]^2 and \Leftrightarrow (x) = x\log x, thenD^{\cdot} f(x)(\mid)_{x=e}$ has the value e (B) 2e (c) 4e (d) none of these

A. e

B. 2e

C. 4e

D. none of these

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7. If $y = \cot^{-1}\left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right](0 < x < \pi/2)$, then $\frac{dy}{dx} =$

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. 3

D. 1

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8. if $y = \frac{(a - x)\sqrt{a - x} - (b - x)\sqrt{x - b}}{\sqrt{a - x} + \sqrt{x - b}}$ then $\frac{dy}{dx}$ wherever it is defined is equal to :

A. $\frac{x + (a + b)}{\sqrt{(a - x)(x - b)}}$

B. $\frac{2x - a - b}{2\sqrt{a - x}\sqrt{x - b}}$

C. $-\frac{(a + b)}{2\sqrt{(a - x)(x - b)}}$

D. $\frac{2x + (a + b)}{2\sqrt{(a - x)(x - b)}}$

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9. the derivative of $y = (1 - x)(2 - x)..........(n - x)$ at $x = 1$ is equal to

A. 0

B. $(-1)(n-1)!$

C. $n!-1$

D. $(-1)^{n-1}(n-1)!$

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10. If $y=\sqrt{\frac{1-x}{1+x}}$, then $\left(1-x^2\right)\frac{dy}{dx}$ is equal to

A. y^2

B. $1/y$

C. $-y$

D. $-y/x$

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11. If $y=\frac{\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}, then\frac{dy}{dx}isequa<o$ $\frac{ay}{x\sqrt{a^2-x^2}}$ (b) $\frac{ay}{\sqrt{a^2-x^2}}$ $\frac{ay}{x\sqrt{a^2-x^2}}$ (d) none of these

A. $\frac{ay}{x\sqrt{a^2-x^2}}$

B. $\frac{ay}{\sqrt{a^2-x^2}}$

C. $\frac{ay}{x\sqrt{x^2-a^2}}$

D. none of these

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12. Let $u(x)andv(x)$ be differentiable functions such that $\frac{u(x)}{v(x)}=7.\frac{u'(x)}{v'(x)}=pand\left(\frac{u(x)}{v(x)}\right)'=q, then\frac{p+q}{p-q}$ has the value of to 1 (b) 0 (c) 7 (d) -7

A. 1

B. 0

C. 7

D. -7

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13. If $\sin^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = \log a, then \frac{dy}{dx}$ is equal to $\frac{x}{y}$ (b) $\frac{y}{x^2}$ $\frac{x^2 - y^2}{x^2 + y^2}$ (d) $\frac{y}{x}$

A. $\frac{x}{y}$

B. $\frac{y}{x^2}$

C. $\frac{x^2 - y^2}{x^2 + y^2}$

D. $\frac{y}{x}$

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14. Let $h(x)$ be differentiable for all x and let $f(x) = \left(kx + e^x\right)h(x)$, where k is some constant. If $h(0) = 5, h^{'}(0) = -2, and f^{'}(0) = 18,$ then the value of k is

A. 5

B. 4

C. 3

D. 2.2

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15. If $\lim_{t \to x} \frac{e^t f(x) - e^x f(t)}{(t - x)(f(x))^2} = 2$ and $f(0) = \frac{1}{2},$ then find the value of $f'(0)$.

A. 4

B. 2

C. 0

D. 1

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16. If $f(0)=0, f'(0)=2,$ then the derivative of $y=f(f(f(x)))$ at $x=0$ is 2 (b) 8 (c) 16 (d) 4

A. 2

B. 8

C. 16

D. 4

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17. If $f(x)=\sqrt{1+\cos^2\left(x^2\right)}, \, then f'\left(\frac{\sqrt{\pi}}{2}\right)$ is $\frac{\sqrt{\pi}}{6}$ (b) -$\sqrt{\pi/6}$ $1/\sqrt{6}$ (d) $\pi/\sqrt{6}$

A. $\sqrt{\pi/6}$

B. $-\sqrt{\pi/6}$

C. $1/\sqrt{6}$

D. $\pi/\sqrt{6}$

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18. $\frac{d}{dx}\cos^{-1}\sqrt{\cos x}$ is equal to

A. $\frac{1}{2}\sqrt{1+\sec x}$

B. $\sqrt{1+\sec x}$

C. $-\frac{1}{2}\sqrt{1+\sec x}$

D. $-\sqrt{1+\sec x}$

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19. if $y = \tan^{-1}\left(\frac{2^x}{1 + 2^{2x+1}}\right)$ then $\frac{dy}{dx} at x = 0$ is

A. 1

B. 2

C. In 2

D. $-\frac{1}{10} In 2$

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20. If $y = \left(x + \sqrt{x^2 + a^2}\right)^n$, $then \frac{dy}{dx}$ is $\frac{ny}{\sqrt{x^2 + a^2}}$ (b) $-\frac{ny}{\sqrt{x^2 + a^2}}$ $\frac{nx}{\sqrt{x^2 + a^2}}$ (d) $-\frac{nx}{\sqrt{x^2 + a^2}}$

A. $\frac{ny}{\sqrt{x^2 + a^2}}$

B. $-\frac{ny}{\sqrt{x^2 + a^2}}$

C. $\frac{nx}{\sqrt{x^2 + a^2}}$

D. $-\frac{nx}{\sqrt{x^2 + a^2}}$

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21. if $y = \log_{\sin x}\tan x$ then $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}}$ is

A. $\frac{4}{\log 2}$

B. $-4\log 2$

C. $\frac{-4}{\log 2}$

D. none of these

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22. $\frac{d}{dx}\left[\sin^2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]$ is

A. -1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 1

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23. The differential coefficient of $f\left(\log_e x\right) w.r.t. x,$ where $f(x) = \log_e x,$ is (i) $\frac{x}{\ln x}$ (ii) $\frac{\ln x}{x}$ (iii) $\frac{1}{x\ln x}$ (iv) $x\ln x$

A. $\frac{x}{\log_e x}$

B. $\frac{1}{x}\log_e x$

C. $\frac{1}{x\log_e x}$

D. none of these

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24. if $f'(x) = \sqrt{2x^2 - 1}$ and $y = f\left(x^2\right)$ then $\frac{dy}{dx}$ at $x = 1$ is:

A. 2

B. 1

C. -2

D. none of these

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25. If $u = f\left(x^3\right), v = g\left(x^2\right), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is

$a\left.\right)\frac{3}{2}x\cos x^3 cosec x^2\ b\left.\right)\frac{2}{3}\sin x^3 \sec x^2\ c)\tan x$ (d) none of these

A. $\frac{3}{2}x \cos x^2 \operatorname{cosec} x^2$

B. $\frac{3}{2} \sin x^3 \sec x^2$

C. $\tan x$

D. none of these

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26. A function f, defined for all positive real numbers, satisfies the equation $f(x^2) = x^3$ for every $x > 0$. Then the value of $f'(4)$ is 12 (b) 3 (c) 3/2 (d) cannot be determined

A. 12

B. 3

C. 3/2

D. cannot be determined

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27. Let $f:(-5,5)\to R$ be a differentiable function of with $f(4)=1, f'(4)=1, f(0)=-1$ and $f'(0)=I f g(x)=\left(f\left(2f^2(x)+2\right)\right)^2,$ then g'(0) equals

A. 4

B. -4

C. 8

D. -8

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28. The function $f(x)=e^x+x,$ being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}\left(f^{-1}\right)$ at the point $f(\log 2)$ is $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

A. $\frac{1}{In2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. none of these

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29. If $f(x) = x + \tan x$ and f is the inverse of g, then $g'(x)$ is equal to

A. $\frac{1}{1 + [g(x) - x]^2}$

B. $\frac{1}{2 - [g(x) - x]^2}$

C. $\frac{1}{2 + [g(x) - x]^2}$

D. none of these

Answer: C

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30. If $f(x) = x^3 + 3x + 4$ and g is the inverse function of f(x), then the value of $\frac{d}{dx}\left(\frac{g(x)}{g(g(x))}\right)$ at x = 4 equals

A. $\frac{-1}{6}$

B. 6

C. $\frac{-1}{3}$

D. non-existent

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31. If $y = \frac{\sin^{-1}x}{\sqrt{1 - x^2}}$, *then* $\frac{\left(1 - x^2\right)dy}{dx}$ is equal to $x + y$ (b) $1 + xy$ $1 - xy$ (d) $xy - 2$

A. x+y

B. 1+xy

C. 1-xy

D. xy-2

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32. If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+\ldots\infty}}}$, then $\frac{dy}{dx}$ is

A. $\frac{x}{2y-1}$

B. $\frac{x}{2y+1}$

C. $\frac{1}{x(2y-1)}$

D. $\frac{1}{x(1-2y)}$

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33. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]=$ (a) $\frac{1}{2(1+x)\sqrt{x}}$ (b) $\frac{3}{(1+x)\sqrt{x}}$ $\frac{2}{(1+x)\sqrt{x}}$ (d) $\frac{3}{2(1+x)\sqrt{x}}$

A. $\frac{1}{2(1+x)\sqrt{x}}$

B. $\frac{3}{(1+x)\sqrt{x}}$

C. $\frac{2}{(1+x)\sqrt{x}}$

D. $\frac{3}{2(1+x)\sqrt{x}}$

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34. Suppose the function f(x)-f(2x) has the derivative 5 at x=1 and derivative 7 at x= 2. The derivative of the function f(x)-f(4x) at x=1 has the value equal to

A. 19

B. 9

C. 17

D. 14

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35. If $y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$, $show tx^{2} \hat{} + 1\frac{dy}{dx}+xy+1=0$

A. 0

B. 1

C. 2

D. none of these

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36.

