MATHS

BOOKS - CENGAGE MATHS (ENGLISH)

PRINCIPLE OF MATHEMATICAL INDUCTION

Sovled Examples

1. Prove the following by using the principle of mathematical induction for all $n \in N$:$1^3 + 2^3 + 3^3 + \ldots\ldots\ldots. + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

2. Using the principle of mathematical induction prove that $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \quad + \frac{1}{1+2+3+ \quad +n} = \frac{2n}{n+}$

for all $n \in N$

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3. Using the principle of mathematical induction, prove that $1.3 + 2.3^2 + 3.3^2 + ... + n.3^n = \frac{(2n-1)(3)^{n+1} + 3}{4}$ for all $n \in N$.

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4. Using principle of mathematical induction, prove that for all $n \in N, n(n+1)(n+5)$ is a multiple of 3.

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5. Prove the following by the principle of mathematical induction: $3^{2n+2} - 8n - 9$ is divisible 8 for all $n \in N$.

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6. Using the principle of mathematical induction prove that $41^n - 14^n$ is a multiple of 27 .

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7. Prove the following by using the principle of mathematical induction for all $n \in N : (2n+7) < (n+3)^2$.

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8. Using the principle of mathematical induction , prove that for $n \in N, \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots\ldots + \frac{1}{3n+1} > 1.$

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9. A sequence $a_1, a_2, a_3, \ldots$ is defined by letting $a_1 = 3$ and $a_k = 7a_{k-1}$, for all natural numbers $k \leq 2$. Show that $a_n = 3 \cdot 7^{n-1}$ for natural numbers.

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10. Let

$A_n = a_1 + a_2 + \ldots\ldots + a_n, B_n = b_1 + b_2 + b_3 + \ldots. + b_n, D_n = c_1 + c_2$

and $c_n = a_1 b_n + a_2 b_{n-1} + \ldots\ldots + a_n b_1 Aan \in N$. Using mathematical induction , prove that

(a)

$D_n = a_1 B_n + a_2 B_{n-1} + \ldots.. + a_n B_1 = b_1 A_n + b_2 A_{n-1} + \ldots\ldots + b_n A_1 \forall n$

(b) $D_1 + D_2 + \ldots\ldots + D_n = A_1 B_n + A_2 B_{n-1} + \ldots.. + A_n B_1 \forall n \in N$

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11. Let $U_1 = 1, \ U_2 = 1 \, and \, U_{n+2} = U_{n+1} + U_n f \text{ or } n \geq 1.$ use mathematical induction to show that:

$U_n = \frac{1}{\sqrt{5}} \left\{ \left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right\} f \text{ or } \ all \, n \geq 1.$

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12. If p is a fixed positive integer, prove by induction that $p^{n+1}+(p+1)^{2n-1}$ is divisible by P^2+p+1 for all $n \in N$.

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13. Let $0 < A_i < \pi$ for $i=1,2,\ldots\ldots n$. Use mathematical induction to prove that $\sin A_1 + \sin A_2 + \ldots\ldots + \sin A_n \leq n \sin\left(\frac{A_1 + A_2 + \ldots\ldots + A_n}{n}\right)$ where $n \geq 1$ is a natural number.

[You may use the fact that $p \sin x + (1-p)\sin y \leq \sin[px + (1-p)y]$, where $0 \leq p \leq 1$ and $0 \leq x, y \leq \pi$]

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14. Prove the following by the principle of mathematical induction:

$$1.3 + 2.4 + 3.5 + \quad + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$$

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15. Using the principle of mathematical induction prove that

$$\frac{1}{1.\,2.\,3}+\frac{1}{2.\,3.\,4}+\frac{1}{3.\,4.\,5}+\;+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

for all $n \in N$

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16. Using the principle of mathematical induction, prove that $\left(2^{3n}-1\right)$ is divisible by 7 for all $n \in N$.

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17. Using the principle of mathematical induction. Prove that $(x^n - y^n)$ is divisible by (x-y) for all $n \in N$.

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18. Using principle of mathematical induction prove that $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{n}}$ for all natural numbers $n \geq 2$.

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19. Show that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number, for all n $\in N$

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20. Using principle of mathematical induction, prove that $7^{4^n} - 1$ is divisible by 2^{2n+3} for any natural number n.

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21. Prove by mathematical induction that n^5 and n have the same unit digit for any natural number n.

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22. A sequence $b_0, b_1, b_2, \ldots$ is defined by letting $b_0 = 5$ and $b_k = 4 + b_{k-1}$, for all natural number k. Show that $b_n = 5 + 4n$, for all natural number n using mathematical induction.

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