



## MATHS

### BOOKS - CENGAGE MATHS (ENGLISH)

# MONOTONOCITY AND MAXIMA-MINIMA OF FUNCTIONS

#### Single Correct Answer Type

1. If  $x \in \left(0, \frac{\pi}{2}\right)$ , then the function  $f(x) = x \sin x + \cos x + \cos^2 x$  is (a) Increasing (b) Decreasing (c) Neither increasing nor decreasing (d) None of these

A. increasing

B. Decreasing

C. Neither increasing nor decreasing

D. None of these

**Answer: B**



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2. The function  $f: (a, \infty) \rightarrow R$  where  $R$  denotes the range corresponding to the given domain, with rule  $f(x) = 2x^3 - 3x^2 + 6$ , will have an inverse provided

A.  $a \leq 1$

B.  $a \geq 0$

C.  $a \leq 0$

D.  $a \geq 1$

**Answer: D**

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3. Let  $f(x) = 1 - x - x^3$ . Then, the real values of  $x$  satisfying the inequality,

$$1 - f(x) - f^3(x) > f(1 - 5x), \text{ are}$$

A.  $(-2, 0)$

B.  $(2, \infty)$

C.  $(0, 2)$

D. None of these

**Answer: C**



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4. If  $g(x) = 2f(2x^3 - 3x^2) + f(6x^2 - 4x^3 - 3) \forall x \in R$  and  $f''(x) > 0 \forall x \in R$  then  $g(x)$  is increasing in the interval

A.  $\left(-\infty, -\frac{1}{2}\right) \cup (0, 1)$

B.  $\left(-\frac{1}{2}, 0\right) \cup (1, \infty)$

C.  $(0, \infty)$

D.  $(-\infty, 1)$

**Answer: B**



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5. Find the set of all values of the parameter 'a' for which the function,  $f(x) = \sin 2x - 8(a + b)\sin x + (4a^2 + 8a - 14)x$  increases for all  $x \in R$  and has no critical points for all  $a \in R$ .

A.  $(-\infty, -\sqrt{5}, -2)$

B.  $(1, \infty)$

C.  $(\sqrt{5}, \infty)$

D. None of these

**Answer: B**



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6. if  $f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$  monotonically increases for  $\forall x \in R$  then the minimum value of 'a' is

A. 2

B. 1

C. 0

D. -1

**Answer: C**



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7. If the function  $f(x) = 2 \cot x + (2a + 1) \ln |\cos ec x| + (2 - a)x$  is strictly decreasing in  $\left(0, \frac{\pi}{2}\right)$  then range of a is

A.  $[0, \infty)$

B.  $(-\infty, 0]$

C.  $(-\infty, \infty)$

D. None of these

**Answer: A**



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8. If  $x_1, x_2 \in \left(0, \frac{\pi}{2}\right)$ , then  $\frac{\tan x_2}{\tan x_1}$  is (where  $x_1 < x_2$ )

A.  $< \frac{x_1}{x_2}$

B.  $= \frac{x_1}{x_2}$

C.  $< x_1 x_2$

D.  $> \frac{x_2}{x_1}$

**Answer: D**



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9. If  $f(x)$  is a differentiable real valued function satisfying  $f''(x) - 3f'(x) > 3 \forall x \geq 0$  and  $f'(0) = -1$ , then  $f(x) + x \forall x > 0$  is

- A. decreasing function of  $x$
- B. increasing function of  $x$
- C. constant function
- D. none of these

**Answer: B**



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10. The roots of  $(x - 41)^{49} + (x - 49)^{41} + (x - 2009)^{2009} = 0$  are

- A. all necessarily real
- B. non-real except one positive real root
- C. non-real except three positive real roots
- D. non-real except for three real roots of which exactly one is positive

**Answer: B**



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11. Let  $h$  be a twice continuously differentiable positive function on an open interval  $J$ . Let  $g(x) = \ln(h(x))$  for each

$x \in J$  Suppose  $(h'(x))^2 > h''(x)h(x)$  for each  $x \in J$ . Then

- A.  $g$  is increasing on  $H$
- B.  $g$  is decreasing on  $H$
- C.  $g$  is concave up on  $H$
- D.  $g$  is concave down on  $H$