Let $e^y=\frac{\sqrt{1+\alpha}+\sqrt{1-\alpha}}{\sqrt{1+\alpha}-\sqrt{1-\alpha}}$ and $\tan\frac{x}{2}=\sqrt{\frac{1-\alpha}{1+\alpha}}$, $\alpha\in[-1,0]\cup(0,1]$. Then $\left(\frac{dy}{dx}\right.$

A. 1/2

B. 1

C. 2

D. $1/3$

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37. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$ is $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

A. $1/8$

B. $1/4$

C. $1/2$

D. 1

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38. If $\ln\left((e-1)e^{xy}+x^2\right)=x^2+y^2$ then $\left(\frac{dy}{dx}\right)_{1,0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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39. If $y=x^{\left(x^x\right)}$ then $\frac{dy}{dx}$ is

A. $y\left[x^x(\log ex)\log x+x^x\right]$

B. $y\left[x^x(\log ex)\log x+x\right]$

C. $y\left[x^x(\log ex)\log x+x^{-1}\right]$

D. $y\left[x^x\left(\log_e x\right)\log x+x^{-1}\right]$

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40. The first derivative of the function $\left[\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right)+x^{x}\right]$ with respect to x at x = 1 is

A. $3/4$

B. 0

C. $1/2$

D. $-1/2$

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41. Let $f(x) = x^3, x \in (0, \infty)$ and let g(x) be inverse of f(x), then g'(x) must be

A. $x(1 + \log x)$

B. $x(1 + \log(x))$

C. $\dfrac{1}{x(1 + \log g(x))}$

D. non-existent

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42. If $y = ax^{n+1} + bx^{-n}$, then $x^2 \dfrac{d^2y}{dx^2}$ is equal to

A. n(n-1) y

B. n(n+1)y

C. ny

D. n^2y

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43. If $y = ax^{n+1} + bx^{-n}$, then $x^2\frac{d^2y}{dx^2}$ is equal to

A. $m^2\left(ae^{mx} - be^{-mx}\right)$

B. 1

C. 0

D. none of these

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44. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the value of ab is equal to:

A. 25

B. 9

C. -15

D. -9

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45. $\frac{d^{20}y}{dx^{20}}(2\cos x\cos 3x) is equal \rightarrow$ $2^{20}\left(\cos 2x - 2^{20} os 3x\right)$ $2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$ $2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$ $2^{20}\left(\sin 2x - 2^{20}\sin 4x\right)$

A. $2^{20}\left(\cos 2x - 2^{20}\cos 3x\right)$

B. $2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$

C. $2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$

D. $2^{20}\left(\sin 2x - 2^{20}\sin 4x\right)$

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46. $\frac{d^n}{dx^n}(\log x) = ?$ (a) $\frac{(n-1)!}{x^n}$ (b) $\frac{n!}{x^n}$ (c) $\frac{(n-2)!}{x^n}$ (d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

A. $\frac{(n-1)!}{x^n}$

B. $\frac{n!}{x^n}$

C. $\frac{(n-2)!}{x^n}$

D. $(-1)^{n-1}\frac{(n-1)!}{x^n}$

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47. The nth derivative of the function $f(x) = \frac{1}{1-x^2}[where x \in (-1, 1)]$ at the point $x = 0$ where n is even is (b) $n!$ (c) n^nC_2 (d) 2^nC_2

A. 0

B. $n!$

C. b^nC_2

D. 2^nC_2

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48. If $y = x\log\{\frac{x}{(a + bx)}]$, then show that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^{2\cdot}$

A. $x\frac{dy}{dx} - y$

B. $\left(x\frac{dy}{dx} - y\right)^2$

C. $y\frac{dy}{dx} - x$

D. $\left(y\frac{dy}{dx} - x\right)^2$

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49. If $ax^2 + 2hxy + by^2 = 1, then\frac{d^2y}{dx^2}$ is $\frac{h^2 - ab}{(hx + by)^2}$ (b) $\frac{ab - h^2}{(hx + by)^2}$ $\frac{h^2 + ab}{(hx + by)^2}$

(d) none of these

A. $\frac{h^2 - ab}{(hx + by)^3}$

B. $\frac{ab - h^2}{(hx + by)^2}$

C. $\frac{h^2 + ab}{(hx + by)^2}$

D. none of these

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50. If $y^{1/m} = \left(x + \sqrt{1 + x^2}\right)$, then $\left(1 + x^2\right)y_2 + xy_1$ is (where y_r represents the rth derivative of y w.r.t. x)

A. m^2y

B. my^2

C. m^2y^2

D. none of these

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51. If $(\sin x)(\cos y) = \frac{1}{2}, then \frac{d^2y}{dx^2} at \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is -4 (b) -2 (c) -6 (d) 0

A. -4

B. -2

C. -6

D. 0

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52. A function f satisfies the condition $f(x) = f^{'}(x) + f''(x) + f'''(x) .... + \in f \in ity, where f(x)$ is a differentiable function indefinitely and dash denotes the order of derivative. If $f(0) = 1, then f(x)$ is $e^{\frac{x}{2}}$ (b) e^x (c) e^{2x} (d) e^{4x}

A. $e^{x/2}$

B. e^x

C. e^{2x}

D. e^{4x}

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53. Let $f(x)$ be a polynomial of degree 3 such that $f(3) = 1, f'(3) = -1, f^3 = 0, and f^3 = 12.$ Then the value of $f'(1)$ is 12 (b) 23 (c) -13 (d) none of these

A. 12

B. 23

C. -13

D. none of these

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54. If $y^2 = ax^2 + bx + c$, then $y^3 \frac{d^2y}{dx^2}$ is (a) a constant (b) a function of x only (c) a function of y only (d) a function of x and y

A. a constant

B. a function of x only

C. a function of y only

D. a function of x and y

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55. If $y = x + e^x$, $then \frac{d^2x}{dy^2} is\ e^x$ (b) - $\frac{e^x}{\left(1 + e^x\right)^3}$ - $\frac{e^x}{\left(1 + e^x\right)^3}$ (d) $\frac{-1}{\left(1 + e^x\right)^3}$

A. $\left(-\sin x + e^x\right)^{-1}$

B. $\frac{\sin x - e^x}{\left(\cos x + e^x\right)^2}$

C. $\frac{\sin x - e^x}{\left(\cos x + e^x\right)^3}$

D. $\frac{\sin x + e^x}{\left(\cos x + e^x\right)^3}$

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56. if y= sin mx, them the value of the determinant

$$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} \text{ Where } y_n = \frac{d^n y}{dx^n} \text{ is}$$

A. 1

B. 0

C. -1

D. none of these

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57. If $f^x = -f(x) and g(x) = f'(x) and \; F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5,$ then $F(10)$ is 5 (b) 10 (c) 0 (d) 15

A. 5

B. 10

C. 0

D. 15

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58. Let $y = 1n(1 + \cos x)^2$. Then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{\frac{y}{2}}}$ equal (a) 0 (b) $\frac{2}{1 + \cos x}$ (c) $\frac{4}{1 + \cos x}$ (d) $\frac{-4}{(1 + \cos x)^2}$

A. 0

B. $\frac{2}{1 + \cos x}$

C. $\frac{4}{1 + \cos x}$

D. $\frac{-4}{(1 + \cos x)^2}$

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59. $x = t\cos t, y = t + \sin t$. $Then$(d^(2)x)/(dy^(2))" at "t=(pi)/(2)` is

A. $\frac{\pi + 4}{2}$

B. $-\frac{\pi + 4}{2}$

C. -2

D. none of these

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60. If $f(x) = (x - 1)^4(x - 2)^3(x - 3)^2(x - 4)$, then the value of $f'''(1) + f''(2) + f'(3) + f'(4)$ equals

A. 0

B. 50

C. 324

D. 648

Answer: 648

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61. let $y = t^{10} + 1$, and $x = t^8 + 1$, then $\frac{d^2y}{dx^2}$ is

A. $\frac{5}{2}t$

B. $20t^8$

C. $\frac{5}{16t^6}$

D. none of these

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62. If x = log p and $y = \frac{1}{p}$, then

A. $\frac{d^2y}{dx^2} - 2p = 0$

B. $\frac{d^2y}{dx^2} + y = 0$

C. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

D. $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

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63. If $x = \phi(t), y = \psi(t),$ then $\frac{d^2y}{dx^2}$ is

A. $\frac{\phi' \psi'' - \psi' \phi''}{(\phi')^2}$

B. $\frac{\phi' \psi'' - \psi' \phi''}{(\phi')^3}$

C. $\frac{\phi''}{\psi''}$

D. $\frac{\psi''}{\phi''}$

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64. If f(x) = $x^4\tan\left(x^3\right)$ - x In $\left(1 + x^2\right)$, then the value of $\frac{d^4(f(x))}{dx^4}$ at $x = 0$ is

A. 0

B. 6

C. 12

D. 24

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65. If graph of $y = f(x)$ is symmetrical about the y-axis and that of $y = g(x)$ is symmetrical about the origin and if $h(x) = f(x)\dot{g}(x), then \frac{d^3h(x)}{dx^3} at x = 0$ is cannot be determined (b) $f(0)\dot{g}(0)$ 0 (d) none of these

A. cannot be determined

B. $f(0) \cdot g(0)$

C. 0

D. none of these

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66. Let $g(x)$ be the inverse of an invertible function $f(x)$, which is differentiable for all real x. Then $g''(f(x))$ equals.

