MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

LIMITS

ILLUSTRATION

1. Find the value of $\lim_{x \to 3^-} \frac{x-2}{x-3}$.

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2. Prove that $\lim_{x \to 2} [x]$ does not exists, where [.] represents the greatest integer function.

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3. $Let f(x) = \begin{cases} x+1 & , \quad \text{if} \quad x \geq 0 \\ x-1 & , \quad \text{if} \quad x < 0 \end{cases}$. Then prove that $\lim_{x \to 0} f(x)$ does not exist.

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4. IF $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1-x & \text{if } x \text{ is irrational} \end{cases}$, then find $\lim_{x \to 1/2} f(x)$ if exists.

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5. Evalate $\lim_{x \to 1^+} 2^{-2^{\frac{1}{1-x}}}$.

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6. Evaluate $\lim_{x \to 0^-} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$.

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7. Evaluate $\lim_{x \to 0} \frac{\sin x - 2}{\cos x - 1}$.

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8. If a and b are positive and [x] denotes greatest integer less than or equal to x, then find $\lim_{x \to 0^+} \frac{x}{a}\left[\frac{b}{x}\right]$.

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9. $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)} & \text{if } x \neq 4 \\ 0 & \text{if } x = 4 \end{cases}$ check limit at $x = 4$ is.

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10. Evaluate the left- and right-hand limits of the function defined $f(x) = \begin{cases} 1 + x^2, \text{if } 0 \leq x < 1 \\ 2 - x, \text{if} x > 1 \end{cases}$ at $x = 1$. *Also, showt*ˆlim_(xto1) f(x)` does not exist.

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11. Let $f(x) = \begin{cases} \cos[x], & x \leq 0 \\ |x| + a, & x < 0 \end{cases}$. Then find the value of a, so that $\lim_{x \to 0} f(x)$ exists, where [x] denotes the greatest integer function less than or equal to x.

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12. Evaluate: $\lim_{x \to \frac{5\pi}{4}} [\sin x + \cos x]$, [.] denotes the greatest integer function.

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13. Let

$$f(x) = \begin{cases} x + 1, & xl > 0 \\ 2 - x, & x \leq 0 \end{cases} \text{and} g(x) = \begin{cases} x + 3, & x < 1 \\ x^2 - 2x - 2, & 1 \leq x < 2 \\ x - 5, & x \geq 2 \end{cases}$$

Find the LHL and RHL of g(f(x)) at x=0 and, hence, find $\lim_{x \to 0} g(f(x))$.

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14. If $\lim_{x \to a} [f(x) + g(x)] = 2$ and $\lim_{x \to a} [f(x) - g(x)] = 1$, then find the value of $\lim_{x \to a} f(x)g(x)$.

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15. Find the following limits.

(i) $\lim_{x \to 2} \frac{4x}{x^3 - 3}$ (ii) $\lim_{x \to 1} \frac{\log_{10} x - 3}{3x - 2}$ (iii) $\lim_{x \to \pi} \frac{3 + \cos}{2 - \sin}$

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16. Evaluate $\lim_{x \to 0} \left(\frac{x + 4}{2 - x} \right)^{\frac{x^2 + 2x - 3}{x - 1}}$

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17. If $a_1 = 1$ and $a_n + 1 = \frac{4 + 3a_n}{3 + 2a_n}, n \geq 1$and if $\lim_{n \to \infty} a_n = a$, then find the value of a.

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18. Evaluate $\lim_{x \to \infty} \frac{\sin x}{x}$.

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19. Find $\lim_{x \to 0} [x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$, (where $[.]$ represents the greatest integer funciton).

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20. In the neighbourhood of $x = 0$ it is known that $1 + |x| < \frac{e^x - 1}{x} < 1 - |x|$ then find $\lim_{x \to 0} \frac{e^x - 1}{x}$.

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21. If $0 < \log_e x < \sqrt{x}$ for all $x < 1$, then find the value of $\lim_{x \to \infty} \frac{\log_e x}{x}$.

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22. If $3 - \left(\frac{x^2}{12}\right) \leq f(x) \leq 3 + \left(\frac{x^3}{9}\right)$ in the neighborhood of x=0, then find the value of $\lim_{x \to 0} f(x)$.

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23. Evaluate $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4}$.

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24. Evaluate $\lim_{x \to 1} \left(\frac{2}{1 - x^2} - \frac{1}{1 - x}\right)$.

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25. Evaluate $\lim_{x \to 1} \frac{x^2 + x \log_e x - \log_e x - 1}{(x^2) - 1}$

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26. The value of $\lim_{x \to \frac{3\pi}{4}} \frac{1 + \sqrt[3]{\tan x}}{1 - 2\cos^2 x}$ is

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27. Evaluate $\lim_{x \to \infty} \frac{\log_e x}{x}$

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28. Evaluate $\lim_{n \to \infty} (4^n + 5^n)^{1/n}$

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29. Evaluate $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$.

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30. Evaluate $\lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}, (a \neq 0)$.

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31. Evaluate `lim_(xto pi//2) tan^(2)x[sqrt(2sin^(2)x+3sinx+4)-sqrt(sin^(2)x+6sinx+2)]

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32. Evaluate $\lim_{n \to \infty} \frac{1^3 + 2^3 + 3^3 + \ldots + n^3}{\sqrt{4n^8 + 1}}$.

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33. If $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 7x + 12}$, then which of the following limits exists?

(i) $\lim_{x \to \infty} \sin^{-1} f(x)$ $\quad$ (ii) $\lim_{x \to \infty} \cos^{-1} f(x)$

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34. Evaluate the following limits :

$$\lim_{x \to \infty} \frac{\sqrt{3x^2 - 1} - \sqrt{2x^2 - 1}}{4x + 3}$$

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35. Evaluate $\lim_{x \to \infty} \sqrt{x}\left(\sqrt{x + c} - \sqrt{x}\right)$.

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36. Find the value of $\lim_{x \to 0^+} \dfrac{3(\log_e x)^2 + 5\log_e x + 6}{1 + (\log_e x)^2}$.

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37. Evaluate $\lim_{x \to \infty} \dfrac{3^{\sin x} + 2x + 1}{\sin x - \sqrt{x^2 + 1}}$.

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38. Evaluate $\lim_{x \to \infty} \dfrac{x + 7 \sin x}{-2x + 13}$

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39. Evaluate $\lim_{x \to \infty} \dfrac{\sqrt{x^2 + 1} - \sqrt[3]{x^3 + 1}}{\sqrt[4]{x^4 + 1} - \sqrt[5]{x^4 + 1}}$

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40. Evaluate $\lim_{x \to \infty} \left(\sqrt{25x^2 - 3x} + 5x\right)$.

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41. Evaluate $\lim_{x \to \infty} \left(\frac{x^2 + x - 1}{3x^2 + 2x + 4} \right)^{\frac{3x^2 + x}{x - 2}}$

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42. Evaluate $\lim_{n \to \infty} \sin^n \left(\frac{2\pi n}{3n + 1} \right), n \in N.$

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43. Evaluate $\lim_{x \to \infty} \left(\sqrt[3]{(x + 1)(x + 2)(x + 3)} - x \right).$

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44. Evaluate $\lim_{n \to \infty} \frac{1}{1 + n^2} + \frac{2}{2 + n^2} + \ldots + \frac{n}{n + n^2}.$

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45. If [x] denotes the greatest integer less than or equal to x, then evaluate $\lim_{n\to\infty} \frac{1}{n^3}\left([1^1x]+[2^2x]+[3^2x]+\ldots+[n^2x]\right)$.

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46. If $\lim_{x\to\infty}\left\{\frac{x^2+1}{x+1}-(ax+b)\right\}=0$, then find the values of a and b.

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47. Evaluate $\lim_{n\to\infty}\frac{n^p\sin^2(n!)}{n+1}$, wher $0<p<1$.

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48. Evaluate $\lim_{n\to\infty}(-1)^{n-1}\sin\left(\pi\sqrt{n^2+0.5n+1}\right)$, where $n\in N$

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49. Evaluate $\lim_{x \to 2} \frac{x^{10} - 1024}{x^5 - 32}$.

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50. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$ and $n \in N$, then find the value of n.

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51. Evaluate $\lim_{x \to 1} \frac{\sqrt{x} + \sqrt{\sqrt{x}} + \sqrt{\sqrt{\sqrt{x}}} + \sqrt{\sqrt{\sqrt{\sqrt{x}}}} - 4}{x - 1}$.

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52. Evaluate $\lim_{x \to a} \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a}$.

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53. Evaluate $\lim_{x \to 2} \dfrac{\sqrt{(x+7)} - 3\sqrt{(2x-3)}}{\sqrt[3]{(x+6)} - 2\sqrt[3]{(3x-5)}}$.

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54. If $\lim_{x \to 0} \dfrac{(4x-1)^{\frac{1}{3}} + a + bx}{x} = \dfrac{1}{3}$ then find the values of a and b.

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55. Evaluate $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$.

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56. Evaluate $\lim_{x \to 0} \dfrac{5\sin x - 7\sin 2x + 3\sin 3x}{x^2 \sin x}$.

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57. Evaluate $\lim_{x \to 0} \frac{\sin x + \log(1 - x)}{x^2}$.

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58. Evaluate $\lim_{x \to 0} \frac{e^{\sin x} - (1 + \sin x)}{(\tan(\sin x))^2}$

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59. Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

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60. If $\lim_{x \to 0} \frac{\cos 4x + a \cos 2x + b}{x^4}$ is finite, find a and b using expansion formula. Also, find the limit.

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61. Find the integral value of n for which $(\lim)_{x \to 0} \frac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^3}{2}}{x^n}$ is a finite nonzero number

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62. Find the limits of the following:

$(i) \lim_{x \to 0} \frac{\sin 3x}{x}$ $(ii) \lim_{x \to 0} \frac{\sin 7x}{\sin 4x}$ $(iii) \lim_{x \to 0} \frac{1 - \cos^2 x}{x^2}$

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63. Find the following limits:

$(i) \lim_{x \to 0} \frac{1}{x} \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$ $(ii) \lim_{x \to 0} \frac{1}{x} \sin^{-1}(3x - 4x^3)$

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64. Evaluate $\lim_{x \to \infty} 2^{x-1} \tan\left(\frac{a}{2^x}\right)$.

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65. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}$.

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66. Evaluate $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 2x - \sin(x - 2)}$.

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67. Find $\lim_{x \to 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ if exists.

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68. Evaluate $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$.

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69. Evaluate $\lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$.

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70. Evaluate $\lim_{x \to \pi} \frac{\sin^{-1}(1 + \cos x).\sec\left(\frac{x}{2}\right)}{(x - \pi)}$.

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71. Evaluate $\lim_{x \to \infty} \left[\frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{\left(1 + |x|^3\right)}\right]$.

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72. Evaluate $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$.

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73. Evaluate $\lim_{x \to \infty} x\left(\tan^{-1}\frac{x+1}{x+4} - \frac{\pi}{4}\right)$.

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74. Evaluate $\lim_{n \to \infty} n\sin\left(2\pi\sqrt{1+n^2}\right), (n \in N)$.

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75. Evaluate $\lim_{x \to -1^+} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{1+x}}$.

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76. Evaluate $\lim_{x \to 0} x\left[\tan^{-1}\left(\frac{x+1}{x+2}\right) - \tan^{-1}\left(\frac{x}{x+2}\right)\right]$.

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77. Evaluate $\lim_{x \to 0} \frac{1 - \cos(1 - \cos x)}{x^4}$.

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78. Using $\lim_{\theta \to 0} \left(\frac{\sin \theta}{\theta} \right) = 1$ prove that the area of circle of radius R is πR^2 (Figure)

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79. Prove that $\left[\lim_{x \to 0} \frac{\sin x}{x} \right] = 0$, where $[.]$ represents the greatest integer function.

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80. Prove that $\left[\lim_{x \to 0} \frac{\tan^{-1} x}{x} \right] = 0$, where $[.]$ represents the greatest integer function.

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81. If $L = \lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ is finite, then find the value of a and L.

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82. If $m, n \in I_0 and (\lim)_{x \to 0} \frac{\tan 2x - n \sin x}{x^3} =$ some integer, then find the value of n and also the value of limit.

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83. Evaluate $\lim_{x \to 0} \frac{3^{2x} - 2^{3x}}{x}$.

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84. Evaluate $\lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{x \tan x}$.

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85. Evaluate $\lim_{x \to 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$.

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86. Evaluate $\lim_{x \to 1} \frac{a^{x-1} - 1}{\sin \pi x}$.

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87. Evaluate $\lim_{x \to 0} \frac{e^x - e^{x \cos x}}{x + \sin x}$.

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88. Evaluate x $\lim_{x \to 2} \frac{x - 2}{\log_a (x - 1)}$.

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89. Evaluate $\lim_{x \to a} \frac{\log x - \log a}{x - a}$.

