



MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

METHODS OF DIFFERENTIATION

Single Correct Answer Type

1. The right hand derivative of $f(x) = [x]\tan\pi x$ at $x = 7$ is (where $[.]$ denotes the greatest integer function) 0 b. 7π c. -7π d. none of these

A. 0

B. 7π

C. -7π

D. None of these

Answer: B



Watch Video Solution

2. If $f(x - y) = f(x) \cdot g(y) - f(y) \cdot g(x)$ and $g(x - y) = g(x) \cdot g(y) + f(x) \cdot f(y)$ for all $x \in R$. If right handed derivative at $x=0$ exists for $f(x)$ find the derivative of $g(x)$ at $x=0$

A. -1

B. 0

C. 1

D. none of these

Answer: B



Watch Video Solution

3. If $xe^{xy} - y = \sin^2 x$ then $\frac{dy}{dx}$ at $x = 0$ is

A. 0

B. 1

C. -1

D. none of these

Answer: B



Watch Video Solution

4. about to only mathematics

A. 2

B. 4

C. 8

D. 16

Answer: C



Watch Video Solution

5. If for a continuous function f , $f(0) = f(1) = 0$, $f'(1) = 2$ and $y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to a. 1 b. 2 c. 0 d. none of these

A. 1

B. 2

C. 0

D. none of these

Answer: B

 [Watch Video Solution](#)

6. The derivative of $\cos\left(2 \tan^{-1} \sqrt{\frac{1-x}{1+x}}\right) - 2 \cos^{-1} \sqrt{\frac{1-x}{2}}$ w.r.t. x is

A. $1 - \frac{1}{\sqrt{1-x^2}}$

B. $1 - \frac{1}{\sqrt{1+x^2}}$

C. $2 - \frac{1}{\sqrt{1-x^2}}$

$$D. 2 - \frac{1}{\sqrt{1+x^2}}$$

Answer: A



Watch Video Solution

7. If $y = \frac{x^2}{2} + \frac{1}{2}x\sqrt{x^2+1} + \ln\sqrt{x + \sqrt{x^2+1}}$ then the value of $xy' + \log y'$ is

A. y

B. $2y$

C. 0

D. $-2y$

Answer: B



Watch Video Solution

8. Let $g(x) = f(x)\sin x$, where $f(x)$ is a twice differentiable function on $(-\infty, \infty)$ such that $f'(-\pi) = 1$. The value of $|g^{-\pi}|$ equals _____

A. 1

B. 2

C. -2

D. 0

Answer: C



[Watch Video Solution](#)

9. If $f(x) = \frac{\log_e(\log_e x)}{\log_e x}$ then $f'(x)$ at $x = e$ is

A. 0

B. 1

C. e

D. $1/2$

Answer: D

 Watch Video Solution

10. Let $g(x) = e^{f(x)}$ and $f(x+1) = x + f(x) \forall x \in \mathbb{R}$. If

$$n \in I^+, \text{ then } \frac{g'(n + \frac{1}{2})}{g(n + \frac{1}{2})} - \frac{g'(\frac{1}{2})}{g(\frac{1}{2})} = 2 \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$$

$$2 \left(1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{2n-1} \right) n$$

A. $2 \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$

B. $2 \left(1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n-n} \right)$

C. n

D. 1

Answer: C

 Watch Video Solution

$$11. \frac{d}{dx} \left[\cos^{-1} \left(x\sqrt{x} - \sqrt{(1-x)(1-x^2)} \right) \right] = \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$$

-1/4 d. none of these

A. $\frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$

B. $\frac{-1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$

C. $\frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x-x^2}}$

D. $\frac{1}{\sqrt{1-x^2}}$

Answer: B



Watch Video Solution

12. If $t(1+x^2) = x$ and $x^2 + t^2 = y$, then at $x = 2$ the value of $\frac{dy}{dx}$ is equal to

A. $\frac{24}{5}$

B. $\frac{101}{125}$

C. $\frac{488}{125}$

D. $\frac{358}{125}$

Answer: C



Watch Video Solution

13. if $x = \frac{1+t}{t^3}$, $y = \frac{3}{2t^2} + \frac{2}{t}$ satisfies $f(x) \cdot \left\{ \frac{dy}{dx} \right\}^3 = 1 + \frac{dy}{dx}$ then

$f(x)$ is:

A. x

B. $\frac{x^2}{1+x^2}$

C. $x + x + \frac{1}{x}$

D. $x - \frac{1}{x}$

Answer: A



Watch Video Solution

14. Let $y = x^3 - 8x + 7$ and $x = f(t)$. If $\frac{dy}{dt} = 2$ and $x = 3$ at $t = 0$, then $\frac{dx}{dt}$ at $t = 0$ is given by (a) 1 (b) $\frac{19}{2}$ (c) $\frac{2}{19}$ (d) none of these