A. $-\dfrac{f''(x)}{(f'(x))^3}$

B. $\dfrac{f'(x)f''(x)-(f(x))^3}{f'(x)}$

C. $\dfrac{f'(x)f''(x)-(f'(x))^2}{(f'(x))^2}$

D. none of these

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67. $f(x) = e^{x} - e^{-x} - 2\sin x - \frac{2}{3}x^{3}$. Then the least value of n for which $\frac{d^{n}}{dx^{n}}f(x)\,|_{x=0}$ is nonzero is

A. 5

B. 6

C. 7

D. 8

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68. Let $y = f(x)$ and $x = \frac{1}{z}$. If $\frac{d^{2}y}{dx^{2}} = \lambda\left(z^{3}\right)\frac{dy}{dz} + z^{4}\frac{d^{2}y}{dz^{2}}$, then the value of λ is

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

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69. Let x=f(t) and y=g(t), where x and y are twice differentiable function. If

f'(0)= g'(0) =f''(0) = 2. g''(0) = 6, then the value of $\left(\frac{d^2y}{dx^2}\right)_{t=0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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70. If $f(x)$ satisfies the relation $f\left(\frac{5x-3y}{2}\right)=\frac{5f(x)-3f(y)}{2}\ \forall x, y \in R,$ and $f(0)=3andf'(0)=2,$ then the period of $\sin(f(x))$ is 2π (b) π (c) 3π (d) 4π

A. 2π

B. π

C. 3π

D. 4π

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71. A function $f: R\vec{R}$ satisfies $\sin x \cos y\left(f(2x+2y)-f(2x-2y)=\cos x \sin y(f(2x+2y)+f(2x-2y))\right.$ If $f'(0)=\frac{1}{2},$ then $a)f''(x)=f(x)=0 \ \ b)4f''(x)+f(x)=0 \ \ c)f''(x)+f(x)=0$ $d)4f''(x)-f(x)=0$

A. $f''(x)=f(x)=0$

B. $4f'(x) + f(x) = 0$

C. $f'(x) + f(x) = 0$

D. $4f'(x) - f(x) = 0$

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72. If $y = a\sin x + b\cos x, then y^2 + \left(\frac{dy}{dx}\right)^2$ is a function of x (b) function of y function of $x and y$ (d) constant

A. function of x

B. function of y

C. function of x and y

D. constant

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73. `d/(dx)sqrt((1-sin2x)/(1+sin2x))i se q u a lto,(0

A. $\sec^2 x$

B. $-\sec^2\left(\frac{\pi}{4} - x\right)$

C. $\sec^2\left(\frac{\pi}{4} + x\right)$

D. $\sec^2\left(\frac{\pi}{4} - x\right)$

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74. If $f(x) = |\cos x| + |\sin x|$, then $\frac{dy}{dx}$ at $x = \frac{2\pi}{3}$ is equal to

A. $\frac{1 - \sqrt{3}}{2}$

B. 0

C. $\frac{1}{2}\left(\sqrt{3} - 1\right)$

D. none of these

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75. If f(x) $= \left|\log_e |x|\right|$, then f'(x) equals

A. $\frac{1}{|x|}$, where $x \neq 0$

B. $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

C. $-\frac{1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

D. $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

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76. If $f(x) = \sqrt{1 - \sin 2x}$, then $f'(x)$ is equal to

(a)$-(\cos x + \sin x)$,for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(b)$\cos x + \sin x$,for $x \in \left(0, \frac{\pi}{4}\right)$

(c)-$(\cos x + \sin x)$,for $x \in \left(0, \frac{\pi}{4}\right)$

(d)$\cos x - \sin x$,for $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

A. -$(\cos x + \sin x)$, for x $\in (\pi/4, \pi/2)$

B. cosx + sin x for x $\in (0, \pi/4)$

C. -$(\cos x + \sin x)$, for $x \in (0, \pi/4)$

D. $\cos x - \sin x$, for $x \in (\pi/4, \pi/2)$

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77. Instead of the usual definition of derivative $Df(x)$, if we define a new kind of derivative $D^{\cdot}F(x)$ by the formula $D \cdot (x) = (\lim)_{h\vec{0}} \frac{f^2(x+h) - f^2(x)}{h}, where f^2(x)$ mean $[f(x)]^2 and \Leftrightarrow (x) = x\log x, then D^{\cdot}f(x)(\mid)_{x=e}$ has the value e (B) 2e (c) 4e (d) none of these

A. e

B. 2e

C. 4e

D. none of these

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78. if $y=\frac{(a-x)\sqrt{a-x}-(b-x)\sqrt{x-b}}{\sqrt{a-x}+\sqrt{x-b}}$ then $\frac{dy}{dx}$ wherever it is defined is equal to :

A. $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$

B. $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$

C. $-\frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$

D. $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

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79. The derivative of y = (1-x) (2-x)...(n-x) at x= 1 is

A. 0

B. $(-1)(n-1)!$

C. $n!-1$

D. $(-1)^{n-1}(n-1)!$

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80. If $y=\sqrt{\frac{1-x}{1+x}}$, then $\left(1-x^2\right)\frac{dy}{dx}$ is equal to

A. y^2

B. $1/y$

C. $-y$

D. $-y/x$

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81. If $y = \frac{\sqrt{a + x} - \sqrt{a - x}}{\sqrt{a + x} + \sqrt{a - x}}, then \frac{dy}{dx} isequa < o$ $\frac{ay}{x\sqrt{a^2 - x^2}}$ (b) $\frac{ay}{\sqrt{a^2 - x^2}}$ $\frac{ay}{x\sqrt{a^2 - x^2}}$ (d) none of these

A. $\frac{ay}{x\sqrt{a^2 - x^2}}$

B. $\frac{ay}{\sqrt{a^2 - x^2}}$

C. $\frac{ay}{x\sqrt{x^2 - a^2}}$

D. none of these

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82. Let $u(x) and v(x)$ be differentiable functions such that $\frac{u(x)}{v(x)} = 7. \frac{u'(x)}{v'(x)} = p and \left(\frac{u(x)}{v(x)}\right)' = q, then \frac{p+q}{p-q}$ has the value of to 1 (b) 0 (c) 7 (d) -7

A. 1

B. 0

C. 7

D. -7

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83. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a, then \frac{dy}{dx}$ is equal to $\frac{x}{y}$ (b) $\frac{y}{x^2}$ $\frac{x^2-y^2}{x^2+y^2}$ (d) $\frac{y}{x}$

A. $\frac{x}{y}$

B. $\frac{y}{x^2}$

C. $\frac{x^2 - y^2}{x^2 + y^2}$

D. $\frac{y}{x}$

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84. Let $h(x)$ be differentiable for all x and let $f(x) = \left(kx + e^x\right)h(x)$, where k is some constant. If $h(0) = 5, h'(0) = -2, and f'(0) = 18,$ then the value of k is

A. 5

B. 4

C. 3

D. 2.2

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85. If $\lim_{t \to x} \dfrac{e^t f(x) - e^x f(t)}{(t - x)(f(x))^2} = 2$ and $f(0) = \dfrac{1}{2}$, then find the value of $f'(0)$.

A. 4

B. 2

C. 0

D. 1

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86. If $f(0) = 0, f'(0) = 2,$ then the derivative of $y = f(f(f(x)))$ at $x = 0$ is 2 (b) 8 (c) 16 (d) 4

A. 2

B. 8

C. 16

D. 4

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87. If $f(x) = \sqrt{1 + \cos^2(x^2)}, then f'\left(\frac{\sqrt{\pi}}{2}\right)$ is $\frac{\sqrt{\pi}}{6}$ (b) -$\sqrt{\pi/6}$ $1/\sqrt{6}$ (d) $\pi/\sqrt{6}$

A. $\sqrt{\pi/6}$

B. -$\sqrt{\pi/6}$

C. $1/\sqrt{6}$

D. $\pi/\sqrt{6}$

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88. $\frac{d}{dx}\cos^{-1}\sqrt{\cos x}, 0 < x < \frac{\pi}{2}$ is equal to

A. $\frac{1}{2}\sqrt{1 + \sec x}$

B. $\sqrt{1 + \sec x}$

C. -$\frac{1}{2}\sqrt{1 + \sec x}$

D. $-\sqrt{1+\sec x}$

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89. If $\tan y = \frac{2^x}{1+2^{2x+1}}, then \frac{dy}{dx} at x = 0 is$

A. 1

B. 2

C. In 2

D. $-\frac{1}{10} In 2$

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90. If $y = \left(x+\sqrt{x^2+a^2}\right)^n, then \frac{dy}{dx}$ is $\frac{ny}{\sqrt{x^2+a^2}}$ (b) $-\frac{ny}{\sqrt{x^2+a^2}}$ $\frac{nx}{\sqrt{x^2+a^2}}$ (d) $-\frac{nx}{\sqrt{x^2+a^2}}$

A. $\frac{ny}{\sqrt{x^2 + a^2}}$

B. $-\frac{ny}{\sqrt{x^2 + a^2}}$

C. $\frac{nx}{\sqrt{x^2 + a^2}}$

D. $-\frac{nx}{\sqrt{x^2 + a^2}}$

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91. if $y = \log_{\sin x}\tan x$ then $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}}$ is

A. $\frac{4}{\log 2}$

B. $-4\log 2$

C. $\frac{-4}{\log 2}$

D. none of these

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92. $\frac{d}{dx}\left[\sin^2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]$ is

A. -1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 1

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93. The differential coefficient of $f\left(\log_e x\right) w.r.t. x,$ where $f(x) = \log_e x,$ is

A. $\frac{x}{\log_e x}$

B. $\frac{1}{x}\log_e x$

C. $\frac{1}{x\log_e x}$

D. none of these

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94. if $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$ then $\frac{dy}{dx}$ at $x = 1$ is:

A. 2

B. 1

C. -2

D. none of these

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95. If $u = f(x^3), v = g(x^2), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is

A. $\frac{3}{2}x \cos x^3 \operatorname{cosec} x^2$

B. $\frac{3}{2} \sin x^3 \sec x^2$

C. $\tan x$

D. none of these

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96. A function f, defined for all positive real numbers, satisfies the equation $f\left(x^2\right) = x^3 = x^3$ for every $> 0.$ Then the value of f'(4) is

A. 12

B. 3

C. $3/2$

D. cannot be determined

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97. Let $f:(-5,5) \to R$ be a differentiable function of with $f(4)=1, f'(4)=1, f(0)=-1$ and $f'(0)=Ifg(x)=\left(f\left(2f^2(x)+2\right)\right)^2,$ then g'(0) equals

A. 4

B. -4

C. 8

D. -8

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98. The function $f(x)=e^x+x,$ being differentiable and one- to -one, has a differentiable inverse $f^{-1}(x).$ The value of $\frac{x}{dx}\left(f^{-1}atf(\log 2)\right.$ is

A. $\frac{1}{In2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. none of these

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99. If $f(x) = x + \tan x$ and f is the inverse of g, then $g'(x)$ is equal to

