MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

INTEGRALS

Others

1. If $\int \dfrac{dx}{x^2 + ax + 1} = f(g(x)) + c,$ then $f(x)$ is inverse trigonometric function for $|a| > 2$ $f(x)$ is logarithmic function for $|a| < 2$ $g(x)$ is quadratic function for $|a| > 2$ $g(x)$ is rational function for $|a| < 2$

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2. If $f(x) = \int \dfrac{x^8 + 4}{x^4 - 2x^2 + 2} dx$ and $f(0) = 0, then$ (a)$f(x)$ is an odd function (b) $f(x)$ has range R (c)$f(x)$ has at least one real root (d)$f(x)$ is a monotonic

function.

3. $\int \dfrac{dx}{(x + 1)(x - 2)} = A\log(x + 1) + B\log(x - 2) + C$, where

4. Each question has four choices, a,b,c and d, out of which only one is correct. Each question contains STATEMENT 1 and STATEMENT 2. (a) If both the statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1. (b) If both the statements are TRUE but STATEMENT 2 is NOT the correct explanation of STATEMENT 1. (c) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE. (d) If STATEMENT 1 is FALSE and STATEMENT 2 is TRUE. Statement 1: $\int e^x \sin x \, dx = \dfrac{e^x}{2}(\sin x - \cos x) + c$ Statement 2: $\int e^x \Big(f(x) + f'(x)\Big) dx = e^x f(x) + c$

5. If $\int \dfrac{x^2 - x + 1}{\left(x^2 + 1\right)^{\frac{3}{2}}} e^x dx = e^x f(x) + c$, then (a) $f(x)$ is an even function (b) $f(x)$ is a bounded function (c) the range of $f(x)$ is $(0, 1)$ (d) $f(x)$ has two points of extrema

6. If $\int \dfrac{x^4 + 1}{x^6 + 1} dx = \tan^{-1} f(x) - \dfrac{2}{3}\tan^{-1} g(x) + C$, *then* both $f(x) and g(x)$ are odd functions $f(x)$ is monotonic function $f(x) = g(x)$ has no real roots $\int \dfrac{f(x)}{g(x)} dx = -\dfrac{1}{x} + \dfrac{3}{x^3} + c$

7. If $\int \sin^{-1}x \cos^{-1}x\, dx = f^{-1}(x)\left[\dfrac{\pi}{2}x - xf^{-1}(x) - 2\sqrt{1 - x^2}\right] + 2x + C$, *then* $f(x) = \sin x$ (b) $f(x) = \cos x$ $A = \dfrac{\pi}{4}$ (d) $A = \dfrac{\pi}{2}$

8. If $\int \dfrac{\cos 4x + 1}{\cot x - \tan x} dx = A\cos 4x + B$, *then* (a)$A = -\dfrac{1}{8}$ (b) $B = \dfrac{1}{2}$ (c) $f(x)$ has fundamental period $\dfrac{\pi}{2}$ (d)$f(x)$ is an odd function

9. Statement 1: $\int \dfrac{dx}{x^3\sqrt{1 + x^4}} = -\dfrac{1}{2}\sqrt{1 + \dfrac{1}{x^4}} + C$ Statement 2: For integration by parts, we have to follow ILATE rule. Which of the following Statements is/are correct ?

10. Statement 1: $\int \dfrac{\sin x dx}{x}$, $(x > 0)$, cannot be evaluated. Statement 2: Only differentiable functions can be integrated.

11. $\int \dfrac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B\log\left(9e^{2x} - 4\right) + C,\ then\ A=$_____ _, B=_____,

C=_____

12. The value of the integral $\int \dfrac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx\ is\ \ \sin x - 6\tan^{-1}(\sin x) + C$

$\sin x - 2(\sin x)^{-1} + C$ $\qquad\qquad\qquad\qquad \sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + C$

$\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + C$

13. Evaluate: $\int \sin^{-1}\left(\dfrac{2x + 2}{\sqrt{4x^2 + 8x + 13}}\right) dx$

14. Evaluate: for $m \in N$, $\int x^{3m} + x^{2n} + x^m \Big)\Big(2x^{2m} + 3x^m + 6\Big)^{\frac{1}{m}} dx, x > 0$

15. Evaluate: $\int \dfrac{\cos^4 x}{\sin^3 x \Big(\sin^5 x + \cos^5 x\Big)^{\frac{3}{5}}} dx$

16. Evaluate: $\int \sqrt{1 + \cosec x}\, dx$, $\Big(0 < x < \dfrac{\pi}{2}\Big)$

17. $\int \dfrac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx$ is equal to (a) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + C$ (b) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$ (c) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + C$ (d) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$

18. The integral $\int \dfrac{\sec^2 x}{(\sec x + \tan x)^{\frac{9}{2}}} dx$ equals (for some arbitrary constant K).

$-\dfrac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \dfrac{1}{11} - \dfrac{1}{7}(\sec x + \tan x)^2 \right\} + K$

$\dfrac{1}{(\sec x + \tan x)^{\frac{1}{11}}} \left\{ \dfrac{1}{11} - \dfrac{1}{7}(\sec x + \tan x)^2 \right\} + K$

$-\dfrac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \dfrac{1}{11} + \dfrac{1}{7}(\sec x + \tan x)^2 \right\} + K$

$\dfrac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \dfrac{1}{11} + \dfrac{1}{7}(\sec x + \tan x)^2 \right\} + K$

19. Evaluate: $\int \dfrac{dx}{x^3 \sqrt{x^2 - 1}}$

20. Evaluate : $\int \dfrac{2x}{\left(1 - x^2\right)\sqrt{x^4 - 1}}dx$

21. Evaluate: $\int \sqrt{\sec x - 1}\,dx$

22. Evaluate: $\int \sqrt{\dfrac{3 - x}{3 + x}}\sin^{-1}\left(\dfrac{1}{6}\sqrt{3 - x}\right)dx$

23. Evaluate: $\int \dfrac{(1 - x\sin x)dx}{x\left(1 - x^3 e^{3\cos x}\right)}$

24. Evaluate: If $\int \cos^n x\, dx$ prove that $I_n = \dfrac{1}{n}\left(\cos^{n-1}x\sin x\right) + \left(\dfrac{n-1}{n}\right)I_{n-2}$

25. Evaluate: $\displaystyle\int \dfrac{x^2 - 1}{x\sqrt{\left(x^2 + \alpha x + 1\right)\left(x^2 + \beta x + 1\right)}}\, dx$

26. Evaluate: $\displaystyle\int \dfrac{e^{\tan^{-1}x}}{\left(1 + x^2\right)}\left[\left(\sec^{-1}\sqrt{1 + x^2} + \cos^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)\right)\right] dx,\ (x > 0)$

27. $\displaystyle\int \dfrac{\sqrt{x - 1}}{x\sqrt{x + 1}}\, dx\, isequa < o$
$\qquad\qquad\qquad\qquad \ln\left|x - \sqrt{x^2 - 1}\right| - \tan^{-1}x + c$

$\ln\left|x + \sqrt{x^2 - 1}\right| - \tan^{-1}x + c$
$\qquad\qquad\qquad\qquad \ln\left|x - \sqrt{x^2 - 1}\right| - \sec^{-1}x + c$

$\ln\left|x + \sqrt{x^2 - 1}\right| - \sec^{-1}x + c$

28. If $I = \int \dfrac{dx}{\left(2ax + x^2\right)^{\frac{3}{2}}}$, then I is equal to (a)- $\dfrac{x + a}{\sqrt{2ax + x^2}} + c$ (b)

$-\dfrac{1}{a}\dfrac{x + a}{\sqrt{2ax + x^2}} + c$ (c)- $\dfrac{1}{a^2}\dfrac{x + a}{\sqrt{2ax + x^2}} + c$ (d) $-\dfrac{1}{a^3}\dfrac{x + a}{\sqrt{2ax + x^3}} + c$

29. If $f'(x) = \dfrac{1}{-x + \sqrt{x^2 + 1}}$ and $f(0) = \dfrac{1 + \sqrt{2}}{2}$ then $f(1)$ is equal to- (a)

$\log\left(\sqrt{2} + 1\right)$ (b) 1 (c)$1 + \sqrt{2}$ (d) none of these

30. $\int e^x\left(\dfrac{2\tan x}{1 + \tan x} + \cot^2\left(x + \dfrac{\pi}{4}\right)\right)dx$ is equa $< o$ $e^x\tan\left(\dfrac{\pi}{4} - x\right) + c$

$e^x\tan\left(x - \dfrac{\pi}{4}\right) + c\ e^x\tan\left(\dfrac{3\pi}{4} - x\right) + c$ (d) none of these

31. The value of the integral $\int \left(x^2 + x\right)\left(x^{-8} + 2x^{-9}\right)^{\frac{1}{10}} dx$ is

$\frac{5}{11}\left(x^2 + 2x\right)^{\frac{11}{10}} + c$ (b) $\frac{5}{6}(x + 1x)^{\frac{11}{10}} + c$ $\frac{6}{7}(x + 1)^{\frac{11}{10}} + c$ (d) none of these

32. If$\int \dfrac{dx}{(x + 2)\left(x^2 + 1\right)} = a\ln\left(1 + x^2\right) + b\tan^{-1}x + \dfrac{1}{5}\ln|x + 2| + C$ Then (a)

$a = \dfrac{1}{10}, b = -\dfrac{2}{5}$ (b) $a = \dfrac{1}{10}, b = -\dfrac{2}{5}$ (c) $a = -\dfrac{1}{10}, b = \dfrac{2}{5}$ (d) $a = \dfrac{1}{10}, b = \dfrac{2}{5}$

33. If $\int \dfrac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx = ax + b\ln|2\sin x + 3\cos x| + C,$ then (a)

$a = -\dfrac{12}{13}, b = \dfrac{15}{39}$ (b) $a = -\dfrac{7}{13}, b = \dfrac{6}{13}$ (c) $a = \dfrac{12}{13}, b = \dfrac{15}{39}$ (d)

$a = -\dfrac{7}{13}, b = -\dfrac{6}{13}$

34. If $\int \dfrac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx = ax + b\ln\left(4e^x + 5e^{-x}\right) + C$, then (a) $a = -\dfrac{1}{8}, b = \dfrac{7}{8}$ (b) $a = \dfrac{1}{8}, b = \dfrac{7}{8}$ (c) $a = -\dfrac{1}{8}, b = -\dfrac{7}{8}$ (d) $a = \dfrac{1}{8}, b = -\dfrac{7}{8}$

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35. $\int\sqrt{\dfrac{\cos x - \cos^3 x}{1 - \cos^3 x}} dx$ is equal to (a) $-\dfrac{2}{3}\sin^{-1}\left(\cos^{\frac{3}{2}}x\right) + C$ (b) $\dfrac{3}{2}\sin^{-1}\left(\cos^{\frac{3}{2}}x\right) + C$ (c) $\dfrac{2}{3}\cos^{-1}\left(\cos^{\frac{3}{2}}x\right) + C$ (d) none of these

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36. If $l'(x)$ means $\log\log\log x$, the log being repeated r times, then $\int\left[xl(x)l^2(x)l^3(x)l'(x)\right]^{-1} ds$ is equal to $l^{r+1}(x) + C$ (b) $\dfrac{l^{r+1}(x)}{r+1} + C$ (c) $l^r(x) + C$ (d) none of these

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37. $\int \dfrac{\cos 4x - 1}{\cot x - \tan x} dx$ is equal to

(A) $\dfrac{1}{2}\ln|\sec 2x| - \dfrac{1}{4}\cos^2 2x + c$

(B) $\dfrac{1}{2}\ln|\sec 2x| + \dfrac{1}{4}\cos^2 x + c$

(C) $\dfrac{1}{2}\ln|\cos 2x| - \dfrac{1}{4}\cos^2 2x + c$

(D) $\dfrac{1}{2}\ln|\cos 2x| + \dfrac{1}{4}\cos^2 x + c$

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38. If $\int \dfrac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + c,$ then (a)$a = -1, b = \dfrac{1}{3}$ (b) $a = -3, b = \dfrac{2}{3}$ (c)$a = -2, b = \dfrac{4}{3}$ (d) none of these

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39. $\int e^{\tan^{-1}x}\left(1 + x + x^2\right)d\left(\cot^{-1}x\right)$ is equal to (a) $-e^{\tan^{-1}x} + c$ (b) $e^{\tan^{-1}x} + c$ (c) $-xe^{\tan^{-1}x} + c$ (d) $xe^{\tan^{-1}x} + c$

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40. $\int \dfrac{x^9 dx}{\left(4x^2 + 1\right)^6}$ *isequal* $\rightarrow$

(a) $\dfrac{1}{5x}\left(4 + \dfrac{1}{x^2}\right)^{-5} + c$

(b) $\dfrac{1}{5}\left(4 + \dfrac{1}{x^2}\right)^{-5} + c$

(c) $\dfrac{1}{10}\left(1 + 4x^2\right)^{-5} + c$

(d) $\dfrac{1}{10}\left(4 + \dfrac{1}{x^2}\right)^{-5} + c$

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41. $\int \dfrac{2\sin x}{3 + \sin 2x} dx$ is equal to

A. (a) $\dfrac{1}{2}\ln\left|\dfrac{2 + \sin x - \cos x}{2 - \sin x + \cos x}\right| - \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{\sin x + \cos x}{\sqrt{2}}\right) + c$

B. (b) $\dfrac{1}{2}\ln\left|\dfrac{2 + \sin x - \cos x}{2 - \sin x + \cos x}\right| - \dfrac{1}{2\sqrt{2}}\tan^{-1}\left(\dfrac{\sin x + \cos x}{\sqrt{2}}\right) + c$

C. (c) $\dfrac{1}{4}\ln\left|\dfrac{2 + \sin x - \cos x}{2 - \sin x + \cos x}\right| - \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{\sin x + \cos x}{\sqrt{2}}\right) + c$

D. (d) None of these

Answer: null

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42. $\int\left(\dfrac{\ln(\tan x)}{\sin x \cos x}\right)dx$ i se q u a lto (a)$\dfrac{1}{2}\ln(\tan x) + c$ (b) $\dfrac{1}{2}\ln\left(\tan^2 x\right) + c$ (c) $\dfrac{1}{2}(\ln(\tan x))^2 + c$ (d) none of these

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43. $\int \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)dx$ is e q u a lto

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44. If $\int \dfrac{1-x^7}{x\left(1+x^7\right)}dx = a\log_e|x| + b\log_e\left|x^7+1\right| + c$, then

45. if $\int \dfrac{dx}{x^2\left(x^n + 1\right)^{\left(\frac{n-1}{n}\right)}} = -[f(x)]^{\frac{1}{n}} + c, then f(x) is$ $\left(1 + x^n\right)$ (b) $1 + x^{-1}$

$x^n + x^{-n}$ (d) none of these

46. If $\int \dfrac{1}{x\sqrt{1 - x^3}}dx = a\log\left|\dfrac{\sqrt{1 - x^3} - 1}{\sqrt{1 - x^3} + 1}\right| + b, then a is equal$ $\dfrac{1}{3}$ (b) $\dfrac{2}{3}$ (c) $-\dfrac{1}{3}$ (d0

$-\dfrac{2}{3}$

47. Evaluate: $\int \dfrac{\sqrt{\cos 2x}}{\sin x}dx$

48. Evaluate: $\int\left(\sqrt{\dfrac{1+x^2}{x^2-x^4}}\right)dx$

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49. Evaluate: $\int\dfrac{x^2}{(a+bx)^2}dx$

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50. Evaluate: $\int\sqrt{1+\sin\left(\dfrac{x}{2}\right)}dx$

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51. Evaluate: $\int\dfrac{x^2}{\sqrt{1-xdx}}$

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52. Evaluate: $\int \left(e^{\log x} + \sin x\right)\cos x\, dx$

53.

If $f(x) = \sqrt{x}$, $g(x) = e^x - 1$, and $\int fog(x)dx = Afog(x) + B\tan^{-1}(fog(x)) + C$, then A

54. If $\int \dfrac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2}dx = A\ln|\cos x + \sin x - 2| + Bx + C$, then the value of

$A + B + |\lambda|$ is_____

55. If $\int \left[\left(\dfrac{x}{e}\right)^x + \left(\dfrac{e}{x}\right)^x\right]\ln x\, dx = A\left(\dfrac{x}{e}\right)^x + B\left(\dfrac{e}{x}\right)^x + C$ then the value of

$A + B$ is____

56. Evaluate: $\int \dfrac{\sin x}{\sin x - \cos x} dx$

57. Evaluate: $\int \dfrac{(x-1)e^x}{(x+1)^3} dx$

58. Evaluate: $\int \dfrac{dx}{x^2\left(x^4 + 1\right)^{\frac{3}{4}}}$

59. If $\int x\log\left(1 + \dfrac{1}{x}\right)dx = f(x)\log(x+1) + g(x)x^2 + Ax + C,$ then (a)$f(x) = \dfrac{1}{2}x^2$ (b) $g(x) = \log x$ (c)$A = 1$ (d) none of these

60. If $I = \int \dfrac{dx}{x^3\sqrt{x^2 - 1}}$, then I equals a. $\dfrac{1}{2}\left(\dfrac{\sqrt{x^2 - 1}}{x^3} + \tan^{-1}\sqrt{x^2 - 1}\right) + C$ b.

$\dfrac{1}{2}\left(\dfrac{\sqrt{x^2 - 1}}{x^2} + x\tan^{-1}\sqrt{x^2 - 1}\right) + C$ c. $\dfrac{1}{2}\left(\dfrac{\sqrt{x^2 - 1}}{x} + \tan^{-1}\sqrt{x^2 - 1}\right) + C$ d.

$\dfrac{1}{2}\left(\dfrac{\sqrt{x^2 - 1}}{x^2} + \tan^{-1}\sqrt{x^2 - 1}\right) + C$

61. The value of $\int \dfrac{ax^2 - b}{x\sqrt{c^2x^2 - \left(ax^2 + b\right)^2}}$ (a) $\dfrac{1}{c}\sin^{-1}\left(ax + \dfrac{b}{x}\right) + k$ (b)

$c\sin^{-1}\left(a + \dfrac{b}{x}\right) + e$ (c) $\sin^{-1}\left(\dfrac{ax + \dfrac{b}{x}}{c}\right) + k$ (d) none of these

62. If $\int \dfrac{dx}{\cos^3 x\sqrt{\sin 2x}} = a\left(\tan^2 x + b\right)\sqrt{\tan x} + c$, then (a) $a = \dfrac{\sqrt{2}}{5}, b = \dfrac{1}{\sqrt{5}}$ (b) $a = \dfrac{\sqrt{2}}{5}, b = 5$ (c) $a = \dfrac{\sqrt{2}}{5}, b = -\dfrac{1}{\sqrt{5}}$ (d) $a = \dfrac{\sqrt{2}}{5}, b = \sqrt{5}$

63. $4\int \dfrac{\sqrt{a^6 + x^8}}{x} dx$ is equal → (a) $\sqrt{a^6 + x^8} + \dfrac{a^3}{2}\ln\left|\dfrac{\sqrt{a^6 + x^8} + a^3}{\sqrt{a^6 + x^8} - a^3}\right| + c$ (b) $a^6\ln\left|\dfrac{\sqrt{a^6 + x^8} - a^3}{\sqrt{a^6 + x^8} + a^3}\right| + c$ (c) $\sqrt{a^6 + x^8} + \dfrac{a^3}{2}\ln\left|\dfrac{\sqrt{a^6 + x^8} - a^3}{\sqrt{a^6 + x^8} + a^3}\right| + c$ (d) $a^6\ln\left|\dfrac{\sqrt{a^6 + x^8} + a^3}{\sqrt{a^6 + x^8} - a^3}\right| + c$

64. If $I = \int e^{-x}\log\left(e^x + 1\right)dx$, then I equals

65. If $I_{m,n} = \int \cos^m x \sin nx \, dx$, then $7I_{4,3} - 4I_{3,2}$ is equal to   (a) constant   (b)

$-\cos^2 x + C$ (c) $-\cos^4 x \cos 3x + C$ (d) $\cos 7x - \cos 4x + C$

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66.   if $\int \dfrac{dx}{x^2 \left(x^n + 1 \right) \left(\frac{n-1}{n} \right)} = -[f(x)]^{\frac{1}{n}} + c, \; then f(x) is$ (a) $\left(1 + x^n \right)$ (b)

$1 + x^{-1}$ (c)$x^n + x^{-n}$ (d) none of these

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67. Evaluate: $\int \left(\sqrt{\tan x} + \sqrt{\cot x} \right) dx$

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68. Evaluate: $\int \dfrac{\sqrt{\cos 2x}}{\sin x} dx$

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69. $If \int xe^x \cos x \, dx = ae^x(b(1 - x)\sin x + cx\cos x) + d, \ then$ (a)

