MATHS

BOOKS - JEE ADVANCED PREVIOUS YEAR

JEE Advanced

Maths

1. A line y=mx+1 meets the circle $(x-3)^2+(y+2)^2=25$ at point P and Q. if mid point of PQ has abscissa of $-\frac{3}{5}$ then value of m satisfies

A. $6 \leq m < 8$

B. $2 \leq m < 4$

C. $-3 \leq m < -1$

D. $4 \leq m < 6$

Answer: B

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2. if z is a complex number belonging to the set $S = \{z : |z - 2 + i| \geq \sqrt{5}\}$ and $z_0 \in S$ such that $\frac{1}{|z_0 - 1|}$ is maximum then arg $\left(\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}\right)$ is

A. $\frac{\pi}{4}$

B. $\frac{3\pi}{4}$

C. $-\frac{\pi}{2}$

D. $\frac{\pi}{2}$

Answer: C

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3. Area bounded the point (x,y) in certesian plane satesfying $xy \leq 8$ and $1 \leq y \leq x^2$ wll be

A. $16 \ln 2 - \frac{14}{3}$

B. $8 \ln 2 - \frac{7}{3}$

C. $8 \ln 2 - \frac{14}{3}$

D. $16 \ln 2 - 6$

Answer: A

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4. $M = \begin{bmatrix} \sin^4 \theta & -1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{bmatrix} = \alpha I + \beta M^{-1}$

Where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ ar real numbers and I is an identity matric of 2×2

if $\alpha^* =$ min of set $\{\alpha(\theta) : \theta \in [0.2\pi)\}$

and $\beta^* =$ min of set $\{\beta(\theta) : \theta \in [0.2\pi)\}$

Then value of $\alpha^* + \beta^*$ is

A. $\frac{-37}{16}$

B. $\frac{-17}{16}$

C. $\frac{-31}{16}$

D. $\frac{-29}{16}$

Answer: D

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5. if $a_{n=\frac{\alpha^n - \beta^n}{\alpha - \beta}}$ where α and β are roots of equation $x^2 - x - 1 = 0$ and $b_n = a_{n+1} + a_{n-1}$ then

A. $b_n = \alpha^n + \beta^n$

B. $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

C. $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

D. $a_1 + a_2 + \ldots a_n = a_{n+2} - 1$

Answer: A::C::D

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6. if a matrix M is given by $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ and if $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ then

A. $adj(M^{-1}) + (adjM)^{-1} = -M$

B. $|adj(M^2)| = 81$

C. $\alpha + 2\beta + 3\gamma = 2$

D. $\beta + 2\gamma = 3$

Answer: A::C

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7. There are three bags B_1, B_2, B_3, B_1 contians 5 red and 5 green balls. B_2 contains 3 red and 5 green balls and B_3 contains 5 red and 3 green balls, bags B_1, B_2 and B_3 have probabilities 3/10, 3/10, and 4/10 respectively of bieng chosen. A bag is selected at randon and a ball is randomly chosen from the bag. then which of the following options is/are correct?

A. Probability that the chosen ball is green equals $\frac{39}{80}$

B. Probability that the chosen all is green, gen that selected bag is B_3 equals $\frac{3}{8}$

C. Probability that the selected bag is B_3, given that the chosen ball is green equals $\frac{4}{13}$

D. Probability that the selected bag is B_3 given that the chosen ball is green equals $\frac{3}{10}$

Answer: A::B::C

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8. Let L_1 and L_2 denote the lines $\overrightarrow{r} = \overrightarrow{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right), \lambda \in R$ and $\overrightarrow{r} = \mu\left(2\hat{i} - \hat{j} + 2\hat{k}\right), \mu \in R$

Respectively if L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?

A. $\overrightarrow{r} = t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

B. $\vec{r} = \frac{2}{9}\left(4\hat{i} + \hat{j} + \hat{k}\right), t\left(2\hat{i} + 2\hat{j} - \hat{j}\right), t \in R$

C. $\vec{r} = \frac{1}{3}\left(2\hat{i} + \hat{j}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

D. $\vec{r} = \frac{2}{9}\left(2\hat{i} - \hat{j} + 2\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

Answer: B::C::D

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9. Equation of ellipse E_1 is $\frac{x^2}{9} + \frac{y^2}{4} = 1$, A rectangle R_1, whose sides are parallel to co-ordinate axes is inscribed in E_1 such that its area is maximum now E_n is an ellipse inside R_{n-1} such that its axes is along co-ordinate axes and has maxmim possible area $\forall n \geq 2, n \in N$, further R_n is a rectangle whose sides are parallel to co-ordinate axes and is inscribed in E_{n-1}. Having maximum area $\forall n \geq 2, n \in N$

A. $\sum_{n=1}^{m}$ area of rectangle $(R_n) < 24 \forall m \in N$

B. Length of latus rectum of $E_9 = \frac{1}{6}$

C. Distance between focus and centre of $E_9 = \frac{\sqrt{5}}{32}$

D. The eccentricities of E_{18} and E_{19} are not equal.

Answer: A::B

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10. In a non right angled triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angle P,Q,R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. if $p = \sqrt{3}, q = 1$ and the radius of the circumcircle of the ΔPQR equals to 1, then which of the followign options is/are correct?

A. length of $RS = \frac{\sqrt{7}}{2}$

B. length of $OE = \frac{1}{6}$

C. Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}(2 - \sqrt{3})$

D. Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

Answer: A::B::C

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11. let T denote a curve $y = f(x)$ which is in the first quadrant and let the point (1,0) lie on it. Let the tangent to T at a point P intersect the y-axis at Y_P and PY_P has length 1 for each poinit P on T. then which of the following option may be correct?

A. $y = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

B. $xy' - \sqrt{1-x^2} = 0$

C. $y = -\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

D. $xy' + \sqrt{1+x^2} = 0$

Answer: A::B::C::D

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12. Let $f: R \to R$ be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 3x + 1 & x < 0 \\ x^2 - x + 1 & 0 \le x < 1 \\ (2/3)x^3 - 4x^2 + 7x - (8/3) & 1 \le x < 3 \\ (x-2)\ln(x-2) - x + (10/3) & x \ge 3 \end{cases}$$

Then which of the following options is/are correct?