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90. Evaluate $\lim_{x \to 0} \frac{\log(5+x) - \log(5-x)}{x}$.

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91. Evaluate $\lim_{h \to 0} \frac{\log_e(1+2h) - 2\log_e(1+h)}{h^2}$.

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92. Let $P_n = a^{P_{n-1}} - 1, \forall n = 2, 3, \ldots,$ and let $P_1 = a^x - 1,$ where $a \in R^+$. Then evaluate $\lim_{x \to 0} \frac{P_n}{x}$.

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93. If $\lim_{x \to 0} \frac{ae^x - b}{x}$=2, then find the values of a and b.

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94. Find the following limits:

$(i)\ \lim_{x \to 0} (1-x)^{\frac{1}{x}}$ $(ii)\ \lim_{x \to 1} (1+\log_e x)^{\frac{1}{\log_e x}}$

$(iii)\ \lim_{x \to 0} (1+\sin x)^{\frac{1}{x}}$

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95. Evaluate $\lim_{x \to 0} (\cos x)^{\cot x}$.

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96. The population of a country increases by 2% every year. If it increases k times in a century, then prove that $[k] = 7, where[.]$ represents the greatest integer function.

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97. If $\lim_{x \to 0} \left(1 + ax + bx^2\right)^{2/x} = e^3$, then find the values of a and b.

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98. Evaluate $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\left(\frac{\sin x}{x - \sin x}\right)}$.

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99. Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{2/x}, (a, b, c > 0)$

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100. If $f(n) = \lim_{x \to 0} \left\{\left(1 + \sin\frac{x}{2}\right)\left(1 + \sin\frac{x}{2^2}\right)\ldots\left(1 + \sin\frac{x}{2^n}\right)\right\}^{\frac{1}{x}}$ then find $\lim_{n \to \infty} f(n)$.

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101. Find the following using L'Hospital's rule

$(i)\ \lim_{x \to 0} \frac{(16+5x)^{1/4}-2}{(32+3x)^{1/5}-2}$ $(ii)\ \lim_{x \to \pi/2} [x \tan x - (\pi/2)\sec x]$

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102. Let $f(x)$ be a twice-differentiable function and $f''(0) = 2$. Then evaluate $\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$.

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103. Let $f(a) = g(a) = k$ and their nth derivatives exist and be not equal for some n.

If $\lim_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$ then find the value of k.

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104. Evaluate $\lim_{x \to 0} \left(\log_{\tan^2 x}\left(\tan^2 2x\right)\right.$.

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105. Evaluate $\lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$.

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106. The graph of the function $y = f(x)$ has a unique tangent at the point (a,0) through which the graph passes. Then evaluate

$\lim_{x \to a} \frac{\log_e\{1 + 6f(x)\}}{3f(x)}$.

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107. Evaluate $\lim_{x \to \infty} x \log_e \left\{\frac{\sin\left(a + \frac{1}{x}\right)}{\sin a}\right\}, 0 < a < \frac{\pi}{2}$.

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108. Find the value of a so that $\lim_{x \to 0} (e^{ax} - e^x - x) = \frac{3}{2}$.

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109. Find the value of $\lim_{x \to 0} \frac{\sin x + \log_e\left(\sqrt{1 + \sin^2 x} - \sin x\right)}{\sin^3 x}$.

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110. Evaluate $\lim_{x \to \infty} x^{\frac{1}{x}}$.

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111. Evaluate $\lim_{x \to \frac{\pi}{2}^-} (\cos x)^{\cos x}$.

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112. Evaluate $\lim_{x \to 0^+} (x)^{\frac{1}{\log_e \sin x}}$.

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Solved Examples

1. Evaluate

$$\lim_{n \to \infty} n^2 \left\{ \sqrt{\left(1 - \cos\frac{1}{n}\right)\sqrt{\left(1 - \cos\frac{1}{n}\right)\sqrt{\left(1 - \cos\frac{1}{n}\right)\ldots}}} \right\}.$$

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2. Evaluate $\lim_{n \to \infty} \left\{ \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\ldots\cos\left(\frac{x}{2^n}\right) \right\}$.

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3. Evaluate $\lim_{x \to 1} \sec\frac{\pi}{2^x} . \log_e x$.

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4. Evaluate $\lim_{x \to 0^+} \frac{1}{x} \cos^{-1}\left(\frac{\sin x}{x}\right)$.

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5. If $f(x) = \frac{\tan x}{x}$, then find $\lim_{x \to 0} \left([f(x)] + x^2\right)^{\frac{1}{\{f(x)\}}}$, where $[.]$ and $\{.\}$ denotes greatest integer and fractional part function respectively.

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6. Evaluate $\lim_{n \to \infty} \frac{1}{n^{2(\log_e n - \log_e (n+1))} + n}$.

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7. Evaluate $\lim_{x \to 0} \frac{(1+x)^{1/x} - e + \frac{1}{2}ex}{x^2}$.

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8. Evaluate $\lim_{x \to 0^+} x^m (\log x)^n, m, n, \in N.$

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9. If $\alpha_1, , \alpha_2, , \alpha_n$ are the roots of equation $x^n + nax - b = 0,$ show that $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)...(\alpha_1 - \alpha_n) = n\alpha_1^{n-1} + n\alpha$

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10. Evaluate $\lim_{x \to 0} \left\{1^{1/\sin^2 x} + 2^{1/\sin^2 x} + ... + n^{1/\sin^2 x}\right\}^{\sin^2 x}.$

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11. Evaluate $\lim_{x \to \pi/2} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \log_e \sin x}.$

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12. Slove $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$

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13. Evaluate

$\lim_{n \to \infty} n^{-n^2}\left[\left(n+2^0\right)\left(n+2^{-1}\right)\left(n+2^{-2}\right)\ldots\left(n+2^{-n+1}\right)\right]^n.$

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14. ABC is an isosceles triangle inscribed in a circle of radius r. If $AB = AC$ and h is the altitude form A to BC. If P is perimeter and A is the area of the triangle then find the value of $\lim_{h \to 0} \frac{A}{P^3}$.

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15. At the endpoint and midpoint of a circular are AB, tangent lines are drawn, and the points, A and B are jointed with a chord. Prove that the

ratio of the areas of the triangles thus formed tends to 4 as the arc AB decreases infinitely.

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EXERCISE 2.1

1. Evaluate $\lim_{x \to -2^+} \frac{x^2 - 1}{2x + 4}$.

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2. Evaluate $\lim_{x \to 2^+} \frac{[x - 2]}{\log(x - 2)}$, where $[.]$ represents the greatest integer function.

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3. Evaluate $\lim_{x \to 0} \frac{\sin[\cos x]}{1 + [\cos x]}$ ($[.]$ denotes the greatest integer function).

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4. If $f(x)=\begin{cases}\frac{x-|x|}{x}, x\neq 0\\ 2, \quad x=0\end{cases}$, show that $\lim_{x\to 0} f(x)$ does not exist.

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5. Show that $\lim_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1}$ does not exist.

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6. Evaluate $\lim_{x\to 0}\frac{3x+|x|}{7x-5|x|}$.

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7. If $f(x)=\begin{cases}x, & x<0\\ 1, & x=0\\ x^2, & x>0\end{cases}$, then find $\lim_{x\to 0} f(x)$ if exists.

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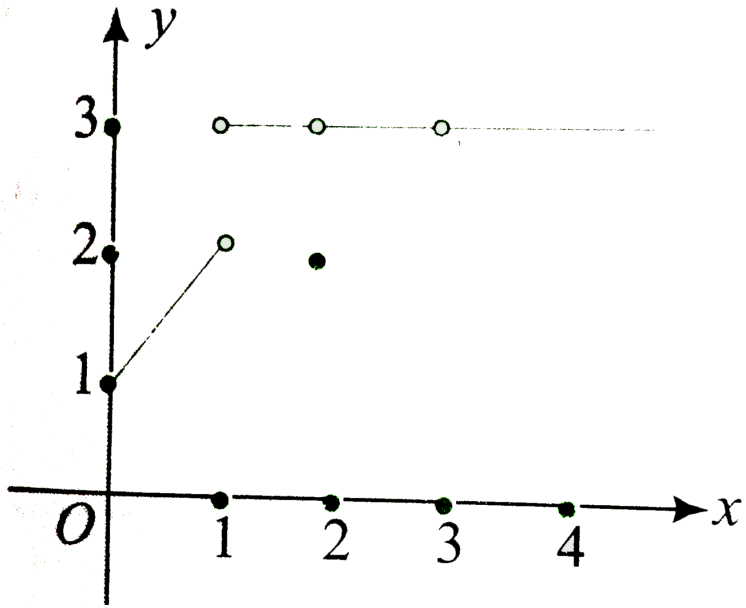
8. Consider the following graph of the function y=f(x). Which of the following is//are correct?



(a) $\lim_{x \to 1} f(x)$ does not exist.

(b) $\lim_{x \to 2} f(x)$ does not exist.

(c) $\lim_{x \to 3} f(x) = 3$.

(d) $\lim_{x \to 1.99} f(x)$exists.

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9. Evaluate $\lim_{x \to 0} \frac{\tan(sgn(x))}{sgn(x)}$ if exists.

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10. If $f(x) = \begin{cases} \sin x, & x \neq n\pi, \ n \in I \\ 2, & \text{otherwise} \end{cases}$ and

$g(x) = \begin{cases} x^2 + 1, & x \neq 0, \ 2 \\ 4, & x = 0 \\ 5, & x = 2 \end{cases}$ then find $\lim_{x \to 0} g\{f(x)\}$.

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EXERCISE 2.2

1. If $|f(x)| \leq x^2$, then prove that $\lim_{x \to 0} \frac{f(x)}{x} = 0$.

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2. If $f(x) = sgn(x)$ and $g(x) = x^3$,then prove that $\lim_{x \to 0} f(x).\, g(x)$ exists though $\lim_{x \to 0} f(x)$ does not exist.

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3. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & \text{for } [x] \neq 0 \\ 0, & \text{for } [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x. Then find $\lim_{x \to 0} f(x)$.

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4. Find the value of $\lim_{x \to 0^+} (\sin x)^{\frac{1}{x}}$.

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5. Let the sequence $< b_n >$ of real numbers satisfy the recurrence relation $b_{n+1} = \frac{1}{3}\left(2b_n + \frac{125}{b_n^2}\right), b_n \neq 0$. Then find $\lim_{n \to o0} b_n$.

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6. Let $f:(1,2)\overrightarrow{R}$ satisfies the inequality $\frac{\cos(2x-4)-33}{2} < f(x) < \frac{x^2|4x-8|}{x-2} \forall x \in (1,2)$. Then find $\lim_{x \to 2^-} f(x)$.

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7. If $\frac{x^2+x-2}{x+3} \leq \frac{f(x)}{x^2} \leq \frac{x^2+2x-1}{x+3}$ holds for a certain interval containing the value of $\lim_{x \to -1} f(x)$.

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EXERCISE 2.3

1. Evaluate $\lim_{x \to 1} \frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3}$.

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2. Evauate $\lim_{x \to 1} \frac{x^4 - 3x^4 + 2}{x^3 - 5x^2 + 3x + 1}$.

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3. Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{1 - \sin 2x}{1 + \cos 4x}$.

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4. Evaluate $\lim_{x \to \pi/4} \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$.

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5. Evaluate $\lim_{x \to a} \frac{\sqrt{3x - a} - \sqrt{x + a}}{x - a}$.

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6. Evaluate $\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$.

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7. Evaluate $\lim_{x \to \sqrt{10}} \frac{\sqrt{7 + 2x} - \left(\sqrt{5} + \sqrt{2}\right)}{x^2 - 10}$.

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8. Evaluate $\lim_{n \to \infty} \frac{\left(1^2 - 2^2 + 3^2 - 4^2 + 5^2 + \ldots n \text{ terms}\right)}{n^2}$.

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9. Evaluate $\lim_{x \to \infty} \left[\sqrt{a^2x^2 + ax + 1} - \sqrt{a^2x^2 + 1}\right]$.

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10. If $[x]$ denotes the greatest integer less than or equal to x, then evaluate $\lim_{n \to \infty} \frac{1}{n^2}([1.x] + [2.x] + [3.x] + \ldots + [n.x])$.

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11. Evaluate $\lim_{x \to \infty} x^3 \left\{ \sqrt{x^2 + \sqrt{1 + x^4}} - x\sqrt{2} \right\}$.

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12. Evaluate $\lim_{x \to \infty} \left(\frac{7x^2 + 1}{5x^2 - 1} \right)^{\frac{x^5}{1 - x^3}}$.

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13. Evaluate $\lim_{n \to \infty} \cos\left(\pi\sqrt{n^2 + n}\right)$ when n is an integer.

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14. Evaluate $\lim_{x \to 1} \frac{\sum_{k=1}^{100} x^k - 100}{x - 1}$.