$a = 1, b = 1, c = -1$ (b) $a = \dfrac{1}{2}, b = -, c = 1$ (c) $a = 1, b = -1, c = 1$ (d)

$a = \dfrac{1}{2}, b = 1, c = -1$

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70. If $I = \int \sqrt{\dfrac{5 - x}{2 + x}} \, dx,$ then equal

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71. Evaluate: $\int \dfrac{x + 1}{x\left(1 + xe^x\right)^2} \, dx$

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72. Evaluate: $\int \cos 2\theta \ln\left(\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right) d\theta$

73. Evaluate: $\int \left(\dfrac{1}{x^3 + x^4} + \dfrac{\ln\left(1 + x^{\frac{1}{6}}\right)}{x^{\frac{1}{3}} + \sqrt{x}} \right) dx$

74. Evaluate: $\int \left(\sqrt{\tan x} + \sqrt{\cot x} \right) dx$

75. If $I = \int \dfrac{dx}{\left(a^2 - b^2 x^2\right)^{\frac{3}{2}}}$, then I equal (a) $\dfrac{x}{\sqrt{a^2 - b^2 x^2}} + C$ (b) $\dfrac{x}{2\sqrt{a^2 - b^2 x^2}} + C$

(c) $\dfrac{ax}{\sqrt{a^2 - b^2 x^2}} + C$ (d) none of these

76. If $I = \int\left(\sqrt{\cot x} - \sqrt{\tan x}\right)dx,$ then I equal $\quad\quad \sqrt{2}\log\left(\sqrt{\tan x} - \sqrt{\cot x}\right) + C$

$\sqrt{2}\log|\sin x|\cos x + \sqrt{\sin 2x}\ | + C \quad\quad \sqrt{2}\log\left|s \in x - \cos x + \sqrt{2}s \in x\cos x\right| + C$

$\sqrt{2}\log\left|\sin\left(x + \dfrac{\pi}{4}\right) + \sqrt{2}\sin x\cos x\right| + C$

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77. $\int x\left(\left(\dfrac{\ln a^{\frac{x}{2}}}{3a^{\frac{5x}{2}}b^{3x}} + \dfrac{\ln b^b \wedge x}{2a^{2x}b^{4x}}\right)dx\left(where\ a, b \in R^+\right)\ is\ equa < o\right.$

$\dfrac{1}{6\ln a^2 b^3}a^{2x}b^{3x}\dfrac{\ln\left(a^{2x}b^{3x}\right)}{e} + k \quad\quad\quad \dfrac{1}{6\ln a^2 b^3}\dfrac{1}{a^{2x}b^{3x}}\dfrac{\ln 1}{ea^{2x}b^{3x}} + k$

$\dfrac{1}{6\ln a^2 b^3}\dfrac{1}{a^{2x}b^{3x}}\ln\left(a^{2x}b^{3x}\right) + k - \dfrac{1}{6\ln a^2 b^3}\dfrac{1}{a^{2x}b^{3x}}\ln\left(a^{2x}b^{3x}\right) + k$

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78. $\left(\int e^x \wedge 4(x+x\char`\^3+2x\char`\^5)e^x \wedge 2dx\right)$ is equa $< o1/2x \quad\quad e\char`\^x 2e\char`\^x\char`\^4+c(b)$

$1/2x\char`\^2e\char`\^x\char`\^4+c1/2e\char`\^x\char`\^2e\char`\^x\char`\^4+c(d)1/2x\char`\^2e\char`\^x\char`\^2e\char`\^x\char`\^4+c$

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79. $\int \dfrac{\cosec^2 x - 2005}{\cos^{2005} x} dx$ is equal to (a) $-\dfrac{\cot x}{(\cos x)^{2005}} + c$ (b) $\dfrac{\tan x}{(\cos x)^{2005}} + c$ (c)

$-\dfrac{\tan x}{(\cos x)^{2005} + c}$ (d) none of these

80. If $\int \dfrac{x\ln\left(x + \sqrt{1 + x^2}\right)}{\sqrt{1 + x^2}} dx = a\sqrt{1 + x^2}\ln\left(x + \sqrt{1 + x^2}\right) + bx + c$, then (A)

$a = 1, b = -1$ (B) $a = 1, b = 1$ (C) $a = -1, b = 1$ (D) $a = -1, b = -1$

81. If $\int f(x)\sin x\cos x\, dx = \dfrac{1}{2\left(b^2 - a^2\right)}\ln f(x) + c$,then f(x) is equal to

A. (a) $\dfrac{1}{a^2\sin^2 x + b^2\cos^2 x}$

B. (b) $\dfrac{1}{a^2\sin^2 x - b^2\cos^2 x}$

C. (c) $\dfrac{1}{a^2\cos^2 x + b^2\cos^2 x}$

D. (d) $\dfrac{1}{a^2\cos^2 x - b^2\cos^2 x}$

Answer: null

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82. If $xf(x) = 3f^2(x) + 2$, then $\displaystyle\int \dfrac{2x^2 - 12xf(x) + f(x)}{(6f(x) - x)\left(x^2 - f(x)\right)^2}\,dx$ equal. (A) $\dfrac{1}{x^2 - f(x)} + c$

(B) $\dfrac{1}{x^2 + f(x)} + c$ (C) $\dfrac{1}{x - f(x)} + c$ (D) $\dfrac{1}{x + f(x)} + c$

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83. $\displaystyle\int \dfrac{dx}{\left(1 + \sqrt{x}\right)\sqrt{\left(x - x^2\right)}}$ is equal to (a) $\dfrac{1 + \sqrt{x}}{(1 - x)^2} + c$ (b) $\dfrac{1 + \sqrt{x}}{(1 + x)^2} + c$ (c)

$\dfrac{1 - \sqrt{x}}{(1 - x)^2} + c$ (d) $\dfrac{2\left(\sqrt{x} - 1\right)}{\sqrt{(1 - x)}} + c$

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84. The value of integral $\int e^x \left(\dfrac{1}{\sqrt{1+x^2}} + \dfrac{1-2x^2}{\sqrt{\left(1+x^2\right)^5}} \right) dx$ is equal to

(a) $e^x \left(\dfrac{1}{\sqrt{1+x^2}} + \dfrac{x}{\sqrt{\left(1+x^2\right)^3}} \right) + c$

(b) $e^x \left(\dfrac{1}{\sqrt{1+x^2}} - \dfrac{x}{\sqrt{\left(1+x^2\right)^3}} \right) + c$

(c) $e^x \left(\dfrac{1}{\sqrt{1+x^2}} + \dfrac{x}{\sqrt{\left(1+x^2\right)^5}} \right) + c$ (d) none of these

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85. $\int \sqrt{1+cosecx}\ dx$ equals

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86. If $I = \int sec^2 x cosec^4 x dx = A cot^3 x + B tanx + C cotx + D,$ then find the values of A,B,C and D.

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87. A curve $g(x) = \int x^{27}\left(1 + x + x^2\right)^6\left(6x^2 + 5x + 4\right)dx$ is passing through origin. Then (a)$g(1) = \dfrac{3^7}{7}$ (b) $g(1) = \dfrac{2^7}{7}$ (c)$g(-1) = \dfrac{1}{7}$ (d)$g(-1) = \dfrac{3^7}{14}$

88. If $\int\sqrt{\cosec x + 1}\,dx = kfog(x) + c, where\, k$ is a real constant, then (a).

$k = -2,\ f(x) = \cot^{-1}x,\ g(x) = \sqrt{\cosec x - 1}$ (b)

$k = -2,\ f(x) = \tan^{-1}x,\ g(x) = \sqrt{\cosec x - 1}$ (c)

$k = 2,\ f(x) = \tan^{-1}xg(x) = \dfrac{\cot x}{\sqrt{\cosec x - 1}}$ (d)

$k = 2,\ f(x) = \cot^{-1}xg(x) = \dfrac{\cot x}{\sqrt{\cosec x - 1}}$

89. $\int\dfrac{x^4 - 1}{x^2\sqrt{x^4 + x^2 + 1}}dx = $ (a) $\sqrt{x^2 + \dfrac{1}{x^2} + 1} + C$ (b) $\dfrac{\sqrt{x^4 + x^2 + 1}}{x^2} + C$ (c) $\dfrac{\sqrt{x^4 + x^2 + 1}}{x} + C$ (d) none of these

90. $\int \dfrac{x^2 - 1}{x^3\sqrt{2x^4 - 2x^2 + 1}}dx$ is equal to (a) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + C$ (b)

$\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$ (c) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + C$ (d) $\dfrac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$

91. $\int \sqrt{e^x - 1}dx\,is\,equa < o$ $2\left[\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}\right] + c$

$\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1} + c$ $\sqrt{e^x - 1} + \tan^{-1}\sqrt{e^x - 1} + c$

$2\left[\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}\right] + c$

92. If $\int \sin x d(\sec x) = f(x) - g(x) + c,\, then\,\, f(x) = \sec x$ (b) $f(x) = \tan x\,\, g(x) = 2x$

(d) $g(x) = x$

93. If $I = \int \dfrac{\sin x + \sin^3 x}{\cos 2x} dx = P\cos x + Q\log|f(x)| + R,$ then   (a)

$P = \dfrac{1}{2}, Q = -\dfrac{3}{4\sqrt{2}}$ (b) $P = \dfrac{1}{4}, Q = \dfrac{1}{\sqrt{2}}$ (c)$f(x) = \dfrac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$ (d)

$f(x) = \dfrac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

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94. If $\int \dfrac{e^{x-1}}{\left(x^2 - 5x + 4\right)} 2x\,dx = AF(x-1) + BF(x-4) + C$ and $F(x) = \int \dfrac{e^x}{x} dx,$ then

A. (a)$A = -\dfrac{2}{3}$

B. (b) $B = \left(\dfrac{4}{3}\right)e^3$

C. (c)$A = \dfrac{2}{3}$

D. (b) $A = \left(\dfrac{4}{3}\right)e^3$

Answer: null

95. Evaluate: $\int \dfrac{1}{x}\sqrt{\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}}\,dx$

96. Evaluate: $\int \dfrac{x^3 + 3x + 2}{\left(x^2 + 1\right)^2(x + 1)}\,dx$

97. $\int \dfrac{3 + 2\cos x}{(2 + 3\cos x)^2}\,dx$ is equal to (a) $\left(\dfrac{\sin x}{3\cos x + 2}\right) + c$ (b) $\left(\dfrac{2\cos x}{3\sin x + 2}\right) + c$ (c) $\left(\dfrac{2\cos x}{3\cos x + 2}\right) + c$ (d) $\left(\dfrac{2\sin x}{3\sin x + 2}\right) + c$

98. $\int \left(\dfrac{x+2}{x+4}\right)^2 e^x dx$ is equal to

A. (a) $e^x \left(\dfrac{x}{x+4}\right) + c$

B. (b) $e^x \left(\dfrac{x+2}{x+4}\right) + c$

C. (c) $e^x \left(\dfrac{x-2}{x+4}\right) + c$

D. (d) $\left(\dfrac{2xe^2}{x+4}\right) + c$

Answer: null

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99. $If\, I = \int \dfrac{\sin 2x}{(3+4\cos x)^3} dx,\, then\, I\, equals$ (a) $\dfrac{3\cos x + 8}{(3+4\cos x)^2} + C$ (b)

$\dfrac{3+8\cos x}{16(3+4\cos x)^2} + C$ (c) $\dfrac{3\cos x}{(3+4\cos x)^2} + C$ (d) $\dfrac{3 - 8\cos x}{16(3+4\cos x)^2} + C$

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100. If $I = \int \dfrac{dx}{secx + cosecx}$, then equals

101. $\int \dfrac{x^3 dx}{\sqrt{1 + x^2}}$ is equal to (A) $\dfrac{1}{3}\sqrt{1 + x^2}\left(2 + x^2\right) + C$ (B)

$\dfrac{1}{3}\sqrt{1 + x^2}\left(x^2 - 1\right) + C$ (C) $\dfrac{1}{3}\left(1 + x^2\right)^{\frac{3}{2}} + C$ (D) $\dfrac{1}{3}\sqrt{1 + x^2}\left(x^2 - 2\right) + C$

102. $\int e^{tanx}(secx - sinx)dx$ is equal to

103. $\int e^x \dfrac{\left(x^2 + 1\right)}{(x + 1)^2} dx$ is equal $\rightarrow$ $\left(\dfrac{x - 1}{x + 1}\right)e^x + c$ (b) $e^x\left(\dfrac{x + 1}{x - 1}\right) + c$

$e^x(x + 1)(x - 1) + c$ (d) none of these

104. $\int x \sin x \sec^3 x\, dx$ is equal to

105. $If\ I = \int \dfrac{dx}{x^3\sqrt{x^2 - 1}}, then\ I\ equals$ $\quad \dfrac{1}{2}\left(\dfrac{\sqrt{x^2 - 1}}{x^3} + \tan^{-1}\sqrt{x^2 - 1}\right) + C$

$\dfrac{1}{2}\left(\dfrac{\sqrt{x^2 - 1}}{x^2} + x\tan^{-1}\sqrt{x^2 - 1}\right) + C$ $\quad \dfrac{1}{2}\left(\dfrac{\sqrt{x^2 - 1}}{x} + \tan^{-1}\sqrt{x^2 - 1}\right) + C$

$\dfrac{1}{2}\left(\dfrac{\sqrt{x^2 - 1}}{x^2} + \tan^{-1}\sqrt{x^2 - 1}\right) + C$

106. $\int \dfrac{\ln\left(\dfrac{x-1}{x+1}\right)}{x^2 - 1}dx$ is equal to (a) $\dfrac{1}{2}\left(\ln\left(\dfrac{x-1}{x+1}\right)\right)^2 + C$ (b) $\dfrac{1}{2}\left(\ln\left(\dfrac{x+1}{x-1}\right)\right)^2 + C$

(c) $\dfrac{1}{4}\left(\ln\left(\dfrac{x-1}{x+1}\right)\right)^2 + C$ (d) $\dfrac{1}{4}\left(\ln\left(\dfrac{x+1}{x-1}\right)\right)^2 + C$

107. Let $g(x) = \int \dfrac{1 + 2\cos x}{(\cos x + 2)^2} dx$ and $g(0) = 0.$ then the value of $8g\left(\dfrac{\pi}{2}\right)$ is

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108.

Let $k(x) = \int \dfrac{\left(x^2 + 1\right) dx}{\sqrt[3]{x^3 + 3x + 6}}$ and $k(-1) = \dfrac{1}{\sqrt[3]{2}}.$ Then the value of $k(-2)$ is - .

109. Column I a) $\int \dfrac{x^2 - x + 1}{x^3 - 4x^2 + 4x} dx$ b) $\int \dfrac{x^2 - 1}{x(x - 2)^3} dx$ c) $\int \dfrac{x^3 + 1}{x(x - 2)^2} dx$ d) $\int \dfrac{x^5 + 1}{x(x - 2)^3} dx$ COLUMN II (which of the following functions appear in integration of functions in columnI) p) $\log|x|$ q) $\log|x - 2|$ r.) $\dfrac{1}{(x - 2)}$ s) x

110. Let $f(x) = \int x^{\sin x}\left(1 + x\cos x \ln x + \sin x\right)dx$ and $f\left(\dfrac{\pi}{2}\right) = \dfrac{\pi^2}{4}$ Then the value of $|\cos(f(\pi))|$ is____

111. Each question contains statements given in two columns which have to be matched. Statements (a,b,c,d) in column I have to be matched with statements (p,q,r,s) in column II. If the correct match are a-p, s, b-r c-p, q. and d-s, then the correctly bubbled 4x4 matrix should be as follows:

figure Column I, a) If $\int \dfrac{2^x}{\sqrt{1 - 4^x}}dx = k\sin^{-1}(f(x)) + C$, then k is greater than,

b) If $\int \dfrac{\left(\sqrt{x}\right)^5}{\left(\sqrt{x}\right)^7 + x^6}dx = a\dfrac{\ln\left(x^k\right)}{x^k + 1} + c$, then a is less than, c) If

$\int \dfrac{x^4 + 1}{x\left(x^2 + 1\right)^2}dx = k\ln|x| + \dfrac{m}{1 + x^2} + n$, where n is the constant of integration, then m is greater than, d) If

$$\int \frac{dx}{5 + 4\cos x} = k\tan^{-1}\left(m\tan\frac{x}{2}\right) + C,$$ then k/m is greater than, COLUMN II p)

0 q) 1 r) 3 s)4

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112. The value of $\int e^2 x\left(\dfrac{1}{x} - \dfrac{1}{2x^2}\right)dx$ is

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113. Statement 1: If the primitive of $f(x) = \pi\sin\pi x + 2x - 4$ has the value 3

for $x = 1$, then there are exactly two values of x for which primitive of $f(x)$

vanishes. Statement 2: $\cos\pi x$ has period 2.

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114. Let $f(x)=2^{2x-1}$ and $\phi(x) =-2^x + 2x\log 2$. If $f'(x) > \phi'(x)$, then value of x is-

115. Evaluate: $\int \dfrac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$

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116. If $\int x^2 e^{-2x} dx = e^{-2x}\left(ax^2 + bx + c\right) + d,$ then the value of $\left|\dfrac{a}{bc}\right|$ is_____

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117. If $f(x) = \int \dfrac{3x^2 + 1}{\left(x^2 - 1\right)^3} dx \, and \, f(0) = 0,$ then the value of $\left|\dfrac{2}{f(2)}\right|$ is___

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118. Evaluate: $\int \sqrt{\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}} dx$

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119. If $\int x^5 \left(1 + x^3\right)^{2/3} dx = A\left(1 + x^3\right)^{8/3} + B\left(1 + x^3\right)^{5/3} + c$, then

120. The value of the integral $\int \dfrac{(1 - \cos\theta)^{\frac{2}{7}}}{(1 + \cos\theta)^{\frac{9}{7}}} d\theta$ is

A. (a) $\dfrac{7}{11}\left(\tan\dfrac{\theta}{2}\right)^{\frac{11}{7}} + C$ (b) $\dfrac{7}{11}\left(\dfrac{\cos\theta}{2}\right)^{\frac{11}{7}} + C$ (c) $\dfrac{7}{11}\left(s\int h\dfrac{\eta}{2}\right)^{\frac{11}{7}} + C$ (d)

none of these

B. null

C. null

D. null

Answer: null

121. Let $f(x) = \int \dfrac{x^2 dx}{\left(1 + x^2\right)\left(1 + \sqrt{x^2 + 1}\right)}$ $and f(0) = 0$. Then value of $f(1)$ will

be

122. If $y = \int \sqrt{x}\left(1 + x^{\frac{1}{3}}\right)^4 dx$ i se q u a lto

123. If $\int \sqrt{1 + \sin x} f(x) dx = \dfrac{2}{3}(1 + \sin x)^{\frac{3}{2}} + c$, $then f(x) equal$ cos$x$ (b) sin$x$ (c) tan$x$ (d) 1

124. Let $\int e^x \left\{ f(x) - f'(x) \right\} dx = \varphi(x)$ Then $\int e^x f(x) dx$ is

125. Evaluate: $\int \dfrac{\sin2x}{\sin^4 + \cos^4 x} dx$

126. $\int \dfrac{\sec x}{\sqrt{\cos2x}} dx$ is equal to

127. Evaluate : $\int \dfrac{x^2 + 20}{(x\sin x + 5\cos x)^2} dx$

128. $\int \dfrac{x + 2}{\left(x^2 + 3x + 3\right)\left(\sqrt{x + 1}\right)} . dx$