**Answer: D**



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12. If  $\sin x + x \geq |k|x^2, \forall x \in \left[0, \frac{\pi}{2}\right]$ , then the greatest value of  $k$  is

A.  $\frac{-2(2 + \pi)}{\pi^2}$

B.  $\frac{2(2 + \pi)}{\pi^2}$

C. can't be determined finitely

D. zero

**Answer: B**



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**13.** If

$$4x + 8 \cos x + \tan x - 2 \sec x - 4 \log\{\cos x(1 + \sin x)\} \geq 6$$

for all  $x \in [0, \lambda)$  then the largest value of  $\lambda$  is

A.  $\pi/3$

B.  $\pi/6$

C.  $\pi/4$

D.  $3\pi/4$

**Answer: B**



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14. The greatest possible value of the expression  $\tan x + \cot x + \cos x$  on the interval  $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$  is (a)  $\frac{12}{5}\sqrt{2}$  (b)  $\frac{11}{6}\sqrt{2}$  (c)  $\frac{12}{5}\sqrt{3}$  (d)  $\frac{11}{6}\sqrt{3}$

A.  $\frac{12}{5}\sqrt{2}$

B.  $\frac{11}{6}\sqrt{2}$

C.  $\frac{12}{5}\sqrt{3}$

D.  $\frac{11}{6}\sqrt{3}$

**Answer: D**



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15. Let  $f(x) = \begin{cases} (x + 1)^3 & -2 < x \leq -1 \\ x^{2/3} - 1 & -1 < x \leq 1 \\ -(x - 1)^2 & 1 < x < 2 \end{cases}$ . The total

number of maxima and minima of  $f(x)$  is

A. 4

B. 3

C. 2

D. 1

**Answer: B**



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16. Consider the graph of the function  $f(x) = x + \sqrt{|x|}$

Statement-1: The graph of  $y = f(x)$  has only one critical point

Statement-2:  $f'(x)$  vanishes only at one point

- A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
- B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
- C. Statement 1 is true, Statement 2 is false.
- D. Statement 1 is false, Statement 2 is true.

**Answer: D**



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17. The minimum value of the function  $f(x) = \frac{\tan\left(x + \frac{\pi}{6}\right)}{\tan x}$

is:

A. 1

B. 0

C.  $\frac{1}{2}$

D. 3

**Answer: D**



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18. Let  $f(x) = \frac{x^2 + 2}{[x]}$ ,  $1 \leq x \leq 3$ , where  $[.]$  is the greatest

integer function. Then the least value of  $f(x)$  is

A. 2

B. 3

C. 3/2

D. 1

**Answer: B**



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19. If  $f(x) = \begin{cases} 3 - x^2, & x \leq 2 \\ \sqrt{a + 14} - |x - 48|, & x > 2 \end{cases}$  and if  $f(x)$  has a

local maxima at  $x = 2$ , then greatest value of  $a$  is

A. 2013

B. 2012

C. 2011



D. 2010

**Answer: C**

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20. The function  $f(x) = x^5 - 5x^4 + 5x^3$  find maximum and minimum value

- A. One minima and two maxima
- B. Two minima and one maxima
- C. Two minima and two maxima
- D. One minima and one maxima

**Answer: D**

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21.

If

$$f(x) = |x - 1| + |x + 4| + |x - 9| + \dots + |x - 2500| \quad \forall x \in \mathbb{R}$$

, then all the values of  $x$  where  $f(x)$  has minimum values lie in

A. (600, 700)

B. (576, 678)

C. (625, 678)

D. none of these

**Answer: C**



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22. Slope of tangent to the curve

$$y = 2e^x \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \cos\left(\frac{\pi}{4} - \frac{x}{2}\right), \text{ where } 0 \leq x \leq 2\pi \text{ is}$$

minimum at  $x =$

A. 0

B.  $\pi$

C.  $2\pi$

D. none of these

**Answer: B**



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23. The value of  $a$  for which all extremum of function

$$f(x) = x^3 + 3ax^2 + 3(a^2 - 1)x + 1, \text{ lie in the interval } (2, 4)$$

is

A. (3, 4)

B. ( - 1, 3)

C. ( - 3, - 1)

D. none of these

**Answer: B**



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24. If  $f(x) = \begin{cases} x^3(1-x), & x \leq 0 \\ x \log_e x + 3x, & x > 0 \end{cases}$  then which of the following is not true?