A. 1

B. $\frac{19}{2}$

C. $\frac{2}{19}$

D. None of these

Answer: C



Watch Video Solution

15. If $x = \sec \theta - \cos \theta$ and $y = \sec^n \theta - \cos^n \theta$ then show that

$$(x^2 + 4) \left(\frac{dy}{dx} \right)^2 = n^2 (y^2 + 4)$$

A. 8

B. 16

C. 64

Answer: C



[Watch Video Solution](#)

16. The derivative of the function represented parametrically as

$x = 2t = |t|, y = t^3 + t^2|t|$ is a. -1 b. 1 c. 0 d. does not exist

A. -1

B. 0

C. 1

D. does not exist

Answer: B



[Watch Video Solution](#)

17. If $y = \tan^{-1}\left(\frac{u}{\sqrt{1-u^2}}\right)$ and $x = \sec^{-1}\left(\frac{1}{2u^2-1}\right)$,
 $u \in \left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$, prove that $2\frac{dy}{dx} + 1 = 0$.

A. y

B. xy

C. 0

D. 1

Answer: C



Watch Video Solution

18. The differential coefficient of $\sin^{-1}\left(\frac{5 \cos x - 4s \in x}{\sqrt{41}}\right)$ is -2 b. -1 c.

1 d. 2

A. -2

B. -1

C. 1

D. 2

Answer: D



[View Text Solution](#)

19. $xy = (x + y)^6$ and $\frac{dy}{dx} = \frac{y}{x}$ then $\frac{y}{x} =$ 1 b. 2 c. 3 d. 4

A. 1

B. 2

C. 3

D. 4

Answer: B



[Watch Video Solution](#)

20. If $x + y = 3e^2$ then $\frac{d}{dx}(x^y) = 0$ for $x = e^2$ b. e^e c. e d. $2e^2$

A. e

B. e^2

C. e^e

D. $2e^2$

Answer: B



Watch Video Solution

21. If $f(x) = (x - 1)^{100}(x - 2)^{2(99)}(x - 3)^{3(98)} \dots (x - 100)^{100}$, then

the value of $\frac{f'(101)}{f(101)}$ is

A. 5050

B. 2575

C. 3030

D. 1250

Answer: A

 [Watch Video Solution](#)

22. The function $f: \overrightarrow{RR}$ satisfies $f(x^2)\dot{f}^x = f'(x)\dot{f}'(x^2)$ for all real x . Given that $f(1) = 1$ and $f^1 = 8$, then the value of $f'(1) + f^1$ is 2 b. 4 c. 6 d. 8

A. 2

B. 4

C. 6

D. 8

Answer: C

 [Watch Video Solution](#)

23. The second derivative of a single valued function parametrically represented by $x = \phi(t)$ and $y = \psi(t)$ (where $\phi(t)$ and $\psi(t)$ are different function and $\phi'(t) \neq 0$) is given by

$$\text{A. } \frac{d^2y}{dx^2} = \frac{\left(\frac{dx}{dt}\right)\left(\frac{d^2y}{dt^2}\right) - \left(\frac{d^2x}{dt^2}\right)\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)^3}$$

$$\text{B. } \frac{d^2y}{dx^2} = \frac{\left(\frac{dx}{dt}\right)\left(\frac{d^2y}{dt^2}\right) - \left(\frac{d^2x}{dt^2}\right)\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)^2}$$

$$\text{C. } \frac{d^2y}{dx^2} = \frac{\left(\frac{d^2x}{dt}\right)\left(\frac{dy}{dt}\right) - \frac{dx}{dt}\left(\frac{d^2y}{dt^2}\right)}{\left(\frac{dx}{dt}\right)^3}$$

$$\text{D. } \frac{d^2y}{dx^2} = \frac{\left(\frac{d^2x}{dt}\right)\left(\frac{dy}{dt}\right) - \left(\frac{d^2y}{dt^2}\right)\left(\frac{dy}{dt}\right)}{\left(\frac{dy}{dt}\right)^3}$$

Answer: A



Watch Video Solution

24. For the curve $\sin x + \sin y = 1$ lying in first quadrant. If $\lim_{x \rightarrow 0} x^\alpha \frac{d^2y}{dx^2}$ exists and non-zero then $2\alpha =$