129. Evaluate: $\int \dfrac{1}{\left(x^2\right)\sqrt{1-x^2}}\,dx$

130. Evaluate: $\int x \cdot \sqrt{1-x^4}\,dx$

131. Evaluate: $\int \dfrac{1}{(x-3)\sqrt{x+1}}\,dx$

132. Evaluate: $\int (x-5)\sqrt{x^2+x}\,dx$

133. Evaluate: $\int \dfrac{x + 1}{(x - 1)\sqrt{x + 2}} dx$

134. Evaluate: $\int \sec^3 x \, dx$

135. Evaluate: $\int \dfrac{x^2 - 1}{\left(x^2 + 1\right)\sqrt{x^4 + 1}} dx$

136. Evaluate: $\int \dfrac{1}{(x + 1)\sqrt{x^2 - 1}} dx$

137. Evaluate: $\int \dfrac{1}{\left(x^2 - 4\right)\sqrt{x + 1}}\,dx$

138. Evaluate: $\int \dfrac{1}{(x + 1)\sqrt{x^2 + x + 1}}\,dx$

139. Evaluate: $\int \dfrac{x}{\left(x^2 + 4\right)\sqrt{x^2 + 1}}\,dx$

140. Evaluate: $\int \dfrac{x}{(x - 1)\left(x^2 + 4\right)}\,dx$

141. Evaluate: $\int e^{\sin^{-1}x}dx$

142. Evaluate: $\int \dfrac{x^2+1}{(x-1)^2(x+3)}dx$

143. $\int \sin^3 x \cos^5 x \ dx$

144. Find: $\int \dfrac{dx}{\sin x \cos^3 x}$

145. Find $\int \dfrac{e^x(1 + x)}{\cos^2\left(xe^x\right)}dx$

146. Evaluate: $\int \dfrac{\sin^3 x dx}{\left(\cos^4 x + 3\cos^2 x + 1\right)\tan^{-1}(\sec x + \cos x)}$

147. Evaluate: $\int \dfrac{e^{\sqrt{x}}\cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}}dx$

148. Evaluate: $\int \dfrac{\tan x}{a + b\tan^2 x}dx$

149. Evaluate: $\int \dfrac{1}{\left(\sqrt{e^{5x}}\left(e^{2x} + e^{-2x}\right)^3\right)^{\frac{1}{4}}}$

150. Find: $\int \sin^5 x \, dx$

151. Evaluate: $\int \dfrac{1}{\left(x^2 + 2x + 2\right)} dx$

152. Evaluate: $\int \left(\left(\dfrac{e}{x}\right)^x + \left(\dfrac{x}{e}\right)^x\right) \ln x \, dx.$

153. Evaluate: $\int \dfrac{x}{x^4 + x^2 + 1} dx$

154. Evaluate: $\int \dfrac{x^2 - 1}{\left(x^4 + 3x^2 + 1\right)\tan^{-1}\left(x + \dfrac{1}{x}\right)} dx$

155. Evaluate: $\int \dfrac{x^2 - 1}{\left(x^2 + 1\right)\sqrt{1 + x^4}}$

156. Evaluate: $\int \dfrac{1}{2x^2 + x - 1} dx$

157. Evaluate: $\int \dfrac{x^2 + 4}{x^4 + 16}dx$

158. Evaluate: $\int \sqrt{\tan\theta}d\theta$

159. Evaluate: $\int \dfrac{3\sin x + 2\cos x}{3\cos x + 2\sin x}dx$

160. Evaluate: $\int \dfrac{x^2 + 1}{x^4 + 1}dx$

161. Evaluate: $\int e^x \left(\dfrac{1}{x} - \dfrac{1}{x^2} \right) dx$

162. Evaluate: $\int \dfrac{1}{x^4 + 1} dx$

163. Evaluate: $\int \dfrac{1}{\sin^4 x + \cos^4 x} dx$

164. Evaluate: $\int [f(x)g''(x) - f''(x)g(x)] dx$

165. Evaluate: $\int \left(\log(\log x) + \dfrac{1}{(\log x)^2} \right) dx$

166. Evaluate: $\int x^3 d\left(\tan^{-1} x \right)$

167. Evaluate: $\int \sin^2(\log x) dx$

168. Evaluate: $\int \left(\dfrac{\cos x}{x} - \log x^{\sin x} \right) dx$

169. Evaluate: $\displaystyle \int \frac{dx}{x^2 \left(1 + x^5\right)^{\frac{4}{5}}}$

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170. Evaluate: $\displaystyle \int \frac{1 + x^4}{\left(1 - x^4\right)^{\frac{3}{2}}} dx$

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171. Evaluate: $\displaystyle \int \frac{dx}{\sqrt[3]{\sin^{11} x \cos x}}$

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172. Evaluate: $\displaystyle \int \frac{\cos 2x}{2 + \sin 2x} dx$

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173. Evaluate $\int \dfrac{\log_e\left(x + \sqrt{x^2 + 1}\right)}{\sqrt{x^2 + 1}}\,dx.$

174. Evaluate: $\int \dfrac{\sqrt{\tan x}}{\sin x \cos x}\,dx$

175. Evaluate: $\int \dfrac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4}\,dx$

176. Evaluate: $\int \dfrac{1}{\left[(x - 1)^3(x + 2)^5\right]^{\frac{1}{4}}}\,dx$

177. Evaluate: $\int \dfrac{x^3 + 1}{x^2 + x} dx$

178. Evaluate: Evaluate: $\int x^{-11}\left(1 + x^4\right)^{-\frac{1}{2}} dx$

179. Evaluate: $\int \dfrac{2x - \sqrt{\sin^{-1}x}}{\sqrt{1 - x^2}} dx$

180. Evaluate: $\int \dfrac{e^{2x} - 1}{e^{2x} + 1} dx$

181. Evaluate: $\int \dfrac{\sin x}{\sin(x - a)} dx$

182. Evaluate: $\int \dfrac{dx}{\cos x - \sin x}$

183. Evaluate: $\int \sin\left(e^x\right) d\left(e^x\right)$

184. Evaluate: $\int \tan^3 x \, dx$

185. Evaluate : $\int \dfrac{\sin 2x}{a^2 + b^2 \sin^2 x} dx$

186. Evaluate: $\int \cos^3 x \sqrt{\sin x} dx$

187. Evaluate: $\int \dfrac{dx}{x^{\frac{1}{2}} + x^{\frac{1}{3}}}$

188. Evaluate: $\int \left(2^{2^{2^x}} \left(2^{2^x} \right) \left(2^x \right) \right) dx$

189. Evaluate: $\int \dfrac{e^x}{e^{2x} + 6e^x + 5} dx$

190. The number of irrational roots of the equation

$$\frac{4x}{x^2 + x + 3} + \frac{5x}{x^2 - 5x + 3} = -\frac{3}{2}$$ is

A. *a.* 4

B. *b.* 0

C. *c.* 1

D. *d.* 2

Answer: null

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191. Evaluate: $\int \sqrt{\frac{1 + x}{x}} dx$

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192. Evaluate: $\int \dfrac{4x + 1}{x^2 + 3x + 2} dx$

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193. Evaluate: $\int \dfrac{dx}{\left(a^2 + x^2\right)^{\frac{3}{2}}}$

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194. Evaluate: $\int \dfrac{1}{x^2\sqrt{1 + x^2}} dx$

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195. Evaluate: $\int \dfrac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$

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196. Evaluate: $\int \dfrac{1}{\sqrt{(x-1)(x-2)}} dx$

197. Evaluate: $\int \sqrt{\dfrac{x}{a^3 - x^3}} dx$

198. Evaluate: $\int \dfrac{x^2}{\sqrt{1 - x^6}} dx$

199. Evaluate: $\int \dfrac{\left[\sqrt{1 + x^2} + x\right]^n}{\sqrt{1 + x^2}} dx$

200. Evaluate: $\int \dfrac{dx}{(x - p)\sqrt{(x - p)(x - q)}}$

201. Evaluate: $\int \sec^5 x \cos ec^3 x \, dx$

202. Evaluate: $\int \dfrac{dx}{\cos^3 x \sqrt{\sin 2x}}$

203. $\int \left(\left(x^4 - x \right)^{\frac{1}{4}} \right) \dfrac{dx}{x^5}$

204. $\displaystyle\int \frac{1}{x^2\left(x^4 + 1\right)^{\frac{3}{4}}}\,dx$

205. Evaluate: $\displaystyle\int x^x 1n(ex)\,dx$

206. Evaluate: $\displaystyle\int \frac{x + 1}{(x - 1)\sqrt{x + 2}}\,dx$

207. Evaluate: $\displaystyle\int \{1 + 2\tan x(\tan x + \sec x)\}^{\frac{1}{2}}\,dx$

208. Evaluate: $\int \dfrac{dx}{(1 + \sin x)^{\frac{1}{2}}}$

209. Find $\int \dfrac{\sin^6 x}{\cos^8 x} dx.$

210. Evaluate: $\int \sin 2x \, d(\tan x)$

211. Evaluate: $\int \dfrac{1}{\sqrt{3}\sin x + \cos x} dx$

212. Evaluate: $\int \dfrac{1}{\sin(x - a)\sin(x - b)} dx$

213. Evaluate: $\int \left(1 + 2x + 3x^2 + 4x^3 + \dots \right) dx$, $(|x| < 1)$

214. Evaluate: $\int \dfrac{\ln(\ln x)}{x \ln x} dx$, $(x > 0)$

215. Evaluate: $\int \dfrac{dx}{x + x \log x}$

216. Evaluate: $\int \sec^p x \tan x \, dx$

217. Evaluate: $\int \csc^4 x \, dx$

218. Evaluate: $\int \dfrac{\cos x - \sin x}{\cos x + \sin x}(2 + 2\sin 2x)dx$

219. Evaluate: $\int \tan^2 x \sin^2 x \, dx$

220. Evaluate: $\int \dfrac{e^{3x} + e^{5x}}{e^x + e^{-x}} dx$

221. Evaluate: $\int \dfrac{\cos2x - \cos2\theta}{\cos x - \cos\theta}\,dx$

222. Evaluate: $\int \dfrac{(1 + 1nx)^5}{x}\,dx$

223. Evaluate: $\int \sin x \cos x \cos 2x \cos 4x \cos 8x\,dx$

224. Evaluate: $\int \dfrac{\sin 2x}{(a + b\cos x)^2}\,dx$

225. Evaluate: $\dfrac{1}{\sqrt{1 - e^{2x}}} dx$

226. Evaluate: $\displaystyle\int \dfrac{2x + 3}{\sqrt{x^2 + 4x + 1}} dx$

227. Evaluate: $\displaystyle\int \dfrac{dx}{\sqrt{x} + \sqrt{x - 2}}$

228. Evaluate: $\displaystyle\int \dfrac{x^3}{x + 1} dx$

229. Evaluate: $\int x\sin 3x\,dx$

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230. Evaluate: $\int x\log x\,dx$

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231. Evaluate: $\int \sin^{-1}x\,dx$

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232. Evaluate: $\int \dfrac{x - \sin x}{1 - \cos x}\,dx$

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233. Evaluate: $\int \sqrt{x^2 + 2x + 5}\ dx$

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234. Evaluate: $\int \sqrt{1 + 3x - x^2}\ dx$

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235. Evaluate: $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\ dx$

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236. Evaluate: $\int \frac{\cot x}{\sqrt{\sin x}}\ dx$

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237. Evaluate: $\int \dfrac{\left(\sin^{-1}x\right)^3}{\sqrt{1-x^2}}dx$

238. Evaluate: $\int \left(\dfrac{x+1}{x}\right)(x+\log x)^2 dx$

239. Evaluate: $\int \tan^4 x\,dx$

240. Evaluate: $\int (\tan x - x)\tan^2 x\,dx$

241. Evaluate: $\displaystyle \int \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{\sin x} dx$

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242. Evaluate: $\displaystyle \int \frac{\sqrt{2 + \log x}}{x} dx$

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243. Evaluate: $\displaystyle \int \frac{\log x}{(1 + \log x)^2} dx$

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244. $\displaystyle \int [\sin(\log x) + \cos(\log x)] dx$

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245. Evaluate: $\int e^x \left(\dfrac{1 - \sin x}{1 - \cos x} \right) dx$

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246. Evaluate: $\int \dfrac{e^x x}{(x + 1)^2} dx$

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247. If $f(x)$ is polynomaial function of degree n, prove that

$\int e^x f(x) dx = e^x \left[f(x) - f'(x) + f''(x) - f'''(x) + \ldots\ldots + (-1)^n f^n(x) \right]$ where

$f^n(x) = \dfrac{d^n f}{dx^n}$

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248. Evaluate: $\int e^x \left(f(x) + f'(x) \right) dx = e^x f(x) + C$

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249. Evaluate: $\int \dfrac{dx}{x^2\left(x^4 + 1\right)^{\frac{3}{4}}}$ (A) $-\left(1 + \dfrac{1}{x^4}\right)^{\frac{1}{4}} + c$ (B) $\left(1 + \dfrac{1}{x^4}\right)^{\frac{1}{4}} + c$ (C)

$-\left(1 - \dfrac{1}{x^4}\right)^{\frac{1}{4}} + c$ (D) $-\left(1 + \dfrac{1}{x^4}\right)^{\frac{1}{4}} + c$

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250. Evaluate: $\int \dfrac{dx}{\sqrt{2ax - x^2}}$

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251. Evaluate: $\int \sin^{-1}\sqrt{\dfrac{x}{a + x}}\, dx$

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252. Evaluate: $\int \dfrac{x^{\frac{5}{2}}}{\sqrt{1 + x^7}} dx$

253. Evaluate: $\int \dfrac{\sec^2 x}{3 + \tan x} dx$

254. Evaluate: $\int \dfrac{x}{\sqrt{x + 2}} dx$

255. Evaluate: $\int \dfrac{1 - \tan x}{1 + \tan x} dx$

256. Evaluate: $\int \dfrac{e^x - e^{-x}}{e^x + e^{-x}} dx$

257. Evaluate: $\int \dfrac{1}{\sqrt{3x + 4} - \sqrt{3x + 1}} dx$.

258. Evaluate: $\int \dfrac{dx}{(2x - 7)\sqrt{(x - 3)(x - 4)}}$

259. Evaluate: $\int \left(x + \dfrac{1}{x} \right)^{\frac{3}{2}} \left(\dfrac{x^2 - 1}{x^2} \right) dx$

260. Find the values of a and b such that $\int \dfrac{dx}{1 + \sin x} = \tan\left(\dfrac{x}{2} + a\right) + b$

261. Evaluate: $\int \dfrac{\sin 2x}{a^2\sin^2 x + b^2\cos^2 x}\,dx$

262. Evaluate: $\int \dfrac{1}{1 + e^{-x}}\,dx$

263. Evaluate: $\int \dfrac{e^x\left(2 - x^2\right)dx}{(1 - x)\sqrt{1 - x^2}}$

264. Evaluate: $\int e^x \left(1 + \tan x + \tan^2 x\right) dx$

265. If $\int g(x)dx = g(x)$, then evaluate $\int g(x)\left\{f(x) + f'(x)\right\}dx$

266. Evaluate: $\int \cos\sqrt{x}dx$

267. Evaluate: $\int x\sin^2 x dx$

268. Evaluate: If $\int f(x)dx = g(x)$, then $\int f^{-1}(x)dx$

269. Evaluate: $\int \cos x \log\left(\tan\dfrac{x}{2}\right) dx$

270. Evalaute: $\int \left(\dfrac{\log x - 1}{1 + (\log x)^2}\right)^2 dx$

271. Evaluate: $\int \dfrac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx$

272. Evaluate: $\int \tan^{-1}\sqrt{x}\, dx$

273. Evaluate $\int \sin^3 x \, dx$

274. Evaluate $\int \sin 2x \sin 3x \, dx$.

275. Evaluate: $\int a^{mx} b^{nx} \, dx$

276. Evaluate $\int \dfrac{\tan x}{\sec x + \tan x} \, dx$

277. If $\int \dfrac{1}{x + x^5} dx = f(x) + c$,then the value of $\int \dfrac{x^4}{x + x^5} dx$.

278. If $\int \dfrac{\left(x^3 + 8\right)(x - 1)}{x^2 - 2x + 4} dx$

279. Integrate the functions $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

280. Evaluate: $\int \tan^{-1}(\sec x + \tan x) dx, -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

281. Evaluate: $\int \dfrac{x + 2}{(x + 1)^2} dx.$

282. Evaluate $\int \left(\dfrac{8x + 13}{\sqrt{4x + 7}} \right) dx$

A. (A) $\dfrac{1}{3}(4x + 7)^{\frac{3}{2}} - \dfrac{1}{2}(4x + 7)^{\frac{1}{2}} + c$

B. (B) $\dfrac{1}{6}(4x + 7)^{\frac{5}{2}} - \dfrac{2}{3}(4x + 7)^{\frac{3}{2}} + c$

C. (C) $\dfrac{1}{3}(4x + 7)^{\frac{5}{2}} - \dfrac{1}{2}(4x + 7)^{\frac{3}{2}} + c$

D. (D) $(4x + 7)^{\frac{3}{2}} - \dfrac{1}{2}(4x + 7)^{\frac{1}{2}} + c$

Answer: null

283. Evaluate: $\int \dfrac{\sec x \, dx}{\sqrt{\cos 2x}}$

284. Evaluate: $\int \sin^3 x \cos^2 x \, dx$

285. Evaluate: $\int \dfrac{x \, dx}{\sqrt{\left(1 + x^2\right)} + \sqrt{\left(1 + x^2\right)^3}}$

286. Evaluate: $\int \dfrac{2x + 1}{x^4 + 2x^3 + x^2 - 1} \, dx$

287. Evaluate: $\int \dfrac{1}{x^2 - x + 1} \, dx$

288. Evaluate: $\int \dfrac{1}{2x^2 + x - 1}\,dx$

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289. Evaluate: $\int \dfrac{\cos x}{\sin\left(x - \dfrac{\pi}{6}\right)\sin\left(x + \dfrac{\pi}{6}\right)}\,dx$

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290. Evaluate: $\int \dfrac{\sin x}{\sin 3x}\,dx$

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291. Evaluate: $\int \dfrac{1}{3 + \sin 2x}\,dx$

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292. Evaluate: $\int \dfrac{1}{1 + \sin x + \cos x} dx$

293. Evaluate: $\int \dfrac{2x - 1}{(x - 1)(x + 2)(x - 3)} dx$

294. Evaluate: $\int \tan^{-1} \sqrt{\dfrac{1 - x}{1 + x}} dx$

295. Evaluate: $\int \dfrac{1}{\sin x - \sin 2x} dx$

296. Evaluate: $\int \dfrac{2x}{\left(x^2 + 1\right)\left(x^2 + 2\right)} dx$

297. Evaluate: $\int \dfrac{(x - 1)(x - 2)(x - 3)}{(x - 4)(x - 5)(x - 6)} dx$

298. Evaluate: $\int \dfrac{1 - \cos x}{\cos x (1 + \cos x)} dx$

299. Evaluate : $\int (1 - \cos x) \cosec^2 x \, dx$

300. Evaluate: $\int (\sec x + \tan x)^2 dx$

301. Evaluate: $\int \dfrac{\sec x}{\sec x + \tan x} dx$

302. Evaluate: $\int \tan^{-1}\left\{\sqrt{\left(\dfrac{1 - \cos 2x}{1 + \cos 2x}\right)}\right\}$

303. Evaluate: $\int \dfrac{1}{1 + \sin x} dx$

304. Evaluate $\int \dfrac{1 - \cos x}{1 + \cos x} dx.$

305. Evaluate: $\int \sec^2 x \cosec^2 x \, dx$

306. Evaluate $\int \dfrac{2^{x+1} - 5^{x-1}}{10^x}$

307. Evaluate: $\int \dfrac{(1 + x)^3}{\sqrt{x}} dx$

308. Evaluate $\int \dfrac{\cos 2x \sin 4x \, dx}{\cos^4 x \left(1 + \cos^2 2x\right)}$

309. Evaluate: $\int \dfrac{ax^3 + bx}{x^4 + c^2} \, dx$

310. Evaluate $\int \dfrac{e^{2x} - 2e^x}{e^{2x} + 1} \, dx$

311. Evaluate: $\int \dfrac{dx}{9 + 16\sin^2 x}$

312. Find $\int \dfrac{dx}{\sqrt{x} + x}$

313. Evaluate: $\int \dfrac{\cot x}{\sqrt{\sin x}} dx$

314. Evaluate: $\int \dfrac{\sqrt{x} dx}{1 + x}$

315. Evaluate: $\int \dfrac{x^2 \tan^{-1} x^3}{1 + x^6} dx$

316. Evaluate: $\int \sin^2 x \cos^2 x\, dx$

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317. Evaluate: $\int \dfrac{1}{x^4 - 1}\, dx$

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318. If the product of n positive numbers is n^n. Then their sum is

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319. Evaluate: $\int \dfrac{dx}{\sin x \left(3 + \cos^2 x\right)}$

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320. Evaluate: $\int \dfrac{x^2 + 1}{x\left(x^2 - 1\right)} dx$

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321. Evaluate: $\int \dfrac{\sin x}{\sin 4x} dx$

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322. Evaluate $\int \dfrac{\log_{ex} e \cdot \log_{e^2 x} e \cdot \log_{e^3 x} e}{x} dx.$

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323. Evaluate: $\int e^{3\log x}\left(x^4 + 1\right)^{-1} dx$

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324. Evaluate: $\displaystyle\int \dfrac{dx}{x^{\frac{2}{3}}\left(1 + x^{\frac{2}{3}}\right)}$

325. $\displaystyle\lim_{x \to 0} \dfrac{1}{x}\left[\int_y^a e^{\sin^2 t}dt - \int_{x+y}^a e^{\sin^2 t}dt\right]$ is equal to

