MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

GRAPHS OF POLYNOMIAL AND RATIONAL FUNCTIONS

Illustration

1. Draw the rough sketch of the curve $y = (x-1)^2(x-3)^3$.

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2. Draw the rough sketch of the curve $y = (x-1)^2(x-2)(x-3)^3$.

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3. Draw the rough sketch of the curve $y = x^4 - x^2$.

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4. Draw the graph of the function $y = 3x^4 - 4x^3$. Discuss the points of local extremum, inflection and intervals of monotonicity.

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5. Draw the graph of $f(x) = (x-1)|(x-2)(x-3)|$.

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6. Draw the graph of $y = x^3 - x^2 + x - 2$ and find the number of real root(s) of the equation $x^3 - x^2 + x - 2 = 0$. Also locate the root.

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7. Sketch the graph of the following functions $y = f(x)$ and find the number of real roots of the corresponding equation $f(x) = 0$.

$(i) f(x) = 2x^3 - 9x^2 + 12x - (9/2)$ $(ii) f(x) = 2x^3 - 9x^2 + 12x - 3$

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8. Draw the graph of $y = x^4 + 2x^2 - 8x + 3$

Find the number of real roots of the equation $x^4 + 2x^2 - 8x + 3 = 0$.

Also find the sum of the integral parts of all real roots.

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9. Let a $\in$ R and f : $R \to R$ be given by $f(x) = x^5 - 5x + a$, then

(a) $f(x) = 0$ has three real roots if $a > 4$

(b) $f(x) = 0$ has only one real root if $a > 4$

(c) $f(x) = 0$ has three real roots if $a < -4$

(d) $f(x) = 0$ has three real roots if $-4 < a < 4$

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10. Find the values of p for which the equation $x^4 - 14x^2 + 24x - 3 - p = 0$ has

(a) Two distinct negative real roots

(b) Two real roots of opposite sign

(c) Four distinct real roots

(d) No real roots

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11. Find the area bounded by the curves $y = \sqrt{1 - x^2}$ and $y = x^3 - x$ without using integration.

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12. Draw the graph of $f(x) = 4x^3 - 3x$ and hence draw the graph of $g(x) = \cos^{-1}(4x^3 - 3x)$.

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13. Let $f(x) = 1 + 4x - x^2,\ \forall x \in R$

$g(x) = \max\{f(t), x \le t \le (x+1), 0 \le x < 3\} = \min\{(x+3), 3 \le x$

Verify conntinuity of g(x), for all $x \in [0, 5]$

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14. Find the value of k if $x^3 - 3x + a = 0$ has three real distinct roots.

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15. If t is a real number satisfying the equation $2t^3 - 9t^2 + 30 - a = 0,$ then find the values of the parameter a for which the equation $x + \frac{1}{x} = t$ gives six real and distinct values of x .

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16. Let $f(x) = x^3 - 9x^2 + 24x + c = 0$ have three real and distinct roots α, β and λ.

(i) Find the possible values of c.

(ii) If $[\alpha] + [\beta] + [\lambda] = 8$, then find the values of c, where $[\cdot]$ represents the greatest integer function.

(ii) If $[\alpha] + [\beta] + [\lambda] = 7$, then find the values of c, where $[\cdot]$ represents the greatest integer function.s

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17. Draw the graph of $f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$.

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18. Draw the graph of $y = \frac{x - 1}{x^2}$ and hence the graph of $y = \frac{|x - 1|}{x^2}$.

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19. Write a possible rational function f that has a vertical asymptote at x = 2, a horizontal asymptote y = 3 and a zero at x = -5. Also draw the graph of the function.

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20. Write a rational function g with vertical asymptotes at x = 3 and x = -3, a horizontal asymptote at y = -4 and with no x-intercept.

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21. Draw the graph of $y = f(x) = \frac{x+1}{x^2+1}$

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22. Draw the graph of the function $f: R - \{-1, 1\} \to R.\ f(x) = \frac{x}{1-|x|}$.