A. f is onto

B. f' is not differentiable at x=1

C. f' has a local maximum at x=1

D. f is increasing on $(-\infty, 0)$

Answer: A::B::C

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13. $I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{\sin x})(2 - \cos 2x)}$ then find $27I^2$

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14. let the point B be the reflection of the point A(2,3) with respect to the line 8x-6y-23=0. let T_A and T_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles T_A and T_B such that both the circles are on the same side of T. if C is the point of intersection of T and the line passing through A and B then the length of the line segment AC is

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15. if (a,d) denotes an A.P with first term a and common different d. if the A.P formed by intersection of three A.P's given (1,3), (2,5),and (3,7) is a new A.P (A,D). Then the value of A+D is

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16. Let S be the set of matrices of order 3×3 such that all elemtns of the matrix belong to $\{0, 1\}$

let $E_1 = \{A \in S : |A| = 0\}$ where |A| denotes determinant of matrix A

$E_2 = \{A \in S :$ sum of elements of $A = 7\}$ find $P(E_1 / E_2)$

A. 0.1

B. 0.9

C. 1.2

D. 0.5

Answer: D

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17. Equation of three lines $\vec{r} = \lambda\hat{i}, \vec{r} = \mu(\hat{i} + \hat{j}), \vec{r} = \gamma(\hat{i} + \hat{j} + \hat{k})$

and a plane $x + y + z = 1$ are given

then area of triangle formed by point of intersectioin of line and plane is

Δthen $(6\Delta)^2$equals

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18. What $\omega \neq 1$ be a cube root of unity. Then minimum value of set $\left\{\left|a+b\omega+c\omega^2\right|^2\right.$, a,b,c are distinct non zero intergers) equals

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19. Three lines L_1, L_2, L_3 are given by $L_1 : \vec{r} = \lambda\hat{i}, L_2 : \vec{r} = \mu\hat{j}+\hat{k}, L_3 : \vec{r} = \hat{i}+\hat{j}+\gamma\hat{k}$ which of the following point Q can be taken on L_2 so that the point P on line L_1 point Q on L_2 and point R on L_2 are collinear

A. $\hat{k}-\frac{1}{2}\hat{j}$

B. $\hat{k}$

C. $\hat{k}+\hat{j}$

D. $\hat{k}+\frac{1}{2}\hat{j}$

Answer: A::D

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20. $\lim_{n \to \infty} \dfrac{\sqrt[3]{1} + \sqrt[3]{2} + \dots + \sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(na+a)^2} + \frac{1}{(na+2)^2} + \dots + \frac{1}{(na+n)^2}\right)} = 54$ then

possible values a is/zer

A. -9

B. 8

C. 7

D. -6

Answer: A::B

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21. Let $f(x) = \dfrac{\sin \pi x}{x^2}, x > 0$

The $x_1 < x_2 < x_3 \dots < x_n < \dots$ be all points of local maximum of f(x) and $y^1 < y_2 < y_3 \dots < y_n < \dots$ be all the points of lcal minimum of f(x) then correct options is/are

A. $|x_n - y_n| > 1$ for every n

B. $x_1 < y_1$

C. $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n

D. $x_{n+1} - x_n > 2$ for every n

Answer: A::C::D

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22. $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$ and $R = PQP^{-1}$ then which are correct

A. det R=det $\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$ for all $x \in R$

B. for x=1 thre exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which are $R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

C. for x=0 if $R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$ then a+b=5

D. There exists a real number x such that $PQ = QP$

Answer: A:C

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23. Let $f R \rightarrow R$ be a function we say that f has property 1 if $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exist and is finite.

Property 2 if $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h^2}$ exist and is finite. Then which of the following options is/are correct?

A. $f(x) = x|x|$ has property 2

B. $f(x) = x^{2/3}$ has property 1

C. $f(x) = \sin x$ has property 2

D. $f(x) = |x|$ has property 1

Answer: B::D

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24. For non-negative inger n, let

$$f(n) = \sum_{k=}^{n} \frac{\sin\left(\frac{k+1}{n+1}\pi \sin\left(\frac{k+2}{n+1}\pi\right)\right)}{\sum_{k=0}^{n} \sin^2\left(\frac{k+1}{n+1}\pi\right)}$$

Assuming $\cos^{-1} x$ takes values in $[0, \pi]$ which of the following options is/are correct?

A. if $\alpha = \tan\left(\cos^{-1} f(6)\right)$, then $\alpha^2 + 2\alpha - 1 = 0$

B. $\lim_{n \to \infty} f(x) = \frac{1}{2}$

C. $f(4) = \frac{\sqrt{3}}{2}$

D. $\sin\left(7 \cos^{-1} f(5)\right) = 0$

Answer: A::C::D

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25. Let $f: R \to R$ be given $f(x) = (x-1)(x-2)(x-5)$ ltBrgt Define $F(x) = \int_0^x f(t) dt, x > 0$ the following options is/are correct?

A. $F(x) \neq 0, \ \forall x \in (0, 5)$

B. F(x) has two local maxima and one local minima in $(0, \infty)$

C. F(x) has a local maxima at x=2

D. F(x) has a local minima at x=1

Answer: A::C::D

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26. let

$P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},$

and $X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$ Where P_k^T is transpose of matrix P_k.

Then which of the following options is/are correct?

A. X is a symmetric matrix

B. if $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$

C. X-30I is an invertible matrix

D. The sum of diagonal entries of X is 18.

Answer: A::B::D

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27. A set S is given by {1,2,3,4,5,6}. |X| is number of elements in set X. If A and B are independent eventsassociated with S are chosen such that each elements is equally likely and $1 \leq |B| \leq |A|$ then the number of ordered pairs of (A,B) are

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28. Suppose det $\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} .^{n}C_{k}k^{2} \\ \sum_{k=0}^{n} .^{n}C_{k}k^{k} & \sum_{k=0}^{n} .^{n}C_{k}3^{2} \end{bmatrix} = 0$ holds for some positive integer n. then $\sum_{k=0}^{n} \frac{.^{n}C_{k}}{k+1}$ equals ....