A. (a) $(0, 1]$

B. (b) $[1, \infty)$

C. (c) $(0, \infty)$

D. (d) none of these

Answer: null

326. A function f is continuous for all x (and not everywhere zero) such that $f^2(x) = \int_0^x f(t) \dfrac{\cos t}{2 + \sin t} dt$ Then $f(x)$ is (a) $\dfrac{1}{2} \ln\left(\dfrac{x + \cos x}{2}\right)$; $x \neq 0$ (b) $\dfrac{1}{2} \ln\left(\dfrac{3}{x + \cos x}\right)$; $x \neq 0$ (c) $\dfrac{1}{2} \ln\left(\dfrac{2 + \sin x}{2}\right)$; $x \neq n\pi, n \in I$ (d) $\dfrac{\cos x + \sin x}{2 + \sin x}$; $x \neq n\pi + \dfrac{3\pi}{4}, n \in I$

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327. If $\int_{\cos x}^1 \left(t^2 f(t) dt = 1 - \cos x \ \forall x \in \left(0, \dfrac{\pi}{2}\right)\right.$, then the value of $\left[f\left(\dfrac{\sqrt{3}}{4}\right)\right]$ is

[.] denotes the greatest integer function) 4 (b) 5 (c) 6 (d) -7

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328. If $\int_0^y \cos t^2 dt = \int_0^{x^2} \dfrac{\sin t}{t} dx$, theprovethat $\dfrac{dy}{dx} = \dfrac{2 \sin x^2}{x \cos y^2}$

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329. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of $f(1)$

330. Evaluate: $\lim x \to \infty \dfrac{\left(\int_0^x e^{x^2}dx\right)^2}{\int_0^x e^{2x^2}dx}$

331. If $f(x) = 1 + \dfrac{1}{x}\int_1^x f(t)dt$, then the value of $f\left(e^{-1}\right)$ is

332. If $\int_0^1 e^{-x^2}dx = a$, then find the value of $\int_0^1 x^2 e^{-x^2}dx$ in terms of a.

333. If $\int_0^{f(x)} t^2 dt = x\cos\pi x$, then $f'(9)$ is $-\dfrac{1}{9}$ (b) $-\dfrac{1}{3}$ (c) $\dfrac{1}{3}$ (d) non-existent

334. If $I_n = \int_0^\pi x^n\sin x\,dx$, then find the value of $I_5 + 20I_3$

335. The value of $\int_0^1 \lim_{n \to \infty} \sum_{k=0}^n \dfrac{x^{k+2}2^k}{k!}dx$ is: (a) $e^2 - 1$ (b) 2 (c) $\dfrac{e^2 - 1}{2}$ (d) $\dfrac{e^2 - 1}{4}$

336. If $I_n = \int_0^{\frac{\pi}{2}} \sin^n x\,dx$, then show that $I_n = ((n - 1)n)I_{n-2}$.

Hence prove that

$$
I_n = \begin{cases}
\left(\dfrac{n-1}{n}\right)\left(\dfrac{n-3}{n-2}\right)\left(\dfrac{n-5}{n-4}\right)\cdots\cdots\left(\dfrac{1}{2}\right)\dfrac{\pi}{2} & \text{if } n \text{ is even} \\[4mm]
\left(\dfrac{n-1}{n}\right)\left(\dfrac{n-3}{n-2}\right)\left(\dfrac{n-5}{n-4}\right)\cdots\cdots\left(\dfrac{2}{3}\right)1 & \text{if } n \text{ is odd}
\end{cases}
$$

337. If $A = \int_0^1 x^{50}(2 - 2x)^{50}dx$, $B = \int_0^1 x^{50}(1 - x)^{50}dx$, which of the following is true? (A) $A = 2^{50}B$ (B) $A = 2^{-50}B$ (C) $A = 2^{100}B$ (D) $A = 2^{-100}B$

338. If $I_n = \int_0^1 \dfrac{dx}{\left(1 + x^2\right)^n}$; $n \in N,$ then prove that $2nI_{n+1} = 2^{-n} + (2n - 1)I_n$

339. Given that f satisfies $|f(u) - f(v)| \le |u - v|$ f or u and v in $[a, b]$ Then $\left|\int_a^b f(x)dx - (b - a)f(a)\right| \le$ (a) $\dfrac{(b - a)}{2}$ (b) $\dfrac{(b - a)^2}{2}$ (c) $(b - a)^2$ (d) none of these

340. If $I_n = \int_0^1 x^n \left(\tan^{-1} x \right) dx$, thenprovethat $(n + 1)I_n + (n - 1)I_{n-2} = -\frac{1}{n} + \frac{\pi}{2}$

341. If $I = \int_{-20\pi}^{20\pi} |\sin x| [\sin x] dx$ (where[.] denotes the greatest integer function), then the value of I is -40 (b) 40 (c) 20 (d) -20

342. If $I_m = \int_1^x (\log x)^m dx$ satisfies the relation $\left(I_m \right) = k - lI_{m-1}$ then

343. The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ i s (a) $\frac{\pi}{2} + 1$ (b) $\frac{\pi}{2} - 1$ (c)-1 (d) 1

344. If $I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx$, Then show that

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n} (m, n \in N)$$ Hence, prove that

$$I_{m,n} = f(x) = \left\{ \frac{(n-1)(n-3)(m-5)(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)} \frac{\pi}{4} \right. \text{ when both } m \text{ and } n$$

are even

$$\left. \frac{(m-1)(m-3)(m-5)(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)} \right\}$$

345. Evaluate: $\int_0^{\pi} \left| x \sin 2x \frac{\sin\left(\frac{\pi}{2} \cos x\right)}{2x - \pi} dx \right.$

346. If $I_K = \int_1^e (\ln x)^k dx \left(k \in I^+ \right) dx \left(k \in I^+ \right)$, then find the value of I_4

347. If $\int_0^x \left\{ \int_0^u f(t)dt \right\} du$ is equal to (a) $\int_0^x (x - u)f(u)$ (b) $\int_0^x uf(x - u)du$ (c) $x\int_0^x f(u)du$ (d) $x\int_0^x uf(u - x)du$

348. Prove that: $I_n = \int_0^\infty x^{2n+1} e^{-x^2} dx = \dfrac{n!}{2}, n \in N.$

349. The value of the integral

$$\int_{-1}^{3} \left(\tan^{-1}\dfrac{x}{x^2 + 1} + \tan^{-1}\dfrac{x^2 + 1}{x} \right) dx \text{ is equal to}$$

350. It is known that $f(x)$ is an odd function in the interval $\left[-\dfrac{p}{2}, \dfrac{p}{2} \right]$ and has a period p, Prove that $\int_a^x f(t)dt$ is also periodic function with the same period.

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351. Evaluate: $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4}(3 - f(x))dx = 7$, then find the value of $\int_{-1}^{2} f(x)dx$

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352. If $\int_0^{\frac{\pi}{2}} \log\sin\theta\, d(\theta) = k$, then find the value of $\int_0^{\frac{\pi}{2}} \left(\dfrac{\theta}{\sin\theta} \right)^2 d(\theta)$ in terms of k

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353. Evaluate: $\int_0^{\frac{\pi}{2}} |\sin x - \cos x| \, dx$

354. Let $f(x)$ is continuous and positive for $x \in [a, b]$, $g(x)$ is continuous for $x \in [a, b]$ and $\int_a^b |g(x)| dx > \left| \int_a^b g(x) dx \right|$ STATEMENT 1 : The value of $\int_a^b f(x) g(x) dx$ can be zero. STATEMENT 2 : Equation $g(x) = 0$ has at least one root for $x \in (a, b)$.

355. Evaluate: $\int_{-5}^{5} x^2 \left[x + \dfrac{1}{2} \right] dx$ (where [.] denotes the greatest integer function).

356. STATEMENT 1 : The value of $\int_{-4}^{-5}\sin\left(x^2 - 3\right)dx + \int_{-2}^{-1}\sin\left(x^2 + 12x + 33\right)$ is zero. STATEMENT 2 : $\int_{-a}^{a}f(x)dx = 0 \Leftrightarrow f(x)$ is an odd function.

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357. Evaluate: $\int_{0}^{2\pi}[\sin x]dx$,where[.] denotes the greatest integer function.

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358. STATEMENT 1 : The value of $\int_{0}^{1}\tan^{-1}\left(\dfrac{2x - 1}{1 + x - x^2}\right)dx = 0$ STATEMENT 2 :

$\int_{a}^{b}f(x)dx = \int_{0}^{b}f(a + b - x)dx$ then Which of the following statement is correct ?

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359. Evaluate: $\int_{0}^{\infty}\left[2e^{-x}\right]dx,$ $where[x]$ represents greatest integer function.

360. STATEMENT 1 : On the interval $\left[\dfrac{5\pi}{4}, \dfrac{4\pi}{3}\right]$ the least value of the function $f(x) = \int_{\frac{5x}{4}}^{x}(3\sin t + 4\cos t)dt$ is 0 STATEMENT 2 : If $f(x)$ is a decreasing function on the interval $[a, b]$, then the least value of $f(x)$ is $f(b)$

361. Evaluate: $\int_{0}^{10\pi}\left[\tan^{-1}x\right]dx, where [x]$ represents greatest integer function.

362. Consider the function $f(x)$ satisfying the relation $f(x + 1) + f(x + 7) = 0 \,\forall x \in R$ STATEMENT 1 : The possible least value of t for which $\int_{a}^{a+t}f(x)dx$ is independent of a is 12. STATEMENT 2 : $f(x)$ is a periodic function.

363. Evaluate: $\int_0^{\frac{5\pi}{2}} [\tan x]dx$, where [.] denotes the greatest integer function.

364. Consider $I_1 = \int_0^{\pi/4} e^{x^2}dx, I_2 = \int_0^{\pi/4} e^x dx, I_3 = \int_0^{\pi/4} e^{x^2}\cos x dx$,
$I_4 = \int_0^{\pi/4} e^{x^2}\sin x dx$.

STATEMENT 1 : $I_2 > I_1 > I_3 > I_4$

STATEMENT 2 : For $x \in (0, 1), x > x^2 and \sin x > \cos x$.

365. Evaluate: $\int_0^2 \left[x^2 - x + 1\right]dx$, $where$[.] denotos the greatest integer function.

366. If $f(x) = A\sin\left(\dfrac{\pi x}{2}\right) + b, f'\left(\dfrac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \dfrac{2A}{\pi}$ the constants A

and B are (a) $\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$ (b) $\dfrac{2}{\pi}$ and $\dfrac{3}{\pi}$ (c) 0 and $-\dfrac{4}{\pi}$ (d) $\dfrac{4}{\pi}$ and 0

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367. If $[x]$ denotes the greatest integer less than or equal to $x,$ then find

the value of the integral $\int_0^2 x^2[x]dx$

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368. The value of $\int_0^{2\pi}[2\sin x]dx,$ $where[.]$ represents the greatest integral

function, is (a) $\dfrac{-5\pi}{3}$ (b) $-\pi$ (c) $\dfrac{5\pi}{3}$ (d) -2π

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369. $f(x) = \int_1^x \dfrac{\tan^{-1}(t)}{t}dt$ $\forall x \in R^+,$ then find the value of $f\left(e^2\right) - f\left(\dfrac{1}{e^2}\right)$

370. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} xf([x(1-x)])dx$,

$I_2 = \int_{1-k}^{k} f[x(1-x)]dx$, where $2k-1 > 0$. Then $\dfrac{I_1}{I_2}$ is 2 (b) k (c) $\dfrac{1}{2}$ (d) 1

371. Find the points of minima for $f(x) = \int_0^x t(t-1)(t-2)dt$

372. If $f(x) = e^{g(x)}$ and $g(x) = \int_2^x \dfrac{tdt}{1+t^4}$, then find the value of $f'(2)$

373. If $g(x) = \int_0^x \cos^4 t\, dt$, then $g(x + \pi)$ equals (a)$g(x) + g(\pi)$ (b) $g(x) - g(\pi)$ (c) $g(x)g(\pi)$ (d) $\dfrac{g(x)}{g(\pi)}$

374. Evaluate $\left(\lim \right)_{x \to 4} 4\int_4^x \frac{(4t - f(t))}{(x - 4)} dt$

375. The Integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$ is equal to:

376. If $I_n = \int_0^1 \left(1 - x^5\right)^n dx$, then $\frac{55}{7} \frac{I_{10}}{I_{11}}$ is equal to ___

377. Let f be a one-to-one continuous function such that $f(2) = 3$ and $f(5) = 7$. Given $\int_2^5 f(x)dx = 17$, then find the value of $\int_3^7 f^{-1}(x)dx$.

378. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x] + 1n\left(\frac{1+x}{1-x} \right) \right) dx$ is equal to (where [.] represents the greatest integer function)

A. (a) $-\frac{1}{2}$

B. (b) 0

C. (c) 1

D. (d) $2\ln\left(\frac{1}{2}\right)$

Answer: null

379. Evaluate: $\int_0^{e-1} \frac{e^{\frac{x^2+2x-1}{2}}}{x+1} dx + \int_1^e x\log x e^{\frac{x^2-2}{2}} dx$

380. Evaluate: $\lim_{x \to 0} \dfrac{x - \left(\int_0^x \cos t^2 dt\right)}{x^3 - 6x}$

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381. The value of $2^{2010} \dfrac{\int_0^1 x^{1004}(1 - x)^{1004} dx}{\int_0^1 x^{1004}\left(1 - x^{2010}\right)^{1004} dx}$

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382. If $\int_0^1 \dfrac{e^t}{1 + t} dt = a$ then find the value of $\int_0^1 \dfrac{e^t}{(1 + t)^2} dt$ in terms of a.

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383. If $f(x) = x + \int_0^1 t(x + t)f(t)dt$,then the value of $\dfrac{23}{2}f(0)$ is equal to _____

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384. Let $f:(0,\infty) \xrightarrow{} 0, \infty$ be a differentiable function satisfying,

$\xi nt_0^x(1-t)f(t)dt = \int_0^x tf(t)dtx \in R^+ and f(1) = 1.$ Determine $f(x)$

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385. If $f(x) and g(x)$ are continuous functions, then

$$\int_{1n\lambda}^{1n\frac{1}{\lambda}} \frac{f\left(\dfrac{x^2}{4}\right)[f(x) - f(-x)]}{g\left(\dfrac{x^2}{4}\right)[g(x) + g(-x)]} dxis$$ (a)dependent on λ (b) a none-zero constant

(c)zero (d) none of these

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386. The value of $\int_{-\pi}^{\pi} \dfrac{\cos^2 x}{1 + a^x} dx, a > 0$ is

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387. If $\int_0^1 \dfrac{e^t dt}{t+1} = a$, then evaluate $\int_{b-1}^b \dfrac{e^t dt}{t-b-1}$

388. If $f(x) = x + \sin x$, then find the value of $\int_\pi^{2\pi} f^{-1}(x)dx$.

389. Let $F(x) = \int_x^{x^2 + \frac{\pi}{6}} \left(2\cos^2 t\right)dt$ for all $x \in R$ and $f: \left[0, \dfrac{1}{2}\right] \rightarrow [0, \infty)$ be a

continuous function.For $a \in \left[0, \dfrac{1}{2}\right]$, if F'(a)+2 is the area of the region

bounded by x=0,y=0,y=f(x) and x=a, then f(0) is

390. If $f(x)$ is a function satisfying $f\left(\dfrac{1}{x}\right) + x^2 f(x) = 0$ for all nonzero x , then

evaluate $\int_{\sin\theta}^{\csc\theta} f(x)dx$

391.

$I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$, $I_2 = \int_0^{2\pi} \cos^6 x \, dx$, $I_3 = \int_{\frac{\pi}{2}}^{\pi} \sin^3 x \, dx$, $I_4 = \int_0^1 \ln\left(\frac{1}{x} - 1\right) dx$ Then

$I_2 = I_3 = I_4 = 0, I_1 \neq 0$ $I_1 = I_2 = I_3 = 0, I_4 \neq 0$ $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

$I_1 = I_4 = I_3 = 0, I_2 \neq 0$

392. The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{(\log)_e x}{x}\right| dx$ is (A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) 3 (D) 5

393. If $f(x) = \int_1^x \frac{\log t}{1 + t + t^2} dx \, \forall x \leq 1$, then prove that $f(x) = f\left(\frac{1}{x}\right)$.

394. If $f(x) = \int_0^\pi \dfrac{t\sin t\, dt}{\sqrt{1 + \tan^2 x \sin^2 t}}$ for $0 < x < \dfrac{\pi}{2}$, then (a) $f\left(0^+\right) = -\pi$ (b)

$f\left(\dfrac{\pi}{4}\right) = \dfrac{\pi^2}{8}$ (c) f is continuous and differentiable in $\left(0, \dfrac{\pi}{2}\right)$ (d) f is

continuous but not differentiable in $\left(0, \dfrac{\pi}{2}\right)$

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395. Evaluate: $\int_{\sqrt{2}-1}^{\sqrt{2}+1} \dfrac{\left(x^2 - 1\right)}{\left(x^2 + 1\right)^2}\, dx$

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396. $\int_0^x \dfrac{2^t}{2^{[t]}}\, dt$, where[.] denotes the greatest integer function and $x \in R^+$

, is equal to

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397. If $y = \int_0^x f(t)\sin\{k(x - t)\}dt$, then prove that $\dfrac{d^2y}{dx^2} + k^2y = kf(x)$.

398. $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x)dx = \dfrac{n^2}{2}$ $\forall n \in I$ Then $\int_{-3}^5 f(|x|)dx$ is equal to (a) $\dfrac{19}{2}$ (b) $\dfrac{35}{2}$ (c) $\dfrac{17}{2}$ (d) none of these

399. If $x = \int_0^y \dfrac{dt}{\sqrt{1 + 9t^2}}$ and $\dfrac{d^2y}{dx^2} = ay$, then find a

400. The value of $\int_{\frac{1}{e}}^{\tan x} \dfrac{t\,dt}{1 + t^2} + \int_{\frac{1}{e}}^{\cot x} \dfrac{dt}{t\left(1 + t^2\right)}$, where $x \in \left(\dfrac{\pi}{6}, \dfrac{\pi}{3}\right)$, is equal to: (a)0 (b) 2 (c) 1 (d) none of these

401. Prove that: $y = \int_{\frac{1}{8}}^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_{\frac{1}{8}}^{\cos^2 x} \cos^{-1}\sqrt{t}$, where $0 \le x \le \dfrac{\pi}{2}$, is the equation of a straight line parallel to the x-axis. Find the equation.

402. $f(x) > 0 \,\forall x \in R$ and is bounded. If

$$\lim_{n \to \infty} \left[\int_0^a \frac{f(x)dx}{f(x) + f(a-x)} + a\int_a^{2a} \frac{f(x)dx}{f(x) + f(3a-x)} \right.$$
$$\left. + a^2\int_{2a}^{3a} \frac{f(x)dx}{f(x) + f(5a-x)} + \dots + a^{n-1}\int_{(n-1)a}^{na} \frac{f(x)dx}{f(x) + f[(2n-1)a-x]} \right] = 7/5$$

(where $a < 1$), then a is equal to

A. (a) $\dfrac{2}{7}$

B. (b) $\dfrac{1}{7}$

C. (c) $\dfrac{14}{19}$

D. (d) $\dfrac{9}{14}$

Answer: null

403. Evaluate: $\int_1^a x a^{\cdot - \left[(\log)_a x \right]} dx, (a > 1)$

404. $\int_0^\infty \frac{\sin^2 x}{x^2} dx$ must be same as:

A. (a) $\int_0^\infty \frac{\sin x}{x} dx$

B. (b) $\left(\int_0^\infty \frac{\sin x}{x} \right)^2$

C. (c) $\int_0^\infty \frac{\cos^2 x}{x^2}$

D. (d) none of these

Answer: null

405. Evaluate: $\int_1^{e^6}\left[\dfrac{\log x}{3}\right]dx$, where[.] denotes the greatest integer function.

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406. If $\int_0^\infty \dfrac{\sin x}{x}dx = \dfrac{\pi}{2}$, then $\int_0^\infty \dfrac{\sin^3 x}{x}dx$ is equal to

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407. Find the value of $\int_{-1}^1\left[x^2 + \{x\}\right]dx$, where [.] and {. } denote the greatest function and fractional parts of x, respectiely.