A.  $f(x)$  has point of maxima at  $x = 0$

B.  $f(x)$  has point minima at  $x = e^{-4}$

C.  $f(x)$  has range  $R$

D. none of these

**Answer: D**



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25. The coordinates of the point on the curve  $x^3 = y(x - a)^2$  where the ordinate is minimum is

A.  $\left(3a, \frac{27}{4}a\right)$

B.  $(2a, 8a)$

C.  $(a, 0)$

D. None of these

**Answer: A**



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26. The fraction exceeds its  $p^{\text{th}}$  power by the greatest number possible, where  $p \geq 2$  is

A.  $\left(\frac{1}{p}\right)^{1/(p-1)}$

B.  $\left(\frac{1}{p}\right)^{p-1}$

C.  $p^{1/p-1}$

D. none of these

**Answer: A**



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27. If  $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - e^{x-1}, & 1 < x \leq 2 \\ x - e, & 2 < x \leq 3 \end{cases}$  and

$g'(x) = f(x), x \in [1, 3]$ , then`

A.  $g(x)$  has no local maxima

B.  $g(x)$  has no local minima

C.  $g(x)$  has local maxima at  $x = 1 + \ln 2$  and local minima

at  $x = e$

D.  $g(x)$  has local minima at  $x = 1 + \ln 2$  and local maxima

at  $x = e$

**Answer: C**



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28. If  $g(x) = \max (y^2 - xy) (0 \leq y \leq 1)$ , then the minimum value of  $g(x)$  (for real  $x$ ) is

A.  $\frac{1}{4}$

B.  $3 - \sqrt{3}$

C.  $3 + \sqrt{8}$

D.  $\frac{1}{2}$

**Answer: B**



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29. If  $a, b \in \mathbb{R}$  distinct numbers satisfying  $|a-1| + |b-1| = |a| + |b| = |a+1| + |b+1|$ , Then the minimum value of  $|a-b|$  is :

A. 3



B. 0

C. 1

D. 2

**Answer: D**



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30. If equation  $2x^3 - 6x + 2\sin a + 3 = 0$ ,  $a \in (0, \pi)$  has only one real root, then the largest interval in which  $a$  lies is

A.  $\left(0, \frac{\pi}{6}\right)$

B.  $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

C.  $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

D.  $\left(\frac{5\pi}{6}, \pi\right)$

**Answer: C**



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**31.** Let  $f$  be a continuous and differentiable function in

$(x_1, x_2)$ . If  $f(x) \cdot f'(x) \geq x\sqrt{1 - (f(x))^4}$  and

$\lim_{x \rightarrow x_1} (f(x))^2 = 1$  and  $\lim_{x \rightarrow x_2} (f(x))^2 = \frac{1}{2}$ , then minimum

value of  $(x_1^2 - x_2^2)$  is

A.  $\frac{\pi}{6}$

B.  $\frac{2\pi}{3}$

C.  $\frac{\pi}{3}$

D. none of these

**Answer: C**



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32. If  $ab = 2a + 3b$ ,  $a > 0$ ,  $b > 0$ , then the minimum value of  $ab$  is

A. 12

B. 24

C.  $\frac{1}{4}$

D. none of these

**Answer: B**



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33. Let  $a, b, c, d, e, f, g, h$  be distinct elements in the set  $\{-7, -5, -3, -2, 2, 4, 6, 13\}$ . The minimum value of

$(a + b + c + d)^2 + (e + f + g + h)^2$  is: (1) 30 (2) 32 (3) 34 (

4) 40

A. 30

B. 32

C. 34

D. 40

**Answer: B**



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**34.** The perimeter of a sector is  $p$ . The area of the sector is maximum when its radius is