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15. Evaluate $\lim_{h \to 0} \left[\frac{1}{h^3 \sqrt{8+h}} - \frac{1}{2h} \right]$.

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EXERCISE 2.4

1. Evaluate $\lim_{x \to 0} \left\{ \frac{\sin x - x + \frac{x^3}{6}}{x^5} \right\}$.

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2. Evaluate $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$.

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3. Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$.

Watch Video Solution

4. If $\lim_{x \to 0} \frac{1 - \cos x}{e^{ax} - bx - 1} = 2$ then find the values of a and b.

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5. Find the values of a and b in order that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1.$$

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EXERCISE 2.5

1. Evaluate $\lim_{x \to \infty} \frac{\sin x^0}{x}$.

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2. Evaluate $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx}$.

Watch Video Solution

3. Evaluate $\lim_{x \to 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x}$.

Watch Video Solution

4. Evaluate $\lim_{x \to 0} \frac{\tan 2x - x}{3x - \sin x}$.

Watch Video Solution

5. Evaluate $\lim_{n \to \infty} n \cos\left(\frac{\pi}{4n}\right) \sin\left(\frac{\pi}{4n}\right)$.

Watch Video Solution

6. Evaluate $\lim_{x \to 0} \frac{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)}{\sin^{-1} x}$.

Watch Video Solution

7. Evaluate $\lim_{h \to 0} \frac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right) - \cos\left(\frac{\pi}{6}+h\right)\right]}{\sqrt{3}h\left(\sqrt{3}\cosh - \sinh\right)}$.

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8. Evaluate $\lim_{x \to 0} \frac{8}{x^8}\left\{1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right\}$.

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9. Evaluate $\lim_{y \to 0} \frac{y^2 + \sin x}{x^2 + \sin y^2}$, where $(x, y) \to (0, 0)$ along the curve $x = y^2$.

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10. Evaluate $\lim_{x \to 1} (1-x)\tan\frac{\pi x}{2}$.

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11. Evaluate $\lim_{x \to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$.

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12. If $\lim_{x \to 2} \frac{\tan(x-2).\left(x^2+(k-2)x-2k\right)}{(x^2-4x+4)} = 5$, then find the value of k.

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EXERCISE 2.6

1. Evaluate $\lim_{x \to \infty} \left[x\left(a^{1/x}-1\right)\right], a > 1$

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Watch Video Solution

2. Evalaute $\lim_{x \to 0} \frac{x2^x - x}{1 - \cos x}$

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3. Evaluate $\lim_{x \to 2} \frac{\sin\left(e^{x-2} - 1\right)}{\log(x - 1)}$

Watch Video Solution

4. Evaluate $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$

Watch Video Solution

5. Evaluate $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$

Watch Video Solution

6. $\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)}$

Watch Video Solution

7. Evaluate $\lim_{x \to 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}, a > 0$

Watch Video Solution

8. $\lim_{x \to 0} \frac{(1 - 3^x - 4^x + 12^x)}{\sqrt{(2\cos x + 7)} - 3}$

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9. $\lim_{x \to 0} \frac{(729)^x - (243)^x - (81)^x + 9^x + 3^x - 1}{x^3}$

Watch Video Solution

EXERCISE 2.7

1. Evaluate $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x$

A. 0

B. 1

C. -1

D. does not exist

Answer: e^2

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2. Evaluate $\lim_{x \to \infty} (\log_3 3x)^{\log_x 3}$.

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3. Evaluate $\lim_{x \to \infty} \left(\frac{x+2}{x+1}\right)^{x+3}$.

View Text Solution

4. Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{a + bx}\right)^{c + dx}$, where a, b, c, and d are positive.

A. 4

B. 2

C. -1

D. 0

Answer: $e^{d/b}$

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5. Evaluate $\lim_{x \to 7/2} \left(2x^2 - 9x + 8\right)^{\cot(2x - 7)}$.

A. both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist

B. $\lim_{x \to a} f(x)$ need not exist but $\lim_{x \to a} g(x)$ exists

C. neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ may exist

D. $\lim_{x \to a} f(x)$ exists but $\lim_{x \to a} g(x)$ need not exist

Answer: $e^{5/2}$

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6. If x_1 and x_2 are the real and distinct roots of $ax^2 + bx + c = 0$ then prove that $\lim_{x \to x1} \left(1 + \sin\left(ax^2 + bx + c\right)\right)^{\frac{1}{x - x_1}} = e^{a(x_1 - x_2)}$.

A. does not exist

B. 1

C. ∞

D. $\frac{1}{2}$

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7. $\lim_{x \to 0} \sin^2\left(\frac{\pi}{2 - px}\right)^{\sec^2\left(\left(\frac{\pi}{2 - px}\right)\right)}$

A. -1

B. 2

C. $\sqrt{5}$

D. e^{-p^2/q^2}

Answer: D

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EXERCISE 2.8

1. Evaluate $\lim_{x \to 1} \frac{\cos \frac{\pi}{2} x}{1 - \sqrt{x}}$

A. -1

B. 1

C. 0

D. does not exist

Answer: π

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2. Evalaute $\lim_{x \to 1} \frac{1 + \log x - x}{1 - 2x + x^2}$

A. 0

B. 1

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer: $-1/2$

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3. Evaluate $\lim_{x \to \pi/2} \tan x \log \sin x$

A. -2

B. -1

C. $-\frac{2}{7}$

D. 0

Answer: 0

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4. Evaluate $\lim_{x \to 0} \frac{\log \cos x}{x}$

A. 0

B. $2/3$

C. $-1/4$

D. $3/2$

Answer: 0

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5. Evaluate $\lim_{x \to 0} \dfrac{2^x - 1}{(1+x)^{1/2} - 1}$

A. $\dfrac{1}{8\sqrt{3}}$

B. $\dfrac{1}{4\sqrt{3}}$

C. 0

D. none of these

Answer: $\log 4$

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6. Evaluate $\lim_{x \to \pi/4} (2 - \tan x)^{1/\ln(\tan x)}$

A. 16

B. 8

C. 4

D. 2

Answer: e^{-1}

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7. Evaluate $\lim_{x \to 0^+} x^x$ and $\lim_{x \to 0^+} x^{x^x}$

A. $1/2$

B. 2

C. 1

D. None of these

Answer: $1 \quad \text{and} \quad 0$

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8. If $\lim_{x \to a} \frac{a^x - x^a}{x^x - a^a} = -1 \quad \text{and} \quad a > 0, \quad$ then find the value of a.

A. 43529

B. 2

C. $\sin\frac{2}{5}$

D. $\sin\frac{1}{5}$

Answer: $a = 1$

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Exercises (Single Correct Answer Type)

1. $\lim_{x \to 0} \left[\frac{\sin(sgn(x))}{(sgn(x))}\right]$, where $[.]$ denotes the greatest integer function, is equal to

A. $\hat{}\ (2n)p_n$

B. $\hat{}\ (2n)C_n$

C. $(2n)!$

D. none of these

Answer: A

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2. Let $\lim_{x \to 0} \frac{[x]^2}{x^2} = m$, where $[.]$ denotes greatest integer. Then, m equals to a. $-\frac{1}{\sqrt{2}}$ b. $\frac{1}{\sqrt{2}}$ c. $\sqrt{2}$ d. Limit doesn't exist

A. $-\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: B

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3. $\lim_{x \to 1} \left[\cos ec \frac{\pi x}{2}\right]^{1/(1-x)}$ (where $[.]$ represents the greatest integer function) is equal to

A. (i) exists, (ii) does not exist

B. (i) does not exist, (ii) exists

C. both (i) and (ii) exist

D. neither (i) nor (ii) exists

Answer: B

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4. The value of the limit $\lim_{x \to 0} \frac{a^{\sqrt{x}} - a^{1/\sqrt{x}}}{a^{\sqrt{x}} + a^{1/\sqrt{x}}}, a > 1$, is

A. does not exist

B. $1/3$

C. 0

D. $2/9$

Answer: C

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5. If $\lim_{x \to a} \left\{ \frac{f(x)}{g(x)} \right\}$ exists, then which one of the following correct ?

A. both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist

B. $\lim_{x \to a} f(x)$need not exist $\lim_{x \to a} g(x)$ exist

C. neither $\lim_{x \to a} f(x)$ nor $\lim_{x \to a} g(x)$ may exist

D. $\lim_{x \to a} f(x)$ exist but $\lim_{x \to a} g(x)$ need not exist

Answer: C

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6. $\lim_{x \to -1} \frac{1}{\sqrt{|x| - \{-x\}}}$ (where $\{x\}$ denotes the fractional part of x) is equal to

A. 16

B. 24

C. 32

D. 8

Answer: A

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7. If $x_1 = 3$ and $x_{n+1} = \sqrt{2 + x_n}$, $n \geq 1$, then $\lim_{n \to \infty} x_n$ is

A. 0

B. $\frac{1}{2}$

C. log2

D. e^4

Answer: B

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8. $\lim_{x \to 0^-} \frac{\sum_{r=1}^{2n+1} [x^r] + (n+1)}{1 + [x] + |x| + 2x}$, where $n \in N$ and $[.]$ denotes the greatest integer function, equals

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9. $\lim_{x \to \infty} \frac{\sin^4 x - \sin^2 x + 1}{\cos^4 x - \cos^2 x + 1}$ is equal to

A. 0

B. 1

C. 10

D. 100

Answer: B

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10. If $f(x) = \frac{2}{x-3}, g(x) = \frac{x-3}{x+4}$, and $h(x) = -\frac{2(2x+1)}{x^2+x-12}$, then $\lim_{x}$

A. 1

B. ∞

C. $\sqrt{2}$

D. none of these

Answer: C

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11. The value of $\lim_{x \to \pi} \frac{1+\cos^3 x}{\sin^2 x}$ is

A. [2, 5)

B. (1, 5)

C. (-1, 5)

D. $(-\infty, \infty)$

Answer: D

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12. The value of $\lim_{x \to 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$ is

A. 0

B. e^x

C. $\log_e x$

D. none of these

Answer: A

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13. The value of $\lim_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$ is

A. $|2x| > \sqrt{3}$

B. $|2x| < \sqrt{3}$

C. $|2x| \geq \sqrt{3}$

D. $|2x| \leq \sqrt{3}$

Answer: B

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14. The value of $\lim_{x \to 2} \left(\left(\frac{x^3 - 4x}{x^3 - 8} \right)^{-1} - \left(\frac{x + \sqrt{2x}}{x - 2} - \frac{\sqrt{2}}{\sqrt{x} - \sqrt{2}} \right)^{-1} \right)$ is

A. 1

B. $1/2$

C. 2

D. none of these

Answer: A

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15. If $\lim_{x \to -2^-} \frac{ae^{1/|x+2|} - 1}{2 - e^{1/|x+2|}} = \lim_{x \to -2^+} \sin\left(\frac{x^4 - 16}{x^5 + 32}\right)$, then a is

A. 1

B. -1

C. 0

D. none of these

Answer: C

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16. $\lim_{x \to 1} \frac{(1-x)(1-x^2)...(1-x^{2n})}{\{(1-x)(1-x^2)...(1-x^n)\}^2}$, ninN," equals"`

A. 0

B. π

C. 2π

D. none of these

Answer: B

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17. The value of $\lim_{x \to \frac{1}{\sqrt{2}}} \frac{x - \cos\left(\sin^{-1} x\right)}{1 - \tan\left(\sin^{-1} x\right)}$ is

A. $-\frac{1}{\sqrt{2}}$

B. -1

C. non-existent

D. none of these

Answer: A

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18. Among (i) $\lim_{x \to \infty} \sec^{-1}\left(\frac{x}{\sin x}\right)$ and (ii) $\lim_{x \to \infty} \sec^{-1}\left(\frac{\sin x}{x}\right)$.