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29. $\sec^{-1}\left|\frac{1}{4}\sum_{k=0}^{10}\left(\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+(k+1)\left(\frac{\pi}{2}\right)\right)\right]\right.$ will be

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30. if $I=\int_0^{\pi/2}\frac{3\sqrt{\cos\theta}}{\left(\sqrt{\sin\theta}+\sqrt{\cos\theta}\right)^5}d\theta$, then I^2 is equal to

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31. Five persons A, B, C, D & E are seated in a circular arrangement. If each of the is given a hat of one of the three colours red, blue & green, then the numbers of ways of distributing the hats such that the person seated in adjacent seat gets different coloured hats is

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32. Let $\overrightarrow{a} = 2\hat{i} + \hat{j} - \hat{k} \& \overrightarrow{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\overrightarrow{C} = \alpha\overrightarrow{a} + \beta\overrightarrow{b}, \alpha, \beta \in R$. If the projection of $\overrightarrow{c}$ on the vector $\left(\overrightarrow{a} + \overrightarrow{b}\right)$ is $3\sqrt{2}$ then the minimum value of $\left(\overrightarrow{c} - \left(\overrightarrow{a} \times \overrightarrow{b}\right)\right).\overrightarrow{c}$ equal to

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33. Let the circle $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$ intersect at the point X and Y. Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions

(i). Centre of C_3 is collinear with the center of $C_1 \& C_2$

(ii). $C_1 \& C_2$ both lie inside C_3 and

(iii). C_3 touches C_1 at M and C_2 at N

Let hte line through X and Y intersect C_3 at Z and W and let a common tangent of C_1 & C_3 be a tangent to the parabola $x^2 = 8\alpha y$

There are some expressions given in the following lists

List I		List II	
(I)	$2h+k$	(P)	6
(II)	$\frac{\text{length of ZW}}{\text{length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

Q. Which of the following is the only correct combination?

(A) (I)-(S)

(B) (II)-(Q) ltBrgt (C) (I)-(U)

(D) (II)-(T)

A. (I)-(S)

B. (II)-(Q)

C. (I)-(U)

D. (II)-(T)

Answer: B

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34. Let the circle $C_1: x^2+y^2=9$ and $C_2: (x-3)^2+(y-4)^2=16$ intersect at the point X and Y. Suppose that another circle $C_3: (x-h)^2+(y-k)^2=r^2$ satisfies the following conditions

(i). Centre of C_3 is collinear with the center of $C_1 \& C_2$

(ii). $C_1 \& C_2$ both lie inside C_3 and

(iii). C_3 touches C_1 at M and C_2 at N

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(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

Q. Which of the following is the only incorrect combination?

A. (IV)-(U)

B. (III)-(R)

C. (IV)-(S)

D. (I)-(P)

Answer: C

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35. Let f(x)= $\sin(\pi \cos x)$ and g(x)=$\cos(1\pi \sin x)$ e two function defined for $x > 0$ define the following sets whose elements are written in increasing order.

$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$

List I		List II	
(I)	X	(P)	$\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithmetic progression
(III)	Z	(R)	not an arithmetic progression
(IV)	W	(S)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Q. Which of th following is the only correct combination

A. IV-(P),(R),(S)

B. III-(R),(U)

C. III-(P),(Q),(U)

D. IV-(Q),(T)

Answer: A

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36. Let f(x)= $\sin(\pi \cos x)$ and g(x)=$\cos(1\pi \sin x)$ e two function defined for $x > 0$ define the following sets whose elements are written in increasing order.

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$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$

List I		List II	
(I)	X	(P)	$\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
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(III)	Z	(R)	not an arithmetic progression
(IV)	W	(S)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Q.

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Question

1. Consider a triangle Δ whose two sides lies on the x-axis and the line $x+y+1=0$. If the orthocenter of Δ is (1,1), then the equation of the circle passing through the vertices of the triangle is

A. $x^2+y^2-3x+y=0$

B. $x^2+y^2+x+3y=0$

C. $x^2+y^2+2y-1=0$

D. $x^2 + y^2 + x + y = 0$

Answer:

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2. The area of the region $\left\{(x, y) : 0 \leq x \leq \frac{9}{4}, 0 \leq y \leq 1, x \geq 3y, x + y \geq 2\right\}$ is

A. $\frac{11}{32}$

B. $\frac{35}{96}$

C. $\frac{37}{96}$

D. $\frac{13}{32}$

Answer:

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3. Consider three sets $E_1 = \{1, 2, 3\}, F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements. Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

A. $\frac{1}{5}$

B. $\frac{3}{5}$

C. $\frac{1}{2}$

D. $\frac{2}{5}$

Answer:

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4. Let $\theta_1, \theta_2, \ldots, \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + \ldots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}$, $z_k = z_{k-1}e^{i\theta_k}$ for $k = 2, 3, \ldots, 10$, where $i = \sqrt{-1}$. Consider the statements ? and ? given below:

$P: |z_2 - z_1| + |z_3 - z_2| + \ldots + |z_{10} - z_9| + |z_1 - z_{10}| \leq 2\pi$

$Q: |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \leq 4\pi$ Then

A. P is TRUE and Q id FALSE

B. Q is TRUE and P id FALSE

C. both P and Q are TRUE

D. both P and Q are FALSE

Answer:

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5. Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \ldots, 100\}$. Let P_1 be the probability that the maximum of chosen numbers is at least 81 and P_2 be the probability that the minimum of chosen numbers is at most 40. then the vaue of $\frac{625}{4}P_1$ is

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6. Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \ldots, 100\}$. Let P_1 be the probability that the maximum of chosen numbers is at least 81 and P_2 be the probability that the minimum of chosen numbers is at most 40. then the vaue of $\frac{125}{4}P_2$ is

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7. Let α, β and γ be real numbers such that the system of linear equations

$x + 2y + 3z = \alpha$

$4x + 5y + 6z = \beta$

$7x + 8y + 9z = \gamma - 1$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point $(0, 1, 0)$ from the plane P.

The value of $|M|$ is _____

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8. Let α, β and γ be real numbers such that the system of linear equations

$x + 2y + 3z = \alpha$

$4x + 5y + 6z = \beta$

$7x + 8y + 9z = \gamma - 1$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point $(0, 1, 0)$ from the plane P.

The value of D is _____

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9. Consider the lines L_1 and L_2 defined by

$L_1 : x\sqrt{2} + y - 1 = 0$ and $L_2 : x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2.

The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the square of the distance between R' and S'.

The value of λ^2 is

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10. Consider the lines L_1 and L_2 defined by

$L_1 : x\sqrt{2} + y - 1 = 0$ and $L_2 : x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2.

The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S' . Let D be the square of the distance between R' and S'.

The value of D is

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11. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order 3×3, then which of the following statements is (are) TRUE ?

A. $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

B. $\left|EQ + PFQ^{-1}\right| = |EQ| + \left|PFQ^{-1}\right|$

C. $\left|(EF)^3\right| > |EF|^2$

D. Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

Answer:

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12. Let $f: R \to R$ be definded by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Then which of the following statements is(are) TRUE?