A. $\frac{1}{1 + [g(x) - x]^2}$

B. $\frac{1}{2 - [g(x) - x]^2}$

C. $\frac{1}{2 + [g(x) - x]^2}$

D. none of these

Answer: C

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100. If $f(x) = x^3 + 3x + 4$ and g is the inverse function of f(x), then the value of $\frac{d}{dx}\left(\frac{g(x)}{g(g(x))}\right)$ at x = 4 equals

A. $\frac{-1}{6}$

B. 6

C. $\frac{-1}{3}$

D. non-existent

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101. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, $then \frac{\left(1-x^2\right)dy}{dx}$ is equal to $x + y$ (b) $1 + xy$ $1 - xy$ (d) $xy - 2$

A. x+y

B. 1+xy

C. 1-xy

D. xy-2

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102. If $y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+\ldots\infty}}}$, then $\frac{dy}{dx}$ is

A. $\frac{x}{2y-1}$

B. $\frac{x}{2y+1}$

C. $\frac{1}{x(2y-1)}$

D. $\frac{1}{x(1-2y)}$

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103. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]=$

A. $\frac{1}{2(1+x)\sqrt{x}}$

B. $\frac{3}{(1+x)\sqrt{x}}$

C. $\frac{2}{(1+x)\sqrt{x}}$

D. $\frac{3}{2(1+x)\sqrt{x}}$

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104. Suppose the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative $7atx = 2.$ The derivative of the function $f(x) - f(4x)atx = 1$ has the value equal to 19 (b) 9 (c) 17 (d) 14

A. 19

B. 9

C. 17

D. 14

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105. If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right), showthat\left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$

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106.

Let $e^y = \frac{\sqrt{1+\alpha}+\sqrt{1-\alpha}}{\sqrt{1+\alpha}-\sqrt{1-\alpha}}$ and $\tan\frac{x}{2} = \sqrt{\frac{1-\alpha}{1+\alpha}}, \alpha \in [-1, 0] \cup (0, 1]$. Then $\left(\frac{dy}{dx}\right.$

A. $1/2$

B. 1

C. 2

D. $1/3$

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107. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$ is $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

A. $1/8$

B. $1/4$

C. $1/2$

D. 1

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108. If $\ln\left((e-1)e^{xy}+x^2\right)=x^2+y^2$ then $\left(\frac{dy}{dx}\right)_{1,0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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109. If $y = \left(x^{x}\right)^{x}$ then $\frac{dy}{dx}$ is

A. $y\left[x^{x}(\log ex)\log x + x^{x}\right]$

B. $y\left[x^{x}(\log ex)\log x + x\right]$

C. $y\left[x^{x}(\log ex)\log x + x^{-1}\right]$

D. $y\left[x^{x}\left(\log_{e}x\right)\log x + x^{-1}\right]$

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110. The first derivative of the function $\left[\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right)+x^{x}\right]$ with respect to x at $x=1$ is

A. $3/4$

B. 0

C. $1/2$

D. $-1/2$

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111. $f(x)=x^{x}, x \in (0,\infty)$ and let $g(x)$ be inverse of f(x) , then $g(x)'$ must be

A. $x(1+\log x)$

B. $x(1+\log(x))$

C. $\frac{1}{x(1+\log g(x))}$

D. non-existent

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112. If $y = ax^{n+1} + bx^{-n}, then x^2 \frac{d^2y}{dx^2}$ is equal to $n(n-1)y$ (b) $n(n+1)y$ ny (d) n^2y

A. n(n-1) y

B. n(n+1)y

C. ny

D. n^2y

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113. If $y = ax^{n+1} + bx^{-n}, then x^2 \frac{d^2y}{dx^2}$ is equal to $n(n-1)y$ (b) $n(n+1)y$ ny (d) n^2y

A. $m^2\left(ae^{mx} - be^{-mx}\right)$

B. 1

C. 0

D. none of these

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114. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that f''(x) -2f'(x)-15f(x)=0 for all x. Then the product ab is

A. 25

B. 9

C. -15

D. -9

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115. $\frac{d^{20}y}{dx^{20}}(2\cos x\cos 3x) is equal \rightarrow$ $a)2^{20}\left(\cos 2x - 2^{20}os3x\right)$ $b)2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$ $c)2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$ $d)2^{20}\left(\sin 2x - 2^{20}\sin 4x\right)$

A. $2^{20}\left(\cos 2x - 2^{20}\cos 3x\right)$

B. $2^{20}\left(\cos 2x + 2^{20}\cos 4x\right)$

C. $2^{20}\left(\sin 2x + 2^{20}\sin 4x\right)$

D. $2^{20}\left(\sin 2x - 2^{20}\sin 4x\right)$

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116. $\frac{d^n}{dx^n}(\log x) = \frac{(n-1)!}{x^n}$ (b) $\frac{n!}{x^n}$ $\frac{(n-2)!}{x^n}$ (d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

A. $\frac{(n-1)!}{x^n}$

B. $\frac{n!}{x^n}$

C. $\frac{(n-2)!}{x^n}$

D. $(-1)^{n-1}\frac{(n-1)!}{x^n}$

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117. If $y = x\log\left(\frac{x}{a+bx}\right)$, then $x^3\frac{d^2y}{dx^2} =$

A. $x\frac{dy}{dx} - y$

B. $\left(x\frac{dy}{dx} - y\right)^2$

C. $y\frac{dy}{dx} - x$

D. $\left(y\frac{dy}{dx} - x\right)^2$

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118. If $ax^2 + 2hxy + by^2 = 1$, then $\frac{d^2y}{dx^2}$ is $\frac{h^2 - ab}{(hx+by)^2}$ (b) $\frac{ab - h^2}{(hx+by)^2}$ $\frac{h^2 + ab}{(hx+by)^2}$

(d) none of these

A. $\dfrac{h^2 - ab}{(hx + by)^3}$

B. $\dfrac{ab - h^2}{(hx + by)^2}$

C. $\dfrac{h^2 + ab}{(hx + by)^2}$

D. none of these

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119. If $y^{1/m} = \left(x + \sqrt{1 + x^2}\right)$, then $\left(1 + x^2\right)y_2 + xy_1$ is (where y_r represents the rth derivative of y w.r.t. x)

A. m^2y

B. my^2

C. m^2y^2

D. none of these

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120. If (sin x) (cos y) = $1/2$, then d^2y/dx^2 at $(\pi/4, \pi/4)$ is

A. -4

B. -2

C. -6

D. 0

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121. A function f satisfies the condition $f(x) = f'(x) + f''(x) + f'''(x) + \ldots,$ where f(x) is a differentiable function indefinitely and dash denotes the order the derivative. If f(0) = 1, then f(x) is

A. $e^{x/2}$

B. e^x

C. e^{2x}

D. e^{4x}

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122. Let f(x) be a polynomial of degree 3 such that $f(3) = 1, f'(3) = -1, f''(3) = 0$, and $f'''(3) = 12$. Then the value of f'(1) is

A. 12

B. 23

C. -13

D. none of these

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123. If $y^2 = ax^2 + bx + c$, then $y^3 \frac{d^2y}{dx^2}$ is (a) a constant (b) a function of x only (c) a function of y only (d) a function of x and y

A. a constant

B. a function of x only

C. a function of y only

D. a function of x and y

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124. If $y = \sin x + e^x$, then $\frac{d^2x}{dy^2} =$

A. $\left(-\sin x + e^x\right)^{-1}$

B. $\frac{\sin x - e^x}{\left(\cos x + e^x\right)^2}$

C. $\frac{\sin x - e^x}{\left(\cos x + e^x\right)^3}$

D. $\frac{\sin x + e^x}{\left(\cos x + e^x\right)^3}$

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125. If f''(x) =- f(x) and g(x) = f'(x) and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that F(5) =5, then F(10) is

A. 5

B. 10

C. 0

D. 15

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126. Let $y = \ln(1 + \cos x)^2$. The the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{y/2}}$ equals

A. 0

B. $\frac{2}{1 + \cos x}$

C. $\frac{4}{1 + \cos x}$

D. $\frac{-4}{(1 + \cos x)^2}$

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127. $x = t\cos t, y = t + \sin t$ Then $\frac{d^{2x}}{dy^2}a = \frac{\pi}{2}$ is $\frac{\pi + 4}{2}$ (b) - $\frac{\pi + 4}{2}$ -2 (d) none of these

A. $\frac{\pi + 4}{2}$

B. - $\frac{\pi + 4}{2}$

C. -2

D. none of these

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128. Let $y = t^{10} + 1$ and $x = t^8 + 1$. Then $\frac{d^2y}{dx^2}$ is

A. $\frac{5}{2}t$

B. $20t^8$

C. $\frac{5}{16t^6}$

D. none of these

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129. If x = log p and $y = \frac{1}{p}$, then

A. $\frac{d^2y}{dx^2}$ - $2p = 0$

B. $\frac{d^2y}{dx^2} + y = 0$

C. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

D. $\frac{d^2y}{dx^2}$ - $\frac{dy}{dx} = 0$

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130. If $x = \varphi(t), y = \psi(t), then \frac{d^2y}{dx^2}$ is $\frac{\varphi' \psi - \psi' \varphi''}{(\varphi')^2}$ (b) $\frac{\varphi' \psi - \psi' \varphi''}{(\varphi')^3}$ φ'/ψ (d) ψ'/φ

A. $\frac{\phi' \psi'' - \psi' \phi''}{(\phi')^2}$

B. $\frac{\phi' \psi'' - \psi' \phi''}{(\phi')^3}$

C. $\frac{\phi''}{\psi''}$

D. $\frac{\psi''}{\phi''}$

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131. If $f(x) = x^4\tan(x^3) - x1n(1 + x^2)$, then the value of $\frac{d^4(f(x))}{dx^4}$ at $x = 0$ is 0 (b) 6 (c) 12 (d) 24

A. 0

B. 6

C. 12

D. 24

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132. Let g(x) be the inverse of an invertible function f(x), which is differentiable for all real x. Then g''(f(x)) equals