A. $-\frac{1}{2\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. does not exist

Answer: A

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19. $\lim_{x \to \infty}\left(\frac{x^3}{3x^2-4} - \frac{x^2}{3x+2}\right)$ is equal to

A. $a + c = b$

B. $b + c = a$

C. $a + b = c$

D. none of these

Answer: D

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20. $\lim_{n \to \infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$ is equal to

A. 1

B. 0

C. 2

D. none of these

Answer: C

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21. $\lim_{x \to \infty} \frac{(2x+1)^{40}(4x+1)^5}{(2x+3)^{45}}$ is equal to

A. $\frac{1}{2\pi}$

B. $\frac{-1}{\pi}$

C. $\frac{-2}{\pi}$

D. none of these

Answer: C

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22. $\lim_{x \to \infty} \left[\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}\right]$ is equal to

A. 1

B. 0

C. 2

D. none of these

Answer: B

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23. $\lim_{x \to \infty} \frac{2 + 2x + \sin 2x}{(2x + \sin 2x)e^{\sin x}}$ is equal to

A. $\sec x(x \tan x + 1)$

B. $x \tan x + \sec x$

C. $x \sec x + \tan x$

D. none of these

Answer: D

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24. $\lim_{x \to \infty} \frac{(x+1)^{10} + (x+2)^{10} + \ldots + (x+100)^{10}}{x^{10} + 10^{10}}$ is equal to

A. 0

B. 1

C. 2

D. 4

Answer: D

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25. $\lim_{x \to \infty} \frac{2\sqrt{x} + 3\sqrt[3]{x} + 4\sqrt[4]{x} + \ldots + n\sqrt[n]{x}}{\sqrt{(2x-3)} + \sqrt[3]{(2x-3)} + \ldots + \sqrt[n]{(2x-3)}}$ is equal to

A. 0

B. 1

C. $\sqrt{2}$.

D. $2\sqrt{2}$.

Answer: C

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26. If $\lim_{n \to \infty} \frac{n.3^n}{n(x-2)^n + n.3^{n+1} - 3^n} = \frac{1}{3}$, then the range of x is (where $n \in N$)

A. 1

B. $(2/3)^{1/2}$

C. $(3/2)^{1/2}$

D. $e^{1/2}$

Answer: A

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27. $(\lim)_{x \underset{\infty}{\to}} n^2\left(x^{\frac{1}{n}} - x^{\frac{1}{(n+1)}}\right), x > 0, isequa < o$ 0 (b) e^x (c) $(\log)_e x$ (d) none of these

A. π

B. 2π

C. $\pi/2$

D. none of these

Answer: C

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28. $Let f(x) = \lim_{n \to \infty} \frac{1}{\left(\frac{3}{\pi}\tan^{-1} 2x\right)^{2n} + 5}$. Then the set of values of x for which $f(x) = 0$ is

A. 199

B. 198

C. 0

D. none of these

Answer: A

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29. $f(x) = \frac{\ln\left(x^2 + e^x\right)}{\ln\left(x^4 + e^{2x}\right)}$. Then $\lim_{x \to \infty}$ f(x) is equal to

A. $\frac{2a}{\pi}$

B. $-\frac{2a}{\pi}$

C. $\frac{4a}{\pi}$

D. $-\frac{4a}{\pi}$

Answer: B

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30. The value of $\lim_{n\to\infty}\left[\frac{2n}{2n^2-1}\cos\frac{n+1}{2n-1}-\frac{n}{1-2n}.\frac{n}{n^2+1}\right]$ is

A. 5

B. 6

C. 7

D. none of these

Answer: C

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31. If $f(x) = 0$ is a quadratic equation such that $f(-\pi) = f(\pi) = 0$ and $f\left(\frac{\pi}{2}\right) = -\frac{3\pi^2}{4}$, then $\lim_{x \to -\pi^+} \frac{f(x)}{\sin(\sin x)}$ is equal to

A. $-\frac{3}{2}\pi$

B. $-\frac{1}{2}$

C. (2π)

D. $\frac{3}{2}\pi$

Answer: C

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32. $\lim_{x \to 1} \frac{x \sin(x - [x])}{x - 1}$, where $[.]$ denotes the greatest integer function, is equal to

A. $\frac{2}{\pi - 1}$

B. $\frac{\pi + 1}{2}$

C. $\frac{2}{\pi + 1}$

D. $\frac{2(\pi+1)}{\pi-1}$

Answer: C

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33. $\lim_{x \to \infty} \frac{x^2 \tan\frac{1}{x}}{\sqrt{8x^2+7x+1}}$ is equal to

A. $-\frac{1}{2\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. 2

D. $1/4$

Answer: A

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34. $\lim_{x \to 0} \frac{x^a \sin^b x}{\sin(x^c)}$, where $a, b, c \in R\text{~}\{0\}$, exists and has non-zero value. Then,

A. 0

B. $\pi / 2$

C. π

D. 2π

Answer: C

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35. $\lim_{x \to 0} \left(x^4 \frac{\cot^4 x - \cot^2 x + 1}{\tan^4 x - \tan^2 x + 1} \right)$ is equal to (a) 1 (b) 0 (c) 2 (d) none of these

A. -1

B. 1

C. 0

D. none of these

Answer: A

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36. $\lim_{x \to 1} \frac{1 - x^2}{\sin 2\pi x}$ is equal to

A. 0

B. ∞

C. -2

D. 2

Answer: B

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37. $\lim_{x \to 0} \frac{1}{x} \cos^1\left(\frac{1-x^2}{1+x^2}\right)$ is equal to

A. $f(x)f(y)$

B. $f(x) + f(y)$

C. $f(x) - f(y)$

D. none of these

Answer: D

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38. $\lim_{y \to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y}$ is equal to

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39. $\lim_{x \to 1} \frac{1 + \sin\pi\left(\frac{3x}{1+x^2}\right)}{1 + \cos\pi x}$ is equal to

A. $\log n\left(\frac{2}{3}\right)$

B. 0

C. $n \log n\left(\frac{2}{3}\right)$

D. not defined

Answer: A

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40. $(\lim)_{x \overrightarrow{\infty}} \sum_{x=1}^{20} \cos^{2n}(x-10) isequa < o$ 0 (b) 1 (c) 19 (d) 20

A. (a) 2

B. (b) -2

C. (c) 1

D. (d) -1

Answer: B

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41. $(\lim)_{x \to 1} \frac{\left(\frac{x^4+x^2+x+1}{x^2-x+1}\right)^{1-\cos(x+1)}}{(x+1)^2} isequa < o\,1$ (b) $\left(\frac{2}{3}\right)^{\frac{1}{2}}$ (c) $\left(\frac{3}{2}\right)^{\frac{1}{2}}$ (d) $e^{\frac{1}{2}}$

A. 1

B. -1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: B

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42. $\lim_{x \to \infty} \{(x+5)\tan^{-1}(x+5) - (x+1)\tan^{-1}(x+1)\}$ is equal to

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43. The value of $\lim_{x \to 0} \left(\left[\frac{100x}{\sin x} \right] + \left[\frac{99 \sin x}{x} \right] \right)$ (where $[.]$ represents the greatest integral function) is

A. 199

B. 198

C. 0

D. none of these

Answer: B

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44. The value of $\lim_{x \to a} \sqrt{a^2 - x^2} \cot \frac{\pi}{2} \sqrt{\frac{a - x}{a + x}}$ is

A. $\frac{5050}{\pi e}$

B. $100 \frac{)}{\pi e}$

C. $-\frac{5050}{\pi e}$

D. $-\frac{4950}{\pi e}$

Answer: C

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45. $\lim_{x \to 0} \left[\min \left(y^2 - 4y + 11\right) \frac{\sin x}{x} \right]$ (where $[.]$ denotes the greatest integer function) is

A. -1

B. 1

C. 0

D. 2

Answer: B

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46. The value of $\lim_{x \to 0} \frac{1 - (\cos x)\sqrt{\cos 2x}}{x^2}$ is

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{1}{6}$

D. $\frac{5}{12}$

Answer: D

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47. $\lim_{x \to \infty} \frac{1}{x+1} \tan\left(\frac{\pi x + 1}{2x + 2}\right)$ is equal to

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 0

D. none of these

Answer: A

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48. The value of $(\lim)_{x \overrightarrow{1}^{-}} \frac{1-\sqrt{x}}{(os^{-1}x)^2}$ is 4 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$

A. $1/6$

B. $-1/3$

C. $1/2$

D. 1

Answer: D

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49. $(\lim)_{x \overrightarrow{\frac{\pi}{2}}} \left(\frac{\sin(x \cos x)}{\cos(x \sin x) isequa < o} \right.$ 0 (b) $\frac{p}{2}$ (c) p (d) $2p$

A. $a = 3$ and $b = 9/2$

B. $a = 3$ and $b = 9/2$

C. $a = -3$ and $b = -9/2$

D. $a = 3$ and $b = -9/2$

Answer: B

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50. $\lim_{x \to 0} \left[(1 - e^x) \frac{\sin x}{|x|} \right]$ is (where $[.]$ represents the greatest integer function)

A. 1

B. 2

C. 3

D. none of these

Answer: A

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51. Evaluate $\lim_{x \to 0} \frac{x(e^x - 1)}{1 - \cos x}$ is equal to

A. e

B. $\frac{1}{e}$

C. 1

D. none of these

Answer: D

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52. If $f(x) = \lim_{n \to \infty} n\left(x^{1/n} - 1\right)$, then for $x > 0, y > 0$, $f(xy)$ is equal to

A. f(x)f(y)

B. f(x) + f(y)

C. f(x) - f(y)

D. none of these

Answer: B

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53. $\lim_{x \to 0} \left\{(1+x)^{\frac{2}{x}}\right\}$ (where $\{.\}$ denotes the fractional part of x) is equal to

A. 1

B. e

C. e^{-1}

D. none of these

Answer: A

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54. The value of $\lim_{x \to \infty} \frac{\left(2^{x^n}\right)e^{\frac{1}{x}} - \left(3^{x^n}\right)e^{\frac{1}{x}}}{x^n}$ (where $n \in N$) is

A. e

B. e^2

C. e^{-1}

D. 1

Answer: B

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55. $(\lim)_{x \overrightarrow{0}} \frac{\sin\left(x^2\right)}{1n(\cos(2x^2 - x))}$ is equal to (a) 2(b) -2 (c) 1 (d) -1

A. e^a

B. $-a$

C. e^{1-a}

D. e^{1+a}

Answer: B

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56. $\lim_{x \to \infty} \frac{e^{1/x^2} - 1}{2\tan^{-1}(x^2) - \pi}$ is equal to

A. $e^{(1-e)}$

B. $e^{\left(\frac{1-e}{e}\right)}$

C. $e^{\left(\frac{e}{1-e}\right)}$

D. $e^{\frac{1+e}{e}}$

Answer: D

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57. $\lim_{x \to 0} \frac{(2^m + x)^{1/m} - (2^n + x)^{1/n}}{x}$ is equal to

A. $(n!)^n$

B. $(n!)^{1/n}$

C. $n!$

D. $\ln(n!)$

Answer: C

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58. The value of $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{e^{1/n}}{n}+\frac{e^{2/n}}{n}+\ldots+\frac{e^{(n-1)/n}}{n}\right]$ is

A. $\frac{p+q}{2}$

B. $\frac{pq}{2}$

C. $\frac{p-q}{2}$

D. $\sqrt{\frac{p}{q}}$

Answer: C

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59. $\lim_{x \to 1} \frac{nx^{n-1} - (n+1)x^n + 1}{(e^x - e)\sin \pi x}$, where $n = 100$,is equal to

A. 0

B. -1

C. 1

D. does not exist

Answer: C

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60. $\lim_{x \to 0} \frac{\log\left(1 + x + x^2\right) + \log\left(1 - x + x^2\right)}{\sec x - \cos x} =$

A. 2

B. 1

C. $\log_a 2$

D. 0

Answer: B

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61. The value of $\lim_{x \to \infty} \left(\sqrt[3]{x^3 + 2x^2} - \sqrt{x^2 + x} \right)$ is

A. e

B. e^2

C. $\sqrt{€}$

D. e^{-1}

Answer: C

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62. The value of $\lim_{x \to 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x^3}$ is

A. 1

B. -1

C. 2

D. -2

Answer: B

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63. If $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$ then $\lim_{x \to a} \{f(x)\}^{g(x)} = e^{\lim_{x \to a}(f(x)-1)xg(x)}$

$\lim_{x \to 0} \left(\frac{x - 1 + \cos x}{x} \right)^{\frac{1}{x}}$ is equal to

A. $f(1+0) = -1, f(1-0) = 0$

B. $f(1+0) = 0 = f(1-0)$

C. $\lim_{x \to 1} f(x)$ exists

D. $f(x)$ does not exist
$x \to 1$

Answer: B

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64. If $(\lim)_{x\ 0}\left(x^{-3}\sin 3x + ax^{-2} + b\right)$ exists and is equal to 0, then $a = -3 and b = \frac{9}{2}$ $a = 3 and b = \frac{9}{2}$ $a = -3 and b = -\frac{9}{2}$ $a = 3 and b = -\frac{9}{2}$

A. $a = -3$

B. $a = 0$

C. $b = 1$

D. $b = -1$

Answer: A

Watch Video Solution

65. If $\lim_{x \to 0} \frac{x^n - \sin x^n}{x - \sin^n x}$ is non-zero finite, then n must be equal to 4 (b) 1 (c) 2 (d) 3

A. $1, \quad \text{if} \quad n = m$

B. $0, \quad \text{if} \quad n > m$

C. $\infty, \quad \text{if} \quad n < m$

D. $n/m, \quad \text{if} \quad n < m$

Answer: B

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66. $\lim_{x \to 0} \left(\frac{1 + \tan x}{1 + \sin x} \right)^{\cos ecx}$ is equal to