A. f is decreasing in the interval $(-2, -1)$

B. f is increasing in the interval (1,2)

C. f is onto

D. Range of f is $\left[-\frac{3}{2}, 2\right]$

Answer:

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13. Let E,F and G be three events having probabilities

$P(E)=\frac{1}{8}, P(F)=\frac{1}{6}$ and $P(G)=\frac{1}{4}$, and let $P(E\cap F\cap G)=\frac{1}{10}$

For any event H, if H^c denotes its complement, then which of the following statements is (are) TRUE ?

A. $P(E\cap F\cap G^c)\leq\frac{1}{40}$

B. $P(E^c\cap F\cap G)\leq\frac{1}{15}$

C. $P(E\cup F\cup G)\leq\frac{13}{24}$

D. $P(E^c\cap F^c\cap G^c)\leq\frac{5}{12}$

Answer:

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14. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that

$(I - EF)$ is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) TRUE ?

A. $|FE| = |I - FE||FGE|$

B. $(1 - FE)(1 + FGE) = I$

C. $EFG = GEF$

D. $(I - FE)(I - FGE) = I$

Answer:

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15. For any positive integer n . let $S_n : (0, \infty) \rightarrow R$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1 + k(k+1)x^2}{x}\right)$$

where for any $x \in R$, $\cot^{-1} x \in (0, \pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

.Then which of the following statement is(are TRUE ?

A. $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1 + 11x^2}{10x}\right)$ for all $x > 0$

B. $\lim_{n \to \infty} \cot(S_n(x)) = x$ for all $x > 0$

C. The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0, \infty)$

D. $\tan(S_n(x)) \leq \frac{1}{2}$ for all $n \geq 1$ and $x \geq 0$

Answer:

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16. For any complex number $w = c + id$ let $arg(w) \in (-\pi, \pi], wherei = \sqrt{-1}$. Let α and β be real number such that all complex number $z = x + iy$ satisfying $arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair (x,y) lies on the circle $\left(x^2 + y^2 + 5x - 3y + 4 = 0\right)$

Then which of the following statement is(are) TRUE?

A. $\alpha = -1$

B. $\alpha . \beta = 4$

C. $\alpha . \beta = -4$

D. $\beta = 4$

Answer:

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17. For $x \in R$ the number of real roots of the equation

$3x^2 - 4|x^2 - 1| + x - 1 = 0$ is ______

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18. In a triangle ABC, let $AB = \sqrt{23}, BC = 4$ and $CA = 5$. Then the value of $\frac{\cot A + \cot B}{\cot C}$ is

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19. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u}.\vec{w} = 1, \vec{v}.\vec{w} = 1, \vec{w}.\vec{w} = 4$

If the volume of the parallelepiped, whose adjacent sides are represented by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ $is\sqrt{2}$ then the value of $\left|3u + 5\vec{v}\right|$ is

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20. Let $s_1 = \{(i, j, k) : i, j, k \in \{1, 2, \ldots, 10\}\}$

,$s_2 = \{(i, j) : 1 \leq i < j + 2 \leq 10, i, j \in \{1, 2, \ldots, 10\}\}$,

$s_3 = \{(i, j, k, l) : 1 \leq i < j < k < l, i, j, k, l \in \{1, 2, \ldots, 10\}\}$ and

$s_4 = \{(i, j, k, l) : i, j, k$ and l are distinct elements in $\{1, 2, \ldots, 10\}\}$. If the total number of elements in the set s_r is $n_r, r = 1, 2, 3, 4,$ then which of the following statements is (are) TRUE ?

A. n_1 = 1000

B. n_2 = 44

C. n_3 = 220

D. $\frac{n_4}{12}$ = 420

Answer:

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21. Consider a triangle PQR having sides of lengths p,q,r opposite to the angles P,Q,R respectively. Then which of the following statements is (are) true?

A. $\cos P \geq 1 - \frac{p^2}{2qr}$

B. $\cos R \geq \left(\frac{q-r}{p+q}\right)\cos P + \left(\frac{p-r}{p+q}\right)\cos Q$

C. $\frac{q+r}{p} < 2\frac{\sqrt{\sin Q \sin R}}{\sin P}$

D. if $p < q$ and $p < r$, then $\cos Q > \frac{p}{r}$ and $\cos R > \frac{p}{q}$

Answer:

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22. Let $f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R$ be a continuous function such that $f(0) = 1$ and $\int_0^{\frac{\pi}{3}} f(t)dt = 0$

Then which of the following statement is(are) TRUE?

A. The equation $f(x) - 3\cos 3x = 0$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

B. The equation $f(x) - 3\cos 3x = -\frac{6}{\pi}$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

C. $\lim_{x \to 0} x \frac{\int_0^x f(t)dt}{1 - e^{x^2}} = -1$

D. $\lim_{x \to 0} \frac{\sin x \left(\int_0^x f(t)dt\right)}{x^2} = -1$

Answer:

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23. For any real number α and β, let $y_{\alpha,\beta}(x) : \alpha, \beta \in R\}$ Then which of the following functions belong(s) to the set S ?

A. $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$

B. $f(x) = -\frac{x^2}{2}e^{-x} + \left(e + \frac{1}{2}\right)e^{-x}$

C. $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{4}\right)e^{-x}$

D. $f(x)=\frac{e^x}{2}\left(\frac{1}{2}-x\right)+\left(e+\frac{e^2}{4}\right)e^{-x}$

Answer:

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24. Let O be the origin and $\overrightarrow{OA}=2\hat{i}+2\hat{j}+\hat{k}, \overrightarrow{OB}=\hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{OC}=\frac{1}{2}\left(\overrightarrow{OB}-\lambda\overrightarrow{OA}\right)$ for same $\lambda>0. If\left|\overrightarrow{OB}\times\overrightarrow{OC}\right|=\frac{9}{2}$, then which of the following is(are) TRUE ?

A. Projection of $\overrightarrow{OC} on \overrightarrow{OA}$ is $-\frac{3}{2}$

B. Area of triangle OAB is $\frac{9}{2}$

C. Area of triangle ABC is $\frac{9}{2}$

D. The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{t}(OA)$ and $\overrightarrow{OC} is \frac{\pi}{3}$

Answer:

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25. Let E denote the parabola $y^2 = 8x$. Let $P = (-2, 4)$, and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is (are) TRUE ?