A.  $\sqrt{p}$

B.  $\frac{1}{\sqrt{p}}$

C.  $\frac{p}{2}$

D.  $\frac{p}{4}$

**Answer: D**



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**35.** Minimum integral value of  $k$  for which the equation

$e^x = kx^2$  has exactly three real distinct solution,

A. 1

B. 2

C. 3

D. 4

**Answer: B**



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**36.** Let  $f(x) = x^3 - 3x + 1$ . Find the number of different real solutions of the equation  $f(f(x)) = 0$

A. 2

B. 4

C. 5

D. 7

**Answer: D**



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## Multiple Correct Answer Type

1. Which of the following statement(s) is/are true?

A. Differentiable function satisfying  $f(-1) = f(1)$  and

$f'(x) \geq 0$  for all  $x$  must be a constant function on the interval  $[-1, 1]$ .

B. There exists a function with domain  $\mathbb{R}$  satisfying  $f(x) > 0$ ,

for all  $x$ ,  $f'(x) > 0$  for all  $x$  and  $f''(x) > 0$  for all  $x$ .

C. If  $f''(x) = 0$  then  $(c, f(c))$  is an inflection point.

D. Suppose  $f(x)$  is a function whose derivative is the

function  $f'(x) = 2x^2 + 2x - 12$ . Then  $f(x)$  is decreasing

for  $-3 < x < 2$  and concave up for  $x > -\frac{1}{2}$ .

**Answer: A::D**



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2. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x + \log_e(1 + x^2)$ . Then  $f(x)$  is what kind of function

A.  $f$  is injective

B.  $f$  is surjective

C. there is a point on the graph of  $y = f(x)$  where tangent is not parallel to any of the chords

D. inversed of  $f(x)$  exists.

**Answer: A::B::C::D**



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3. Let  $f(x) = x - \frac{1}{x}$  then which one of the following statements is true?

- A.  $f(x)$  is one-one function.
- B.  $f(x)$  is increasing function.
- C.  $f(x) = k$  has two distinct real roots for any real  $k$ .
- D.  $x = 0$  is point inflection.

**Answer: B::C::D**



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4. Let  $f(x)$  be an even function in  $\mathbb{R}$ . If  $f(x)$  is monotonically increasing in  $[2, 6]$ , then

- A.  $f(3) < f(-5)$

B.  $f(4) < f(-3)$

C.  $f(2) > f(-3)$

D.  $f(-3) < f(5)$

**Answer: A::D**



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5. If  $f(x) = \begin{cases} -e^{-x} + k & , x \leq 0 \\ e^x + 1 & , 0 < x < 1 \\ ex^2 + \lambda & , x \geq 1 \end{cases}$  is one-one and

monotonically increasing  $\forall x \in R$ , then

A. maximum value of  $k$  is 1

B. maximum value of  $k$  is 3

C. minimum value of  $\lambda$  is 0

D. minimum value of  $\lambda$  is 1

**Answer: B::D**



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6. If the function  $f(x) = axe^{-bx}$  has a local maximum at the point (2,10), then

A.  $a = 5e$

B.  $a = 5$

C.  $b = 1$

D.  $b = 1/2$

**Answer: A::D**



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7. Let  $f(x) = \frac{e^x}{1+x^2}$  and  $g(x) = f'(x)$ , then

- A.  $g(x)$  has two local maxima and two local minima points
- B.  $g(x)$  has exactly one local maxima and one local minima point
- C.  $x = 1$  is a point of local maxima for  $g(x)$
- D. There is a point of local maxima for  $g(x)$  in the interval  $(-1,0)$

**Answer: B::D**



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8. If  $f'(x) = (x - a)^{2010}(x - b)^{2009}$  and  $a > b$ , then

- A.  $f(x)$  has relative maxima at  $x = b$

B.  $f(x)$  has relative minima at  $x = b$

C.  $f(x)$  has relative maxima at  $x = a$

D.  $f(x)$  has neither maxima, nor minima at  $x = a$

**Answer: B::D**



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9. If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} [f(x)]$  ( $[.]$  denotes the greatest integer function) and  $f(x)$  is non-constant continuous function, then

A.  $\lim_{x \rightarrow a} f(x)$  is an integer

B.  $\lim_{x \rightarrow a} f(x)$  is non-integer

C.  $f(x)$  has local maximum at  $x = a$

D.  $f(x)$  has local minimum at  $x = a$

**Answer: A::D**

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10. Consider the function  $f(x) = \ln(\sqrt{1-x^2} - x)$  then which of the following is/are true?