A. $\lim_{x \to 0} f(x)$ exists for $n > 0$

B. $\lim_{x \to 0} f(x)$ does not exists for $n < 0$

C. $\lim_{x \to 0} f(x)$ does not exists for any value of n

D. $\lim_{x \to 0} f(x)$ exists for any value of n

Answer: C

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67. The value of $\lim_{x \to 1} (2-x)^{\tan\left(\frac{\pi x}{2}\right)}$ is

A. $a = 1/4$

B. $b = 3/4$

C. $L = -1/32$

D. $L = 1/32$

Answer: C

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68. The value of $\lim_{m \to \infty} \left(\cos\frac{x}{m}\right)^m$ is

A. $\lim_{x \to \infty} \frac{\log_e x}{\{x\}} = \infty$

B. $\lim_{x \to 2^+} \frac{x}{x^2 - x - 2} = \infty$

C. $\lim_{x \to -1^-} \frac{x}{x^2 - x - 2} = \infty$

D. $\lim_{x \to \infty} \frac{\log_{0.5} x}{\{x\}} = \infty$

Answer: A

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69. $\lim_{n \to \infty} \left(\frac{n^2 - n + 1}{n^2 - n - 1} \right)^{n(n-1)}$ is equal to

A. $\lim_{x \to 0} \frac{[x + |x|]}{x} = 0$, where $[x]$ denotes the greatest integer functions.

B. $\lim_{x \to 0} \frac{x e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}} = 0$

C. $\lim_{x \to 3} (x - 3)^{\frac{1}{5}} sgn(x - 3) = 0$, where sgn stands for signum function.

D. $\lim_{x \to 0} \frac{\tan^{-1}|x|}{x} = 0$

Answer: B

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70. $\lim_{n \to \infty} \left\{ \left(\frac{n}{n+1} \right)^{\alpha} + \sin \frac{1}{n} \right\}^{n}$ (where $\alpha \in Q$) is equal to

A. $a = 1/3, b = 1$

B. $a = 1, b = -1$

C. $a = 9, b = -9$

D. $a = 2, b = 2/3$

Answer: C

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71. $\lim_{x \to \infty} \left[\left(\frac{e}{1-e} \right) \left(\frac{1}{e} - \frac{x}{1+x} \right) \right]^{x}$ is

A. limit does not exist when $a = \pi/6$

B. $L = -1$ when $a = \pi$

C. $L = 1$ when $a = \pi/2$

D. $L = 1$ when $a = 0$

Answer: C

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72. $\lim_{x \to 0} \left(\frac{1^x + 2^x + 3^x + ... + n^x}{n} \right)^{\frac{1}{x}}$

A. $f(1^+) + f(1^-) = 0$

B. $f(1^+) + f(1^-) + f(1) = 3/2$

C. $f(-1^+) + f(-1^-) = -1$

D. $f(1^+) + f(-1^-) = 0$

Answer: B

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73. The value of $\lim_{x \to 1} \left(\frac{p}{1 - x^p} - \frac{q}{1 - xq} \right), p, q, \in N,$ equals

A. $-\frac{3}{4}$

B. 0 if n is even

C. $-\frac{3}{4}$ if n is odd

D. none of these

Answer: C

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74. $\lim_{x \to \infty} \frac{x(\log x)^3}{1 + x + x^2}$ equals

A. $\lim_{x \to 0^+} f(x) = 1$

B. $\lim_{x \to 0^-} f(x) = \cot 1$

C. $\cot^{-1}\left(\lim_{x \to 0^-} f(X)\right)^2 = 1$

D. $\tan^{-1}\left(\lim_{x \to 0^+} f(x)\right) = \frac{\pi}{4}$

Answer: A

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75. $\lim_{x \to \infty} \frac{\cot^{-1}(x^{-a} \log_a x)}{\sec^{-1}(a^x \log_x a)}, (a > 1)$is equal to

(a)2

(b)1

(c) $(\log)_a 2$

(d) 0

A. $\lim_{x \to 1} f(x)$ exists $\Rightarrow a = -2$

B. $f(x)$ exists $\Rightarrow a = 13$
$x \to -2$

C. $f(x) = 4/3$ if it exists
$x \to 1$

D. $\lim_{x \to -2} f(x) = -1/3$ if it exists

Answer: B

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76. The value of $\lim_{n \to \infty} \frac{e^n}{\left(1 + \frac{1}{n}\right)^{n^2}}$is (a) -1 (b) 0 (c) 1 (d) ∞

A. -1

B. 0

C. 1

D. ∞

Answer: C

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Multiple Correct Answers Type

1. Let $f(x) = \begin{cases} 1 + \frac{2x}{a}, & 0 \leq x < 1 \\ ax, & 1 \leq x < 2 \end{cases}$. If $\lim_{x \to 1} f(x)$ exists, then a is

A. $\lim_{x \to 5^-} f(x) = 0$

B. $\lim_{x \to 5^+} f(x) = 1$

C. $\lim_{x \to 5} f(x)$ does not exist

D. none of these

Answer: B::C

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2. If $f(x) = |x - 1| - [x]$, where $[x]$ is the greatest integer less than or equal to x, then

A. $\lim_{x \to 0} [f(x)] = 0$

B. $\lim_{x \to 0} [f(x)] = 1$

C. $\lim_{x \to 0} \left[\frac{f(x)}{x}\right]$ does not exist

D. $\lim_{x \to 0} \left[\frac{f(x)}{x}\right]$ exists

Answer: A::D

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3. $(\lim)_{n \to \infty} \left(an - \frac{1 + n^2}{1 + n}\right) = b,$ where a is a finite number, then $a = 1$ (b) $a = 0$ (c) $b = 1$ (d) $b = -1$

A. $f(0) = 1$

B. $f\left(\frac{\pi}{2}\right) = 1$

C. $f(a) = (\cos a)^{\cos^2 a} . (\sin a)^{\sin^2 a}$ if $a \in \left(0, \frac{\pi}{2}\right)$

D. $f(a) = \frac{(\sin a)^{\sin^2 a}}{(\cos a)^{\cos^2 a}}$ if $a \in \left(0, \frac{\pi}{2}\right)$

Answer: A::C

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4. If $m, n \in N$, $\lim_{x \to 0} \frac{\sin x^n}{(\sin x)^m}$ is

A. $\frac{p}{2}$

B. $\frac{\pi}{2\sqrt{2}}$

C. $\frac{\pi}{\sqrt{2}}$

D. $\sqrt{2}\pi$

Answer: A::B::C

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5. Let $f(x) = \begin{cases} x^n \sin\left(1/x^2\right), x \neq 0 \\ 0, \qquad\qquad x = 0 \end{cases}, (n \in I)$. Then

A. $\frac{\pi}{2}$

B. $\frac{\pi}{2\sqrt{2}}$

C. $\frac{\pi}{\sqrt{2}}$

D. $\sqrt{2}\pi$

Answer: A::B

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6. If $L = \lim_{x \to 0} \frac{1}{x^3}\left(\frac{1}{\sqrt{1+x}} - \frac{1+ax}{1+bx}\right)$ exists,then

A. $\cos L < \cos R$

B. $\tan(2L) < an < 2R$

C. $\sin L > \sin R$

D. `None of these

Answer: A::B::C

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7. Which of the following is true ($\{.\}$ denotes the fractional part of the function)?

A. always 1

B. always -1

C. $(-1)^{n-m+1}$

D. $(-1)^{n-m}$

Answer: A::B::C

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8. Which of the following is/are correct?

A. always 1

B. always -1

C. $(-1)^{m+1}$

D. $(-1)^{n-m}$

Answer: A::B::C

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9. If $\lim_{x \to 1} (2 - x + a[x-1] + b[1+x])$ exists, then a and b can take the values (where $[.]$ denotes the greatest integer function)

A. is always equal to -1

B. is always equal to +1

C. does not exist None of these

D.

Answer: B::C

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10. $L = (\lim)_{x \to a} \frac{|2\sin x - 1|}{2\sin x - 1} Then$ limit does not exist when $a = \frac{\pi}{6}$

$L = -1 when a = \pi\ L = 1 when a = \frac{\pi}{2}\ L = 1 when a = 0$

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11. Let $f(x) = \lim_{n \to \infty} \frac{x}{x^{2n} + 1}$. Then f has

A. real and equal roots

B. complex roots

C. unequal positive real roots

D. unequal roots

Answer: B::C::D

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12. $\lim_{n \to \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$ is equal to $(n \in N)$

A. $[-2, 2]$

B. $[0, 2]$

C. [-1,1]`

D. $[-2, 1]$

Answer: A::C

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13. Given a real-valued function f such that $f(x) = \begin{cases} \frac{\tan^2\{x\}}{\left(x^2 - [x]^2\right)\sqrt{\{x\}\cot\{x\}}}, f \text{ or } x < 0, fx > 0 \end{cases}$ Where $[x]$ is the integral part and $\{x\}$ is the fractional part of x, then $(\lim)_{x \to 0^+} f(x) = 1$, $(\lim)_{x \to 0^-} f(x) = \cot 1$, $\cot^{-1}\left((\lim)_{x \to 0^-} f(x)\right)^2 = 1, \tan^{-1}\left((\lim)_{x \to 0^+} f(x)\right) = \frac{\pi}{4}$

A. $p_1 \ \ln \ a_1 + p_2 \ \ln \ a_2 + \ldots + p_n \ \ln \ a_n$

B. $a_1^{p_1} + a_2^{p_2} + \ldots + a_n^{p_n}$

C. $a_1^{p_1} . a_2^{p_2} \ldots a_n^{p_n}$

D. $\sum_{r=1}^{n} a_r p_r$

Answer: A::B::C::D

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14. If $f(x) = \frac{3x^2 + ax + a + 1}{x^2 + x - 2}$, then which of the following can be correct (a) $(\lim)_{x \to 1} f(x) f$ or $a = -2$ (b) $(\lim)_{x \to -2} f(x) f$ or $a = 13$ (c) $(\lim)_{x \to 1} f(x) = \frac{4}{3}$ (d) $(\lim)_{x \to -2} f(x) = -\frac{1}{3}$

A. $\ln \ a_1$

B. e^{a_n}

C. a_(1)`

D. a_n

Answer: A::B::C::D

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15. The value of $\lim_{n \to \infty} \frac{1}{1 + n \sin^2 nx}$ can be $(n \in N)$

A. $\ln \ a_n$

B. e^{a_1}

C. a_1

D. a_n

Answer: B::C

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16. Lim x tending 5f(x) = (x^2-9x+20)/(x-[x])$where$[x] $de \neg esgreatest \int e \geq r \leq ssthan \text{ or } equal \to$ x`), then

A. -1

B. $1/2$

C. 1

D. $3/2$

Answer: A::B::C

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17. Given $\lim_{x \to 0} \frac{f(x)}{x^2} = 2$, where $[.]$ denotes the greatest integer function, then

A. (a) $\lim_{x \to 0} (f(x)) = 0$

B. (b) $\lim_{x \to 0} (f(x)) = 1$

C. (a) $\lim_{x \to 0} \frac{f(x)}{x} = does \neg e\xi st$

D. (a) $\lim_{x \to 0} \frac{f(x)}{x} = e\xi ts$

Answer: A::C

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18. If $f(a) = \lim_{x \to 2} (\sin^x a + \cos^x a)^{\frac{1}{(x-2)}}$ for $a \in \left[0, \frac{\pi}{2}\right]$, then

A. $f(0) = 1$

B. $f\left(\frac{\pi}{2}\right) = 1$

C. $f(\alpha) = (\cos\alpha)^{\cos^2\alpha} \cdot (\sin\alpha)^{\sin^2\alpha}$ if $\alpha \in \left(0, \frac{\pi}{2}\right)$

D. $f(\alpha) = \frac{(\sin\alpha)^{\sin^2\alpha}}{(\cos\alpha)^{\cos^2\alpha}}$ if $\alpha \in \left(0, \frac{\pi}{2}\right)$

Answer: A::B::C

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19. Let $L = (\lim)_{x \vec{0}} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0. IfLi \in ite, then\ a = 2$

(b) $a = 1\ L = \frac{1}{64}$ (d) $L = \frac{1}{32}$

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20. Let $f(x) = \left(\frac{1 - x(1 + |1 - x|)}{|1 - x|}\right)\cos\left(\frac{1}{1 - x}\right)$ for $x \neq 1$

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21. If $A = \lim_{x \to 0} \frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$ and $B = \lim_{x \to 0} \frac{[|x|]}{x}$, then

A. A = 1

B. A does not exist

C. B = 0

D. B = 1

Answer: B::C

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22. If $f(x) = x\left(\frac{e^{|x|+[x]} - 2}{|x| + [x]}\right)$ then (where [.] represent the greatest integer function)

A. $\lim_{x \to 0^+} f(x) = -1$

B. $\lim_{x \to 0^-} f(x) = 0$

C. $\lim_{x \to 0} f(0) = -1$

D. $\lim_{x \to 0} f(x) = 0$

Answer: A::B

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23. Assume that $\lim_{\theta \to -1} f(\theta)$ exists and $\frac{\theta^2 + \theta - 2}{\theta + 3} \leq \frac{f(\theta)}{\theta^2} \leq \frac{\theta^2 + 2\theta - 1}{\theta + 3}$ holds for certain interval containing the point $\theta = -1$ then $\lim_{\theta \to -1} f(\theta)$

A. is equal to $f(-1)$

B. is equal to 1

C. is non-existent

D. is equal to -1

Answer: A::D

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24. Let $f(x) = \lim_{n \to \infty} \frac{\tan^{-1}(\tan x)}{1 + (\log_x x)^n}, x \neq (2n+1)\frac{\pi}{2}$ then

A. $\forall 1 < x < \frac{\pi}{2}$, $f(x)$ is an identity function

B. $\forall \frac{\pi}{2} < x < \pi$, the graph of f(x) is a straight line having y intercept of $-\pi$

C. $\forall \frac{\pi}{2} < x < e$, the graph of f(x) is a straight line having y intercept of $-\pi$

D. $\forall x > e$, $f(x)$ is a constant function

Answer: A::C::D

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Linked Comprehension Type

1. $Let f(x) = \frac{\sin^{-1}(1-\{x\}) \times \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \times (1-\{x\})}$, where $\{x\}$ denotes the fractional part of x.