A. The triangle PFQ is a right-angled triangle

B. The triangle QPQ' is a right-angled triangle

C. The distance between P and F is $5\sqrt{2}$

D. F lies on the line joining Q and Q`

Answer:

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26. Consider the region $R = \{(x, y) \in R \times R : x \geq 0 \text{ and } y^2 \leq (4 - x)\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

The radius of the circle C is ___

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27. Consider the region $R = \{(x, y) \in R \times R : x \geq 0 \text{ and } y^2 \leq (4 - x)\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

The value of α is ___ .

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28. Let $f_1 : (0, \infty) \to R$ and $f_2 : (0, \infty) \to R$ be defined by $f_1(x) = \int_0^x \prod_{j=1}^{21} \left((t - j)^j\right) dt, x > 0$ and

$f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450, x > 0,$ where, for any positive integer n and real numbers $a_1, a_2, \ldots, a_n, \prod_{i=1}^{n}(a_i)$ denotes the product of $a_1, a_2, \ldots, a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function $f_i, i = 1, 2,$ in the interval $(0, \infty)$

The value of $2m_1 + 3n_1 + m_1n_1$ is ___.

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29. Let $f_1 : (0, \infty) \to R$ and $f_2 : (0, \infty) \to R$ be defined by $f_1(x) = \int_0^x \prod_{j=1}^{21}\left((t-j)^j\right)dt, x > 0$ and $f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450, x > 0,$ where, for any positive integer n and real numbers $a_1, a_2, \ldots, a_n, \prod_{i=1}^{n}(a_i)$ denotes the product of $a_1, a_2, \ldots, a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function $f_i, i = 1, 2,$ in the interval $(0, \infty)$

The value of $6m_2 + 4n_2 + 8m_2n_2$ is ___.

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30. Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R, i = 1, 2,$ and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R$ be function such that $g_1(x) = 1, g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$ for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$ Define

$$S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x).\, g_i(x) dx, i = 1, 2$$

The value of $\frac{16S_1}{\pi}$ is

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31. Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R, i = 1, 2,$ and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R$ be function such that $g_1(x) = 1, g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$ for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$ Define

$$S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x).\, g_i(x) dx, i = 1, 2$$

The value of $\frac{48S_2}{\pi}$ is

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32. Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circle C_n that are inside M. Let l be the maximum possible number of circle amaong these k circles such that no two circle intersect .Then

A. $k + 2l = 22$

B. $2k + l = 26$

C. $2k + 3l = 34$

D. $3k + 2l = 40$

Answer:

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33. Consider M with $r = \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$. The number of all those circle D_n that are inside M is

A. 198

B. 199

C. 200

D. 201

Answer:

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34. Let

$\psi_1 : [0, \infty] \to R, \psi_2 : [0, \infty) \to R, f : [0, \infty) \to R$ and $g : [0, \infty) \to R$

be functions such that $f(0) = g(0) = 0$

$\psi_1(x) = e^{-x} + x, x \geq 0$,

$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$

$f(x) = \int_{-x}^{x} (|t| - t^2) e^{-t^2} dt, x > 0$

$g(x) = \int_0^{x^2} (\sqrt{t}) e^{-t} dt, x > o$

Which of the following statements is TRUE

A. $f(\sqrt{\ln 3}) + g(\sqrt{\ln 3}) = \frac{1}{3}$

B. For every $x > 1$ there exists an $\alpha \in (1, x)$ such that

$\psi_1(x) = 1 + ax$

C. For every $x > 0$ there exists $\alpha\beta \in (0, x)$ such that

$\psi_2(x) = 2x(\psi_1(\beta) - 1)$

D. f is an increasing function on the interval $\left[0, \frac{3}{2}\right]$

Answer:

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35. Let

$\psi_1 : [0, \infty] \to R, \psi_2 : [0, \infty) \to R, f : [0, \infty) \to R$ and $g : [0, \infty) \to R$

be functions such that $f(0) = g(0) = 0$

$\psi_1(x) = e^{-x} + x, x \geq 0$,

$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$

$f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2} dt, x > 0$

$g(x) = \int_0^{x^2} (\sqrt{t})e^{-t} dt, x > o$

Which of the following statements is TRUE

A. $\psi_1(x) \leq 1$ for all $x > 1$

B. $\psi_2(x) \leq 0$ for all $x > 1$

C. $f(x) \geq 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all $x \in \left(0, \frac{1}{2}\right)$

D. $g(x) \leq \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

Answer:

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36. A number is chosen at random from the set $\{1, 2, 3, \ldots 2000\}$ Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7 Then the value of $500p$ is

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37. Let E be the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. For For any three distinct points P, Q and Q' on E, let $M(P, Q)$ be the mid-point of the line segment joining P and Q, and $M(P, Q')$ be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between $M(P, Q)$ and $M(P, Q')$, as P, Q and Q' vary on E, is ___ .

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38. For any real number x, let $[x]$ denote the largest integer less than or equal to x. If

$\int_0^{10}\left[\sqrt{\frac{10x}{x+1}}\right]dx$ then the value of $9I$ is

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Mathematics Section 1

1. Let α and β be real numbers such that $-\frac{\pi}{4}<\beta<0<\alpha<\frac{\pi}{4}$ .If $\sin(\alpha+\beta)=\frac{1}{3}$ and $\cos(\alpha-\beta)=\frac{2}{3}$, then the greatest integer less than or equal to

$\left(\frac{\sin\alpha}{\cos\beta}+\frac{\cos\beta}{\sin\alpha}+\frac{\cos\alpha}{\sin\beta}+\frac{\sin\beta}{\cos\alpha}\right)^2$ is ________.

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2. If y (x) is the solution of the differential equation

$xdv - (y^2 - 4y)dx = 0$ for $x > 0$, $y(1) = 2$ and the slope of the curve y = y (x) is never zero , then the value of $10y(\sqrt{2})$ is _______.

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3. The greatest integer less than or equal to

$\int_1^2 \log_2(x^3 + 1)dx + \int_1^{\log_2 9} (2^x - 1)^{\frac{1}{3}} dx$ is ________.

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4. The product of all positive real values of x satisfies the equation

$x^{\left(16(\log_5 x)^3 - 68\log_5 x\right)} = 5^{-16}$ is ________.

