MATHS

BOOKS - KUMAR PRAKASHAN

SURFACE AREAS AND VOLUMES

1. Rasheed got a playing top (lattau) as his birthday present which surprisingly had no colour on it. He wanted to colour it with his

cryons. The top is shaped like a cone surmounted by a hemisphere (see the given figure). The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (Take $\pi = \dfrac{22}{7}$)

2. the decorative block shown in the given figure is made of two solids -a cube and a hemisphere. The base of the block is acube

with ege 5 cm, and the hemisphere fixed on the top a diameter of 4.2 cm. Find the total surface area of the block (Take $\pi = \dfrac{22}{7}$)

3. A wooden toy rocke is in the shape of a cone mounted on a cylinder, as shown in the given figure. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm, the base of the concial protion has a diameter

of 5 cm, while the base diameter ofthe cylinderical protion is 3 cm. if the concial portion is tobe painted organe and the cylinderical portion yellow. Find the area of the rocke painted with each of these colours. (Take $\pi = 3.14$)

4. Mayank made new a bird-bath for his garden in the shape of a cylinder with a

hemispherical depression at one end [see the given figure]. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird- bath. (Take $\pi = \dfrac{22}{7}$)

5. Shanta runs an industry in a shed which isin the shape of a cuboid surmounted by a half cylinder (see the given figure). If the base of the shed is of dimension $7m \times 15m$, and the

hegiht of the coboidal portion is 8m find the volume of air that the shed can hold. Further suppose te machinery in the shed occupies a total space of $300m^3$, and there are 20 workers, each of whom occupy about $0.08m^3$ space on an average . Then how much air is in the shed ? (Take $\pi = \dfrac{22}{7}$)

6. A juice seller was swerving his customers using glasses as shown in the given figure. The inner diameter of the cylindrical glass was 5 cm. but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glas.s If the height of the glass was 10 cm, find the appartent capacity of the glass and its actualcapacity. (use $\pi = 3.14$)

7. A solid toy isinthe form of a hemisphere surmounted by a right circular conc. The height of the cone is 2 cm and the diameter of the base is 4 cm . Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (take $\pi = 3.14$) [see the given figure)

8. A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

9. Selvi's house has an overhead tank in the sphase of a cylinder. This is filled by pumping water from a sump (an underground tank) which is in the shape of a cuboid. The sump has dimensions $1.57m \times 1.44m \times 95cm$. The

overhead tank has its radius 60cm and height 95 cm. Find the hieght of the water left in the sump after the overhead tank has been completely filled with water from the sump which had been full. Compare the capacity of the tank with that of the sump (Use $\pi = 3.14$)

10. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 148 m of

uniform thickness. Find the thickness of the wire :

11. a hemispherical tank full of water is emptied by a pipe at the rate of $3\frac{4}{7}$ litres per second. How much time will it take to empty half the tank. If the diameter of the base of the tank is 3m? $\left(\text{Take } \pi = \frac{22}{7}\right)$

12. The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm (sec the given figure). Find the volume, the curved surface area and the total surface area (take $\pi = \dfrac{22}{7}$)

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13. Hanumappa and his wif Gangamma are busy making jaggery out of sugarcane juice.

They have processed the sugrcane juice to make the molasses, which is poured into moulds in the shape of a frustum of a cone having the diameters of its two circular faces as 30 cm and 35 cm and the vertical height of the mould is 14 cm (see the give figure). If each cm^3 of molasses has mass about n1.2g find the mass of the molasses that can be poured into each mould, (Take $\pi = \dfrac{22}{7}$)

14. An open metal bucket is in the shape of a frustum of a conce. Mounted on a hollow cylinderical base made of the same metallic sheet (see the given figure) the diameters of the two circular ends of the bucket are 45 cm and 25 cn. The vertical height of the bucket is 40 cm and that of the cylinderical base is 6 cm. find the area of the metallic sheet used to make the bucket, hwere we do not tak einto account the handle of the bucket. also, find the volu8me of water the bucket can hold

$$\left(\text{Take } \pi = \frac{22}{7}\right)$$

Other Important Examples

1. The internal and external diameters of a hollow hemispherical veseel are 12 cm and 16 cm respectively. If the cost of painting $1cm^2$ of the surface area is Rs 5.00 find the total cost

of painting the vessel all over. (Use $\pi = 3.14$)

2. The cost of painting the total outer surface of a closed cylinderical oil tank at 60 paise per sq cm is Rs 237.60 . The length of the tank is 6 times the radius of the base of the tank. Find the height and radius of the tank.

3. The sum of the radius of the base and height of a solid right circular cylinder is 37 cm. if its total surface area is $1628cm^2$. Find its volume.

4. A tent is in the form of a cylinder of diameter 4.2 m and height 8 m surmounted by a conc of equal base and heitght 6m, find the volume of air in the tent.