$R = \lim_{x \to 0+} f(x)$ is equal to

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2. $Let f(x) = \frac{\sin^{-1}(1-\{x\}) \times \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \times (1-\{x\})}$, where $\{x\}$ denotes the fractional part of x.

$L = \lim_{x \to 0-} f(x)$ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{2\sqrt{2}}$

C. (π)

D. $\frac{1}{2\sqrt{2}}$

Answer: B

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3. let $f(x)=\frac{\cos^{-1}(1-\{x\})\sin^{-1}(1-\{x\})}{\sqrt{2\{x\}(1-\{x\})}}$ where $\{x\}$ denotes the fractional part of x then

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4. $A_i=\frac{x-a_i}{|x-a_i|}, i=1,2,...,n,$ and $a_1<a_2<a_3<...<a_n.$

If $1\le m\le n,$ min $N,$ then the value of $L=\lim_{x\to a_m}(A_1A_2...A_n)$is (a) 2 (b) -1 (c) not exist (d) 1

A. 2

B. -1

C. not exist

D. 1

Answer: C

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5. $A_i = \frac{x - a_i}{|x - a_i|}, i = 1, 2, ..., n, \quad \text{and} \quad a_1 < a_2 < a_3 < ... < a_n.$

If $1 \leq m \leq n, \ \min N, \ \text{then the value of} \ R = \lim_{x \to a_m +} (A_1 A_2 ... A_n) \text{is}$

A. $e^{-\frac{1}{4}}$

B. $e^{-\frac{1}{2}}$

C. e^{-2}

D. e^{-4}

Answer: D

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6. $A_i = \frac{x - a_i}{|x - a_i|}, i = 1, 2, ..., n, \quad \text{and} \quad a_1 < a_2 < a_3 < ... < a_n.$

If $1 \leq m \leq n, \ \min N, \ \text{then} \ \lim_{x \to a_m} (A_1 A_2 ... A_n)$

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7. If $L = \lim_{x \to 0} \frac{\sin x + ae^x + be^{-x} + c\log_e(1+x)}{x^3}$ exists finitely, then

The value of L is

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8. If $L = \lim_{x \to 0} \frac{\sin x + ae^x + be^{-x} + c\log_e(1+x)}{x^3}$ exists finitely, then

Equation $ax^2 + bx + c = 0$ has

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9. If $L = \lim_{x \to 0} \frac{\sin x + ae^x + be^{-x} + c\log_e(1+x)}{x^3}$ exists finitely, then

The solutions set of $||x + c| - 2a| < 4b$ is

A. $a \quad b \quad c \quad d$
$s \quad r \quad q \quad p$

B. $a \quad b \quad c \quad d$
$q \quad s \quad r \quad p$

C. $a \quad b \quad c \quad d$
$s \quad r \quad p \quad q$

D. $a \quad b \quad c \quad d$
$s \quad p \quad q \quad r$

Answer: C

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10. Let $a_1 > a_2 > a_3 > \ldots > a_n > 1$.

$p_1 > p_2 > p_3 > \ldots > p_n > 0$ such that $p_1 + p_2 + p_3 + \ldots + p_n = 1$.

Also, $F(x) = \left(p_1 a_1^x + p_n a_n^x\right)^{1/x}$.

$\lim_{x \to 0^+} F(x)$ equals

A. $\begin{matrix} a & b & c & d \\ s & r & q & p \end{matrix}$

B. $\begin{matrix} a & b & c & d \\ q & p & s & p \end{matrix}$

C. $\begin{matrix} a & b & c & d \\ s & r & p & q \end{matrix}$

D. $\begin{matrix} a & b & c & d \\ p & p & q & r \end{matrix}$

Answer: C

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11. Let $a_1 > a_2 > a_3 > \ldots > a_n > 1$.

$p_1 > p_2 > p_3 > \ldots > p_n > 0$ such that $p_1 + p_2 + p_3 + \ldots + p_n = 1$.

Also, $F(x) = \left(p_1 a_1^x + p_n a_n^x\right)^{1/x}$.

$\lim_{x \to 0^+} F(x)$ equals

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12. Let $a_1 > a_2 > a_3 > \ldots > a_n > 1$.

$p_1 > p_2 > p_3 > \ldots > p_n > 0$ such that $p_1 + p_2 + p_3 + \ldots + p_n = 1$.

Also, $F(x) = \left(p_1 a_1^x + p_n a_n^x\right)^{1/x}$.

$\lim_{x \to 0^+} F(x)$ equals

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13. If $L = \lim_{x \to \infty} \left(x + 1 - \sqrt{ax^2 + x + 3}\right)$ exists infinetely then

The value of a is

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14. If $L = \lim_{x \to \infty} \left(x + 1 - \sqrt{ax^2 + x + 3}\right)$ exists finitely then

The value of a is

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15. Let $f: R \to R$ be a real function. The function f is double differentiable. If there exists $n \in N$ and $p \in R$ such that $\lim_{x \to \infty} x^n f(x) = p$ and there exists $\lim_{x \to \infty} x^{n+1} f(x)$, then $\lim_{x \to \infty} x^{n+1} f'(x)$ is equal to

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16. Let $f: R \to R$ be a real function. The function f is double differentiable. If there exists $n \in N$ and $p \in R$ such that $\lim_{x \to \infty} x^n f(x) = p$ and there exists $\lim_{x \to \infty} x^{n+1} f(x)$, then $\lim_{x \to \infty} x^{n+1} f'(x)$ is equal to

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17. Let $f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^2 f(x)}{2x^5 + 3} = 6$ and $f(1) = 3, f(3) = 7$ and $f(5) = 11$. Then The value of $f(0)$ is

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18. Let $f(x)$ be a polynomial satisfying $\lim_{x \to \infty} \frac{x^2 f(x)}{2x^5 + 3} = 6$ and $f(1) = 3, f(3) = 7$ and $f(5) = 11$. Then The value of $f(0)$ is

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19. If $\lim_{x \to 0} \frac{f(x)}{\sin^2 x} = 8, \lim_{x \to 0} \frac{g(x)}{2\cos x - ye^x + x^3 + x - 2} = \lambda$ and $\lim_{x \to 0} (1 +$ then $\lambda =$

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20. If $\lim_{x \to 0} \frac{f(x)}{\sin^2 x} = 8$, $\lim_{x \to 0} \frac{g(x)}{2\cos x - xe^x + x^3 + x - 2} = \lambda$ and $\lim_{x \to 0} (1 + 2f(x))^{\frac{1}{g(x)}} = \frac{1}{e}$, then

$\lim_{x \to 0} (1 + f(x))^{\frac{1}{2g(x)}}$ is equal to

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Matrix Match Type

1. Match the following lists:

List I	List II
a. If $L = \lim_{x \to -1} \frac{\sqrt[3]{(7-x)} - 2}{(x+1)}$, then $12L =$	**p.** -2
b. If $L = \lim_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$, then $-L/4 =$	**q.** 2
c. If $L = \lim_{x \to 1} \frac{(2x-3)(\sqrt{x} - 1)}{2x^2 + x - 3}$, then $20L =$	**r.** 1
d. If $L = \lim_{x \to \infty} \frac{\log x^n - [x]}{[x]}$, where $n \in N$, ($[x]$ denotes greatest integer less than or equal to x), then $-2L =$	**s.** -1

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2. Match the following lists:

List I	List II
a. If $f(x)$ is an integrable function for $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $I_1 = \int_{\pi/6}^{\pi/3} \sec^2\theta f(2\sin 2\theta)\, d\theta$, and $I_2 = \int_{\pi/6}^{\pi/3} \operatorname{cosec}^2\theta f(2\sin 2\theta) d\theta$, then $I_1/I_2 =$	**p.** 3
b. If $f(x+1) = f(3+x)\ \forall\ x$, and the value of $\int_a^{a+b} f(x)dx$ is independent of a, then the value of b can be	**q.** 1
c. The value of $2\int_1^4 \frac{\tan^{-1}[x^2]}{\tan^{-1}[x^2] + \tan^{-1}[25 + x^2 - 10x]} dx$ (where [.]denotes the greatest integer function) is	**r.** 2
d. If $I = \int_0^2 \sqrt{x + \sqrt{x + \sqrt{x + \cdots \infty}}}\ dx$ (where $x > 0$), then [I] is equal to (where [.] denotes the greatest integer function)	**s.** 4

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3. Match the following lists:

List I	List II
a. If $\lim_{x\to\infty}(\sqrt{(x^2-x-1)}-ax-b)=0$, where $a>0$, then there exists at least one a and b for which point $(a, 2b)$ lies on the line.	**p.** $y=-3$
b. If $\lim_{x\to\infty}\frac{(1+a^3)+8e^{1/x}}{1+(1-b^3)e^{1/x}}=2$, then there exists at least one a and b for which point (a, b^3) lies on the line.	**q.** $3x-2y-5=0$
c. If $\lim_{x\to\infty}(\sqrt{(x^4-x^2+1)}-ax^2-b)=0$, then there exists at least one a and b for which point $(a, -4b)$ lies on the line.	**r.** $15x-2y-11=0$
d. If $\lim_{x\to-a}\frac{x^7+a^7}{x+a}=7$, where $a<0$, then there exists at least one a for which point $(-a, 2)$ lies on the line.	**s.** $y=2$

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4. Consider $\lim_{x\to\infty}\left(\frac{x^3+x^2+x+\sin x}{x^2+2\cos x}-a\sin x-bx+c\right)=4.$

Now, match the following lists and then choose the correct code.

List I	List II
a. The value of a is	p. 1
b. The value of b is	q. 3
c. The value of c is	r. 2
d. Number of real roots of equation $cx^2 + bx + a = 0$ is	s. 0

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5. Match the following lists (where $[x]$ represents the greatest integer function) and then choose the correct code.

List I	List II
a. $\lim_{x\to 0} x(-1)^{[1/x]}$	p. Does not exist
b. $\lim_{x\to 2}(-1)^{[x]}$	q. is 0
c. $\lim_{x\to \frac{3}{2}}(x-[x])$	r. is 1
d. $\lim_{x\to 0}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$	s. is 2

Codes :

	a	b	c	d
(1)	s	r	q	p
(2)	q	p	s	p
(3)	s	r	p	q
(4)	p	p	q	r

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Numerical Value Type

1. The reciprocal of the value of

$$\lim_{n \to \infty} \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\ldots\left(1 - \frac{1}{n^2}\right) \text{is}\_\_\_\_\_.$$

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2. $\lim_{x \to \infty} f(x)$, where $\frac{2x-3}{x} < f(x) < \frac{2x^2+5x}{x^2}\ \forall x > 0$, is _____.

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3. If

$f(x) = \{x - 1, x \geq 12x^2 - 2, x < 1, g(x) = \{x + 1, x > 0 - x^2 + 1, x \leq$

$= |x|, then \lim_{x \to 0} f(g(h(x)))$ is___

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4. If $\lim_{x \to \infty} f(x)$ exists and is finite and nonzero and if $\lim_{x \to \infty} \left\{ f(x) + \frac{3f(x) - 1}{f^2(x)} \right\} = 3$, then the value of $\lim_{x \to \infty} f(x)$ is _____.

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5. If $L - (\lim)_{x \vec{2}} \frac{(10 - x)^{\frac{1}{3}} - 2}{x - 2}, then the value of |1(4L)| is\_\_$

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6. If $\lim_{x \to 0} \frac{p \sin 2x + (1 - \cos 2x)}{x + \tan x} = 1$, then the value of p is_____.