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5. If $\beta = \lim_{x \to 0} \frac{e^{x^3} - (1 - x^3)^{\frac{1}{3}} + (1 - x^2)^{\frac{1}{2}} - 1 \sin x}{x \sin^2 x}$ then the value of 6β is

________.

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6. Let β be a real number .Consider the matrix

$$A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix}$$

If $A^7 - (\beta - 1)A^6 - \beta A^5$ is a singular , then the value of 9β is ______.

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7. Consider the hyperbola

$\frac{x^2}{100} - \frac{y^2}{64} = 1$ with foci at S and S_1 ,where S lies on the positive x - axis ,Let P be a point on the hyperbola , in the first quadrant ,let $\angle SPS_1 = \alpha$, with $\alpha < \frac{\pi}{2}$ .The straight line passing through the point S and having the same slope as that of the tangent at P to the hyperbola , intersects the straight line S_1P at P_1 .Let δ be the distance of P from the straight line SP_1 and $\beta = S_1P$ and $\beta = S_1P$. Then the greatest integer less than or equal to $\frac{\beta\delta}{9}\frac{\sin\alpha}{2}$ is _______ .

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8. Consider the functions f , g : $\mathbb{R} \to \mathbb{R}$defined by $f(x) = x^2 + \frac{5}{12}$ and $g(x) = \begin{cases} 2\left(1 - \frac{4|x|}{3}\right) & |x| \leq \frac{3}{4} \\ 0 & |x| > \frac{3}{4} \end{cases}$ If α is the area of the region

$\left\{\left(x, y \in \mathbb{R} \times \mathbb{R} : |x| \leq \frac{3}{4}, 0 \leq y \leq \min\ \{f(x), g(x)\}\right\}$, then the value of 9 α is ________.

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Mathematics Section 2

1. Let PQRS be a quadrilateral in a plane , where QR = 1, $\angle PQR = \angle QRS = 70^\circ, \angle = 15^\circ$ and $\angle PRS = 40^\circ$. If

$\angle RPS = \theta°$, $PQ = \alpha$ and $PS = \beta$, then the interval (s) that contain (s) the value of $4\alpha\beta\sin\theta°$ is/are .

A. $(0, \sqrt{2})$

B. $(1, 2)$

C. $(\sqrt{2}, 3)$

D. $(2, \sqrt{2}, 3\sqrt{2})$

Answer:

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2. Let $\alpha = \sum_{k=1}^{\infty} \sin^{2k}\left(\frac{\pi}{6}\right)$

Let $g: [0, 1] \to \mathbb{R}$ be the function defined by $g(x) = 2^{\alpha x} + 2^{\alpha(1-x)}$ Then, which of the following statements is/are TRUE ?

A. The minimum value of $g(x)$ is $2^{\frac{7}{6}}$

B. The maximum value of $g(x)$ is $1 + 2^{\frac{1}{3}}$

C. The function $g(x)$ attains its maximum at more than one point

D. The function $g(x)$ attains its minimum at more than one point

Answer:

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3. Let $\bar{z}$ denote the complex conjugate of a complex number ?. If ? is a non-zero complex number for which both real and imaginary parts of

$$(\bar{z})^2 + \frac{1}{z^2}$$

are integers, then which of the following is/are possible value(s) of |z| ?

A. $\left(\frac{43 + 3\sqrt{205}}{2}\right)^{\frac{1}{4}}$

B. $\left(\frac{7 + \sqrt{3}}{4}\right)^{\frac{1}{4}}$

C. $\left(\frac{9 + \sqrt{65}}{4}\right)^{\frac{1}{4}}$

D. $\left(\frac{7 + \sqrt{13}}{6}\right)^{\frac{1}{4}}$

Answer:

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4. Let G be a circle of radius $R > 0$. Let $G_1, G_2, \ldots.., G_n$ be n circles of equal radius $r > 0$. Suppose each of the n circles $G_1, G_2 \ldots ...G_n$ touches the circle G externally. Also, for I = 1,2,....,$n - 1$, the circle G_i touches G_{i+1} externally , and G_n touches G_1 externally .Then ,which of the following statements is/are TRUE ?

A. If $n = 4$, then $(\sqrt{2} - 1)r < R$

B. If n = 5, then $r < R$

C. If n = 8, then $(\sqrt{2} - 1)r < R$

D. If n = 12, then $\sqrt{2}(\sqrt{3} + 1)r > R$

Answer:

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5. Let $\hat{i}, \hat{j}$ and $\hat{k}$? be the unit vectors along the three positive coordinate axes. Let $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$,

$\vec{b} = \hat{i} + b_2\hat{j} + b_3\hat{k}, \qquad b_2 b_3 \in \mathbb{R}$

$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}, \qquad c_1, c_2, c_3 \in \mathbb{R}$ be three vectors such that

$b_2 b_3 > 0 \vec{a}.\vec{b} = 0$ and

$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3 - c_1 \\ 1 - c_2 \\ -1 - c_3 \end{pmatrix}$ Then, which of the following is/are TRUE ?

A. $\vec{a}.\vec{c} = 0$

B. $\vec{b}.\vec{c} = 0$

C. $\left|\vec{b}\right| > \sqrt{10}$

D. $\left|\vec{c}\right| \leq \sqrt{11}$

Answer:

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6. For $x \in \mathbb{R}$, let the function y(x) be the solution of the differential equation

$\frac{dy}{dx} + 12y = \cos\left(\frac{\pi}{12}x\right), y(0) = 0$, Then, which of the following statements is/are TRUE ?

A. $y(x)$ is an increasing function

B. $y(x)$ is a decreasing function

C. There exists a real number β such that the line $y = \beta$ intersects the curve $y = y(x)$ at infinitely many points

D. $y(x)$ is a periodic function

Answer:

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Mathematics Section 3

1. Consider 4 boxes, where each box contains 3 red balls and 2 blue balls. Assume that all 20 balls are distinct. In how many different ways can 10 balls be chosen from these 4 boxes so that from each box at least one red ball and one blue ball are chosen ?

A. 21816

B. 85536

C. 12096

D. 156816

Answer:

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2. If $M = \begin{pmatrix} \frac{5}{2} & \frac{3}{2} \\ -\frac{3}{2} & \frac{-1}{2} \end{pmatrix}$, then which of the following matrices is equal to M^{2022} ?