5. A vesel is in the form of a hemispherical bowl surmounted by a hollow cylinder. Thye diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the capacity of the vessel.

6. How many spherical lead shots each of diameter 4.2 cm can be obtained by melting a cuboidal lead plece with dimensions $66cm \times 42cm \times 21cm$.

7. A solid metallic spheres of radius 10.5 cm is melted and reacts into number of small cones. Each of radius 3.5 cm and height 3 cm. Find the number of cones so formed.

8. A well of diameter 2 m is dug 14 m dugs the earth taken out of it is spread evenly all around it to form an embankmenat of heigh 40 cm. find the width of the embankment.

9. A friction elutch is in the form of a frustum of a coce with radii 16 cm and 10 cm and

height 8 cm. find its lateral surface area and its volume in multiples of π.

10. If the slant height of a frustum of a conc is 20 cm and the radii of its circular bases are 20 cm and 8 cm respectively, find the volume of the frustum (Use $\pi = 3.14$)

11. The raddi of the circular ends of a bucket in the form of a fructum of a cone are 14 cm and r cm. where $r < 14$. If the height of the bucket is 15 cm and its capacity is 5.39 litres., find the value of r.

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Exercise 13 1

1. 2 cubes each of volume $64cm^3$ are joined end to end. Find the surface area of the resulting cuboid.

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder the diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. find the total surface area of the toy.

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest

diameter the hemisphere can have ? Find the surface area of the solid.

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

6. A medicine capsule is in the shape of a cylinder with two hermispheres stuck to each of its end [see te given figure]. The length of the entire capsule is 14 mm and the dimaeter of the capsule is 5 mm. find the surface area.

7. A tent is in the shape of cylinde surmounted bya concial top. If the height and diameter of the cylindrical part are 21 m and 4 m respectively. And the slant height of the top is 2.8m find the are of te canvasused for making the tent at the rat of Rs 500 per m^2. [Note that the base of th tent will not be covered with canvas.]

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2 .

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9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. As shown in the given figure. If the

height of the cylinder is 10 cm, and its base is of radius 3.5 cm find the total surface area of the article.

Exercise 13 2

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to

its radius find the volume of the solid in terms of π

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of

air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

3. A gulab jamun. Contains sugar syrup up to about 30% its volume. Find approximately how much syrup . Would be found in 45 gulab jamuns. Each shaped like a cylinder with two hemispherical ends with length 5 cm and

diameter 2.8 cm [see the given figure].

4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm 3.5 cm . The radius of each of the depression is 0.5 cm and the depth is 1.4 cm find the volume of wood in the entire stand

(See the given figure).

5. A vessel is in the form of an inverted cone. Its height is 8 cm and radius of its top, which is open, is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find

the number of lead shots dropped in the vessel.

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm find the mass of the pole. Give that $1cm^3$ of iron has approximately 8 g mass. (Use $\pi = 3.14$)

7. A solid consisting of a right circular conce of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of wate such that it touches the bottom. Find the volume of water left in the cylinder. if the radius of the cylinder is 60 cm and its height is 150 cm.

8. A spherical glass vessel has cylindrical neck 8 cm long 2 cm in diameter the diameter of the spherical part is 8.5 cm . By measuring the amount of water if holds a child finds its volume to be $345cm^3$. Check whether she is correct taking the above as the inside measurements, and $\pi = 3.14$

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Exercise 13 3

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. find the height of the cylinder.

2. Metallic spheres of radii 6cm,8cm and 10cm respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

3. a 20 m deep well with diameters 7 m is dug and the earth digging is evenly spread out to from a platform 22 m by 14 m find the height of the platform.

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of widt 4 m to from on embankment. Find the heigh of the embankment.

5. A container shpaed like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice creqa. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

6. How many silver coins, 1.75cm in diameter and of thickness 2mm, must be melted to form a cuboid of dimensions 5.5cm×10cm×3.5cm?

7. A cylinderical bucket. 32 cm high with radius of base 18 cm is filled with sand . This bucke tis empited on the ground and a coincal heap of sand is formed. If the heigh of the conical beap is 24 cm. Find the radius and slant height of the heap.

8. Water in a canal. 6m wide and 1.5 m dep. Is flowig with a speed of 10 km//h . How much area will it irrigate in 30 minutes if 8 cm of standing water is needed ?

9. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field which is 10 m in diameter and

2 m deep. If water flows through the pipe at the rate of 3 km// h, in how much time will the tank be filled ?

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1. A drinking glass is in the shape of a frustum of a cone of height 14 cm. the diameters of its two circular ends are 4 cm and 2 cm. find the capacity of the glass.

2. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

3. A fez, the cap used by the Turks, is shaped like the frustum of a cone (see the given figure). If its radius on the open side is 10 cm,

radius at the upper side is 4 cm and its slant height is 15 cm, find the area of material used for making it.

4. A container, open from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which

can completely fill the container, at the rate of Rs 20 per litre. also find the cost of metal sheet used to make the container, if it cost Rs 8 per $100cm^2$ (Take $\pi = 3.14$)

5. A metallic right circular cone 20 cm high and whose vertical angle is $60°$ is cut into two parts at the middle of its height by a plane parallel at the middle of its height by a plane parallel to its base. If the frustum so obtained

be drawn into a wire of diameter $\dfrac{1}{16}$ cm, find the length of the wire.

Exercise 13 5

1. A copper wire , 3 mm in diameter, is wound about a cylinde whose length is 12 cm, and diameter 10 cm. so as to cover the curved surface of the cylinder find the length and

mass of the wire, assuming the density of copper to be 8.88g per cm^3 (Use $\pi = 3.14$)

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2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed.

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3. A cistern, internally measuring $150cm \times 120cm \times 110cm$ has $129600cm^3$ of water in it. Porous brickes are placed in the water until the cistern is full to the brim. Each brick absorbs one-seveneenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being $22.5cm \times 7.5cm \times 6.5cm$?

4. In one fortnight of a given month, there was a rainfall fo 10 cm in a river valley. If the area of the valley is $7280ckm^2$, show that the total rainfall was approximately equivalent in the addition. Of the normal water of three rivers each $1072km$ long 75 m wide and 3 m deep.

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5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to

frustum of a cone. If the total height is 22 cm diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm find the area of the tin sheet required to make the funnel (see the given figure).

6. Derive the formula for the volume of the frustum of a conc, given to you in Section 13.5, using the symbols as explained.

Practice Thoroughly

1. The surface area of two sphere are in the ratio 16:9. Find the ratio of their volumes.

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2. If a ball of radius 2.1 cm is put into a cylindrical cup full of water having radius 5 cm

and height 6 cm, then find the volume of water that flows out of the cylindrical cup.

3. A child reshapes a cone made up of china clay of height 24 cm and radius 6 cm into a sphere. Find the radius of the sphere.

4. A solid metallic sphere of radius 5.6 cm is melted and solid cones of radius 2.8 cm and height 3.2 cm are made. Find the number of such cones formed.

5. A 15 cm long test tube having diameter of 3.6 cm has water up to a height of 6 cm. 20 spherical drops of oil of radius 9 mm are

dropped into it. What length of test tube remains empty ?

6. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of each bottle, if 10% liquid is wasted in transfer.

7. From the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the frustum of a cone so formed. (Use $\sqrt{5} = 2.236$)

8. From a cuboidal solid metallic block of dimensions $15cm \times 10cm \times 5cm$ a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. (see the

given figure).

9. A cylindrical bottle with radius 5 cm and height 14 cm has jam packed in it. The jam is spread evenly over bread of length 11 cm and breadth 10 cm. How thick can the layer of jam be spread, if the entire jam is to cover 10 bread pieces ?

10. Water is flowing at the speed of 2.52 km/h through a cylindrical pipe into a cylindrical tank with base radius 40 cm. If the increase in the level of water in the tank in half an hour is 3.15 m, find the internal diameter of the pipe.

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11. A wall 24 m long, 0.4 m thick and 6m high is constructed bricks each of the dimensions $25cm \times 16cm \times 10cm$. If mortar occupies to

$\dfrac{1}{10}$ th of the volume of the wall, find the number of bricks used in constructing the wall.

12. A cone is cut by a plane parallel to the base and upper part is removed. If the curved surface area of the remainder frustum is 8/9 of the curved surface area of the whole cone, find the ratio of the line segments into which the cone's height is divided by the plane.

Objective Questions Fill In The Blanks So As To Make Each Of The Following Statements True

1. The surface areas of two spheres are in the ratio 1:4. Then, the ratio of their volumes is ....

2. Two cubes each of volume $8cm^3$ are joined end to end, then the total surface area of the resulting cuboid is .... cm^2 ?

3. If the volume of a cube is $1728cm^3$ the length of its edge is .... cm.

4. The ratio of the volumes of two cones is 4:5 and the ratio of their radii is 2:3. Then, the ratio of their heights is .....

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5. The volume of a cone with radius 7 cm and slant height 25 cm is .... cm^3

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1. Three metallic spheres with radii 3 cm, 4 cm and 5 cm are melted and recast into a single sphere. Then, the radius of the new sphere formed is .... cm.

A. 4

B. 12

C. 6

D. 8

Answer: 6

2. The volume of the largest right circular cone that can be carved out of a solid hemisphere of radius r is given by

A. $\dfrac{4}{3}\pi r^3$

B. $\dfrac{2}{3}\pi r^3$

C. $4\pi r^3$

D. $\dfrac{1}{3}\pi r^3$

Answer: A::C

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3. From a wooden cube of side a cm, a right circular cylinder of maximum volume is cut. Then, the volume of the cylinder is .... cm.