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7. The value of $\lim_{x \to \infty} \left(\frac{100}{1 - x^{100}} - \frac{50}{1 - x^{50}} \right)$ is _____.

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8. If $L = \lim_{x \to 2} \frac{\sqrt[3]{60 + x^2} - 4}{\sin(x - 2)}$, then the value of $1 / L$ is_____.

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9. The value of $\lim_{x \to \infty} \left(\frac{20^x - 1}{19(5^x)} \right)^{1/x}$ is _____.

A. 3

B. 1

C. $\frac{2}{3}$

D. $\frac{3}{2}$

Answer: (4)

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10. The value of $\lim_{n \to \infty} \left[\sqrt[3]{(n+1)^2} - \sqrt[3]{(n-1)^2}\right]$ is _____.

A. Equals $\frac{1}{\sqrt{2}}$

B. Does not exist

C. Equals $\sqrt{2}$

D. Equals -sqrt(2)`

Answer: (0)

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11. If $L = \lim_{n \to \infty} \left(2 \times 3^2 \times 2^3 \times 3^4 \ldots \times 2^{n-1} \times 3^n\right)^{\frac{1}{(n^2+1)}}$, then the value of L^4 is _____. (a) -1/4 (b) 1/2 (c) 1 (d) none of the above

A. $-\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

Answer: (6)

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12. The value of $\lim_{x \to \infty} \frac{\log_e(\log_e x)}{e\sqrt{x}}$ is _____. (a) π/ 2 (b)0 (c)-π (d)π

A. $\frac{\pi}{2}$

B. 1

C. $-\pi$

D. π

Answer: (0)

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13. about to only mathematics

A. 4

B. 3

C. 2

D. $1/2$

Answer: (6)

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14. The value of $\lim_{x \to \infty} \left(x - x^2 \log_e \left(1 + \frac{1}{x} \right) \right)$ is _____.

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. 2

Answer: (0.5)

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15. Let $S_n = 1 + 2 + 3 + \ldots + n$ and $P_n = \frac{S_2}{S_2 - 1} \cdot \frac{S_3}{S_3 - 1} \cdot \frac{S_4}{S_4 - 1} \ldots \frac{S_n}{S_n - 1}$, where $n \in N, (n \geq 2) Then \lim_{n \to \infty} P_n$=_____.

A. $\frac{1}{4}$

B. $\frac{1}{24}$

C. 3

D. $\frac{1}{8}$

Answer: (3)

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16. If $\lim_{x \to 1} \frac{a \sin(x-1) + b \cos(x-1) + 4}{x^2 - 1} = -2$, then $|a+b|$ is_____.

A. does not exist (in R)

B. is equal to 8

C. is equal to 15

D. is equal to 120

Answer: (8)

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17. Let $\lim_{x \to 1} \frac{x^a - ax + a - 1}{(x-1)^2} = f(a)$. Then the value of $f(4)$ is_____.

A. 4

B. 3

C. 6

D. 2

Answer: (6)

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18. Number of integral values of k for which

$\lim_{x \to 1} \sin^{-1}\left(\frac{k}{\log_e x} - \frac{k}{x-1}\right)$ exists is _____.

A. $a = 1, b = 4$

B. $a = 1 b = -4$

C. $a = 2, b = -3$

D. $a = 2, b = 3$

Answer: (5)

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19. If $\lim_{x \to 1} \left(1 + ax + bx^2\right)^{\frac{e}{(x-1)}} = e^3$, then the value of bc is _____.

A. $a = 2$

B. $a = 1$

C. $L = \frac{1}{64}$

D. L=(1)/(32)`

Answer: (3)

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20. Let $f''(x)$ be continuous at $x = 0$

If $\lim_{x \to 0} \frac{2f(x) - 3af(2x) + bf(8x)}{\sin^2 x}$ exists and $f(0) \neq 0, f'(0) \neq 0,$ then the value of $3a/b$ is_____.

A. $\lim_{x \to 1^+} f(x) = 0$

B. $\lim_{x \to 1^-} f(x)$ does not exist

C.

D.

Answer: (7)

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21. If $L = \lim_{x \to 0} \frac{e^{-x^2/2} - \cos x}{x^3 \sin x}$, then the value of $1/(3L)$ is _____.

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22. The integer n for which $(\lim)_{x \to 0}\left((\cos x - 1)\frac{\cos x - e\widehat{x}}{x^n}\right.$ is finite nonzero number is _____

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23. If $\lim_{x \to 0}\left[1 + x + \frac{f(x)}{x}\right]^{1/x} = e^3$, then the value of $\ln\left(\lim_{x \to 0}\left[1 + \frac{f(x)}{x}\right]^{1/x}\right)$ is _____.

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24. The largest value of the non-negative integer a for which

$$\lim_{x \to 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4} \text{ is } \_\_\_\_\_.$$

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25. about to only mathematics

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26. Let $\alpha, \beta \in R$ be such that $\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals_____.

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Archives JEE MAIN

1. Let $f: R \to R$ be a positive, increasing function with $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x \to \infty} \frac{f(2x)}{f(x)}$ is equal to

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2. Show that the $\lim_{x \to 2} \left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x - 2} \right)$ doesnot exist.

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3. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

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4. $\lim_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to

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5. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to

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6. Let $p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$. Then $\log_e p$ is equal to

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7. The $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

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8. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then $\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$

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Archives JEE ADVANCED

1. If

$$\lim_{x \to 0} \left[1 + x \ln\left(1 + b^2\right)\right]^{1/x} = 2b \sin^2 \theta, b > 0, \quad \text{and} \quad \theta \in (-\pi, \pi],$$

then the value of θ is

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2. If $\lim_{x \to \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4,$ then

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Single Correct Answer Type

1. If $f: R \overrightarrow{R}$ is defined by $f(x) = [x - 3] + |x - 4|$ for $x \in R,$ then $(\lim)_{x \overrightarrow{3}} f(x)$ is equal to (where [.] represents the greatest integer function) a. b. c. d. -1

A. -2

B. -1

C. 0

D. 1

Answer: C

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2. $\lim_{x-\frac{\pi}{2}}\left[\frac{[\sin x]-[\cos x]+1}{3}\right]=$ (where $[.]$ denotes the greatest integer integer function)

A. 0

B. 1

C. -1

D. does not exist

Answer: A

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3. $\lim_{x \to \frac{-1}{3}} \frac{1}{x}\left[\frac{-1}{x}\right] =$ (where [.] denotes the greatest integer function)

A. -9

B. -12

C. -6

D. 0

Answer: C

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4. If $f(x) = \begin{cases} x + \frac{1}{2}, x < 0 \\ 2x + \frac{3}{4}, x > +0 \end{cases}$, then $\left[\left(\lim\right)_{x \to 0} f(x)\right] =$ (where [.] denotes the greatest integer function) a. b. c. does not exist d. none of these

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. does not exist

D. none of these

Answer: C

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5. $\lim_{x \to -7} \frac{[x]^2 + 15[x] + 56}{\sin(x+7)\sin(x+8)} =$ (where [.] denotes the greatest integer function)

A. is 0

B. is 1

C. is -1

D. does not exist

Answer: A

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6. Which of the following limits exists finitely?

A. $\lim_{x \to 0^+} (x)^{\log_e x}$

B. $\lim_{x \to 1^+} \frac{x^2 - 9 - \sqrt{x^2 - 6x + 6}}{|x - 1| - 2}$

C. $\lim_{x \to 1^+} ([x])^{\frac{1}{x-1}} =$ (where [.] denotes the greatest integer function)

D. none of these

Answer: C

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7. Let $L_1 = \lim_{x \to 4} (x - 6)^x$ and $L_2 = \lim_{x \to 4} (x - 6)^4$.

Which of the following is true?

A. Both L_1 and L_2 exists

B. Neither L_1 nor L_2 exists

C. L_1 exists but L_2 does not exist

D. L_2 exists but L_1 does not exist

Answer: D

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8. Set of all values of x such that $\lim_{n \to \infty} \frac{1}{1 + \left(\frac{4 \tan^{-1}(2\pi x)}{\pi} \right)^{4n}}$ is non-zero and finite number when $n \in N$ is

A. $\left(0, \frac{1}{2\pi}\right)$

B. $\left(-\frac{1}{\pi}, \frac{1}{\pi}\right)$

C. $\left[-\frac{1}{2\pi}, \frac{1}{2\pi}\right]$

D. $\left(-\frac{1}{2\pi}, 0\right)$

Answer: C

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9. $\lim_{x \to \infty} \left[x - \log_e \left(\frac{e^x + e^{-x}}{2} \right) \right] =$

A. $\log_e 4$

B. 0

C. $\log_e 2$

D. ∞

Answer: C

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10. $\lim_{x \to \infty} \left\{ (e^x + \pi^x)^{\frac{1}{x}} \right\}$= (where {.} denotes the fractional part of x) is equal to

A. $\pi - e$

B. $\pi - 3$

C. $e - 2$

D. $3 - e$

Answer: B

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11. If $\frac{\cos x}{\sin ax}$ is periodic function, then $\lim_{m \to \infty} \left(1 + \cos^{2m} n!\pi a\right)$ is equal to

A. 0

B. 1

C. 2

D. -1

Answer: C

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12. The value of $\lim_{x \to 0} \frac{\sqrt{1 - \cos x^2}}{1 - \cos x}$ is

A. $\frac{1}{2}$

B. 2

C. $\sqrt{2}$

D. none of these

Answer: C

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13. $\lim_{x \to \frac{\pi}{2}} (1 - \sin x)\tan x =$

A. $\frac{\pi}{2}$

B. 1

C. 0

D. ∞

Answer: C

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14. The value of $\lim_{x \to \infty} x^2 \left(1 - \cos. \frac{1}{x}\right)$ is

A. 0

B. $1/4$

C. $1/2$

D. 1

Answer: C

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15. $\lim_{x \to \infty} \sqrt[3]{x}\left(\sqrt[3]{(x+1)^2} - \sqrt[3]{(x-1)^2}\right) =$

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. $\frac{4}{3}$

Answer: D

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16. $\lim_{n \to \infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^n + 7.5^n} =$

A. 0

B. 43529

C. $-4/7$

D. $-20/7$

Answer: D

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17. $\lim\limits_{(X \to 2^+)} \{x\} \frac{\sin(x-2)}{(x-2)^2} =$ (where $\{.\}$ denotes the fractional part function)

A. 0

B. 2

C. 1

D. does not exist

Answer: C

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18. $\lim\limits_{x \to \infty} \frac{\cot^{-1}\left(\sqrt{x+1}+\sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^x\right\}} =$

A. 1

B. 0

C. $\pi/2$

D. non existent

Answer: A

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19. $\lim_{x \to 0} \frac{3\tan 3x - 4\tan 2x - \tan x}{4x^2 \tan x}$

A. 0

B. 1

C. 3

D. 4

Answer: D

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20. $\lim_{x \to 0} \left[\frac{\sin^{-1} x}{\tan^{-1} x} \right] =$ (where $[.]$ denotes the greatest integer function)

A. 0

B. 1

C. -1

D. none of these

Answer: B

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21. The value of $\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{1 - \sqrt{\sin 2x}}}{\pi - 4x}$ is

A. $\frac{1}{4}$

B. $-\frac{1}{4}$

C. 1

D. does not exist

Answer: D

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22. The value of $\lim_{x \to \infty} \left(e^{\sqrt{x^4 +}} - e^{(x^2+1)} \right)$ is

A. 0

B. e

C. 1/e

D. $-\infty$

Answer: D

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23. The value of $\lim_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is

A. 8

B. 4

C. -8

D. -2

Answer: C

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24. $\left(\lim\right)_{x \to \frac{\pi}{2}} \dfrac{(1-\sin x)\left(8x^3-\pi^3\right)\cos x}{(\pi-2x)^4}$

a. $\dfrac{\pi^2}{6}$ b. $\dfrac{3\pi^2}{16}$ c. $\dfrac{\pi^2}{16}$ d. $-\dfrac{3\pi^2}{16}$

A. $-\dfrac{\pi^2}{16}$

B. $\dfrac{3\pi^2}{16}$

C. $\dfrac{\pi^2}{16}$

D. $-\dfrac{3\pi^2}{16}$

Answer: D

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25. $\lim_{x \to \infty} \frac{\sum_{r=1}^{10} (x+r)^{2010}}{(x^{1006}+1)(2x^{1004}+1)} =$

A. 5

B. 2010

C. $\frac{502}{1005}$

D. 0

Answer: A

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26. If $\lim_{x \to 0} \frac{f(x)}{x^2} = a$ and $\lim_{x \to 0} \frac{f(1-\cos x)}{g(x)\sin^2 x} = b$ (where $b \neq 0$), then $\lim_{x \to 0} \frac{g(1-\cos 2x)}{x^4}$ is