A. 3

B. 4

C. 5

D. 1

Answer: A



Watch Video Solution

25. If $y = \left(\frac{\alpha x + \beta}{\gamma x + \delta} \right)$, then $2 \frac{dy}{dx} \cdot \frac{d^3y}{dx^3}$ is

A. $7 \left(\frac{d^2y}{dx^2} \right)^2$

B. $5 \left(\frac{d^2y}{dx^2} \right)^2$

C. $3 \left(\frac{d^2y}{dx^2} \right)^2$

D. $\left(\frac{d^2y}{dx^2} \right)^2$

Answer: C



Watch Video Solution

26. If $f(1) = 3$, $f'(1) = 2$, $f''(1) = 4$, then $(f^{-1})''(3) =$ (where $f^{-1} =$ inverse of $y = f(x)$)

A. 1

B. $-\frac{1}{2}$

C. -2

D. None of these

Answer: B



Watch Video Solution

27. If the third derivative of $\frac{x^4}{(x-1)(x-2)}$ is $\frac{-12k}{(x-2)^4} + \frac{6}{(x-1)^4}$,

then the value of k is

A. 2

B. 4

C. 8

D. 16

Answer: C



Watch Video Solution

28. If $(a + bx)e^{\frac{y}{x}} = x$, Prove that $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$

A. $\left(\frac{dy}{dx} + x\right)^2$

B. $\left(x \frac{dy}{dx} - y\right)^2$

C. $\left(\frac{dy}{dx} - y\right)^2$

D. $\left(x \frac{dy}{dx} + y\right)^2$

Answer: B



Watch Video Solution

29. If $R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-3/2}}{\frac{d^2y}{dx^2}}$, then $R^{2/3}$ can be put in the form of

a. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{1}{\left(\frac{d^2x}{dy^2}\right)^{2/3}}$

b. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} - \frac{1}{\left(\frac{d^2x}{dy^2}\right)^{2/3}}$

c.

d. $\frac{2}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{2}{\left(\frac{d^2x}{dy^2}\right)^{2/3}}$

e. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} \cdot \frac{1}{\left(\frac{d^2x}{dy^2}\right)^{2/3}}$

A. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{1}{\left(\frac{d^2y}{dy^2}\right)^{2/3}}$

B. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{1}{\left(\frac{d^2y}{dy^2}\right)^{2/3}}$

C. $\frac{2}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} + \frac{2}{\left(\frac{d^2y}{dy^2}\right)^{2/3}}$

D. $\frac{1}{\left(\frac{d^2y}{dx^2}\right)^{2/3}} \cdot \frac{1}{\left(\frac{d^2y}{dy^2}\right)^{2/3}}$

Answer: A



Watch Video Solution

30. If $x = 2 \cos t - \cos 2t$, $y = 2 \sin t - \sin 2t$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$.

A. $1/2$

B. $5/2$

C. $-3/2$

D. 2

Answer: C



Watch Video Solution

31. If $y^3 - y = 2x$, then $\left(x^2 - \frac{1}{27}\right) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y$ b. $\frac{y}{3}$ c. $\frac{y}{9}$ d. $\frac{y}{27}$

A. y

B. $\frac{y}{3}$

C. $\frac{y}{9}$

D. $\frac{y}{27}$

Answer: C



Watch Video Solution

32. Let $f(x) = \frac{g(x)}{x}$ when $x \neq 0$ and $f(0) = 0$. If $g(0) = g'(0) = 0$ and $g''(0) = 17$ then $f'(0) =$ 3/4 b. $-1/2$ c. $17/3$ d. $17/2$

A. $3/4$

B. $-1/2$

C. $17/3$

D. $17/2$

Answer: D



Watch Video Solution

33. Let $f: (-\infty, \infty) \rightarrow \mathbb{R}$ be a continuous function such that $f(x+y) = f(x) + f(y) + f(x)f(y), \forall x \in \mathbb{R}$. Also $f'(0) = 1$. Then $[f(2)]$ equal ($[.]$ represents the greatest integer function) 5 b. 6 c. 7 d. 8

A. 5

B. 6

C. 7

D. 8

Answer: B



Watch Video Solution

34. Let $f: R \rightarrow R$ be a function satisfying

$$f(x + y) = f(x) + \lambda xy + 3x^2y^2 \text{ for all } x, y \in R$$

If $f(3)=4$ and $f(5)=52$, then $f'(x)$ is equal to

A. $10x$

B. $-10x$

C. $20x$

D. $128x$

Answer: B



Watch Video Solution

35. A function $f: \overrightarrow{R1, \infty}$ satisfies the equation $f(xy) = f(x)f(y) - f(x) - f(y) + 2$. If differentiable on $R - \{0\}$ and $f(2) = 5$, $f'(x) = \frac{f(x) - 1}{x}$. Then $\lambda = 2' f(1)$ b. $3f'(1)$ c. $\frac{1}{2}f'(1)$ d. $f'(1)$