A. πa^3

B. $\dfrac{\pi \alpha^3}{4}$

C. $\dfrac{\pi \alpha^3}{4}$

D. $\dfrac{4}{3}\pi \alpha^3$

Answer: A::C::D

4. The number of conical bottles of radius 2 cm and height 3.6 cm required to empty the liquid from a cylindrical bottle of radius 6 cm and height 10 cm is ....

A. 100

B. 25

C. 75

D. 20

Answer:

5. The surface areas of two spheres are in the

ratio 1:2. Then, the ratio of their volumes is

....

A. $\sqrt{2}:1$

B. $1:2:\sqrt{2}$

C. $1:8$

D. $1:4$

Answer: A::B

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6. The radii of two cylinders are in the ratio 2:3

and their heights are in the ratio 5:3. Then, the

ratio of their volumes is

A. $27 : 20$

B. $20 : 27$

C. $9 : 4$

D. $1 : 4$

Answer: B

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7. Two metallic spheres of radii 9 cm and 10 cm are melted and recast to make two new spheres. If the radius of one of the new

spheres is 12 cm, the radius of the other new sphere is .... cm.

A. 7

B. 5

C. 3

D. 1

Answer: A

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8. If S_1, denotes the surface area of a sphere with radius r and S_2, denotes the total surface area of a cylinder with radius r and height 2r, then .... holds good. a. $S_1 = S_2$ b. $S_1 > S_2$ c. $S_1 < S_2$ d. $S_1 = 2S_2$

A. $S_1 = S_2$

B. $S_1 > S_2$

C. $S_1 < S_2$

D. $S_1 = 2S_2$

Answer: A::B

9. Eight solid spheres of the same size are made by melting a solid metallic cylinder of diameter 6 cm and height 32 cm. Then, the diameter of each sphere is .... cm.

A. 3

B. 6

C. 12

D. 8

Answer:

10. Two solid right circular cones have the same height and radii r_1 cm and r_2 cm. They are melted and recast into a right circular cylinder of the same height. Then, the radius of the cylinder is .... cm.

A. $\dfrac{r_1^2 + r_2^2}{3}$

B. $\sqrt{\dfrac{r_1^2 + r_2^2}{3}}$

C. $\sqrt{r_1^2 + r_2^2}$

D. $\sqrt{r_1 r_2}$

Answer: A::B::C

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Objective Questions Answer The Following By A Number Or A Word Or A Sentence

1. The volumes of two spheres are in the ratio

27:8. Find the ratio of their diameters.

2. The diameter of the base of a right circular cylinder is 28 cm and its height is 21 cm. Find its curved surface area.

3. If the radius of a sphere is increased by 20%, by what percent does its volume increase?

4. A cylinder and a cone are of same base radius and of same height. Find the ratio of the volume of the cylinder to the volume of the cone.

5. If the area of three adjacent faces of a cuboid are X, Y and Z respectively, then find the volume of the cuboid.

1. Two identical solid cubes of side a are joined end to end. Then, the total surface area of the resulting cuboid is $10a^2$.

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2. If a spherical steel ball is melted to make 8 new identical balls then the radius of new ball

is how many times the radius of the original ball?

3. Write 'True' or 'False' and justify your answer:

Two identical solid hemispheres of equal base radius r cm are stuck together along their bases. The total surface area of the combination is $6\pi r^2 cm^2$.

4. A solid ball is exactly fitted inside the cubical box of side a. The volume of the ball is $\frac{1}{6}\pi a^3$.

5. Write 'True' or 'False' and justify your answer:

The volume of the frustum of a cone $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 - r_1 r_2\right)$ where h is the vertical height of the frustum and r_1, r_2 are the radii of the ends.

Test Your Skills

1. The slant height of a frustum of a cone is 4 m and the perimeter of circular ends are 18 m and 16 m. Find the cost of painting its curved surface at 10 per sq. m.

2. The radii of circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Find its volume and total surface area.

3. A bucket is in the form of a frustum of a cone whose radii of bottom and top are 7 cm and 28 cm respectively. If the capacity of the

bucket is 2156 cm, find the whole surface area of the bucket. (Use $\sqrt{445} = 21.095$)

4. A drinking glass is in the shape of a frustum of a cone of height 21 cm. The diameters of its two circular ends are 6 cm and 4 cm. Find the capacity of the glass.

5. The height of a bucket in the form of a frustum of a cone is 16 cm. The diameter of its upper open end is 40 cm and the diameter of its closed base is 16 cm. Find the capacity and the total surface area of the bucket in multiples of π

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