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408. $\int_0^x [\cos t]dt$, where $x \in \left(2n\pi, 2n\pi + \dfrac{\pi}{2}\right), n \in N, and[.]$ denotes the greatest integer function . then the value of $f\left(\dfrac{1}{\pi}\right)$ is

409. Evaluate: $\int_{-\frac{\pi}{2}}^{2\pi} \left[\cot^{-1}x\right]dx$, where [.] denotes the greatest integer function

410. Evaluate: $\int_{-\frac{\pi}{2}}^{2\pi} \left[\cot^{-1}x\right]dx$, where [.] denotes the greatest integer function

411. $f(x)$ is continuous function for all real values of x and satisfies $\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \dfrac{x^{16}}{8} + \dfrac{x^6}{3} + a$ Then the value of a is equal to: (a)$-\dfrac{1}{24}$ (b) $\dfrac{17}{168}$ (c) $\dfrac{1}{7}$ (d) $-\dfrac{167}{840}$

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412. Prove that $\int_0^1 xe^x dx=1$

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413. $I_1 = \int_0^{\frac{\pi}{2}} \ln(\sin x)dx$, $I_2 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(\sin x + \cos x)dx$ Then (a)$I_1 = 2I_2$ (b) $I_2 = 2I_1$ $I_1 = 4I_2$ (d) $I_2 = 4I_1$

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414. Prove that $\int_0^\infty \left[ne^{-x} \right] dx = \ln\left(\dfrac{n^n}{n!} \right)$, where n is a natural number greater than 1 and $[.]$ denotes the greatest integer function..

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415. For $x \in R$ and a continuous function f, let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} xf\{x(2-x)\}dx$

and $I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f\{x(2-x)\}dx$. Then $\dfrac{I_1}{I_2}$ is (a) -1 (b) 1 (c) 2 (d) 3

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416. Evaluate of each of the following integrals $\int_{\pi/6}^{\pi/3} \dfrac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$

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417. Given a function $f:[0,4] \rightarrow R$ is differentiable ,then prove that for some $\alpha, \beta \in (0,2)$, $\int_0^4 f(t)dt = 2\alpha f\left(\alpha^2\right) + 2\beta f\left(\beta^2\right)$.

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418. Evaluate: $\int_0^{\frac{\pi}{2}} \log\left(\dfrac{4 + 3\sin x}{4 + 3\cos x}\right) dx$

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419. The value of $\int_0^1 4x^3 \left\{ \dfrac{d^2}{dx^2} \left(1 - x^2\right)^5 \right\} dx$ is

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420. Evaluate: $\int_{-\pi}^{3\pi} \log(\sec\theta - \tan\theta) d\theta$

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421. The value of the integral $\int_0^1 e^{x^2} dx$ lies in the integral

422. Let $P(x)$ be a polynomial of least degree whose graph has three points of inflection $(-1, -1)$, (1.1) and a point with abscissa 0 at which the curve is inclined to the axis of abscissa at an angle of $60°$. Then find the value of $\int_0^1 P(x)dx$.

423. If $I_n = \int_0^1 \dfrac{dx}{\left(1 + x^2\right)^n}$, $wheren \in N$, which of the following statements hold good? (a) $2nI_{n+1} = 2^{-n} + (2n - 1)I_n$

(b) $I_2 = \dfrac{\pi}{8} + \dfrac{1}{4}$ (c) $I_2 = \dfrac{\pi}{8} - \dfrac{1}{4}$ (d) $I_3 = \dfrac{3\pi}{32} + \dfrac{1}{4}$

424. If $f(0) = 1, f(2) = 3, f'(2) = 5$,then find the value of $\int_0^1 xf''(2x)dx$

425. If $\int_a^b \dfrac{f(x)}{f(x) + f(a + b - x)}dx = 10,\ then\quad b = 22,\ a = 2$ (b) $\ b = 15,\ 1 = -5$

$b = 10,\ a = -10$ (d) $b = 10,\ a = -2$

426. Evaluate: $\int_0^{\pi}\dfrac{dx}{1 + \sin x}$

427. If $I_n = \int_0^{\pi/4}\tan^n x\,dx$, ($n > 1$ is an integer), then (a) $I_n + I_{n-2} = \dfrac{1}{n + 1}$ (b)

$I_n + I_{n-2} = \dfrac{1}{n - 1}$ (c) $I_2 + I_4, I_6, \ldots\ldots$ are in H.P. (d) $\dfrac{1}{2(n + 1)} < I_n < \dfrac{1}{2(n - 1)}$

428. Consider the integral $\int_0^{2\pi} \dfrac{dx}{5 - 2\cos x}$ making the substitution

$\tan\left(\dfrac{x}{2}\right) = t,$ we have $I = \int_0^{2\pi} \dfrac{dx}{5 - 2\cos x} = \int_0^0 \dfrac{2dt}{\left(1 + t^2\right)\left[5 - 2\dfrac{1-t^2}{1+t^2}\right]} = 0$ The

result is obviously wrong, since the integrand is positive and

consequently the integral of this function cannot be equal to zero. Find

the mistake.

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429. If $f(x) = \int_0^x |t - 1| dt$, where $0 \le x \le 2$ then (a) range of $f(x)$ is $[0, 1]$ (b)

$f(x)$ is differentiable at $x = 1$ (c) $f(x) = \cos^{-1}x$ has two real roots (d)

$f'\left(\dfrac{1}{2}\right) = \dfrac{1}{2}$

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430. If $\int_0^1 \dfrac{e^{-x} dx}{1 + e^x} = (\log)_e(1 + e) + k$, then find the value of k.

431. The value of $\int_0^\infty \frac{dx}{1+x^4}$ is

A. (a) same as that of $\int_0^\infty \frac{x^2+1}{1+x}dx$

B. (b) $\frac{\pi}{2\sqrt{2}}$

C. (c) same as that of $\int_0^\infty \frac{x^2+1}{1+x^4}dx$

D. (d) $\frac{\pi}{\sqrt{2}}$

Answer: null

432. Let f be a continuous function on $[a, b]$ Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dx = \int_x^b f(t)dt$

433. If $f(x)$ is continuous for all real values of x, then $\sum\limits_{r=1}^{n}\int_0^1 f(r-1+x)dx$ is

equal to (a) $\int_0^n f(x)dx$ (b) $\int_0^1 f(x)dx$ (c) $\int_0^1 f(x)dx$ (d) $(n-1)\int_0^1 f(x)dx$

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434. F ind the value of $\int_0^1 \log x\, dx$

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435. If $A_n = \int_0^{\frac{\pi}{2}} \dfrac{\sin(2n-1)x}{\sin x}dx$, $B_n = \int_0^{\frac{\pi}{2}} \left(\dfrac{\sin nx}{\sin x}\right)^2 dx$ for $n \in N$, Then (A)

$A_{n+1} = A_n$ (B) $B_{n+1} = B_n$ (C) $A_{n+1} - A_n = B_{n+1}$ (D) $B_{n+1} - B_n = A_{n+1}$

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436. A continuous real function f satisfies

$f(2x) = 3f(x)\ \forall x \in R$ If $\int_0^1 f(x)dx = 1$, then find the value of $\int_1^2 f(x)dx$

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437. The value of $\int_0^1 \dfrac{2x^2 + 3x + 3}{(x + 1)\left(x^2 + 2x + 2\right)} dx$ is

A. (a) $\dfrac{\pi}{4} + 2\log2 - \tan^{-1}2$

B. (b) $\dfrac{\pi}{4} + 2\log2 - \dfrac{\tan^{-1}1}{3}$

C. (c)$2\log2 - \cot^{-1}3$

D. (d) $-\dfrac{\pi}{4} + \log4 + \cot^{-1}2$

Answer: null

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438. Evaluate: $\int_0^{1/\sqrt{2}} \dfrac{\sin^{-1}x}{\left(1 - x^2\right)^{3/2}} dx$

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439. If $f(x)$ is differentiable and $\int_0^{t^2} xf(x)dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals (a)$\frac{2}{5}$

(b) $-\frac{5}{2}$ (c)1 (d) $\frac{5}{2}$

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440. Evaluate: $\int_1^\infty \left(e^{x+1} + e^{3-x}\right)^{-1}dx$

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441. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2}dt$, then $f(x)$ increases in

A. (a) $(0, 2)$

B. (b) no value of x

C. (c) $(0, \infty)$

D. (d) $(-\infty, 0)$

Answer: null

442. Estimate the absolute value of the integral $\int_{10}^{19} \frac{\sin x}{1 + x^8} dx$

443. If $I(m, n) = \int_0^1 t^m (1 + t)^n . dt$, then the expression for $I(m, n)$ in terms of $I(m + 1, n - 1)$ is:

444. Prove that $4 \leq \int_1^3 \sqrt{3 + x^2} dx \leq 4\sqrt{3}$

445. If $I_1 = \int_0^{\pi/2} \frac{\cos^2 x}{1 + \cos^2 x} dx$, $I_2 = \int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin^2 x} dx$ and

$I_3 = \int_0^{\pi/2} \frac{1 + 2\cos^2 x \sin^2 x}{4 + 2\cos^2 x \sin^2 x} dx$, then (a) $I_1 = I_2 > I_3$ (b) $I_3 > I_1 = I_2$ (c)

$I_1 = I_2 = I_3$ (d) none of these

446. Prove that $\int_0^1 \sqrt{(1 + x)\left(1 + x^3\right)}\, dx$ cannot exceed $\sqrt{\dfrac{15}{8}}$.

447. Let $T > 0$ be a fixed real number. Suppose f is continuous function such that for all $x \in R$, $f(x + T) = f(x)$. If $I = \int_0^T f(x)dx$, then the value of $\int_3^{3 + 3T} f(2x)dx$ is (a) $\dfrac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

448. Evaluate: $\int_0^1 \dfrac{1}{\sqrt{1 - x^2}} \sin^{-1}\left(2x\sqrt{1 - x^2}\right)dx$

449. Let $f(x) = \int_1^x \sqrt{2 - t^2} dt$ Then the real roots of the equation $x^2 - f'(x) = 0$ are (a)± 1 (b) $\pm \dfrac{1}{\sqrt{2}}$ (c)$\pm \dfrac{1}{2}$ (d) 0 and 1

450. Evaluate: $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\, dx$

451. If $f(2 - x) = f(2 + x)$ and $f(4 - x) = f(4 + x)$ for all x and $f(x)$ is a function for which $\int_0^2 f(x)dx = 5$, then $\int_0^{50} f(x)dx$ is equal to

452. $\int_{-1}^{2} \left| x^3 - x \right| dx$

453. If $f(x)$ is integrable over $[1, 2]$, then $\int_1^2 f(x)dx$ is equal to (a)

$(\lim)_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (b) $(\lim)_{n \to \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$ (c)

$(\lim)_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ (d) $(\lim)_{n \to \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

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454. Evaluate: $\int_{-\frac{\pi}{2}}^{2\pi} \sin^{-1}(\sin x)dx$

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455. The value of $\int_0^{\frac{\pi}{4}} \sin 2x\,dx$ is

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456. Let: $a_n = \int_0^{\frac{\pi}{2}} (1 - \sin t)^n \sin 2t\,dt$ Then find the value of $\lim_{n \to \infty} na_n$

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457. Q. $\int_0^\pi e^{\cos^2 x}\left(\cos^3(2n+1)x\,dx,\ n \in I\right.$

458. Prove that $\int_0^{102}(x-1)(x-2)..(x-100) \times \left(\dfrac{1}{x-1} + \dfrac{1}{x-2} + ... \ + \right.$

$\left.\dfrac{1}{x-100}\right)dx = 101! - 100!$

459. Let $f:[0,2] \to R$ be a function which is continuous on [0,2] and is differentiable on (0,2) with $f(0) = 1$

$Let:F(x) = \int_0^{x^2} f\left(\sqrt{t}\right)dt$ or $x \in [0,2]IfF'(x) = f'(x)$. for all $x \in (0,2)$, then

$F(2)$ equals (a)$e^2 - 1$ (b) $e^4 - 1$ (c)$e - 1$ (d) e^4

460. Evaluate: $\int_0^1 \dfrac{1}{\left(1 + x^2\right)\sqrt{1 - x^2}}\, dx$

461. If $f(x) = \begin{cases} e^{\cos x}\sin x & |x| \le 2 \\ 2 & \text{otherwise} \end{cases}$. **Then** $\int_{-2}^{3} f(x)\, dx =$ _____

462. Evaluate: $\int_0^{\pi/2} \dfrac{1}{\left(a^2\cos^2 x + b^2\sin^2 x\right)^2}\, dx$

463. Prove that: $\int_0^{2\pi} \dfrac{x\sin^{2n} x}{\sin^{2n} + \cos^{2n} x}\, dx = \pi^2$

464. Prove that $1 < \int_0^2 \left(\dfrac{5 - x}{9 - x^2} \right) dx < \dfrac{6}{5}$

465. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is (1) $(- 3, 3)$ (2) $(- 3, \infty)$ (3) $(3, \infty)$ (4) $(- \infty, - 3)$

466. If m is the least value (global minimum) and M is the greatest value (global maximum) of the function $f(x)$ on the interval $[a, b]$ (estimation of an integral), then $m(b - a) \le \int_a^b f(x)dx \le M(b - a)$.

467. Let $f: R \rightarrow R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$

Then the value of $f(1n5)$ is_____

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468. Show that $\int_a^b \frac{|x|}{x}dx = |b| - |a|$

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469. The value of $\frac{\pi^2}{1n3}\int_{\frac{5}{6}}^{\frac{5}{6}} \sec(\pi x)dxis$ __

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470. Let: $f(x) = \int_0^x |2t - 3|dt$ Then discuss continuity and differentiability of

$f(x)atx = \frac{3}{2}$

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471. If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$ then the value of $f\left(\dfrac{\pi}{6}\right)$ is ___

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472. $Let I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \dfrac{\sin x}{x}dx, I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \dfrac{\sin(\sin x)}{\sin x}dx, I_3 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \dfrac{\sin(\tan x)}{\tan x}dx$ Then arrange in the decreasing order in which values I_1, I_2, I_3 lie.

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473. The value of $\int_0^1 e^{x^2 - x}dx$ is (a) < 1 (b) > 1 (c) $> e^{-\frac{1}{4}}$ (d) `

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474. Evaluate: $\int_0^1 x \dfrac{dx}{\sqrt{1 - x^2}}$

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475. The value of the definite integral $\int_0^1 \left(1 + e^{-x^2}\right) dx$ is

476. Prove that $0 < \int_0^1 \dfrac{x^7 dx}{\left(1 + x^8\right)^{\frac{1}{3}}} < \dfrac{1}{8}$

477. Let a, b, c be nonzero real numbers such that $\int_0^1 \left(1 + \cos^8 x\right)\left(ax^2 + bx + c\right) dx = \int_0^2 \left(1 + \cos^8 x\right)\left(ax^2 + bx + c\right) dx = 0$ Then show that the equation $ax^2 + bx + c = 0$ will have one root between 0 and 1 and other root between 1 and 2.

478. Let $f(x)$ be a continuous and differentiable function such that $f(x) = \int_0^x \sin\left(t^2 - t + x\right)dt$ Then prove that $f''(x) + f(x) = \cos x^2 + 2x\sin x^2$

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479. Evaluate: $5050 \dfrac{\int 01\left(1 - x^{50}\right)^{100}dx}{\int 01\left(1 - x^{50}\right)^{101}dx}$

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480. If $\int_{\frac{\pi}{3}}^{x} \sqrt{\left(3 - \sin^2 t\right)}dt + \int_0^y \cos t \, dt = 0, \, then \, evaluate \dfrac{dy}{dx}$

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481. If $f(x) = \int_0^x (\cos(\sin t) + \cos(\cos t)dt, \, then f(x + \pi) is$ (a)$f(x) + f(\pi)$ (b) $f(x) + 2(\pi)$ (c)$f(x) + f\left(\dfrac{\pi}{2}\right)$ (d) $f(x) + 2f\left(\dfrac{\pi}{2}\right)$

482. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{a - \sin\theta}{a + \sin\theta}\right) d\theta, \ a > 0$

483. Evaluate: $\int_0^1 \frac{dx}{1 + x^2}$

484. If $f(t)$ is an odd function, then prove that $\varphi(x) = \int_a^x f(t)dt$ is an even function.

485. Evaluate: $\int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{\left(x^2 + 1\right)\left(x^4 - 1\right)} dx$

486. Find the value of $\int_0^1 \left\{ \left(\sin^{-1}x \right)/x \right\} dx$.

487. Show that $\int_0^{n\pi+v} |\sin x| dx = 2n + 1 - \cos v$, where n is a positive integer and $, 0 \le v < \pi$

488. Find $I = \int_0^\pi \ln(1 + \cos x) dx$

489. The value of $\int_a^b (x - a)^3 (b - x)^4 dx$ is

A. (a) $\dfrac{(b - a)^4}{6^4}$

B. (b) $\dfrac{(b - a)^8}{280}$

C. (c) $\dfrac{(b - a)^7}{7^3}$

D. (d) none of these

Answer: null

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490. Evaluate: $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \left[\left(\dfrac{x + 1}{x - 1} \right)^2 + \left(\dfrac{x - 1}{x + 1} \right)^2 - 2 \right]^{\frac{1}{2}} dx$

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491. The value of $\displaystyle\int_{-a}^{a} \left(\cos^{-1} x - \sin^{-1} \sqrt{1 - x^2} \right) dx$ is $(a > 0)$ there $\displaystyle\int_{0}^{a} \cos^{-1} x \, dx = A$ is

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492. If f is an odd function, then evaluate $I = \int_{-a}^{a} \dfrac{f(\sin x)dx}{f(\cos x) + f\left(\sin^2 x\right)}$

493. Evaluate : $\int_{0}^{\frac{\pi}{4}} \log(1 + \tan x)dx$.

494. Evaluate: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \dfrac{x^9 - 3x^5 + 7x^3 - x + 1}{\cos^2 x} dx$

495. Let $a + b = 4, where a < 2, and let g(x)$ be a differentiable function. If $\dfrac{dg}{dx} > 0$ for all $x,$ prove that $\int_{0}^{a} g(x)dx + \int_{0}^{b} g(x)dx \in crerasesas(b - a) \in crerases$

496. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left\{ \dfrac{ax^2 + bx + c}{ax^2 - bx + c}(a + b)|\sin x| \right\} dx$

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497. If $U_n = \int_0^\pi \dfrac{1 - \cos nx}{1 - \cos x} dx$, where n is positive integer or zero, then show that $U_{n+2} + U_n = 2U_{n+1}$ Hence, deduce that $\int_0^{\frac{\pi}{2}} \dfrac{\sin^2 n\theta}{\sin^2 \theta} = \dfrac{1}{2} n\pi$

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498. Evaluate: $\int_{-1}^{1} \dfrac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx$

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499. Find the value of $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$

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500. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^2 x (\sin x + \cos x) dx$

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501. $\int_{-1}^{2} \left[\dfrac{[x]}{1 + x^2} \right] dx, where[.]$ denotes the greater integer function, is

equal to (a)-2 (b) -1 (c)0 (d) none of these

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502. Evaluate: $\int_a^b \cos x dx$

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503. Let $f: R \rightarrow R$ be a continuous function and $f(x) = f(2x)$ is true

$\forall x \in R.$ If $f(1) = 3,$ then the value of $\int_{-1}^{1} f(f(x)) dx$ is equal to

A. (a)6

B. (b) 0

C. (c) $3f(3)$

D. (d) $2f(0)$

Answer: null

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504. Evaluate: $(\lim)_{n \to \infty} \dfrac{\left(1^2 + 2^2 + 3^3 + \; + n^2\right)\left(1^3 + 2^3 + 3^3 + \; + n^3\right)}{1^6 + 2^6 + 3^6 + \; + n^6}$

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505. if $\displaystyle\int \dfrac{dx}{x^2\left(x^n + 1\right)^{\left(\frac{n-1}{n}\right)}} = -[f(x)]^{\frac{1}{n}} + c,\, then f(x) is$ $\left(1 + x^n\right)$ (b) $1 + x^{-1}$

$x^n + x^{-n}$ (d) none of these

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506. Evaluate the definite integrals as limit of sums $\int_1^2 x^2 dx$

507. Evaluate: $\int_a^b \sin x dx$ using limit of sum

508. $\int_{-3}^3 x^8 \left\{ x^{11} \right\} dx$

509. Evaluate: $\int_a^b e^x dx$ using limit of sum

510. The value of $\int_{-2}^{1} \left[x \left[1 + \cos\left(\dfrac{\pi x}{2} \right) \right] + 1 \right] dx$, where [.] denotes the greatest integer function, is (a)1 (b) 1/2 (c) 2 (d) none of these

511. Evaluate: $\lim\limits_{n \to \infty} \dfrac{((n + 1)(n + 2)...(n + n))^{\frac{1}{n}}}{n}$

512. If $g(x) = \int_{0}^{x}(|\sin t| + |\cos t|)dt$,then $g\left(x + \dfrac{\pi n}{2} \right)$ is equal to, where $n \in N$,

(a) $g(x) + g(\pi)$ (b) $g(x) + g\left(\dfrac{n\pi}{2} \right)$ (c) $g(x) + g\left(\dfrac{\pi}{2} \right)$ (d) none of these

513. Evaluate: $\lim\limits_{n \to \infty} \left[\dfrac{1}{na} + \dfrac{1}{na + 1} + \dfrac{1}{na + 2} + + \dfrac{1}{nb} \right]$

514. $\int_0^x |\sin t| dt$, where $x \in (2n\pi, (2n + 1)\pi), n \in N$, is equal to (A) $4n - \cos x$ (B) $4n - \sin x$ (C) $4n + 1 - \cos x$ (D) $4n - 1 - \cos x$

515. Evaluate $\lim_{n \to \infty} n \left[\dfrac{1}{(n + 1)(n + 2)} + \dfrac{1}{(n + 2)(n + 4)} + ... + \dfrac{1}{6n^2} \right]$

516. Let f(x) be an odd continous function which is periodic with period 2. If $g(x) = \int_0^x f(t)dt$, then

517. Evaluate $\int_a^b \frac{dx}{\sqrt{x}}$, where $a, b > 0$.

518. Let $f: \vec{R} and g: \vec{RR}$ be continuous function. Then the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) - g(-x)]dx$ is (a)π (b) 1 (c) -1 (d) 0

519. Evaluate: $\int_{-\frac{\pi}{2}}^{-\frac{\pi}{3\pi}}\left[(x + \pi)^3 + \cos^2(x + 3\pi)\right]dx$

520. Evaluate: $\int_0^{\frac{1}{2}} \frac{x\sin^{-1}x}{\sqrt{1 - x^2}}dx$

521. Evaluate: $\int_0^{100} x - [x]dx$ where [.] represents the greatest integer function).

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522. If $S_n = \left[\dfrac{1}{1 + \sqrt{n}} + \dfrac{1}{2 + \sqrt{2n}} + \dots + \dfrac{1}{n + \sqrt{n^2}} \right]$ then $(\lim)_{n \to \infty} S_n$ is equal to (A) log2 (B) log4 (C) log8 (D) none of these

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523. The value of $(\lim)_{n \to \infty} \sum_{r=1}^{4n} \dfrac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r} + \sqrt{n}\right)^2}$ is equal to $\dfrac{1}{35}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{10}$ (d) $\dfrac{1}{5}$

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524. Evaluate: $\dfrac{\int 0 n[x]dx}{\int 0 n\{x\}dx}$ $(where [x]\, and\, \{x\}$ are integral and fractional parts

of $x\, and\, n \in N$ $\Bigg)$.