A. $-\frac{f''(x)}{(f'(x))^3}$

B. $\frac{f'(x)f''(x)-(f(x))^3}{f'(x)}$

C. $\frac{f'(x)f''(x)-(f'(x))^2}{(f'(x))^2}$

D. none of these

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133. $f(x) = e^x - e^{-x} - 2\sin x - \frac{2}{3}x^3$. Then the least value of n for which $\frac{d^n}{dx^n} f(x) \mid_{x=0}$ is nonzero is

A. 5

B. 6

C. 7

D. 8

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134. Let $y = f(x)$ and $x = \frac{1}{z}$. If $\frac{d^2y}{dx^2} = \lambda_{z^3} \frac{dy}{dz} + z^4 \frac{d^2y}{dz^2}$, then the value of λ is

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

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135. Let x=f(t) and y=g(t), where x and y are twice differentiable function. If

f'(0)= g'(0) =f''(0) = 2. g''(0) = 6, then the value of $\left(\frac{d^2y}{dx^2}\right)_{t=0}$ is equal to

A. 0

B. 1

C. 2

D. 3

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136. If $f(x)$ satisfies the relation $f\left(\frac{5x - 3y}{2}\right) = \frac{5f(x) - 3f(y)}{2} \forall x, y \in R,$ and

$f(0) = 3 and f'(0) = 2,$ then the period of $\sin(f(x))$ is 2π (b) π (c) 3π (d) 4π

A. 2π

B. π

C. 3π

D. 4π

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137. A function

$f: R \rightarrow R$ satisfies sin x cos y $(f(2x + 2y) - f(2x - 2y)) = \cos x \sin y (f(2x + 2y) + f(2x$

" If "f'(0)=(1)/(2),` then

A. $f'(x) = f(x) = 0$

B. $4f'(x) + f(x) = 0$

C. $f'(x) + f(x) = 0$

D. $4f'(x) - f(x) = 0$

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Multiple Correct Answers Type

1. If $y=e^{\sqrt{x}}+e^{-\sqrt{x}}$, then $\frac{dy}{dx}$ is equal to (a) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$ (b) $\frac{e^{\sqrt{x}}-e^{-\sqrt{x}}}{2\sqrt{x}}$ $\frac{1}{2\sqrt{x}}\sqrt{y^2-4}$

(d) $\frac{1}{2\sqrt{x}}\sqrt{y^2+4}$

A. $\frac{e^{\sqrt{x}}-e^{-\sqrt{x}}}{2\sqrt{x}}$

B. $\frac{e^{\sqrt{x}}-e^{-\sqrt{x}}}{2x}$

C. $\frac{1}{2\sqrt{x}}\sqrt{y^2-4}$

D. $\frac{1}{2\sqrt{x}}\sqrt{y^2+4}$

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2. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \infty}}}$, $\frac{dy}{dx}$ is equal to $\frac{1}{2y - 1}$ (b) $\frac{x}{x + 2y}$ $\frac{1}{\sqrt{1 + 4x}}$ (d) $\frac{y}{2x + y}$

A. $\frac{1}{2y - 1}$

B. $\frac{x}{x + 2y}$

C. $\frac{1}{\sqrt{1 + 4x}}$

D. $\frac{y}{2x + y}$

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3. If $f(\theta) = \tan\left(\sin^{-1}\sqrt{\frac{2}{3 + \cos 2\theta}}\right)$, then $find f'(\theta)$ and

A. $f\left(\frac{\pi}{4}\right) = 1$

B. $f\left(\frac{\pi}{4}\right) = \sqrt{2}$

C. $\frac{d(f(\theta))}{d(\cos\theta)}$ at $\theta = \frac{\pi}{4} is - 2$

D. $f'\left(\frac{\pi}{4}\right) = \sqrt{2}$

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4. $f(x) = \left|x^2 - 3x\left|x\right| + 2\right|$. Then which of the following is/are true ?

A. $f'(x) = 2x - 3 \text{ for } x \in (0, 1) \cup (2, \infty)$

B. $f'(x) = 2x + 3f \text{ or } x \in (-\infty, -2) \cup (-1, 0)$

C. $f'(x) = -2x - 3 \text{ for } x \in (-2, -1)$

D. None of these

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5. Let $f(x) = \frac{\sqrt{x - 2\sqrt{x - 1}}}{\sqrt{x - 1} - 1}. x$ then

A. $f(10) = 1$

B. $f'(3/2) = -1$

C. domain of $f(x) is x \geq 1$

D. range of $f(x) is (-2, -1)] \cup (2, 00)$

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6. If $x^3 - 2x^2y^2 + 5x + y - 5 = 0 and y(1) = 1, then \quad y'(1) = \frac{4}{3}$ (b) $y^1 = -\frac{4}{3}$ $y^1 = -8\frac{22}{27}$ (d) $y'(1) = \frac{2}{3}$

A. $y'(1) = 4/3$

B. $y''(1) = -4/3$

C. $y''(1) = -8\frac{22}{27}$

D. $y'(1) = 2/3$

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7. If $y = x^{(\log x)^{\log(\log x)}}$ then $\frac{dy}{dx} =$

A. $\frac{y}{x}\left(\left(Inx^{x-1}\right) + 2 \text{ In x In(In x)}\right)$

B. $\frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x) + 1)$

C. $\frac{y}{\text{x In x}}\left[(Inx)^2 + 2In(Inx)\right]$

D. $\frac{y}{x}\frac{\log y}{\log x}[2\log(\log x) + 1]$

Answer: B

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8. If $f(x - y), f(x)f(y),$ and $f(x + y)$ are in A.P. for all x, y, and $f(0) \neq 0,$ then

A. $f(4) = f(-4)$

B. $f(2) + f(-2) = 0$

C. $f'(4) + f'(-4) = 0$

D. $f'(2) = f'(-2)$

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9. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, $then \frac{dy}{dx} is$ $\frac{-2}{1+x^2}$ for all x (b) $\frac{-2}{1+x^2}$ for all $|x| < 1$ $\frac{2}{1+x^2}$ for $|x| > 1$ (d) none of these

A. $\frac{-2}{1+x^2}$ for all x

B. $\frac{-2}{1+x^2}$ for all $|x| < 1$

C. $\frac{2}{1+x^2}$ for $|x| > 1$

D. none of these

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10. $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is

A. $f_n(x)\frac{d}{dx}\left\{f_{n-1}(x)\right\}$

B. $f_n(x)f_{n-1}(x)$

C. $f_n(x)f_{n-1}(x)...f_2(x).\ f_1(x)$

D. None of these

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11. Suppose f and g are functions having second derivatives f' and g' every where, if $f(x).\ g(x) = 1$ for all x and f'', g'' are never zero then $\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)}$ equals

A. $\frac{-2f'(x)}{f(x)}$

B. $\frac{-2g'(x)}{g(x)}$

C. $\frac{-f'(x)}{f(x)}$

D. $\frac{2f'(x)}{f(x)}$

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12. If $y = e^{-x}\cos x$ and $y_n + k_n y = 0$ where $yn = \frac{d^n y}{dx^n}$ and k_n are constant $n \in N$ then

A. $k_4 = 4$

B. $k_8 = -16$

C. $k_{12} = 20$

D. $k_{16} = -24$

Answer: B

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13. If a function is represented parametrically be the equations $x = \frac{1 + (\log)_e t}{t^2}; y = \frac{3 + 2(\log)_e t}{t}$, then which of the following statements are true? $y^{x-2xy'} = y\ yy' = 2x\left(y'\right)^2 + 1\ xy' = 2y\left(y'\right)^2 + 2\ y^{y-4xy'} = \left(y'\right)^2$

A. $y''(x - 2xy') = y$

B. $yy' = 2x(y')^2 + 1$

C. $xy' = 2y(y')^2 + 2$

D. $y''(y - 4xy') = (y')^2$

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14. If $y = \frac{x^2}{2} + \frac{1}{2}x\sqrt{x^2 + 1} + \frac{1}{2}(\log)_e\sqrt{x + \sqrt{x^2 + 1}}$, prove that $2y = xy' + (\log)_e y'$, $where y'$ denotes the derivative w.r.t x

A. $y' = x + \sqrt{x^2 + 1}$

B. $y' = \frac{1}{x + \sqrt{x^2 + 1}}$

C. $2y = xy' + \log_e y'$

D. $2y = xy' - \log_e y'$

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15. A curve parametrically given by $x = t + t^3$ and $y = t^2$, where $t \in R$. For what vlaue(s) of t is $\frac{dy}{dx} = \frac{1}{2}$?

A. $1/3$

B. 2

C. 3

D. 1

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16. If $e^{\sin\left(x^2 + y^2\right)} = \tan\frac{y^2}{4} + \sin^{-1}x$, then y'(0) can be

A. $\frac{1}{3\sqrt{\pi}}$

B. $-\frac{1}{3\sqrt{\pi}}$

C. $-\frac{1}{5\sqrt{\pi}}$

D. - $\frac{1}{3\sqrt{5\pi}}$

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17. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$ then $g'(x)$is equal to (1) $1+x^5$ (2) $5x^4$ (3) $\frac{1}{1+\{g(x)\}^5}$ (4) $1+\{g(x)\}^5$

A. a unique point in the interval $\left(n, n+\frac{1}{2}\right)$

B. a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

C. a unique point in the interval $(n, n+1)$

D. two points in the interval $(n, n+1)$

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18. If for $x\left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}g(x)$, then $g(x)$

equals: $\frac{3x}{1-9x^3}$ (2) $\frac{3}{1+9x^3}$ (3) $\frac{9}{1+9x^3}$ (4) $\frac{3x\sqrt{x}}{1-9x^3}$

A. $\lim_{x \to 0^+} f'\left(\frac{1}{x}\right) = 1$

B. $\lim_{x \to 0^+} xf\left(\frac{1}{x}\right) = 2$

C. $\lim_{x \to 0^+} x^2 f'(x) = 0$

D. $|f(x)| \le 2$ for all $x \in (0, 2)$

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19. Let $f(x) = x\sin\pi x, x > 0$. Then for all natural numbers $n, f'(x)$ vanishes at a unique point in the interval $\left(n, n + \frac{1}{2}\right)$ a unique point in the interval $\left(n + \frac{1}{2}, n + 1\right)$ a unique point in the interval $(n, n+1)$ two points in the interval $(n, n+1)$

A. a unique point in the interval $\left(n, n+\frac{1}{2}\right)$

B. a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

C. a unique point in the interval $(n, n+1)$

D. two points in the interval $(n, n+1)$

Answer: A and C

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20. Let $f:(0, \infty) \rightarrow R$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) = 1$, then

A. $\lim_{x \to 0^+} f'\left(\frac{1}{x}\right) = 1$

B. $\lim_{x \to 0^+} xf\left(\frac{1}{x}\right) = 2$

C. $\lim_{x \to 0^+} x^2 f'(x) = 0$

D. $|f(x)| \leq 2$ for all $x \in (0, 2)$

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21. Let $f : R \rightarrow R$ and $h : R \rightarrow R$ be differentiable functions such that $f(x) = x^3 + 3x + 2, g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then, h'(1) equals.