A.  $f(x)$  increases in the on  $x = \left(-1, -\frac{1}{\sqrt{2}}\right)$

B.  $f$  has local maximum at  $x = -\frac{1}{\sqrt{2}}$

C. Least value of  $f$  does not exist

D. Least value of  $f$  exists

**Answer: A::B::C**

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## Comprehension Type

1.

Let

$f: R \rightarrow R, y = f(x), f(0) = 0, f'(x) > 0$  and  $f''(x) > 0$ .

Three point  $A(\alpha, f(\alpha)), B(\beta, f(\beta)), C(\gamma, f(\gamma))$  on  $y = f(x)$

such that  $0 < \alpha < \beta < \gamma$ .

Which of the following is false ?

A.  $\alpha f(\beta) > \beta(f(\alpha))$

B.  $\alpha f(\beta) < \beta f(\alpha)$

C.  $\gamma f(\beta) < \beta(f(\gamma))$

D.  $\gamma(f(\alpha)) < \alpha f(\gamma)$

**Answer: B**

2.

Let

 $f: R \rightarrow R, y = f(x), f(0) = 0, f'(x) > 0$  and  $f''(x) > 0$ .Three point  $A(\alpha, f(\alpha)), B(\beta, f(\beta)), C(\gamma, f(\gamma))$  on  $y = f(x)$ such that  $0 < \alpha < \beta < \gamma$ .

Which of the following is false ?

A. 
$$\frac{f(\alpha) + f(\beta)}{2} < f\left(\frac{\alpha + \beta}{2}\right)$$

B. 
$$f(\alpha) + f(\beta) \frac{1}{2} > f\left(\frac{\alpha + \beta}{2}\right)$$

C. 
$$f(\alpha) + f(\beta) \frac{1}{2} = f\left(\frac{\alpha + \beta}{2}\right)$$

D. 
$$\frac{2f(\alpha) + f(\beta)}{3} < f\left(\frac{2\alpha + \beta}{3}\right)$$

**Answer: B**



3.

Let

$f: R \rightarrow R, y = f(x), f(0) = 0, f'(x) > 0$  and  $f''(x) > 0$ .

Three point  $A(\alpha, f(\alpha)), B(\beta, f(\beta)), C(\gamma, f(\gamma))$  on  $y = f(x)$

such that  $0 < \alpha < \beta < \gamma$ .

Which of the following is true?

A. (a)  $\gamma f(\gamma + \beta - \alpha) > (\gamma + \beta - \alpha) f(\gamma)$

B. (b)  $\gamma f(\gamma + \beta - \alpha) < (\gamma + \beta - \alpha) f(\gamma)$

C. (c)  $\alpha f(\gamma + \beta - \alpha) > (\gamma + \beta - \alpha) f(\alpha)$

D. (d) None of these

**Answer: A**



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4. Let  $f$  be a twice differentiable function such that  $f''(x) > 0 \forall x \in R$ . Let  $h(x)$  is defined by  $h(x) = f(\sin^2 x) + f(\cos^2 x)$  where  $|x| < \frac{\pi}{2}$ .

The number of critical points of  $h(x)$  are

A. 1

B. 2

C. 3

D. more than 3

**Answer: C**



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5.  $f'(\sin^2 x) < f'(\cos^2 x)$  for  $x \in$

A.  $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

B.  $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

C.  $\left(-\frac{\pi}{4}, 0\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

D.  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

**Answer: A**



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**6.**  $h(x)$  is increasing for  $x \in$

A.  $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

B.  $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

C.  $\left(-\frac{\pi}{4}, 0\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

D.  $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(0, \frac{\pi}{4}\right)$

**Answer: B**



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