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525. Evaluate: $\int_{-\frac{\pi}{4}}^{n\pi - \frac{\pi}{4}} |sinx + cosx|dx$

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526. The value of $\int_{0}^{\frac{\pi}{2}} \dfrac{dx}{1 + tan^3 x}$ **is** (a) 0 (b) 1 (c) $\dfrac{\pi}{2}$ (d) π

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527. Evaluate: $\int_{-\pi}^{\pi} \left(1 - x^2\right) sinx cos^2 x dx$

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528. Let f be a contiuous function on $[a, b]$. If

$$F(x) = \left(\int_a^x f(t)dt - \int_x^b f(t)dt \right)(2x - (a + b)),$$

then prove that there exist some $c\varepsilon(a, b)$ such that

$$\int_a^c f(t)dt - \int_c^b f(t)dt = f(c)(+ b - 2c)$$

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529. Evaluate the following: $\int_{-1}^{1} \dfrac{\sin x - x^2}{3 - |x|}dx$

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530. If $\int_a^b |\sin x|dx = 8$ and $\int_0^{a+b} |\cos x|dx = 9$, then

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531. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos^{2n-1}x - \cos^{2n+1}x}\,dx,\ where\ n \in N$

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532. $f(x)$ is a continuous and bijective function on R. If $\forall t \in R$, then the area bounded by $y = f(x)$, $x = a - t$, $x = a$, and the x-axis is equal to the area bounded by $y = f(x)$, $x = a + t$, $x = a$, and the x-axis. Then prove that $\int_{-\lambda}^{\lambda} f^{-1}(x)dx = 2a\lambda (given\,that\,f(a) = 0)$

533. Evaluate: $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log \left(\frac{1 - x}{1 + x} \right) dx$

534. $f(x) = \int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of $f(1)$ is

535. Evaluate: $\int_0^x [\cos t]\, dt\, where\, x \in \left(2n\pi, \left(4n + 1\dfrac{\pi}{2}\right)\right), n \in N, and [.]$ denotes the greatest integer function.

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536. Suppose f is a real-valued differentiable function defined on $[1, \infty]$ with $f(1) = 1.$ Moreover, suppose that f satisfies $f'(x) = \dfrac{1}{x^2 + f^2(x)} Show that f(x) < 1 + \dfrac{\pi}{4} \forall x \geq 1.$

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537. Let f be a real valued function satisfying $f(x) + f(x + r) = f(x + 2) + f(x + 6)$ Prove that $\int_x^{x+8} f(t)\, dt$ is a constant function.

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538. If $x\int_0^x \sin(f(t))dx = (x + 2)\int_0^1 t\sin(f(t))dt$, where $x > 0$, then show that

$f'(x)\cot f(x) + \dfrac{3}{1 + x} = 0$

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539. Evaluate: $\displaystyle\int_0^2 \dfrac{dx}{\left(17 + 8x - 4x^2\right)\left[e^{6(1-x)} + 1\right]}$

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540. If $f(x) = \displaystyle\int_{\frac{x^2}{16}}^{x^2} \dfrac{\sin x \sin\sqrt{\theta}}{1 + \cos^2\sqrt{\theta}}d\theta$, then find the value of $f'\left(\dfrac{\pi}{2}\right)$.

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541. Find the equation of tangent to the $y = F(x)$ at $x = 1$, where

$F(x) = \displaystyle\int_x^{x^3} \dfrac{dt}{\sqrt{1 + t^2}}$

542. If $\int_0^x [x]dx = \int_0^{[x]} xdx,$ then prove that either x is purely fractional or x is such that $\{x\} = \frac{1}{2}(where[.]and\{.\}$ denote the greatest integer and fractional part, respectively).

543. Show that: $\int_0^{\pi/2} f(\sin 2x)\sin x dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x dx$

544. If $I_1 = \int_0^1 2^{x^2}dx, I_2 = \int_0^1 2^{x^3}dx, I_3 = \int_1^2 2^{x^2}dx, I_4 = \int_1^2 2^{x^3}dx,$ then which of the following is/are true?

A. (a) $I_1 > I_2$

B. (b) $I_2 > I_1$

C. (c) $I_3 > I_4$

D. (d) $I_3 < I_4$

Answer: null

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545. If $af(x)+bf\left(\dfrac{1}{x}\right) = \dfrac{1}{x} - 5, x \neq 0, a \neq b$, then $\int_1^2 f(x)dx$ equals

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546. Find the mistake in the following evaluation of the integral

$I = \int_0^\pi \dfrac{dx}{1 + 2\sin^2 x}$,   then   : $I = \int_0^\pi \dfrac{dx}{\cos^2 x + 3\sin^2 x}$

$= \int_0^\pi \dfrac{\sec^2 x \, dx}{1 + 3\tan^2 x} = \dfrac{1}{\sqrt{3}}\left[\tan^{-1}\left(\sqrt{3}\tan x\right)\right]_\pi^0 = 0$

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547. Find the value of $\int_{-1}^1 \dfrac{d}{dx}\left(\tan^{-1}\left(\dfrac{1}{x}\right)\right)dx$

548. The value of $\int_2^3 \dfrac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ is _____

549. The value of $\int_{-2}^2 \left|1-x^2\right|dx$ is_____

550. If $I_I = \int_0^{\pi/2}\cos(\sin x)dx,\ I_2 = \int_0^{\frac{\pi}{2}}\sin(\cos x)dx,\ and\ I_3 = \int_0^{\frac{\pi}{2}}\cos x dx,$ then find the order in which the values $I_1, I_2, I_3,$ exist.

551. The integral $\int_0^{1.5} \left[x^2 \right] dx, \text{where}[.]$ denotoes the greatest integer function, equals ....

552. Evaluate the following limit:

$$\lim_{n \to \infty} \frac{\sum_{r=1}^{n} \sqrt{r} \sum_{r=1}^{h} \frac{1}{\sqrt{r}}}{\sum_{r=1}^{n} r}$$

553. Evaluate: $(\lim)_{n \to \infty} \left[\frac{n!}{n^n} \right]^{1/n}$

554. If the value of the definite integral $\int_0^1 {}^{207}C_7 x^{200}(1-x)^7 dx$ is equal to $\dfrac{1}{k}$, where $k \in N$, then the value of $\dfrac{k}{26}$ is____

555. Evaluate: $\int_{-1}^0 \dfrac{dx}{x^2 + 2x + 2}$

556. If $f and g$ are continuous function on $[0, a]$ satisfying $f(x) = f(a-x) and g(x) + g(a-x) = 2$, then show that $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx$

557. If $f(x) = \min\left(|x|, 1-|x|, \dfrac{1}{4}\right) \forall x \in R$, then find the value of $\int_{-1}^1 f(x)dx$

558. Evaluate: $\int_0^\pi e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\, dx$

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559. Evaluate: $\int_a^b x^3 dx$

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560. Find the value of $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \dfrac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)}\, dx$

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561. Evaluate: $(\lim)_{n \to \infty}\left(\dfrac{1}{\sqrt{4n^2 - 1}} + \dfrac{1}{\sqrt{4n^2 - 2^2}} + + \dfrac{1}{\sqrt{3n^2}}\right)$

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562. If $y(x) = \int_{\frac{\pi^2}{16}}^{x^2} \frac{\cos x \cos\sqrt{\theta}}{1 + \sin^2\sqrt{\theta}}\, d\theta$, find $\dfrac{dy}{dx}$ at $x = \pi$.

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563. Evaluate: $\displaystyle\lim_{n \to \infty} \left[\frac{1}{n^2}\frac{\sec^2 1}{n^2} + 2/n^2\frac{\sec^2 4}{n^2} + \; + \frac{1}{n}\sec^2 1 \right]$

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564. For $x > 0$, $let f(x) = \int_1^x \frac{\log t}{1 + t}\, dt$ Find the function $f(x) + f\left(\dfrac{1}{x}\right)$ and find the value of $f(e) + f\left(\dfrac{1}{e}\right)$

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565. Evaluate $\displaystyle\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k}{n^2 + k^2}$

566. If" f, is a continuous function with $\int_0^x f(t)dt \to \infty$ as $|x \mapsto \infty$ then show that every line $y = mx$ intersects the curve $y^2 + \int_0^x f(t)dt = 2$

567. Prove that $\dfrac{\pi}{6} < \int_0^1 (dx)\left(\sqrt{4 - x^2 - x^3}\right) < \dfrac{\pi}{4\sqrt{2}}$

568. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)f$ or $all x \in [-1, 2]$ Let $R_1 = \int_{-1}^2 xf(x)dx$, and R_2 be the area of the region bounded by $y = f(x), x = -1, x = 2,$ and the x-axis . Then (a) $R_1 = 2R_2$ (b) $R_1 = 3R_2$ (c)$2R_1$ (d) $3R_1 = R_2$

569. Prove that $\int_0^x e^{xt-t^2}\, dt = e^{\frac{x^2}{4}} \int_0^x e^{-\left(\frac{t^2}{4}\right)} dt$

570. If $\alpha = \int_0^1 \left(e^{9x+3\tan^{-1}x}\right)\left(\dfrac{12 + 9x^2}{1 + x^2}\right) dx$ where $\tan^{-1}x$ takes only principal

values, then, find the value of $\left((\log)_e|1 + \alpha| - \dfrac{3\pi}{4}\right)$

571. Evaluate $\int_{-4}^{-5} e^{(x+5)^2}\, dx + 3\int_{1/3}^{2/3} e^9 \left(x - \dfrac{2}{3}\right)^2 dx.$

572. Prove that for any positive integer

k, $\dfrac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + \ + \cos(2k - 1)x]$ Hence, prove that

$$\int_0^{\frac{\pi}{2}} \sin 2xk \cot x \, dx = \frac{\pi}{2}$$

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573. Compute the integrals: $\int_0^\infty f\left(x^n + x^{-n}\right) \log x \dfrac{dx}{x}$

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574. If for a real number y, $[y]$ is the greatest integral function less, then or equal to y, then the value of the integral $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2\sin x] dx$ is $-\pi$ (b) 0 (c) $-\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{2}$

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575. Compute the integrals: $\int_0^\infty f\left(x^n + x^{-n}\right) \log x \dfrac{dx}{1 + x^2}$

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576. $\int_0^x \left\{ \int_0^u f(t)dx \right\} du$ is equal to (a) $\int_0^x (x-u)f(u)du$ (b) $\int_0^x uf(x-u)du$ (c) $x\int_0^x f(u)du$ (d) $x\int_0^x uf(u-x)du$

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577. Compute the integrals: $\int_{\frac{1}{e}}^{e} \frac{1}{x}\sin\left(x - \frac{1}{x}\right)dx$

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578. STATEMENT 1: $\int_0^{\pi} x\sin x\cos^2 x\,dx = \frac{\pi}{2}\int_0^{\pi}\sin x\cos^2 x\,dx$.

STATEMENT 2: $\int_a^b xf(x)dx = \frac{a+b}{2}\int_a^b f(x)dx$.

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579. Evaluate: $\int_0^{\frac{\pi}{2}} x\cos x\,dx$

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580. Evaluate: $\int_0^{\frac{\pi}{4}} \dfrac{\sin x + \cos x}{9 + 16\sin 2x}\,dx$

581. Evaluate: $\int_{-\pi/4}^{\pi/4}\log(\sin x + \cos x)\,dx$

582. If $\alpha, \beta(\beta > \alpha)$, are the roots of $g(x) = ax^2 + bx + c = 0$ and $f(x)$ is an even function, then $\displaystyle\int_\alpha^\beta \dfrac{e^{f\left(\frac{g(x)}{x-\alpha}\right)}}{e^{f\left(\frac{g(x)}{x-\alpha}\right)} + e^{f\left(\frac{g(x)}{x-\beta}\right)}} = $ (a) $\left|\dfrac{b}{2a}\right|$ (b) $\dfrac{\sqrt{b^2 - 4ac}}{|2a|}$ (c) $\left|\dfrac{b}{a}\right|$ (d) none of these

583. Consider the function $f(x)$ satisfying the relation $f(x + 1) + f(x + 7) = 0 \, \forall x \in R$ STATEMENT 1 : The possible least value of t for which $\int_a^{a+t} f(x)dx$ is independent of a is 12. STATEMENT 2 : $f(x)$ is a periodic function.

584. Let $f: R \vec{R}$ be a differentiable function having $f(2) = 6, f'(2) = \dfrac{1}{48}$. Then evaluate $(\lim)_{x2} \int_6^{f(x)} \dfrac{4t^3}{x-2} dt$

585. If $\int_{\sin x}^1 t^2 f(t)dt = 1 - \sin x, where. \, x \in \left(0, \dfrac{\pi}{2}\right),$ then find the value of $f\left(\dfrac{1}{\sqrt{3}}\right)$

586. Prove that $\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x-c)dx$, and when $f(x)$ is odd function,

$\int_{-a}^a f(x)dx = 0$

587. Evaluate: $\int_0^{100\pi} \sqrt{(1-\cos 2x)}dx$

588. Evaluate: $\int_0^\pi \dfrac{xdx}{1+\cos\alpha\sin x}$, $where 0 < \alpha < \pi$

589. Evaluate : $\int_0^{\frac{\pi}{4}} \dfrac{\sin x\cos x}{\sin^4 x + \cos^4 x}dx$

590. A periodic function with period 1 is integrable over any finite interval. Also, for two real numbers a,b and two unequal non-zero positive integers m and n $\int_a^{a+n} f(x)dx = \int_b^{b+m} f(x)$ calculate the value of $\int_m^n f(x)dx$

591. Evaluate: $\int \cos^3 x e^{\log \sin x} dx$

592. The value of the integral $\int_{-\frac{3\pi}{4}}^{\frac{5\pi}{4}} \left(\frac{\sin x + \cos x}{e^{x-\frac{\pi}{4}} + 1} \right) dx$ (A) 0 (B) 1 (C) 2 (D) none of these

593. If $\int_0^{n\pi} f\left(\cos^2 x\right) dx = k \int_0^{\pi} f\left(\cos^2 x\right) dx$, then $f \in d$ the value k

594. $\int_{2-a}^{2+a} f(x)dx$ is equal to [where $f(2 - \alpha) = f(2 + \alpha) \forall \alpha \in R$

(a) $2\int_{2}^{2+a} f(x)dx$ (b) $2\int_{0}^{a} f(x)dx$ (c) $2\int_{2}^{2} f(x)dx$ (d) none of these

595. Evaluate: $\int_0^a \dfrac{dx}{x + \sqrt{\left(a^2 - x^2\right)}}$ or $\int_0^{\frac{\pi}{2}} \dfrac{d\theta}{1 + \tan\theta}$

596. Suppose that $F(x)$ is an anti derivative of $f(x) = \dfrac{\sin x}{x}$, where $x > 0$.

Then $\int_1^3 \dfrac{\sin 2x}{x} dx$ can be expressed as

597. Evaluate: $\int_{-\pi}^{\pi} \dfrac{x \sin x \, dx}{e^x + 1}$

598. $If \int_0^1 \cot^{-1}\left(1 - x + x^2\right) dx = \lambda \int_0^1 \tan^{-1} x \, dx, \, then \, \lambda \, is \, equal \rightarrow$ 1 (b) 2 (c) 3 (d) 4

599. Show that $\int_0^{\frac{\pi}{2}} \sqrt{(\sin 2\theta)} \sin \theta \, d\theta = \dfrac{\pi}{4}$

600. The number of positive continuous $f(x)$ defined in $[0, 1]$ for with

$I_1 = \int_0^1 f(x) dx = 1, I_2 = \int_0^1 x f(x) dx = a,$

$I_3 = \int_0^1 x^2 f(x) dx = a^2$ is /are

601. $\int_0^{\frac{\pi}{2}} \dfrac{\sin^2 x}{\sin x + \cos x} dx$

602. The value of the definite integral $\int_0^{\frac{\pi}{2}} \sqrt{\tan x}\, dx$ is $\sqrt{2}\pi$ (b) $\dfrac{\pi}{\sqrt{2}} 2\sqrt{2}\pi$ (d) $\dfrac{\pi}{2\sqrt{2}}$

603. For $\theta \in \left(0, \dfrac{\pi}{2}\right)$, prove that $\int_0^{\theta} \log(1 + \tan\theta\tan x)dx = \theta\log(\sec\theta)$

604. A function f is defined by $f(x) = \dfrac{1}{2^{r-1}}, \dfrac{1}{2^r} < x \le 1\left(2^{r-1}\right), r = 1, 2, 3$ then the value of $\int_0^1 f(x)dx$

605. Evaluate $\int_0^1 \dfrac{dx}{\left(5 + 2x - 2x^2\right)\left(1 + e^{2-4x}\right)} dx$

606. If $P(x)$ is a polynomial of the least degree that has a maximum equal to 6 at $x = 1$, and a minimum equal to 2 at $x = 3$, then $\int_0^1 P(x)dx$ equals

607. Evaluate: $\int_0^\pi x\log\sin x\,dx$

608. The value of the definite integral $\int_0^{\overline{1n\left(\frac{\pi}{2}\right)}} \cos e^{x^2} 2xe^{x^2} dx$ is 1 (b)

$1 + (\sin 1)$ $1 - (\sin 1)$ (d) $(\sin 1) - 1$

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609. Evaluate: $\int_0^{\frac{\pi}{2}} \cos 2x + 4\sin 2x \cos 2x dx$

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610. *The value of* $\int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2 + 1}{x^4 - x^2 + 1} \log\left(1 + x - \frac{1}{x}\right) dx$ *is* (a)$\frac{\pi}{8}(\log)_e 2$ (b) $\frac{\pi}{2}(\log)_e 2$

(c)-$\frac{\pi}{2}(\log)_e 2$ (d) none of these

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611. If $f(x)$ is a function satisfying $f(x + a) + f(x) = 0$ for all $x \in R$ and positive constant a such that $\int_b^{c+b} f(x)dx$ is independent of b, then find

the least positive value of c

612. If $f(x)$ is monotonic differentiable function on $[a, b]$, then

$\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx = $ (a)$bf(a) - af(b)$ (b) $bf(b) - af(a)$ (c)$f(a) + f(b)$ (d)

cannot be found

613. Find the value of : $\int_0^{10} e^{2x - [2x]} d(x - [x])$ where $[.]$ denotes the greatest

integer function).