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22. If $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$

B. $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2x}$

C. $\frac{1}{2\sqrt{x}}\sqrt{y^2 - 4}$

D. $\frac{1}{2\sqrt{x}}\sqrt{y^2 + 4}$

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23. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\infty}}}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{2y - 1}$

B. $\frac{x}{x + 2y}$

C. $\frac{1}{\sqrt{1 + 4x}}$

D. $\frac{y}{2x + y}$

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24. If $f(x - y), f(x)f(y)$, and $f(x + y)$ are in A.P. for all x, y, and $f(0) \neq 0$, then

A. $f(4) = f(-4)$

B. $f(2) + f(-2) = 0$

C. $f'(4) + f'(-4) = 0$

D. $f'(2) = f'(-2)$

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25. If a function is represented parametrically by the equations $x = \frac{1 + \log_e t}{t^2}, y = \frac{3 + 2\log_e t}{t}$, then which of the following statements are true ?

A. $y''(x - 2xy') = y$

B. $yy' = 2x(y')^2 + 1$

C. $xy' = 2y(y')^2 + 2$

D. $y''(y - 4xy') = (y')^2$

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26. If $y = \frac{x^2}{2} + \frac{1}{2}x\sqrt{x^2 + 1} + \log_e\sqrt{x + \sqrt{x^2 + 1}}$, then

A. $y' = x + \sqrt{x^2 + 1}$

B. $y' = \dfrac{1}{x + \sqrt{x^2 + 1}}$

C. $2y = xy' + \log_e y'$

D. $2y = xy' - \log_e y'$

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27. A curve parametrically given by $x = t + t^3$ and $y = t^2$, where $t \in R$. For what vlaue(s) of t is $\dfrac{dy}{dx} = \dfrac{1}{2}$?

A. $1/3$

B. 2

C. 3

D. 1

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28. Let $f(x) = x\sin\pi x, x > 0$. Then for all natural numbers $n, f'(x)$ vanishes at a unique point in the interval $\left(n, n+\frac{1}{2}\right)$ a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$ a unique point in the interval $(n, n+1)$ two points in the interval $(n, n+1)$

A. a unique point in the interval $\left(n, n+\frac{1}{2}\right)$

B. a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

C. a unique point in the interval $(n, n+1)$

D. two points in the interval $(n, n+1)$

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29. Let $f:(0, \infty) \rightarrow R$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) = 1$, then f(x) is

A. $\lim_{x \to 0^+} f'\left(\frac{1}{x}\right) = 1$

B. $\lim_{x \to 0^+} x f\left(\frac{1}{x}\right) = 2$

C. $\lim_{x \to 0^+} x^2 f'(x) = 0$

D. $|f(x)| \leq 2$ for all $x \in (0, 2)$

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Linked Comprehension Type

1. f(x) is a polynomial function, $f: R \rightarrow R$, such that $f(2x) = f'(x)f''(x)$.

f(x) is

A. 4

B. 12

C. 15

D. none of these

Answer: B

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2. f(x) is a polynomial function, $f: R \to R$, such that $f(2x) = f'(x)f''(x)$. f(x) is (A) one-one and onto (B) one-one and into (C) many-one and onto (D) many-one and into

A. one-one and onto

B. one-one and into

C. many-one and onto

D. many-one and into

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3. f(x) is a polynomial function, $f: R \to R$, such that $f(2x) = f'(x)f''(x)$. Equation f(x) = x has (A) three real and positive roots (B) three real and negative roots (C) one real root (D) three real roots such that sum of roots is zero

A. three real and positive roots

B. three real and negative roots

C. one real root

D. three real roots such that sum of roots is zero

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4. $f: R \rightarrow R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R$.

The value of f(1) is

A. 2

B. 3

C. -1

D. 4

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5. $f: R \rightarrow R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R$.

f(x) is

A. one-one and onto

B. one-one and into

C. many-one and onto

D. many-one and into

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6. $f: R \rightarrow R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R$.

The value of $f'(1) + f''(2) + f'''(3)$ is

A. 0

B. -1

C. 2

D. 3

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7. Repeated roots : If equation f(x) = 0, where f(x) is a polyno- mial function, has roots $\alpha, \alpha, \beta, \ldots$ or α root is repreated root, then f(x) = 0 is equivalent to $(x - \alpha)^2(x - \beta)\ldots = 0$, from which we can conclude that $f(x) = 0$ or $2(x - \alpha)[(x - \beta)\ldots] + (x - \alpha)^2[(x - \beta)\ldots]' = 0$ or $(x - \alpha)[2\{(x - \beta)\ldots\} + (x$ has root α. Thus, if α root occurs twice in the, equation, then it is common in equations f(x) = 0 and f'(x) = 0. Similarly, if α root occurs thrice in equation, then it is common in the equations f(x)=0, f'(x)=0, and f''(x)=0.

If x-c is a factor of order m of the polynomial f(x) of degree n $(1 < m < n)$, then x=c is a root of the polynomial [where $f^r(x)$ represent rth derivative of f(x) w.r.t. x]

A. $f^{m}(x)$

B. $f^{m-1}(x)$

C. $f^{n}(x)$

D. none of these

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8. Repeated roots : If equation f(x) = 0, where f(x) is a polyno- mial function, has roots $\alpha, \alpha, \beta, \ldots$ or α root is repreated root, then f(x) = 0 is equivalent to $(x-\alpha)^2(x-\beta)\ldots = 0,$ from which we can conclude that $f(x) = 0$ or $2(x-\alpha)[(x-\beta)...] + (x-\alpha)^2[(x-\beta)...]' = 0$ or $(x-\alpha)[2\{(x-\beta)...\} + (x$ has root α. Thus, if α root occurs twice in the, equation, then it is common in equations f(x) = 0 and f'(x) = 0. Similarly, if α root occurs thrice in equation, then it is common in the equations f(x)=0, f'(x)=0, and f'''(x)=0.

If $a_1x^3 + b_1x^2 + c_1x + d_1 = 0$ and $a_2x^3 + b_2x^2 + c_2x + d_2 = 0$ have a pair of repeated roots common, then

$$\begin{vmatrix} 3a_1 & 2b_1 & c_1 \\ 3a_2 & 2b_2 & c_2 \\ a_2b_1 - a_1b_2 & c_1a_2 - c_2a_1 & d_1a_2 - d_2a_1 \end{vmatrix} =$$

A. 0

B. 1

C. -1

D. 2

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9. Repeated roots : If equation f(x) = 0, where f(x) is a polyno- mial function, has roots $\alpha, \alpha, \beta, \dots$ or α root is repreated root, then f(x) = 0 is equivalent to $(x - \alpha)^2(x - \beta)\dots = 0$, from which we can conclude that $f(x) = 0$ or $2(x - \alpha)[(x - \beta)\dots] + (x - \alpha)^2[(x - \beta)\dots]' = 0$ or $(x - \alpha)[2\{(x - \beta)\dots\} + (x -$ has root α. Thus, if α root occurs twice in the, equation, then it is common in equations f(x) = 0 and f'(x) = 0. Similarly, if α root occurs thrice in equation, then it is common in the equations f(x)=0, f'(x)=0, and f''(x)=0.

If x-c is a factor of order m of the polynomial f(x) of degree n $(1 < m < n)$, then x=c is a root of the polynomial [where $f^r(x)$ represent rth derivative of f(x) w.r.t. x]

A. If $p < q < n$, then α and β are two of the roots of the equation $f^{p-1}(x) = 0$.

B. If $q < p < n$, then α and β are two of the roots of the equation $f^{q-1}(x) = 0$.

C. If $p < q < n$, then equations f(x) = 0 and $f^p(x) = 0$ have exactly one root common

D. If $q < p < n$, then equations $f^q(x) = 0$ and $f^p(x) = 0$ have exactly two roots common.

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10. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_{n-1}$.

The value of $\left(1 - a_1\right)\left(1 - a_2\right)\ldots\left(1 - a_{n-1}\right)$ is

A. $n^2/2$

B. n

C. $(-1)^n n$

D. none of these

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11. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_n,$. The value of $\sum_{r=2}^{n} \frac{1}{2 - a_r}$, is

A. $\dfrac{2^{n-1}(n-2)+1}{2^n - 1}$

B. $\dfrac{2^n(n-2)+1}{2^n - 1}$

C. $\dfrac{2^{n-1}(n-1)-1}{2^n - 1}$

D. none of these

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12. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_{n-1}$.

The value of $\sum_{r=1}^{n-1} \frac{1}{2 - a_r}$ is

A. $\frac{n}{4}$

B. $\frac{n(n-1)}{2}$

C. $\frac{n-1}{2}$

D. none of these

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13. $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$.

The value of f(3) is

A. 1

B. 0

C. -1

D. -2

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14. $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$.

The value of g(0) is

A. 0

B. -3

C. 2

D. none of these

Answer: C

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15. $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f'(x)$.