A. $2f'(1)$

B. $3f'(1)$

C. $\frac{1}{2}f'(1)$

D. $f'(1)$

Answer: D



Watch Video Solution

36. Let $\frac{f(x+y) - f(x)}{2} = \frac{f(y) - a}{2} + xy$ for all real x and y . If $f(x)$ is differentiable and $f'(0)$ exists for all real permissible value of a and is equal to $\sqrt{5a - 1 - a^2}$. Then $f(x)$ is positive for all real x $f(x)$ is negative for all real x $f(x) = 0$ has real roots Nothing can be said about the sign of $f(x)$

- A. $f(x)$ is positive for all real x
- B. $f(x)$ is negative for all real x
- C. $f(x)=0$ has real roots
- D. nothing can be said about the sign of $f(x)$

Answer: A

 [Watch Video Solution](#)

37. Let $f(3) = 4$ and $f'(3) = 5$. Then $\lim_{x \rightarrow 3} [f(x)]$ (where $[.]$ denotes the greatest integer function) is

A. 3

B. 4

C. 5

D. non-existent

Answer: D



Watch Video Solution

38. Let $f(x)$ be a function which is differentiable any number of times and

$f(2x^2 - 1) = 2x^3 f(x), \forall x \in R.$ Then $f^{(2010)}(0) =$ (Here

$f^{(n)}(x) = n^{th}$ order derivative of f at x)

A. -1

B. 1

C. 0

D. data is insufficient

Answer: C



Watch Video Solution

39. If $f(x) = \begin{vmatrix} (x-a)^4 & (x-a)^3 & 1 \\ (x-b)^4 & (x-b)^3 & 1 \\ (x-c)^4 & (x-c)^3 & 1 \end{vmatrix}$ then

$f'(x) = \lambda \begin{vmatrix} (x-a)^4 & (x-a)^2 & 1 \\ (x-b)^4 & (x-b)^2 & 1 \\ (x-c)^4 & (x-c)^2 & 1 \end{vmatrix}$. Find the value of λ

A. 1

B. 2

C. 3

D. None of these

Answer: C



Watch Video Solution

40. Suppose $\begin{vmatrix} f'(x) & f(x) \\ f''(x) & f'(x) \end{vmatrix} = 0$ where $f(x)$ is continuously differentiable function with $f'(x) \neq 0$ and satisfies $f(0) = 1$ and $f'(0) = 2$ then $\lim_{x \rightarrow 0} \frac{f(x) - 1}{x}$ is

A. 1

B. 2

C. $1/2$

D. 0

Answer: B



Watch Video Solution

41. A nonzero polynomial with real coefficient has the property that $f(x) = f'(x)f''(x)$. If a is the leading coefficient of $f(x)$, then the value of $1/2a$ is ____

A. $1/3$

B. 6

C. 12

D. 1/18

Answer: D



Watch Video Solution

42. If f is an increasing function from $\mathbb{R} \rightarrow \mathbb{R}$ such that $f'' > 0$ and f^{-1} exists then $\frac{d^2(f^{-1}(x))}{dx^2}$ is

A. < 0

B. > 0

C. 0

D. cannot be determined

Answer: A



Watch Video Solution

43. Vertices of a variable acute angled triangle ABC lies on a fixed circle. Also a, b, c and A, B, C are lengths of sides and angles of triangle ABC, respectively. If x_1, x_2 and x_3 are distances of orthocentre from A, B and C, respectively, then the maximum value of $\left(\frac{dx_1}{da} + \frac{dx_2}{db} + \frac{dx_3}{dc}\right)$ is

- A. $-\sqrt{3}$
- B. $-3\sqrt{3}$
- C. $\sqrt{3}$
- D. $3\sqrt{3}$

Answer: B



[Watch Video Solution](#)

44. In a question a student was given to find the derivative of the product of two functions f and g . The student y misstate thought $(fg)' = f'g'$ for his question $f(x) = x^3$ and he got the correct answer. Given that

$g(4) = 1$. Then which of the following is false? $g(5) = \frac{1}{8}$ b. $f'(x) < 0$ c.

$f(0) < 0$ d. none of these

A. $g(5) = \frac{1}{8}$

B. $f'(x) < 0$

C. $f(0) < 0$

D. None of these

Answer: A



Watch Video Solution

45. f is a strictly monotonic differentiable function with

$f'(x) = \frac{1}{\sqrt{1+x^3}}$. If g is the inverse of f , then $g^x = \frac{2x^2}{2\sqrt{1+x^3}}$ b.