A. $\frac{4a}{b}$

B. $\frac{a}{4b}$

C. $\frac{a}{b}$

D. none of these

Answer: C

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27. If

$$f(x)=\begin{cases}\frac{x}{\sin x}, & x>0\\ 2-x, & x\leq 0\end{cases} \text{ and } g(x)=\begin{cases}x+3, & x<1\\ x^2-2x-2, & 1\leq x<2\\ x-5, & x\geq 2\end{cases}$$

Then the value of $\lim_{x\to 0} g(f(x))$

A. is -2

B. is -3

C. is 1

D. does not exist

Answer: B

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28. If $k \in I$ such that $\lim_{n \to \infty} \left(\cos. \frac{k\pi}{4}\right)^{2n} - \left(\cos. \frac{k\pi}{6}\right)^{2n} = 0,$ then

A. k must not be divisible by 24

B. k is divisible by 24 or k is divisible neither by 4 nor by 6

C. k must be divisible by 12 but not necessarity by 24

D. none of these

Answer: B

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29. If a_n and b_n are positive integers and $a_n + \sqrt{2}b_n = \left(2 + \sqrt{2}\right)^n$, then $\lim_{n \to \infty} \left(\frac{a_n}{b_n}\right)$=

A. $\sqrt{2}$

B. 2

C. $e^{\sqrt{2}}$

D. e^2

Answer: A

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30. The value of $\left(\lim_{x \to 0} \frac{\tan x^{\frac{1}{5}}}{\left(\tan^{-1}\sqrt{x}\right)^2} \frac{\log(1+5x)}{e^{3\sqrt[5]{x}} - 1} \right.$ is

A. $\frac{3}{5}$

B. $\frac{5}{3}$

C. 1

D. none of these

Answer: B

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31. The value of $\lim_{x \to 3} \frac{(x^3 + 27)\log_e(x - 2)}{x^2 - 9}$ is

A. 9

B. 18

C. 27

D. $5/3$

Answer: A

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32. The value of $\lim_{x \to 0^+} \left(\frac{1 - \cos(\sin^2 x)}{x^2} \right)^{\frac{\log_e(1 - 2x^2)}{\sin^2 x}}$ is

A. 0

B. e

C. -1

D. ∞

Answer: D

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33. $\lim_{x \to 0} \frac{1}{x^2} \begin{vmatrix} 1-\cos 3x & \log_e(1+4x) \\ \sin^{-1}(x^x -) & \tan^{-1}(2x) \end{vmatrix}$ is equal to

A. 2

B. -4

C. 6

D. 4

Answer: B

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34. If graph of the function $y = f(x)$ is continuous and passes through point (3, 1) then $\lim_{x \to 3} \frac{\log_e(3f(x) - 2)}{2(1 - f(x))}$ is equal

A. $\frac{3}{2}$

B. $\frac{1}{2}$

C. $-\frac{3}{2}$

D. $-\frac{1}{2}$

Answer: C

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35. Let f(x) be defined for all $x \in R$ such that $\lim_{x \to 0} \left[f(x) + \log\left(1 - \frac{1}{e^{f(x)}}\right) - \log(f(x)) \right] = 0$. Then f(0) is

A. 0

B. 1

C. 2

D. 3

Answer: A

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36. $\lim_{x \to \infty} x^2 \sin\left(\log_e \sqrt{\frac{\cos(\pi)}{x}}\right)$

A. 0

B. $-\frac{\pi^2}{2}$

C. $-\frac{\pi^2}{4}$

D. $-\frac{\pi^2}{8}$

Answer: C

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37. If $\lim_{x \to \infty} \left(\frac{x+c}{x-c}\right)^x = 4$ then the value of e^c is

A. $1/4$

B. $1/2$

C. 1

D. 2

Answer: D

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38. If $\lim_{x \to 0} \left[1 + x + \frac{f(x)}{x}\right]^{1/x} = e^3$, then $\lim_{x \to 0} \left[1 + \frac{f(x)}{x}\right]^{1/x} =$

A. e

B. e^2

C. e^3

D. none of these

Answer: B

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39. $\lim_{x \to \frac{\pi}{2}^-} \left[1 + (\cos x)^{\cos x}\right]^2 =$

A. Does not exist

B. 1

C. e

D. 4

Answer: D

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40. If $a > 0, b > 0$ than $\lim_{n \to \infty} \left(\frac{a - 1 + b^{\frac{1}{n}}}{a}\right)^n =$

A. $b^{\frac{1}{a}}$

B. $a^{\frac{1}{b}}$

C. a^b

D. b^a

Answer: A

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41. If $f(x) = \lim_{n \to \infty} \left(\frac{\cos(x)}{\sqrt{n}} \right)^n$, then the value of $\lim_{x \to 0} \frac{f(x) - 1}{x}$ is

A. 0

B. 1

C. 2

D. $3/2$

Answer: A

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42. $\lim_{x \to 0} \frac{\log\left(e^{x^2} + 2\sqrt{x}\right)}{\tan\sqrt{x}}$ is equal to

A. 0

B. 1

C. e^2

D. 2

Answer: D

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43. Let $f: R \to R$ be such that $f(a) = 1, f(a) = 2$. Then $\lim_{x \to 0} \left(\frac{f^2(a+x)}{f(a)}\right)^{1/x}$ is

A. e^2

B. e^4

C. e^{-4}

D. $1/e$

Answer: B

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44. The value of $\left(\lim\right)_{n\to\infty}\left(\frac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^{2+n}-1}}$ is e b. $1/e$ c. e^2 d. e^{-2}

A. e

B. $1/e$

C. e^2

D. e^{-2}

Answer: B

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45. If $f(n) = \lim_{x \to 0} \left\{ \left(1 + \sin\frac{x}{2}\right)\left(1 + \sin\frac{x}{2^2}\right)...\left(1 + \sin\frac{x}{2^n}\right) \right\}^{\frac{1}{x}}$ then find $\lim_{n \to \infty} f(n)$.

A. 1

B. e

C. 0

D. ∞

Answer: B

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46. $\lim_{n \to \infty} \left(1 - x + x.\sqrt[n]{e}\right)^n$ is equal to

A. e^x

B. e^{-x}

C. e^{2x}

D. none of these

Answer: A

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47. The value of $\lim_{x \to 1} \frac{\sqrt[13]{x} - \sqrt[7]{x}}{\sqrt[5]{x} - \sqrt[3]{x}}$ is

A. $\frac{44}{91}$

B. $\frac{45}{89}$

C. $\frac{45}{89}$

D. $\frac{40}{93}$

Answer: B

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48. The value of $\lim_{x \to 1} \frac{\sqrt[13]{x} - \sqrt[7]{x}}{\sqrt[5]{x} - \sqrt[3]{x}}$ is

A. 1

B. -1

C. 0

D. $-1/2$

Answer: D

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49. The value of $\lim_{x \to 0} \frac{1 - \cos 2x}{e^{x^2} - e^x + x}$ is

A. 0

B. 2

C. 4

D. 8

Answer: C

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50. If $f'(a) = \frac{1}{4}, then(\lim)_{h \to 0} \frac{f(a+2h^2) - f(a-2h^2)}{f(a+h^3-h^2) - f(a-h^3+h^2)} =$

0 b. 1 c. -2 d. none of these

A. 0

B. 1

C. -2

D. none of these

Answer: C

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51. $(\lim)_{x \to 0^+} \frac{1}{x\sqrt{x}} \left(a \tan^{-1} \frac{\sqrt{x}}{a} - b \frac{\tan^{-1}(\sqrt{x})}{b} \right)$ has the value equal to $\frac{a-b}{3}$ b. 0 c. $\frac{(a^2-b^2)}{6a^2b^2}$ d. $\frac{a^2-b^2}{3a^2b^2}$

A. $\frac{a-b}{3}$

B. 0

C. $\frac{(a^2-b^2)}{6a^2b^2}$

D. $\frac{a^2-b^2}{3a^2b^2}$

Answer: D

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52. The value of $\lim_{x\to 0}\left(\frac{1+2x}{1+3x}\right)^{\frac{1}{x^2}} e^{\frac{1}{x}}$ is $e^{\frac{5}{2}}$ b. e^2 c. e^{-2} d. 1

A. $e\left(\frac{5}{2}\right)$

B. e^2

C. e^{-2}

D. 1

Answer: A

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53. Let $f: R \to R$ be a differentiable function at x = 0 satisfying f(0) = 0 and f'(0) = 1, then the value of $\lim_{x \to 0} \frac{1}{x} . \sum_{n=1}^{\infty} (-1)^n . f\left(\frac{x}{n}\right)$, is

A. 0

B. $-\log 2$

C. 1

D. e

Answer: B

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54. The value of $\lim_{x \to \frac{3\pi}{4}} \frac{1 + \sqrt[3]{\tan x}}{1 - 2\cos^2 x}$ is

A. $-1/2$

B. $-2/3$

C. $-3/2$

D. $-1/3$

Answer: D

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55. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}, 0 < x < 2$ m and n integers, $m \neq 0, n > 0$ and. If $\lim_{x \to 1+} g(x) = -1$, then

A. n= 1, m = 1

B. n = 1, m = -1

C. n = 2, m = 2

D. n gt 2, m = n

Answer: C

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56. Number of integral values of λ for which $(\lim)_{x \to 1} \sec^{-1}\left(\frac{\lambda^2}{(\log)_e x} - \frac{\lambda^2}{x-1}\right)$ does not exist is a. 1 b. 2 c. 3 d. 4

A. 1

B. 2

C. 3

D. 4

Answer: C

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57. If $\lim_{x \to 0} \frac{e^{ax} - e^x - x}{x^2} = b$ (finite), then

A. a = 2, b = 0

B. a = 0, $b = \frac{3}{2}$

C. $a = 2, b = \frac{3}{2}$

D. a = 0 , b = 2

Answer: C

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58. If $\lim_{x \to 0} \frac{x^3}{\sqrt{a+x}(bx - \sin x)} = 1, a > 0$, then $a + b$ is equal to

A. 36

B. 37

C. 38

D. 40

Answer: B

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59. If $\lim_{x \to \infty} x \log_e \left(\begin{vmatrix} \alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x \end{vmatrix} \right) = -5.$ where α, β, γ are finite real numbers, then

A. $\alpha = 2, \beta = 1, \gamma \in R$

B. $\alpha = 2, \beta = 2, \gamma = 5$

C. $\alpha \in R, \beta = 1, \gamma \in R$

D. $\alpha \in R, \beta = 1, \gamma = 5$

Answer: D

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ComprehensionType

1. Let $f(x)$ be the fourth degree polynomial such that $f'(0) - 6, f(0) = 2and(\lim)_{x \to 1} \frac{f(x)}{(x-1)^2} = 1$ The value of $f(2)$ is 3 b. 1 c. 0 d.2

A. 1

B. 0

C. 2

D. 3

Answer: C

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2. Let $f(x)$ be the fourth degree polynomial such that $f'(0) - 6, f(0) = 2and(\lim)_{x \to 1} \frac{f(x)}{(x-1)^2} = 1$ The value of $f(2)$ is 3 b. 1 c. 0 d.2

A. 4

B. 5

C. 6

D. 7

Answer: C

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Multiple Correct Answer Type

1. If $\lim_{x \to \infty} x \log_e \left(\begin{vmatrix} \alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x \end{vmatrix} \right) = -5.$ where α, β, γ are finite real numbers, then

A. a = 2

B. b = -4

C. c = 2

D. a + b + c = 8

Answer: A::B::C

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2. If $a \in I$, then value of a for which $\lim_{x \to a} \frac{\tan\left(\left[x^2\right] - [x]^3\right)}{(x-a)^3}$ exists finitely, is /are

A. 0

B. 1

C. -1

D. 2

Answer: A::B

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Comprehension Type

1. $L = \lim_{x \to 0} \frac{\sin(\sin x) - \sin x}{ax^5 + bx^3 + c} = -\frac{1}{12}$

The value/values of a is

A. $\in$ R

B. 2

C. 0

D. 1

Answer: A

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2. $L = \lim_{x \to 0} \frac{\sin(\sin x) - \sin x}{ax^5 + bx^3 + c} = -\frac{1}{12}$

The value/values of b is

A. $\in$ R

B. 2

C. 0

D. 1

Answer: B

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3. If

$$f(x) = \lim_{n \to \infty} \frac{(x^2 + ax + 1) + x^{2n}(2x^2 + x + b)}{1 + x^{2n}} \text{ and } \lim_{x \to \pm 1} f(x)$$

exists, then

The value of a is

A. -1

B. 1

C. 0

D. 2

Answer: B

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4. If

$$f(x) = \lim_{n \to \infty} \frac{(x^2 + ax + 1) + x^{2n}(2x^2 + x + b)}{1 + x^{2n}} \text{ and } \lim_{x \to \pm 1} f(x)$$

exists, then

The value of b is

A. -1

B. 1

C. 0

D. 2

Answer: C

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