614. The range of the function $f(x) = \int_{-1}^{1} \dfrac{sinxdt}{\left(1 - 2tcosx + t^2\right)}$ is

A. (a) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right]$

B. (b) $[0, \pi]$

C. (c)$\{0, \pi\}$

D. (d) $\left\{ -\dfrac{\pi}{2}, \dfrac{\pi}{2} \right\}$

Answer: null

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615. $LetA = \int_0^\infty \dfrac{logx}{1 + x^3} dx$ Then find the value of $\int_0^\infty \dfrac{xlogx}{1 + x^3} dx$ in terms of A

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616. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0) = 0, x \in R$, where $f'(x)$ denotes $\dfrac{dy(x)}{dx}$, and $g(x)$ is a given non-constant differentiable function on R with $g(0) = g(2) = 0$. Then the value of $y(2)$ is_____

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617. Show that : $\int_0^1 \dfrac{\log x}{(1+x)}dx = -\int_0^1 \dfrac{\log(1+x)}{x}dx$

618. The value of $\lim\limits_{n \to \infty} \left[\tan\left(\dfrac{\pi}{2n}\right)\tan\left(\dfrac{2\pi}{2n}\right)\text{........}\tan\left(\dfrac{n\pi}{2n}\right) \right]^{\frac{1}{n}}$ is

619. Evaluate: $\int_0^\pi \dfrac{x\sin x}{1+\cos^2 x}dx$

620. Match the statements/expression given in Column I with the values given in Column II. Column I, Column II (p) The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)dx = 1$, *is* , 8 (q) The number of points in the interval

$\left[-\sqrt{13}, \sqrt{13} \right]$ at which $f(x) = \sin\left(x^2\right) + \cos\left(x^2\right)$ attains its maximum

value, is, 2 (r) $\int_{-2}^{2} \dfrac{3x^2}{1 + e^x} dx$ equals, 4 (s) $\dfrac{\int_{-2}^{2} \cos2x\log\left(\dfrac{1+x}{1-x}\right) dx}{\int_{0}^{1} \cos2x\log\left(\dfrac{1+x}{1-x}\right) dx}$, 0

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621. If $\int_{0}^{t} \dfrac{bx\cos4x - a\sin4x}{x^2} = \dfrac{a\sin4t}{t} - 1$ where $0 < t < \dfrac{\pi}{4}$, then the values of

a, b are, respectively, equal to (A) $\dfrac{1}{4}, 1$ (B) - 1, 4 (C) 2, 2 (D) 2, 4

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622. If $\qquad$ $I_1 = \int_{0}^{\pi} xf\left(\sin^3x + \cos^2x\right) dx$ and

$I_2 = \int_{0}^{\frac{\pi}{2}} f\left(\sin^3x + \cos^2x\right) dx$, then relate I_1 and I_2

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623. $\int_0^{\frac{\pi}{2}} \dfrac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

624. If $\quad U_n = \int_0^1 x^n(2 - x)^n dx$ and $V_n = \int_0^1 x^n(1 - x)^n dx$, $n \in N$, $\quad$ and $\quad$ if

$\dfrac{V_n}{U_n} = 1024,$ then the value of n is_____

625. Find the value of $\int_0^1 2x^3 - 3x^2 - x + 13 dx^{\cdot}$

626. If $\int_0^\infty x^{2n+1} e^{-x^2} dx = 360,$ then the value of n is___

627. Evaluate: $\int_0^\infty \dfrac{dx}{1 + x^2}$

628. If $\lambda = \int_0^{\frac{\pi}{2}} \cos\theta f\left(\sin\theta + \cos^2\theta\right) d\theta\eta$

$and I_2 = \int_0^{\pi/2} \sin2\theta f\left(\sin\theta + \cos^2\theta\right) d\theta\eta$, then $I_1 = -2I_2$ (b) $I_1 = I_2$ $2I_1 = I_2$ (d) $I_1 = -I_2$

629. If $f(a + b - x) = f(x)$, then prove that $\int_a^b xf(x)dx = \dfrac{a + b}{2}\int_a^b f(x)dx$

630. Let $I_1 = \int_{-2}^{2} \dfrac{x^6 + 3x^5 + 7x^4}{x^4 + 2}dx$ and $I_2 = \int_{-3}^{1} \dfrac{2(x + 1)^2 + 11(x + 1) + 14}{(x + 1)^4 + 2}dt$

Then the value of $I_1 + I_2$ is (A) 8 (B) $\dfrac{200}{3}$ (C) $\dfrac{100}{3}$ (D) none of these

631. Find the value of $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$

632. The value of $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

633. prove that $|\sin t|$ is non-differentiable at $t = \pi$.

634. Find the value of $\int_0^1 x(1 - x) \, dx$

635. Let $S_n = \sum\limits_{k=0}^{n} \dfrac{n}{n^2 + kn + k^2}$ and $T_n = \sum\limits_{k=0}^{n-1} \dfrac{n}{n^2 + kn + k^2}$ for n=1,2,3... then

(a)$S_n < \dfrac{\pi}{3}\sqrt{3}$ b) $S_n > \dfrac{\pi}{3}\sqrt{3}$ (c) $T_n < \dfrac{\pi}{3}\sqrt{3}$ (d) $T_n > \dfrac{\pi}{3}\sqrt{3}$

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636. The value of the definite integral $\int_{\sqrt{2}-1}^{\sqrt{2}+1} \dfrac{x^4 + x^2 + 2}{\left(x^2 + 1\right)^2} dx$ equals __

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637. If a continuous function f on $[0, a]$ satisfies $f(x)f(a - x) = 1, a > 0,$ then find the value of $\int_0^a \dfrac{dx}{1 + f(x)}$

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638. $Let J = \int_{-5}^{-4}\left(3 - x^2\right)\tan\left(3 - x^2\right)dx$ and $K = \int_{-2}^{-1}\left(6 - 6x + x^2\right)$

$\tan\left(6x - x^2 - 6\right)dx$ Then (J+K) equals _____

639. Let $g(x)$ be differentiable on R and $\int_{\sin t}^1 x^2 g(x)dx = (1 - \sin t)$, where $t \in \left(0, \dfrac{\pi}{2}\right)$. Then the value of $g\left(\dfrac{1}{\sqrt{2}}\right)$ is____

640. Show that $(\lim)_{n\infty}\left(\dfrac{1}{n+1} + \dfrac{1}{n+2} + \; + \; \dfrac{1}{6n}\right) = \log6$

641. $\int_{-\frac{\pi}{3}}^{0}\left[\cot^{-1}\left(\dfrac{2}{2\cos x - 1}\right) + \cot^{-1}\left(\cos x - \dfrac{1}{2}\right)\right]dx$ is equal to $\dfrac{\pi^2}{6}$ (b) $\dfrac{\pi^2}{3}$ (c) $\dfrac{\pi^2}{8}$ (d) $\dfrac{3\pi^2}{8}$

642. If $\dfrac{\displaystyle\int_{-\frac{\pi}{4}}^{\frac{3\pi}{4}} e^{\frac{\pi}{4}} dx}{\left(e^x + e^{\frac{\pi}{4}}\right)(\sin x + \cos x)} = k\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec x\, dx$, then the value of k is (a) $\dfrac{1}{2}$

(b) $\dfrac{1}{\sqrt{2}}$ (c) $\dfrac{1}{2\sqrt{2}}$ (d) $-\dfrac{1}{\sqrt{2}}$

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643. If $f(x) = \dfrac{e^x}{1 + e^x}, I_1 = \displaystyle\int_{f(-a)}^{f(a)} xg(x(1-x))dx$, and $I_2 = \displaystyle\int_{f(-a)}^{f(a)} g(x(1-x))dx,$
then the value of $\dfrac{I_2}{I_1}$ is

A. (a) -1

B. (b) -2

C. (c) 2

D. (d) 1

Answer: null

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644. If $f(y) = e^y$, $g(y) = y > 0$, and $F(t) = \int_0^t f(t - y)g(y)dy$,then

A. (a) $F(t) = e^t - (1 + t)$

B. (b) $F(t) = te^t$

C. (c) $F(t) = te^{-1}$

D. (d) $F(t) = 1 - e^t(1 + t)$

Answer: null

645. $Given \int_0^{\frac{\pi}{2}} \dfrac{dx}{1 + \sin x + \cos x} = \log 2.$ Then the value of integral $\int_0^{\frac{\pi}{2}} \dfrac{\sin x}{1 + \sin x + \cos x} dx$ is equal to (a) $\frac{1}{2}\log 2$ (b) $is \frac{\pi}{2} - \log 2$ $\frac{\pi}{4} - \frac{1}{2}\log 2$ (d) $\frac{\pi}{2} + \log 2$

646. I f$I_1 = \int_{-100}^{101} \dfrac{dx}{\left(5 + 2x - 2x^2\right)\left(1 + e^{2-4x}\right)}$

and$I_2 = \int_{-100}^{101} \dfrac{dx}{5 + 2x - 2x^2}$, then$\dfrac{I_1}{I_2}$i s (a)2 (b) $\dfrac{1}{2}$ (c) 1 (d) $-\dfrac{1}{2}$

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647. $\int_0^\infty \dfrac{xdx}{(1 + x)\left(1 + x^2\right)}$ is equal to (A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{2}$ (C) π (D) none of these"

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648. $\int_0^\infty \dfrac{dx}{\left[x + \sqrt{x^2 + 1}\right]^3}$ isequa $< o$ $\dfrac{3}{8}$ (b) $\dfrac{1}{8}$ (c) $-\dfrac{3}{8}$ (d) none of these

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649. The value of $\int_1^e \dfrac{1 + x^2 In x}{x + x^2 In x}dx$ is equal (a) $In(1 + e)e + In(1 + e)$ (b) $e - In(1 + e)$

650. The value of integral $\int_0^{\frac{1}{\sqrt{3}}} \dfrac{dx}{\left(1 + x^2\right)\sqrt{1 - x^2}}$ must be (a) $3 + 2\pi$ (b)

$4 - \pi 2 + \pi$ (c) non of these

651. If $\int_0^\infty \dfrac{x^2 dx}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)\left(x^2 + c^2\right)} = \dfrac{\pi}{2(a + b)(b + c)(c + a)}$ then the

value of $\int_0^\infty \dfrac{dx}{\left(x^2 + 4\right)\left(x^2 + 9\right)}$ (a) $\dfrac{\pi}{60}$ (b) $\dfrac{\pi}{20}$ (c) $\dfrac{\pi}{40}$ (d) $\dfrac{\pi}{80}$

652. The value of the integral $\int_0^1 \dfrac{dx}{x^2 + 2x\cos\alpha + 1}$ is equal to (a)$\sin\alpha$ (b)

$\alpha\sin\alpha$ (c)$\dfrac{\alpha}{2\sin\alpha}$ (d) $\dfrac{\alpha}{2}\sin\alpha$

653. Determine a positive integer $n \leq 5$ such that $\int_0^1 e^x(x-1)^n = 16 - 6e$

654. Let f be a continuous function on $[a, b]$. Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dx = \int_x^b f(t)dt$.

655. If $f(x) = \begin{cases} 1 - |x| & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$ and $g(x) = f(x-1) + f(x+1)$, then find the value of $\int_{-3}^{5} g(x)dx$.

656. Evaluate: $\int_0^1 \sin^{-1}\left(\dfrac{2x}{1 + x^2}\right)dx$

657. Evaluate: $\int_0^{\frac{\pi}{4}}\left(\tan^{-1}\left(\frac{2\cos^2\theta}{2-\sin2\theta}\right)\right)\sec^2\theta\,d\theta.$

658. $\quad Let\ f(n) = 1 + \dfrac{1}{2} + \dfrac{1}{3} + ... + \dfrac{1}{n}.$ $\qquad$ Then $\qquad$ show $\qquad$ that

$f(n) = \int_0^{\frac{\pi}{2}}\cot\left(\dfrac{\theta}{2}\right)\left(1 - \cos^n\theta\right)d\theta$

659. Let $f(x)$ be a continuous function $\forall x \in R,$ except at $x = 0,$ such that

$\int_0^a f(x)dx,\ a \in R^+$ exists. If $g(x) = \int_x^a \dfrac{f(t)}{t}dt,$ prove that $\int_0^a f(x)dx = \int_0^a g(x)dx$

660. If $f(x) = \dfrac{\sin x}{x} \ \forall x \in (0, \pi]$, prove that $\dfrac{\pi}{2}\int_0^{\frac{\pi}{2}} f(x) f\left(\dfrac{\pi}{2} - x\right) dx = \int_0^\pi f(x) dx$

661. Determine a positive integer n such that $\int_0^{\frac{\pi}{2}} x^n \sin x \, dx = \dfrac{3}{4}\left(\pi^2 - 8\right)$

662. f, g, h are continuous in $[0, 1]$, $f(a - x) = f(x)$, $g(a - x) = -g(x)$, $3h(x) - 4h(a - x) = 5$. Then prove that $\int_0^a f(x) g(x) h(x) dx = 0$.

663. If $f(x + f(y)) = f(x) + y \ \forall x, y \in R$ and $f(0) = 1$, then prove that $\int_0^2 f(2 - x) dx = 2\int_0^1 f(x) dx$.

664. Let $f(x) = (1 - x)^2 \sin^2 x + x^2$ for all $x \in R$, and let

$g(x) = \int \left(\frac{2(t - 1)}{t + 1} - \ln t \right) f(t) dt$ for $t \in [1, x]$ for all $x \in (1, \infty)$. Which of the following is true ?

665. Evaluate: $\int_3^{10} [\log[x]] dx$

666. If the function $f: [0, 8] \to R$ is differentiable, then for

$0 < \alpha < 1 < \beta < 2, \int_0^8 f(t) dt$ is equal to (a) $3\left[\alpha^3 f\left(\alpha^2\right) + \beta^2 f\left(\beta^2\right)\right]$ (b)

$3\left[\alpha^3 f(\alpha) + \beta^2 f(\beta)\right]$ (c) $3\left[\alpha^3 f\left(\alpha^2\right) + \beta^2 f\left(\beta^3\right)\right]$ (d) $3\left[\alpha^2 f\left(\alpha^3\right) + \beta^2 f\left(\beta^3\right)\right]$

667. The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is/are (a) $\frac{22}{7} - \pi$ (b) $\frac{2}{105}$ (c) 0 (d) $\frac{71}{15} - \frac{3\pi}{2}$

668. Let $\int_x^{x+p} f(t)dt$ be independent of x and $I_1 = \int_0^p f(t)dt$, $I_2 = \int_{10}^{p^n+10} f(z)dz$ for some p, where $n \in N$ Then evaluate $\frac{I_2 \cdot}{I_1}$

669. Evaluate: $\int_{-\pi}^{\pi}\left((2x)\frac{1+\sin x}{1+\cos^2 x}\right)dx$

670. Q. if $\int_0^{100}(f(x)dx = a$, then $\sum_{r=1}^{100}\left(\int_0^1(f(r-1+x)dx)\right)=$

671. Let $f(x)$ be a derivable function satisfying $f(x) = \int_0^x e^t \sin(x - t)dt$ and $g(x) = f''(x) - f(x)$ Then the possible integers in the range of $g(x)$ is_____

672. If $\int_0^1 e^{x^2}(x - \alpha)dx = 0$, then (a)$\alpha < 2$ (b)$\alpha < 0$ (c)$0 < \alpha < 1$ (d) $\alpha = 0$

673. $\int_{\frac{5}{2}}^{5} \dfrac{\sqrt{\left(25 - x^2\right)^3}}{x^4} dx$ is equal to

A. (a) $\dfrac{\pi}{6}$

B. (b) $\dfrac{2\pi}{3}$

C. (c) $\dfrac{5\pi}{6}$

D. (d) $\dfrac{\pi}{3}$

Answer: null

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674. The value of $\int_0^{\frac{3\pi}{2}} \dfrac{\left|\tan^{-1}\tan x\right| - \left|\sin^{-1}\sin x\right|}{\left|\tan^{-1}\tan x\right| + \left|\sin^{-1}\sin x\right|} dx$ is equal to

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675. Let $f(0) = 0 \, and \int_0^2 f'(2t)e^{f(2t)}dt = 5.$ $then value of f(4) is$ log 2 (b) log 7 (c) log 11 (d) log 13

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676. The value of the integral $\int_0^\infty \dfrac{x\log x}{\left(1 + x^2\right)^2} dx$, is (a)0 (b) log 7 (c) 5 log 13 (d) none of these

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677. $\int_0^\pi \dfrac{x \tan x}{\sec x + \cos x} dx$ is $\dfrac{\pi^2}{4}$ (b) $\dfrac{\pi^2}{2}$ (c) $\dfrac{3\pi^2}{2}$ (d) $\dfrac{\pi^2}{3}$

678. If $\int_{\log 2}^x \dfrac{dx}{\sqrt{e^x - 1}} = \dfrac{\pi}{6}$, then x is equal to 4 (b) 1n 8 (c) 1n 4 (d) none of these

679. The solution for x of the equation $\int_{\sqrt{2}}^x \dfrac{dt}{t\sqrt{t^2 - 1}} = \dfrac{\pi}{2}$ is π (b) $\dfrac{\sqrt{3}}{2}$ (c) $2\sqrt{2}$ (d) none of these

680. Which of the following is incorrect? $\int_{a+c}^{b+c} f(x)dx = \int_a^b f(x + c)dx$

$\int_{ac}^{bc} f(x)dx = c\int_a^b f(cx)dx$ $\int_{-a}^a f(x)dx = \frac{1}{2}\int_{-a}^a (f(x) + f(-x)dx$ None of these

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681. The value f the integral $\int_{-\pi}^{\pi} \sin mx \sin nx dx$, for $m \neq n (m, n \in I)$, is 0 (b)

π (c) $\dfrac{\pi}{2}$ (d) 2π

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682. $\int \dfrac{e^x \left(2 - x^2\right)}{(1-x)\sqrt{1-x^2}} dx$ is equal to

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683. $\int_0^\infty \left(\dfrac{\pi}{1 + \pi^2 x^2} - \dfrac{1}{1 + x^2} \right) \log x dx$ is equal to (a) $-\dfrac{\pi}{2}\log \pi$ (b) 0 (c) $\dfrac{\pi}{2}\log 2$ (d)

none of these

684. If $f(x) = \cos\left(\tan^{-1}x\right)$, then the value of the integral $\int_0^1 xf'(x)dx$ is (a) $\dfrac{3-\sqrt{2}}{2}$ (b) $\dfrac{3+\sqrt{2}}{2}$ 1 (d) $1 - \dfrac{3}{2\sqrt{2}}$

685. The equation of the curve is $y = f(x)$ The tangents at $[1, f(1)], [2, f(2)],$ $and[3, f(3)]$ make angles $\dfrac{\pi}{6}, \dfrac{\pi}{3}, and\dfrac{\pi}{4},$ respectively, with the positive direction of x-axis. Then the value of $\int_2^3 f'(x)f''dx + \int_1^3 f''dx$ is equal to (a)$-\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{1}{\sqrt{3}}$ (c)0 (d) none of these

686. The value of $\int_1^e\left(\dfrac{\tan^{-1}x}{x} + \dfrac{\log x}{1 + x^2}\right)dx,$ is (a) $\tan e$ (b) $\tan^{-1}e$ (c)$\tan^{-1}\left(\dfrac{1}{e}\right)$

(d) none of these

687. If $f(\pi) = 2\int_0^\pi \left(f(x) + f^x\right)\sin x\,dx = 5$, then $f(0)$ is equal to (it is given that $f(x)$ is continuous in $[0, \pi]$). (a) 7 (b) 3 (c) 5 (d) 1

688. If $\int_1^2 e^{x^2}\,dx = a$, then $\int_e^{e^4} \sqrt{1nx}\,dx$ is equal to (a)$2e^4 - 2e - a$ (b) $2e^4 - e - a$ (c)$2e^4 - e - 2a$ (d) $e^4 - e - a$

689. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \dfrac{e^{|\sin x|}\cos x}{1 + e^{\tan x}}\,dx$ is equal to (a) $e + 1$ (b) $1 - e$ (c) $e - 1$ (d) none of these

690. The value of the expression $\dfrac{\int_0^a x^4\left(\sqrt{a^2 - x^2}\right) dx}{\int_0^a x^2\left(\sqrt{a^2 - x^2}\right)}dx$ is

691. If $A = \int_0^\pi \dfrac{\cos x}{(x + 2)^2}dx$, $then \int_0^{\frac{\pi}{2}} \dfrac{\sin 2x}{x + 1}dx$ is equal to (a)$\dfrac{1}{2} + \dfrac{1}{\pi + 2} - A$ (b) $\dfrac{1}{\pi + 2} - A$ (c)$1 + \dfrac{1}{\pi + 2} - A$ (d) $A - \dfrac{1}{2} - \dfrac{1}{\pi + 2}$

692. $\int_0^4 \dfrac{\left(y^2 - 4y + 5\right)\sin(y - 2)dy}{\left[2y^2 - 8y + 11\right]}$ is equal to (a) 0 (b) 2 (c) -2 (d) none of these