$\frac{2g^2(x)}{2\sqrt{1+g^2(x)}}$ c. $\frac{3}{2}g^2(x)$ d. $\frac{x^2}{\sqrt{1+x^3}}$

A. $\frac{3x^2}{2\sqrt{1+x^3}}$

B. $\frac{3g^2(x)}{2\sqrt{1+g^2(x)}}$

C. $\frac{3}{2}g^2(x)$

D. $\frac{x^2}{\sqrt{1+x^3}}$

Answer: C



Watch Video Solution

46. Suppose $f: \mathbb{R} \rightarrow \mathbb{R}^+$ be a differentiable function such that $3f(x+y) = f(x)f(y) \forall x, y \in \mathbb{R}$ with $f(1) = 6$. Then the value of $f(2)$ is 6 b. 9 c. 12 d. 15

A. 6

B. 9

C. 12

D. 15

Answer: C



Watch Video Solution

Multiple Correct Answer Type

1. If $y = \cos^{-1} \sqrt{\frac{\sqrt{1+x^2+1}}{2\sqrt{1+x^2}}}$, then $\frac{dy}{dx}$ is equal to $\frac{1}{2(1+x^2)}$, $x \in R$

(b) $\frac{1}{2(1+x^2)}$, $x > 0$ $\frac{-1}{2(1+x^2)}$, $x < 0$ (d) $\frac{1}{2(1+x^2)}$, $x < 0$

A. $\frac{1}{2(1+x^2)}$, $x \in R$

B. $\frac{1}{2(1+x^2)}$, $x > 0$

C. $\frac{1}{2(1+x^2)}$, $x < 0$

D. $\frac{-1}{2(1+x^2)}$, $x < 0$

Answer: B::D



Watch Video Solution

2. Suppose that $f(x)$ is differentiable invertible function $f'(x) \neq 0$ and $h'(x) = f(x)$. Given that $f(1) = f'(1) = 1$, $h(1) = 0$ and $g(x)$ is inverse of $f(x)$. Let $G(x) = x^2g(x) - xh(g(x)) \forall x \in R$.

Which of the following is/are correct? $G'(1) = 2$ b. $G'(1) = 3$ c. $G^1 = 2$

d. $G^1 = 3$

A. $G''(1) = 2$

B. $G'(1) = 3$

C. $G''(1) = 2$

D. $G''(1) = 3$

Answer: A::D



Watch Video Solution

3. A curve parametrically given by

$x = t + t^3$ and $y = t^2$, where $t \in R$. For what value(s) of t is $\frac{dy}{dx} =$

?

A. $\frac{1}{3}$

B. 2

C. 3

D. 1

Answer: A::D

 [Watch Video Solution](#)

4. If $y = x^{(\log x)^{\log(\log x)}}$, then $\frac{dy}{dx}$ is

A. $\frac{y}{x} \left(\ln x^{\log x - 1} \right) + 2 \ln x \ln(\ln x)$

B. $\frac{y}{x} (\log x)^{\log(\log x)} (2 \log(\log x) + 1)$

C. $\left[(\ln x)^2 + 2 \ln(\ln x) \right]$

D. $\frac{y}{x} \frac{\log y}{\log x} (2 \log(\log x) + 1)$

Answer: B::D

 [Watch Video Solution](#)

5. If $y = e^{-x} \cos x$ and $y_n + k_n y = 0$ where $y_n = \frac{d^n y}{dx^n}$ and k_n are constant $n \in \mathbb{N}$ then

A. $k_4 = 4$

B. $k_8 = -16$

C. $k_{12} = 20$

D. $k_{16} = -24$

Answer: A::B



[Watch Video Solution](#)

6. If $y = y(x)$ and it follows the relation $e^{xy} + y \cos x = 2$, then find (i) $y'(0)$ and (ii) $y(0)$.

A. $y'(0) = -1$

B. $y''(0) = 2$

C. $y'(0) = 3/2$

$$D. y''(0) = -2$$

Answer: A::B



Watch Video Solution

7. A twice differentiable function $f(x)$ is defined for all real numbers and satisfies the following conditions $f(0) = 2$; $f'(0) = -5$ and $f''(0) = 3$. The function $g(x)$ is defined by $g(x) = e^{ax} + f(x) \forall x \in R$, where 'a' is any constant. If $g'(0) + g(0) = 0$. Find the value(s) of 'a'

A. 1

B. -1

C. 2

D. -2

Answer: A::D



Watch Video Solution