A. $\begin{pmatrix} 3034 & 3033 \\ -3033 & -3032 \end{pmatrix}$

B. $\begin{pmatrix} 3034 & -3033 \\ 3033 & -3032 \end{pmatrix}$

C. $\begin{pmatrix} 3033 & 3032 \\ -3032 & -3031 \end{pmatrix}$

D. $\begin{pmatrix} 3032 & 3031 \\ -3031 & -3030 \end{pmatrix}$

Answer:

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3. Suppose that

Box-I contains 8 red, 3 blue and 5 green balls,

Box-II contains 24 red, 9 blue and 15 green ball

Box-III contains 1 blue, 12 green and 3 yellow balls

Box-IV contains 10 green, 16 orange and 6 white balls

A ball is chosen randomly from Box-I, call this ball ?. If ? is red then a ball is chosen randomly from Box-II, if ? is blue then a ball is chosen randomly from Box-III, and if ? is green then a ball is chosen randomly from Box-IV. The conditional probability of the event 'one of the chosen balls is white' given that the event ‘at least one of the chosen balls is green’ has happened, is equal to

A. $\frac{15}{256}$

B. $\frac{3}{16}$

C. $\frac{5}{12}$

D. $\frac{1}{8}$

Answer:

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4. For positive integer n, define

$$f(n) = n + \frac{16+5n-3n^2}{4n+3n^2} + \frac{32+n-3n^2}{8n+3n^2} + \frac{48-3n-3n^2}{12n+3n^2} + \ldots\ldots + \cdot$$

,Then the value of $\lim_{n\to\infty} f(n)$ is equal to

A. $3 + \frac{4}{3}\log_e 7$

B. $4 - \frac{3}{4}\log_e\left(\frac{7}{3}\right)$

C. $4 - \frac{4}{3}\log_e\left(\frac{7}{3}\right)$

D. $3+\frac{3}{4}\log_e 7$

Answer:

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Mathematics Section 1

1. Considering only the principal values of the inverse trigonometric functions, the value of

$$\frac{3}{2}\cos^{-1}\sqrt{\frac{2}{2+\pi^2}}+\frac{1}{4}\sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}+\tan^{-1}\frac{\sqrt{2}}{\pi}$$ is ________.

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2. Let α be a positive real number , Let f : $\mathbb{R}\to\mathbb{R}$ and $g:(\alpha,\infty)\to\mathbb{R}$ be the functions defined by

$f(x)=\sin\left(\frac{\pi x}{12}\right)$ and g (x) = $\frac{2\log_e\left(\sqrt{x}-\sqrt{\alpha}\right)}{\log_e\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}$

Then the value of $\lim_{x\to\alpha^+}$ f (g (x)) is ______.

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3. In a study about a pandemic, data of 900 persons was collected. It was found that

190 persons had symptom of fever,

220 persons had symptom of cough,

220 persons had symptom of breathing problem,

330 persons had symptom of fever or cough or both,

350 persons had symptom of cough or breathing problem or both,

340 persons had symptom of fever or breathing problem or both,

30 persons had all three symptoms (fever, cough and breathing problem).

If a person is chosen randomly from these 900 persons, then the probability that the person has at most one symptom is ________.

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4. Let ? be a complex number with non-zero imaginary part. If

$$\frac{2+3z+4z^2}{2-3z+4z^2}$$

is a real number , then the value of $|z|^2$ is ________

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5. Let ?$\bar{z}$ denote the complex conjugate of a complex number z? and let i= $\sqrt{-1}$. In the set of complex numbers, the number of distinct roots of the equation

$\bar{z} - z^2 = i(\bar{z} + z^2)$ is ______.

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6. Let $l_1, l_2, \ldots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1 and let $w_1, w_2, \ldots w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2 , where $d_1 d_2 = 10$. for each i=1,2 ,.... 100 , let R_i be a rectangle with length l_i and w_i and area A_i . if $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is ______.

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7. The number of 4-digit integers in the closed interval [2022,4482] formed by using the digits 0,2,3,4,6,7 is ______.

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8. Let ABC be the triangle with AB = 1 , AC = 3 and $\angle BAC = \frac{\pi}{2}$. If a circle of radius r > 0 touches the sides AB , AC and also touches internally the circumcircle of the triangle ABC , then the value of r is _______

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Mathematics Section 2

1. Consider the equation

$$\int_1^e \frac{(\log_e x)^{\frac{1}{2}}}{x\left(a - (\log_e x)^{\frac{3}{2}}\right)^2} dx = 1 \text{ a in } (-\infty, 0) \cup (1, \infty)$$

Which of the following statements is/are TRUE ?

A. No a satisfies the above equation

B. An integer a satisfies the above equation

C. An irrational number a satisfies the above equation

D. More than one a satisfy the above equation

Answer:

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2. Let a_1, a_2, a_3 ...be an arithmetic progression with $a_1 = 7$ and common difference 8 . Let $T_1, T_2, T_3. \dots,$ be such that $T_1 = 3$ and $T_{n+1} = a_n$ for $n \geq 1$. Then which of the following is/are TRUE ?

A. $T_{20} = 1604$

B. $\sum_{k=1}^{20} T_k = 10510$

C. $T_{30} = 3454$

D. $\sum_{k=1}^{30} = T_k = 35610$

Answer:

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3. Let P_1 and P_2 be two places given by

$P_1 : 10x + 15y + 12z - 60 = 0,$

$P_2 : -2x + 5y + 4z - 20$

Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2

A. $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{5}$

B. $\frac{x-6}{-5} = \frac{y}{2} = \frac{z}{3}$

C. $\frac{x}{-2} = \frac{y-4}{5} = \frac{z}{4}$

D. $\frac{x}{1} = \frac{y-4}{-2} = \frac{z}{3}$

Answer:

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4. Let ? be the reflection of a point ? with respect to the plane given by

$$\overrightarrow{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$$

where ?t, p? are real parameters and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of ?Q and S? are $10\hat{i} + 15\hat{j} + 20\hat{k}$ and $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ respectively, then which of the followi ng is/are TRUE ?

A. $3(\alpha + \beta) = -101$

B. $3(\beta + \gamma) = -71$

C. $3(\gamma + \alpha) = -86$

D. $3(\alpha + \beta + \gamma) = -121$

Answer:

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5. Consider the parabola $y^2 = 4x$. Let S be the focus of the parabola . A pair of tangents drawn to the parabola from the point P = (-2 , 1) meet the parabola at P_1 and P_2 . Let Q_1 and Q_2 be points on the lines SP_1 and SP_2 respectively such that PQ_1 is perpendicular to SP_1 and PQ_2 is perpendicular to SP_2 . then , which of the following is/are TRUE ?