The domain of the function $\sqrt{\frac{f(x)}{g(x)}}$ is

A. $(-\infty, 1] \cup (2, 3]$

B. $(-2, 0] \cup (1, \infty)$

C. $(-\infty, 0] \cup (2/3, 3]$

D. none of these

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16. $g(x+y) = g(x) + g(y) + 3xy(x+y)\ \forall x, y \in R$ and $g'(0) = -4$.

Number of real roots of the equation g(x) = 0 is

A. 2

B. 0

C. 1

D. 3

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17. $g(x+y) = g(x) + g(y) + 3xy(x+y)\ \forall x, y \in R$ and $g'(0) = -4$.

For which of the following values of x is $\sqrt{g(x)}$ not defined ?

A. $[-2, 0]$

B. $[2, \infty)$

C. $[-1, 1]$

D. none of these

Answer: [-1,1]

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18. $g(x+y) = g(x) + g(y) + 3xy(x+y)\ \forall x, y \in R$ and $g'(0) = -4.$

The value of g'(1) is

A. 0

B. 1

C. -1

D. none of these

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19. A curve is represented parametrically by the equations $x = f(t) = a^{In\left(b^t\right)}$ and $y = g(t) = b^{-In\left(a^t\right)} a, b > 0$ and $a \neq 1, b \neq 1$ Where $t \in$

The value of $\frac{d^2y}{dx^2}$ at the point where f(t)=g(t) is

A. $\frac{1}{f(t)^2}$

B. $-(g(t))^2$

C. $\frac{-g(t)}{f(t)}$

D. $\frac{-f(t)}{g(t)}$

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20. A curve is represented parametrically by the equations $x = f(t) = a^{In\left(b^t\right)}$ and $y = g(t) = b^{-In\left(a^t\right)} a, b > 0$ and $a \neq 1, b \neq 1$ Where $t \in$

The value of $\frac{d^2y}{dx^2}$ at the point where f(t)=g(t) is

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

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21. A curve is represented parametrically by the equations $x = f(t) = a^{In(b^t)}$ and $y = g(t) = b^{-In(a^t)}a, b > 0$ and $a \neq 1, b \neq 1$

$Where t \in R.$

The value of $\frac{f(t)}{f'(t)} \cdot \frac{f''(-t)}{f'(-t)} + \frac{f(-t)}{f'(-t)} \cdot \frac{f''(t)}{f'(t)} \forall t \in R$ is

A. -2

B. 2

C. -4

D. 4

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22. Let $f: R \to R$ be a differentiable function satisfying $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y. If $\lim_{x \to 0} \frac{f(x)}{x} = 1$, then

The value of f(9) is

A. 8

B. 10

C. 12

D. 18

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23. Let $f: R \to R$ be a differentiable function satisfying $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y. If

$\lim x \to 0 \frac{f(x)}{x} = 1$, then

The value of f(9) is

A. 240

B. 356

C. 252

D. 730

Answer: C

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24. If roots of an equation $x^n - 1 = 0 are 1, a_{1,2}, a_{n-1},$ then the value of $\left(1 - a_1\right)\left(1 - a_2\right)\left(1 - a_3\right)\left(1 - a_{n-1}\right)$ will be n b. n^2 c. n^n d. 0

A. $n^2/2$

B. n

C. $(-1)^n n$

D. none of these

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Matrix Match Type

1. Match the following lists:

List I	List II
a. If $f(x)$ is an integrable function for $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $I_1 = \int_{\pi/6}^{\pi/3} \sec^2\theta f(2\sin 2\theta)\, d\theta$, and $I_2 = \int_{\pi/6}^{\pi/3} \operatorname{cosec}^2\theta f(2\sin 2\theta) d\theta$, then $I_1/I_2 =$	**p.** 3
b. If $f(x+1) = f(3+x)\ \forall\ x$, and the value of $\int_a^{a+b} f(x)dx$ is independent of a, then the value of b can be	**q.** 1
c. The value of $2\int_1^4 \frac{\tan^{-1}[x^2]}{\tan^{-1}[x^2] + \tan^{-1}[25 + x^2 - 10x]} dx$ (where [.]denotes the greatest integer function) is	**r.** 2
d. If $I = \int_0^2 \sqrt{x + \sqrt{x + \sqrt{x + \cdots\infty}}}\ dx$ (where $x > 0$), then $[I]$ is equal to (where [.] denotes the greatest integer function)	**s.** 4

Watch Video Solution

2. Match the following lists:

List I	List II
a. If $f(x)$ is an integrable function for $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $I_1 = \int_{\pi/6}^{\pi/3} \sec^2\theta f(2\sin 2\theta)\, d\theta$, and $I_2 = \int_{\pi/6}^{\pi/3} \operatorname{cosec}^2\theta f(2\sin 2\theta) d\theta$, then $I_1/I_2 =$	**p.** 3
b. If $f(x+1) = f(3+x)\ \forall\ x$, and the value of $\int_a^{a+b} f(x)dx$ is independent of a, then the value of b can be	**q.** 1
c. The value of $2\int_1^4 \frac{\tan^{-1}[x^2]}{\tan^{-1}[x^2] + \tan^{-1}[25 + x^2 - 10x]} dx$ (where [.]denotes the greatest integer function) is	**r.** 2
d. If $I = \int_0^2 \sqrt{x + \sqrt{x + \sqrt{x + \cdots\infty}}}\, dx$ (where $x > 0$), then $[\,I\,]$ is equal to (where [.] denotes the greatest integer function)	**s.** 4

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3. Match the following lists :

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4. Match Column-I to II

Column-I	Column-II
(A) Tetragonal and Hexagonal	(P) are two crystal systems
(B) Cubic and Rhombohedral	(Q) $\alpha = \beta = \gamma$
(C) Monoclinic and Triclinic	(R) $a \neq b \neq c$
(D) Cubic and Orthorhombic	(S) $a = b = c$

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5. Match List I with List II and choose the correct answer

List I	List II
A Hypothalamus	1. Sperm lysins
B Acrosome	2. Estrogen
C Graafian follicle	3. Relaxin
D Leydig cells	4. GnRH
E Parturition	5. Testosterone

A. $\begin{matrix} a & b & c & d \\ q & p & s & r \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ s & p & q & r \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ r & q & s & p \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ q & p & r & s \end{matrix}$

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6. Match the terms given in Column I with the compound given in Column II.

Column I	Column II
A. Acid rain	1. $CHCl_2$-CHF_2
B. Photochemical smog	2. CO
C. Combination with haemoglobin	3. CO_2
D. Depletion of ozone layer	4. SO_2
	5. Unsaturated hydrocarbons

A. $\begin{matrix} a & b & c & d \\ s & r & q & p \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ q & s & r & p \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ s & r & p & q \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ q & s & p & r \end{matrix}$

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Numerical Value Type

1. $f'(x) = \varphi'(x) = f(x)$ for all $\dot{x}$ Also, $f(3) = 5 and f'(3) = 4.$ Then the value of $[f(10)]^2$– [ϕ(10)]^2is _____

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2. If y=f(x) is an odd differentiable function defined on $(-\infty, \infty) such THAT f'(3) = -2 then f'(-3)$ equal to-

Watch Video Solution

3. If $x^3 + 3x^2 - 9x + c$ is of the form $(x - \alpha)^2(x - \beta)$ then c is equal to

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4. If graph of $y = f(x)$ is symmetrical about the point $(5, 0)$ and $f'(7) = 3$, then the value of $f'(3)$ is________

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5. Let $g(x) = f(x)\sin x, where f(x)$ is a twice differentiable function on $(-\infty, \infty)$ such that $f'(-\pi) = 1$. The value of $g''(-\pi)$ equals ________

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6. Let $f(x) = (x - 1)(x - 2)(x - 3)(x - n), n \in N, and f(n) = 5040$. Then the value of n is_______

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7. $y = f(x)$, where f satisfies the relation $f(x+y) = 2f(x) + xy(y) + y\sqrt{f(x)} \forall x, y \in R and f'(0) = 0$. Then $f(6)$ is equal of $f(-3)$ is _______

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8. If function f satisfies the relation $f(x)\cdot f'(-x) = f(-x)\cdot f'(x) f$ or $all x, and f(0) = 3, and$ if $f(3) = 3$, then the value of $f(-3)$ is ____________

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9. If $y = \frac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}} and y' = 0 at x = 5$, then the value of $\frac{a^2}{b^2}$ is______

Watch Video Solution

10. Prove that $\frac{2^{\log 2^{1/4}x} - 3^{\log_{27}\left(x^2+1\right)^3} - 2x}{7^{4\log_{49}x} - x - 1} > 0, \forall x \in (0, \infty).$

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11. $\lim_{h \to 0} \frac{(e+h)^{In(e+h)} - e}{h}$ is-

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12. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3} and g(x) = f^{-1}(x),$ then the reciprocal of $g'\left(\frac{-7}{6}\right)$ is________

Watch Video Solution

13. Suppose that $f(0) = 0$ and $f'(0) = 2,$ and $g(x) = f(-x + f(f(x))).$ The value of g'(0) is equal to -

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14. Let f(x) be a polynominal with real coefficients such that $f(x) = f'(x) \times f'''(x)$. If f(x)=0 is satisfied x=1,2,3 only, then the value of f'(1)f'(2)f'(3) is

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15. A nonzero polynomial with real coefficients has the property that $f(x) = f'(x) \cdot f''(x)$. If a is the leading coefficient of f(x), then the value of $1/a$ is -

Watch Video Solution

16. A function is reprersented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t} \cdot Thenthevalueo\frac{f(dy)}{dx}$ - $x\left(\frac{dy}{dx}\right)^3$ is________

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17. Let $z = (\cos x)^5$ and $y = \sin x$. Then the value of $\frac{d^2z}{dy^2} at x = \frac{2\pi}{9}$ is__________.