693. The values of a for which the integral $\int_0^2 |x - a|dx \geq 1$ is satisfied are (a) $(2, \infty)$ (b) $(-\infty, 0)$ (c)$(0, 2)$ (d) none of these

694. evaluate: $I = \int \dfrac{2x}{\left(x^2 + 1\right)\left(x^2 + 3\right)} dx$ is

695. Let $I = \int_1^3 \sqrt{3 + x^3} dx$, then the value of I will lie in the interval (a)$[4, 6]$ (b) $[1, 3]$ (c)$\left[4, 2\sqrt{30}\right]$ (d) $\left[\sqrt{15}, \sqrt{30}\right]$

696. If $\int_a^b |sinx|dx = 8 and \int_0^{a+b} |cosx|dx = 9, then$

(a) $a + b = \dfrac{9\pi}{2}$

(b) $|a - b| = 4\pi$

(c)$\dfrac{a}{b}$ = 15

(d) $\int_a^b \sec^2 x \, dx = 0$

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697. If $f(x) = \{0$,where $x = \dfrac{n}{n+1}$, $n = 1, 2, 3....$ and 1 else where then the value of $\int_0^2 f(x) dx$ is

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698. The function f and g are positive and continuous. If f is increasing and g is decreasing, then $\int_0^1 f(x)[g(x) - g(1-x)] dx$

(a)is always non-positive

(b)is always non-negative

(c)can take positive and negative values

(d)none of these

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699. A Function $f(x)$ satisfies the relation $f(x) = e^x + \int_0^1 e^x f(t)dt$ Then (a) $f(0) < 0$ (b)$f(x)$ is a decreasing function. (c)$f(x)$ is an increasing function. (d)$\int_0^1 f(x)dx > 0$

700. Let $f(x)$ be positive, continuous, and differentiable on the interval (a, b) and $\lim x \to a^+ f(x) = 1$, $\lim x \to b^- f(x) = 3^{\frac{1}{4}}$ If $f'(x) \geq f^3(x) + \dfrac{1}{f(x)}$, then the greatest value of $b - a$ is (a)$\dfrac{\pi}{48}$ (b) $\dfrac{\pi}{36}$ (c)$\dfrac{\pi}{24}$ (d) $\dfrac{\pi}{12}$

701. Let$f:(0, \infty) \to R$ and$F\left(x^3\right) = \int_0^{x^3} f(t)dt,$ if $F\left(x^3\right) = x^3\left(x^2 + x + 1\right)$ .Then f(27)=

702. If $g(x) = \int_0^x 2|t|dt$, then (a) $g(x) = x|x|$ (b) $g(x)$ is monotonic (c) $g(x)$ is differentiable at x=0 (d) $g'(x)$ is differentiable at x=0

703. Prove that the value of the integral, $\int_0^{2a} \dfrac{f(x)}{f(x) + f(2a - x)} dx$ is equal to a.

704. Let $f: \left[\dfrac{1}{2}, 1\right] \rightarrow R$ (the set of all real numbers) be a positive, non-constant, and differentiable function such that $f'(x) < 2f(x) and f\left(\dfrac{1}{2}\right) = 1$. Then the value of $\int_{\frac{1}{2}}^1 f(x)dx$ lies in the interval (a)$(2e - 1, 2e)$ (b) $(3 - 1, 2e - 1)$ (c)$\left(\dfrac{e - 1}{2}, e - 1\right)$ (d) $\left(0, \dfrac{e - 1}{2}\right)$

705. Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4 + 1}\,dt$, for all, $x \in (-1, 1)$ and let f^{-1} be the inverse function of f Then $\left(f^{-1}\right)'(2)$ is equal to (a) 1 (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{e}$

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706. $\displaystyle\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{2x}{1+x^2}\right)dx$

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707. The value of $\displaystyle\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin x^2}{\sin x^2 + \sin\left(\ln 6 - x^2\right)}\,dx$ is (A) $\dfrac{1}{4}\ln\left(\dfrac{3}{2}\right)$ (B) $\dfrac{1}{2}\ln\left(\dfrac{3}{2}\right)$ (C) $\ln\left(\dfrac{3}{2}\right)$ (D) $\dfrac{1}{6}\ln\left(\dfrac{3}{2}\right)$

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708. Let f be a non-negative function defined on the interval $[0, 1]$. If

$$\int_0^x \sqrt{1 - \left(f'(t)\right)^2}\, dt = \int_0^x f(t)dt, \quad 0 \le x \le 1, \text{ and } f(0) = 0, \text{ then}$$

A. (A)$f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

B. (B)$f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) > \dfrac{1}{3}$

C. (C)$f\left(\dfrac{1}{2}\right) < \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

D. (D)$f\left(\dfrac{1}{2}\right) > \dfrac{1}{2}$ and $f\left(\dfrac{1}{3}\right) < \dfrac{1}{3}$

Answer: null

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709. $\left(\lim\right)_{n \to \infty} \dfrac{n}{2^n}\int_0^2 x^n dx$ equals___

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710. Let $f: [0, \infty) \to R$ be a continuous strictly increasing function, such that $f^3(x) = \int_0^x tf^{.2}(t)dt$ for every $x \geq 0$. Then value of $f(6)$ is_____

711. A continuous real function f satisfies $f(2x) = 3\left(f(x) \, \forall x \in R If \int_0^1 f(x)dx = 1, \right.$ then find the value of $\int_1^2 f(x)dx$

712. Let $f(x) = x^3 - \dfrac{3x^2}{2} + x + \dfrac{1}{4}$ Then the value of $\left(\int_{\frac{3}{4}}^{\frac{3}{4}} f(f(x))dx \right)^{-1}$ is____

713. Consider the polynomial $f(x) = ax^2 + bx + \cdot$ If $f(0) = 0, f(2) = 2,$ then the minimum value of $\int_0^2 |f'(x)| dx$ is __

714. Column I, Column II If $f(x)$ is an integrable function for

$x \in \left[\dfrac{\pi}{6}, \dfrac{\pi}{3}\right]$ and $\qquad\qquad\qquad I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2\theta f(2\sin 2\theta) d th\eta,$ and

$I_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos e\, c^2\theta f(2\sin 2\theta) dth\eta,\ then \dfrac{I_1}{I_2} = $, p. 3 If $f(x+1) = f(3+x)\,\forall x,$ and

the value of $\int_a^{a+b} f(x) dx$ is independen $\to fa,$ then the value of b can be, q. 1

The value of $\int_0^1 \dfrac{\tan^{-1}\left[x^2\right]}{\tan^{-1}\left[x^2\right] + \tan^{-1}\left[25 + x^2 - 10x\right]}$, r. 2

$(d)If\int_0^2 \sqrt{x + \sqrt{x + \sqrt{x + \infty}}} dx (where x > 0), then [I]$ is equal to (where [.]

denotes the greatest integer function), s. 4

715. STATEMENT 1: If $f(x)$ is continuous on $[a, b]$, then there exists a point

$c \in (a, b)$ such that $\int_a^b f(x) dx = f(c)(b - a)$

STATEMENT 2: For $a < b,$ if m and M are, respectively, the smallest and

greatest values of $f(x)$ on $[a, b]$, then $m(b - a) \le \int_a^b f(x) dx \le (b - a)M$

716. The value of the definite integral, $\int_0^{\frac{\pi}{2}} \frac{\sin 5x}{\sin x} dx$ is

717. The value of $\int_0^{\pi} \dfrac{\sin\left(n + \dfrac{1}{2}\right)x}{\sin\left(\dfrac{x}{2}\right)} dx$ is, (a) $n \in I, n \geq 0 \frac{\pi}{2}$ (b) 0 (c) π (d) 2π

718. If $I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, $(m, n \in I, m, n \geq 0)$, then (a) $I(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m-n}}dx$ (b) $I(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}dx$ (c) $I(m, n) = \int_0^{\infty} \frac{x^{n-1}}{(1+x)^{m+n}}dx$ (d) $I(m, n) = \int_0^{\infty} \frac{x^n}{(1+x)^{m+n}}dx$

719. If $I_n = \int_0^{\sqrt{3}} \dfrac{dx}{1 + x^n}$, $(n = 1, 2, 3..)$, then find the value of $(\lim)_{n \to \infty} I_n$.

(a) 0 (b) 1 (c) 2 (d) $\dfrac{1}{2}$

720. $\int_0^1 \dfrac{\tan^{-1}x}{x} dx$ is equals to (a) $\int_0^{\frac{\pi}{2}} \dfrac{\sin x}{x} dx$ (b) $\int_0^{\frac{\pi}{2}} \dfrac{x}{\sin x} dx$ (c) $\dfrac{1}{2}\int_0^{\frac{\pi}{2}} \dfrac{\sin x}{x} dx$ (d) $\dfrac{1}{2}\int_0^{\frac{\pi}{2}} \dfrac{x}{\sin x} dx$

721. If $\int_0^1 \dfrac{\sin t}{1 + t} dt = \alpha$, then the value of the integral $\int_{4\pi-2}^{4\pi} \dfrac{\sin\left(\dfrac{t}{2}\right)}{4\pi + 2 - t} dt$ is (1) 2α (2) -2α (3) α (4) $-\alpha$

722. $Let I_1 = \int_0^1 \frac{e^x dx}{1+x} and I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}\left(2-x^3\right)}$ tehn $\frac{I_1}{I_2}$ isequal $\rightarrow$ (a)3/e(b)e/3(c)

3e(d)1/(3e)`

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723. $\int_{\sin\theta}^{\cos\theta} f(x\tan\theta)dx\left(where\theta \neq \frac{n\pi}{2}, n \in I\right)$ is equal to (a)

$-\cos\theta\int_1^{\tan\theta} f(x\sin\theta)dx$ (b)$-\tan\theta\int_{\cos\theta}^{\sin\theta}f(x)dx$ (c)$\sin\theta\theta\int_1^{\tan\theta} f(x\cos\theta)dx$ (d)

$\frac{1}{\tan\theta}\theta\int_{\sin\theta}^{\sin\theta\tan\theta} f(x)dx$

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724. Let $g(x) = \int_0^x f(t). dt$,where f is such that $\frac{1}{2} \leq f(t) \leq 1$ for $t \in [0, 1]$ and

$0 \leq f(t) \leq \frac{1}{2}$ for $t \in [1, 2]$.Then g(2) satisfies the inequality

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725. The value of $\int_0^{\frac{\pi}{2}} \sin|2x - \alpha|dx$, where $\alpha \in [0, \pi]$, is (a) $1 - \cos\alpha$ (b) $1 + \cos\alpha$ (c) 1 (d) $\cos\alpha$

726. If $I_n = \int_0^\pi e^x(\sin x)^n dx$, then $\frac{I_3}{I_1}$ is equal to $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{2}{5}$

727. $\int_{-2}^0 \left\{(x + 1)^3 + 2 + (x + 1)\cos(x + 1)\right\}dx$ is equal to (A) -4 (B) 0 (C) 4 (D) 6

728. For any real number x, let $[x]$ denote the largest integer less than or equal to x, Let f be a real-valued function defined on the interval $[- 10, 10]$

be $f(x) = \{x - [x]$, if $[x]$ is odd, $1 + [x] - x$, if $[x]$ is even Then the value

of $\dfrac{\pi^2}{10}\int_{-10}^{10}f(x)\cos\pi x\,dx$ is____

729. evaluate: $\int_0^{\frac{\pi}{4}} 2\tan^3 x\,dx$ is

730. If f is continuous function and $F(x) = \int_0^x \left((2t + 3) \cdot \int_t^2 f(u)du \right) dt$, then

$\left| \dfrac{F''(2)}{f(2)} \right|$ is equal to_____

731. Let $f'(x) = \dfrac{192x^3}{2 + \sin^4 \pi x}$ or allx $\in$ Rwithf$\left(\dfrac{1}{2}\right)$ = 0. Ifm $\le \int_{\frac{1}{2}}^{1} f(x)dx \le M$,

then the possible values of $m$and$M$ are (a)m = 13, M = 24 (b)

$m = \dfrac{1}{4}, M = \dfrac{1}{2}$ (c)m = - 11, M = 0 (d) m = 1, M = 12

732. If the value of $\lim\limits_{n \to \infty} \left(n^{-\frac{3}{2}} \sum\limits_{j=1}^{6n} \sqrt{j} \right)$ is equal to $\sqrt{N}$, then the value of $\dfrac{N}{12}$ is _____

733. $f(x) = \int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of $f(1)$ is

734. If $I = \int_0^{\frac{3\pi}{4}} [(1 + x)\sin x + (1 - x)\cos x]dx$, then value of $\left(\sqrt{2} - 1\right)I$ is____

735. The value of $\int_1^a [x] f'(x)dx$, where $a > 1$, and $[x]$ denotes the greatest integer not exceeding x, is

(A) $af(a) - \{f(1)f(2) + \text{.... .} + f([a])\}$

(B) $[a]f(a) - \{f(1) + f(2) + \text{......} + f([a])\}$

(C) $[a]f(a) - \{f(1) + f(2) + \text{.......} + fA\}$

(D) $af([a]) - \{f(1) + f(2) + \text{......} + fA\}$

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736. The value of $\int_0^x [cost]dt$, $x\varepsilon\left[(4n+1)\dfrac{\pi}{2}, (4n+3)\dfrac{\pi}{2}\right]$ and $n\varepsilon N$ is equal to (where [.] represents greatest integer function.

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737. Let $f(x) = \min(\{x\}, \{-x\}) \forall \varepsilon R$, where $\{.\}$ denotes the fractional part of x. Then $\int_{-100}^{100} f(x)dx$ is equal to

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738. $\int_1^4 (x - 0.4)dx$ equals (where $\{x\}$ is a fractional part of (x)

739. The value of the definite integral $\int_2^4 (x(3 - x)(4 + x)(6 - x)(10 - x) + \sin x)dx$ equal (a)$\cos 2 + \cos 4$ (b) $\cos 2 - \cos 4$ (c)$\sin 2 + \sin 4$ (d) $\sin 2 - \sin 4$

740. If $x = \int_c^{\sin t} \sin^{-1} z\, dz$, $y = \int_k^{\sqrt{t}} \frac{\sin z^2}{z}dz$, then $\frac{dy}{dx}$ is equals

741. If $f(x) = \int_0^1 \frac{dt}{1 + |x - t|}$,then $f'\left(\frac{1}{2}\right)$ is equal to (a)0 (b) $\frac{1}{2}$ (c) 1 (d) none of these

742. Let $f(x) = \int_2^x \dfrac{dt}{\sqrt{1 + t^4}}$ and g be the inverse of f. Then, the value of $g'(0)$

is

743. STATEMENT 1: $\int_0^{\frac{\pi}{4}} \log(1 + \tan\theta)d\theta = \dfrac{\pi}{8}\log 2$. STATEMENT 2:

$\int_0^{\frac{\pi}{2}} \log \sin\theta \, d\theta = -\pi\log 2$.

744. STATEMENT 1:$f(x)$ is symmetrical about $x = 2$. Then $\int_{2-a}^{2+a} f(x)dx$ is

equal to $2\int_2^{2+a} f(x)dx$ STATEMENT 2: If $f(x)$ is symmetrical about $x = b$,

then $f(b - \alpha) = f(b + \alpha) \, \forall \, \alpha \in R$

745. If $f(x) = \cos x - \int_0^x (x - t)f(t)dt$, then $f''(x) + f(x)$ is equal to (a)- $\cos x$ (b) - $\sin x$ (c) $\int_0^x (x - t)f(t)dt$ (d) 0

746. If $f'(x) = f(x) + \int_0^1 f(x)dx$, given $f(0) = 1$, then the value of $f\left((\log)_e 2\right)$ is

(a) $\dfrac{1}{3 + e}$ (b) $\dfrac{5 - e}{3 - e}$ (c) $\dfrac{2 + e}{e - 2}$ (d) none of these

747. STATEMENT 1: $\int_0^\pi \sqrt{1 - \sin^2 x}dx = 0$, STATEMENT 2: $\int_0^\pi \cos x dx = 0$

748. STATEMENT 1: Let m be any integer. Then the value of $I_m = \int_0^\pi \dfrac{\sin 2mx}{\sin x}dx$ is zero.

STATEMENT 2 : $I_1 = I_2 = I_3 = \ = I_m$

749. STATEMENT 1: The value of $\int_0^{2\pi} \cos^{99} x\, dx$ is 0 STATEMENT 2: $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$,if $f(2a - x) = f(x)$

750. For $a \varepsilon R$ (the set of all real numbers)

$$a \neq -1, \ \lim_{n \to \infty} \frac{\left(1^a + 2^a + \ldots\ldots + n^a\right)}{(n + 1)^{a-1}[(na + 1) + (n + 2) + \ldots\ldots + (na + n)]} = \frac{1}{60}$$

Then $a =$

751. Let S be the area of the region enclosed by $y - e^{-x^2}, y = 0, x = 0$ and $x = 1$. Then

752. Let $f:(0, \infty)\vec{R}$ be given by $f(x) = \int_{\frac{1}{x}}^{x} \frac{e^{-\left(t + \frac{1}{t}\right)} dt}{t}$, then (a)$f(x)$ is

monotonically increasing on $[1, \infty)$ (b)$f(x)$ is monotonically decreasing on

$(0, 1)$ (c)$f\left(2^x\right)$ is an odd function of x on R (d) none of these.

753. Show that $\int_0^{\pi} xf(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$

754. Let $f(x)$ be a non-constant twice differentiable function defined on

$(-\infty, \infty)$ such that $f(x) = f(1 - x) and f'\left(\frac{1}{4}\right) = 0$. Then

755. Let $f(x) = \int_2^x \frac{dt}{\sqrt{1 + t^4}}$ and $g(x)$ be the inverse of $f(x)$. Then the value of $g'(0)$

756. Let f be a real-valued function defined on interval $(0, \infty)$,by $f(x) = \ln x + \int_0^x \sqrt{1 + \sin t}. \, dt$. Then which of the following statement(s) is (are) true? (A). f"(x) exists for all $\in (0, \infty)$. (B). f'(x) exists for all x $\in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$. (C). there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all x $\in (\alpha, \infty)$. (D). there exists $\beta > 1$ such that $|f(x)| + |f'(x)| \le \beta$ for all x $\in (0, \infty)$.

757. Let $f: R \to R$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^x f(t)dt$ for all $x \in [-1, 2]$

and $G(x) = \int_{-1}^{x} t|f(f(t))| dt$ for all $x \in [-1, 2]$. If $\lim_{x \to 1} \dfrac{F(x)}{G(x)} = \dfrac{1}{14}$, Then

the value of $f\left(\dfrac{1}{2}\right)$ is

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758. If the value of the definite integral $\int_0^1 \dfrac{\sin^{-1}\sqrt{x}}{x^2 - x + 1} dx$ is $\dfrac{\pi^2}{\sqrt{n}}$ (where

$n \in N$), then the value of $\dfrac{n}{27}$ is

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759. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$. Then

the correct expression (s) is (are) (a) $\int_0^{\frac{\pi}{4}} xf(x)dx = \dfrac{1}{12}$ (b) $\int_0^{\frac{\pi}{4}} f(x)dx = 0$ (c)

$\int_0^{\frac{\pi}{4}} xf(x) = \dfrac{1}{6}$ (d) $\int_0^{\frac{\pi}{4}} f(x)dx = \dfrac{1}{12}$

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760. The option(s) with the values of a and L that satisfy the following

equation is (are) $\dfrac{\int_0^{4\pi} e^t \left(\sin^6 at + \cos^4 at \right) dt}{\int_0^{\pi} e^t \left(\sin^6 at + \cos^4 at \right) dt} = L$

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761. Let $f: R\vec{R}$ be a function defined by $f(x) = \{[x], (x \le 2)\ (0, x > 2)$ where

$[x]$ is the greatest integer less than or equal to x. If

$I = \int_{-1}^{2} \dfrac{xf\left(x^2\right)}{2 + f(x + 1)} dx$, then the value of $(4I - 1)$ is

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762. Prove that $\int_0^1 \tan^{-1}\left(\dfrac{1}{1 - x + x^2} \right) dx = 2\int_0^1 \tan^{-1} x dx$. Hence or otherwise,

evaluate the integral $\int_0^1 \tan^{-1}\left(1 - x + x^2 \right) dx$

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763. Let $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \cos ec x \\ \cos^2 x & \cos^2 x & \csc^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$ then, $\int_{-\pi}^{\pi} f(x) dx$ equals

764. If $f(x) = \int_a^x [f(x)]^{-1} dx$ and $\int_a^1 [f(x)]^{-1} dx = \sqrt{2}$, then

A. (a) $f(2) = 2$

B. (b) $f'(2) = \dfrac{1}{2}$

C. (c) $f'(2) = 2$

D. (d) $\int_0^1 f(x) dx = \sqrt{2}$

Answer: null