A. $5Q_1 = 2$

B. $Q_1Q_2 = \frac{3\sqrt{10}}{5}$

C. $PQ_1 = 3$

D. $5Q_2 = 1$

Answer:

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6. Let |M| denote the determinant of a square matrix M . Let $g: \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$ be the function defined by

$$g(\theta) = \sqrt{f(\theta) - 1} + \sqrt{f\left(\frac{\pi}{2} - \theta\right) - 1}$$

where

List-I	List-II
(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has	(1) a unique solution
(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2) no solution
(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has	(3) infinitely many solutions
(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has	(4) $x = 11$, $y = -2$ and $z = 0$ as a solution
	(5) $x = -15$, $y = 4$ and $z = 0$ as a solution

Let p (x) be a quadratic polynomial whose roots are the maximum and minimum values of the function g (θ) and p (2) = $2 - \sqrt{2}$. then , which of the following is/are TRUE ?

A. $p\left(\frac{3+\sqrt{2}}{4}\right) < 0$

B. $p\left(\frac{1+3\sqrt{2}}{4}\right) > 0$

C. $p\left(\frac{5\sqrt{2}-1}{4}\right) > 0$

D. $p\left(\frac{5-\sqrt{2}}{4}\right) < 0$

Answer:

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1. Consider the following lists :

List-I	List-II
(I) $\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	(P) has two elements
(II) $\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	(Q) has three elements
(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	(R) has four elements
(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	(S) has five elements
	(T) has six elements

The correct option is :

A. $(I) \rightarrow (P), (II) \rightarrow (S), (III) \rightarrow (P), (IV) \rightarrow (S)$

B. $(I) \rightarrow (P), (II) \rightarrow (P), (III) \rightarrow (T), (IV) \rightarrow (R)$

C. $(I) \rightarrow (Q), (II) \rightarrow (P), (III) \rightarrow (T), (IV) \rightarrow (S)$

D. $(I) \rightarrow (Q), (II) \rightarrow (S), (III) \rightarrow (P), (IV) \rightarrow (R)$

Answer:

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2. Two players P_1 and P_2 play a game against each other . In every round of the game , each player rolls a fair the once , where the six faces of the die have six distinct numbers . Let x and y denote the readings on the die rolled by P_1 and P_2 respectively . if $x > y$, then P_1 scores 5 points and P_2 scores 0 point . if x = y , then each player scores 2 points . if $x < y$, then P_1 scores 0 point and P_2 scores 5 points . Let X_i and Y_i be the total scores of P_1 and P_2 respectively , after playing the i^{th} round .

List-I	List-II
(I) Probability of $(X_2 \geq Y_2)$ is	(P) $\frac{3}{8}$
(II) Probability of $(X_2 > Y_2)$ is	(Q) $\frac{11}{16}$
(III) Probability of $(X_3 = Y_3)$ is	(R) $\frac{5}{16}$
(IV) Probability of $(X_3 > Y_3)$ is	(S) $\frac{355}{864}$
	(T) $\frac{77}{432}$

The correct option is :

A. $(I) \to (Q), (II) \to (R), (III) \to (T), (IV) \to (S)$

B. $(I) \to (Q), (II) \to (R), (III) \to (T), (IV) \to (T)$

C. $(I) \to (P), (II) \to (R), (III) \to (Q), (IV) \to (S)$

D. $(I) \to (P), (II) \to (R), (III) \to (Q), (IV) \to (T)$

Answer:

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3. Let p , q , r be nonzero real numbers that are respectively , the $10^{th}, 100^{th}$ and 1000^{th} terms of a harmonic progression . Consider the system of linear equations

$x + y + z = 1$

$10x + 100y + 1000z = 0$

$qrx + pry + pqz = 0$

List-I	List-II
(I) If $\frac{q}{r} = 10$, then the system of linear equations has	(P) $x = 0$, $y = \frac{10}{9}$, $z = -\frac{1}{9}$ as a solution
(II) If $\frac{p}{r} \neq 100$, then the system of linear equations has	(Q) $x = \frac{10}{9}$, $y = -\frac{1}{9}$, $z = 0$ as a solution
(III) If $\frac{p}{q} \neq 10$, then the system of linear equations has	(R) infinitely many solutions
(IV) If $\frac{p}{q} = 10$, then the system of linear equations has	(S) no solution
	(T) at least one solution

The correct option is :

A. $(I) \rightarrow (T), (II) \rightarrow (R), (III) \rightarrow (S), (IV) \rightarrow (T)$

B. $(I) \rightarrow (Q), (II) \rightarrow (S), (III) \rightarrow (S), (IV) \rightarrow (R)$

C. $(I) \rightarrow (Q), (II) \rightarrow (R), (III) \rightarrow (P), (IV) \rightarrow (R)$

D. $(I) \rightarrow (T), (II) \rightarrow (S), (III) \rightarrow (P), (IV) \rightarrow (T)$

Answer:

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4. Consider the ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Let H$(\alpha, 0), 0 < \alpha < 2,$ be a point . A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively , in the first quadrant . The tangent to the ellipse at the point E intersects the positive x-axis at a point G . Suppose the straight line joining F and the origin makes an angle ϕ with the positive x-axis .

List-I	List-II
(I) If $\phi = \frac{\pi}{4}$, then the area of the triangle FGH is	(P) $\frac{(\sqrt{3}-1)^4}{8}$
(II) If $\phi = \frac{\pi}{3}$, then the area of the triangle FGH is	(Q) 1
(III) If $\phi = \frac{\pi}{6}$, then the area of the triangle FGH is	(R) $\frac{3}{4}$
(IV) If $\phi = \frac{\pi}{12}$, then the area of the triangle FGH is	(S) $\frac{1}{2\sqrt{3}}$
	(T) $\frac{3\sqrt{3}}{2}$

The correct option is :

A. $(I) \rightarrow (R), (II) \rightarrow (S), (III) \rightarrow (Q), (IV) \rightarrow (P)$

B. $(I) \rightarrow (R), (II) \rightarrow (T), (III) \rightarrow (S), (IV) \rightarrow (P)$

C. $(I) \rightarrow (Q), (II) \rightarrow (T), (III) \rightarrow (S), (IV) \rightarrow (P)$

D. $(I) \rightarrow (Q), (II) \rightarrow (S), (III) \rightarrow (Q), (IV) \rightarrow (P)$

Answer:

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