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18. Let $g(x) = \begin{cases} \frac{x^2 + x\tan x - x\tan 2x}{ax + \tan x - \tan 3x}, x \neq 0 \\ 0, x = 0 \end{cases}$ If $g'(0)$ exists and is equal to nonzero value b, then $52\frac{b}{a}$ is equal to ____

Watch Video Solution

19. Let $f(x) = x + \cfrac{1}{2x + \cfrac{1}{2x + \cfrac{1}{2x + \dots\dots\infty}}}$.

Then the value of $f(50) \cdot f'(50)$ is -

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20. Let $F(x) = f(x)g(x)h(x)$ for all real $x, where f(x), g(x), and h(x)$ are differentiable functions. At some point $x_0, F'\left(x_0\right) = 21F\left(x_0\right), f'\left(x_0\right) = 4f\left(x_0\right), g'\left(x_0\right) = -7g\left(x_0\right)$, and $h'\left(x_0\right) = kh\left(x_0\right)$, then the value of k is

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21. If $y = \dfrac{\sqrt[3]{1+3x}\sqrt[4]{1+4x}\sqrt[5]{1+5x}}{\sqrt[7]{1+7x}\sqrt[8]{1+8x}}$, then y'(0) is equal to -

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22. Let $f(\theta) = \sin\left(\tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\dfrac{\pi}{4} < \theta < \dfrac{\pi}{4}$ then the value of $\dfrac{d}{d(\tan\theta)}f(\theta)$is

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23. The slope of the tangent to the curve $\left(y - x^5\right)^2 = x\left(1 + x^2\right)^2$ at the point $(1, 3)$ is.

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24. Let $f: R \to R$ be a differentiable function with $f(0) = 1$ and satisfying the equation $f(x + y) = f(x)f'(y) + f'(x)f(y)$ for all $x, \ y \in R$. Then, the value of $(\log)_e(f(4))$ is ______

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25. $f'(x) = \varphi'(x) = f(x)$ for all $\dot{x}$ Also, $f(3) = 5 and f'(3) = 4.$ Then the value of $[f(10)]^2$– [ϕ(10)]^2is ____

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26. If y=f(x) is an odd differentiable function defined on $(-\infty, \infty) such THAT f'(3) = -2 then f'(-3)$ equal to-

Watch Video Solution

27. If $x^3 + 3x^2 - 9x + \lambda$ is of the form $(x - \alpha)^2(x - \beta)$ then λ is equal to

Watch Video Solution

28. If graph of $y = f(x)$ is symmetrical about the point $(5, 0)$ and $f'(7) = 3$, then the value of $f'(3)$ is________

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29. Let $f(x) = (x - 1)(x - 2)(x - 3)(x - n), n \in N, and f(n) = 5040.$ Then the value of n is_______

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30. $y = f(x)$, where f satisfies the relation $f(x+y) = 2f(x) + xy(y) + y\sqrt{f(x)} \forall x, y \in R and f'(0) = 0.$ Then $f(6)$ is equal of $f(-3)$ is ______

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31. If function f satisfies the relation $f(x)xf'(-x) = f(-x)xf'(x)f \text{ or } allx, andf(0) = 3, and \Leftrightarrow (3) = 3,$ then the value of $f(-3)$ is ___________

Watch Video Solution

32. If $y = \frac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}} andy' = 0atx = 5,$ then the value of $\frac{a^2}{b^2}$ is______

Watch Video Solution

33. $\lim_{h \to 0} \frac{(e+h)^{In(e+h)} - e}{h}$ is-

Watch Video Solution

34. If the function $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3} and g(x) = f^{-1}(x)$, then the reciprocal of $g'\left(\frac{-7}{6}\right)$ is________

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35. Suppose that $f(0) = 0 and f'(0) = 2$, and let $g(x) = f(-x + f(f(x)))^{\cdot}$ The value of g' (0) is equal to _____

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36. A nonzero polynomial with real coefficients has the property that $f(x) = f'(x) \cdot f''(x)$. If a is the leading coefficient of f(x), then the value of

$1/a$ is -

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37. A function is reprersented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t} \cdot Thenthevalueof \left|\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3\right|$ is________

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38. Let $z = (\cos x)^5 and y = \sin x^{\cdot}$ Then the value of $2\frac{d^2 z}{dy^2} at x = \frac{2\pi}{9}$ is__________.

Watch Video Solution

39. Let $g(x) = \begin{cases} \frac{x^2 + x\tan x - x\tan 2x}{ax + \tan x - \tan 3x}, x \neq 00, x = 0 \end{cases}$ If $g'(0)$ exists and is equal to nonzero value b, then $52\frac{b}{a}$ is equal to ____

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40. Let $f(x) = x + \dfrac{1}{2x + \dfrac{1}{2x + \dfrac{1}{2x + \infty}}}$ Compute the value of $f(50)f^{.'}(50)$

Watch Video Solution

41. Let $F(x) = f(x)g(x)h(x)$ for all real $x, where f(x), g(x), and h(x)$ are differentiable functions. At some point $x_0, F^{'}\left(x_0\right) = 21F\left(x_0\right), f^{'}\left(x_0\right)4f\left(x_0\right), g^{'}\left(x_0\right) = -7g\left(x_0\right),$ then the value of $g^{'}(1)$ is ______

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42. If $y = \dfrac{\sqrt[3]{1 + 3x}\sqrt[4]{1 + 4x}\sqrt[5]{1 + 5x}}{\sqrt[7]{1 + 7x}\sqrt[8]{1 + 8x}}$, then y'(0) is equal to -

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43. If $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, then the value of $\frac{d}{d(\tan\theta)} f(\theta)$ is

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44. The slope of the tangent to the curve $\left(y - x^5\right)^2 = x\left(1 + x^2\right)^2$ at the point $(1, 3)$ is.

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45. Let $f: R \rightarrow R$ be a differentiable function with $f(0) = 1$ and satisfying the equation $f(x + y) = f(x)f'(y) + f'(x)f(y)$ for all $x, y \in R$. Then, the value of $(\log)_e(f(4))$ is ______

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Archives

1. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals: 1 b. $\log 2$ c. $-\log 2$ d. -1

A. -1

B. 1

C. log 2

D. `-log 2

Answer: A

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2. Let $f:(1,1)\vec{R}$ be a differentiable function with $f(0) = -1 and f'(0) = 1$. Let $g(x) = [f(2f(x)+2)]^2$. Then $g'(0) =$ (1) 4 (2) 0 (3) 2 (4) 4

A. -2

B. 4

C. -4

D. 0

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3. $\frac{d^2x}{dy^2}$ equals:

(1.) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

(2) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(3) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$

(4) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{3}$

A. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

B. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

C. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

D. $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

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4. If $y = \sec\left(\tan^{-1}x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

A. $1/2$

B. 1

C. $\sqrt{2}$

D. $1\sqrt{2}$

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5. If g is the inverse of a function f and $f'(x) = \frac{1}{1 + x^5}$ then $g'(x)$is equal to

(1) $1 + x^5$ (2) $5x^4$ (3) $\frac{1}{1 + \{g(x)\}^5}$ (4) $1 + \{g(x)\}^5$

A. $1 + x^5$

B. $5x^4$

C. $\dfrac{1}{1 + \{g(x)\}^5}$

D. $1 + \{g(x)\}^5$

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6. If for $x\left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1 - 9x^3}\right)$ is $\sqrt{x}g(x)$, then $g(x)$ equals: $\frac{3x}{1 - 9x^3}$ (2) $\frac{3}{1 + 9x^3}$ (3) $\frac{9}{1 + 9x^3}$ (4) $\frac{3x\sqrt{x}}{1 - 9x^3}$

A. $\dfrac{3}{1 + 9x^3}$

B. $\dfrac{9}{1 + 9x^3}$

C. $\dfrac{3x\sqrt{x}}{1 - 9x^3}$

D. $\dfrac{3x}{1 - 9x^3}$

Watch Video Solution

7. Let y be an implicit function of x defined by $x^{2x} - 2x^{x}\cot y - 1 = 0$. Then y'(1) equals

A. -1

B. 1

C. log 2

D. `-log 2

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8. Let $f:(1,1)\vec{R}$ be a differentiable functio with $f(0) = -1 and f'(0) = 1$. Let $g(x) = [f(2f(x)+2)]^2$. Then $g'(0) =$ (1) 4 (2) 0 (3) 2 (4) -4

A. -2

B. 4

C. -4

D. 0

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9. $\frac{d^2x}{dy^2}$ equals: $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

A. -1

B. 1

C. log 2

D. `-log 2

Answer: A

Watch Video Solution

10. If $y = \sec\left(\tan^{-1}x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

A. -2

B. 4

C. -4

D. 0

Watch Video Solution

11. If g is the inverse of a function f and $f'(x) = \frac{1}{1 + x^5}$ then g(x) is equal to

(1) $1 + x^5$ (2) $5x^4$ (3) $\frac{1}{1 + \{g(x)\}^5}$ (4) $1 + \{g(x)\}^5$

A. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

B. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

C. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

D. $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

Watch Video Solution

12. If for $x\left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}g(x)$, then $g(x)$ equals: $\frac{3x}{1-9x^3}$ (2) $\frac{3}{1+9x^3}$ (3) $\frac{9}{1+9x^3}$ (4) $\frac{3x\sqrt{x}}{1-9x^3}$

A. $1/2$

B. 1

C. $\sqrt{2}$

D. $1\sqrt{2}$

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13. Let $f(x) = x\sin\pi x, x > 0$. Then for all natural numbers $n, f'(x)$ vanishes

at a unique point in the interval $\left(n, n + \frac{1}{2}\right)$ a unique point in the interval

$\left(n + \frac{1}{2}, n + 1\right)$ a unique point in the interval $(n, n + 1)$ two points in the

interval $(n, n + 1)$

A. $1 + x^5$

B. $5x^4$

C. $\dfrac{1}{1 + \{g(x)\}^5}$

D. $1 + \{g(x)\}^5$

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14. Let $f: (0, \infty) \rightarrow R$ be a differentiable function such that $f'(x) = 2 - \dfrac{f(x)}{x}$

for all $x \in (0, \infty)$ and $f(1) = 1$, then f(x) is

A. $\dfrac{3}{1 + 9x^3}$

B. $\dfrac{9}{1 + 9x^3}$

C. $\dfrac{3x\sqrt{x}}{1 - 9x^3}$

D. $\dfrac{3x}{1 - 9x^3}$

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