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23. Draw the graph of $f(x) = \dfrac{1}{x^2 - 2x + 2}$.

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24. From the graph of $y = x^2 - 4$, draw the graph of $y = \dfrac{1}{x^2 - 4}$.

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25. Draw the graph of $y = x^2 + \dfrac{1}{x^2}, x \neq 0$.

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26. Draw the graph of $f(x) = \left| \dfrac{x^2 - 2}{x^2 - 1} \right|$.

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27. Draw the graph of $y = \frac{1-x^2}{1+x^2}$ and hence draw the graph of $y = \cos^{-1}.\frac{1-x^2}{1+x^2}$.

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28. Write a rational function f with a slant asymptote $y = x + 4$, a vertical asymptote at x = 5 and one of the zeros at x = 2.

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29. Draw the graph of $y = \frac{(x-1)(6x-1)}{2x-1}$.

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30. Draw the graph of $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

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31. Draw the graph of $y = \frac{x^3}{3(x+1)}$.

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32. Draw the graph of $y = \frac{1}{x} + \frac{1}{x-2}$.

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33. Find the greatest value of $f(x)\frac{1}{2ax - x^2 - 5a^2} \in [-3, 5]$ depending upon the parameter a.

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Exercises

1. Draw the graph of $y = (x-1)\left(x^2 - x + 1\right)$.

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2. Draw the graph of $y = \left(x^2 - x^5\right)(x-2)^3$.

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3. Let $P \, and \, Q$ be any two points. Find the coordinates of the point R which divides PQ externally in the ratio 2:1 and verify that Q is the mid point of PR.

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4. Draw the graphs of

(i) $y = x^2(x-1)|x-2|$

(ii) $y = x^3(x-1)|x-2|$

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5. Write a possible rational function f that has a vertical asymptote at x = 2, a horizontal asymptote y = 3 and a zero at x = -5. Also draw the graph of the function.

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6. Draw the graph of $y = f(x) = \frac{x^2}{x^2 + 1}$.

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7. Draw graph of $y = \frac{x^2 - 6x + 4}{x^2 + 2x + 4}$.

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8. Draw the graph of $f(x) = \frac{x^2 - 8x + 15}{x^2 - 2x}$.

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9. Draw the graph of $f(x) = \frac{5x^2}{(x-1)^3}$.

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10. Draw the graph of $f(x) = \frac{2|x-1|}{x^2+1}$.

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11. Draw the graph of $y = \frac{1}{x+1} + \frac{1}{x} + \frac{1}{x-2}$.

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12. Draw the graph of $y = x + \frac{1}{x}$

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13. Draw the graph of $y = \dfrac{(x+6)(x+2)x(x-2)}{(x-3)(x^2-x+1)}$.

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14. Draw graph of $y = \dfrac{1}{x^2} - x$.

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15. Draw graph of $y = \dfrac{x^3 - 2x^2}{3(x+1)^2}$.

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16. Draw graph of $y = \dfrac{x^3 - 5x}{x^2+1}$.

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17. Given $C_1 < C_2 < C_3 < C_4 < C_5$ and the function $y = f(x)$ is twice differentiable .

$f'(x) > 0$ for $x \in (C_2, C_4)$, $f'(C_2) = f'(C_4) = 0$ and $f'(x) < 0$ for all the remaining values. Also $f''(C_1) = f''(C_3) = f''(C_5) = 0$ and $f''(x) > 0$ for $x \in (C_1, C_3) \cup (C_5, \infty)$ and $f''(x) < 0$ for remaining values. Answer the following:

(i) What is the minimum number of asymptotes parallel to the x-axis for $y = f(x)$?

(ii) What is the maximum number of asymptotes parallel to the x-axis of $y = f(x)$?

(iii) If the range of $y = f(x)$ is $[a, b]$, $a, b \in R$, then what is the minimum number of asymptotes parallel to the x-axis of $y = f(x)$?

(iv) If the range of $y = f(x)$ is non-finite interval, then what is the maximum number of asymptotes parallel to the x-axis ?

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