



MATHS

BOOKS - NAGEEN MATHS (HINGLISH)

Continuity and Differentiability

Solved Example

1. Show that the function $f(x) = x^2 + 3x + 5$, is continuous at $x=1$.

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2. If $f(x) = \begin{cases} x^2 & x \neq 0, \\ 4 & x = 0 \end{cases}$ then find whether $f(x)$ is continuous at $x = 0$

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3. Show that $f(x) = |x|$ is continuous at $x=0$

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4. Discuss the continuity of $f(x) = |x| + |x - 1|$ at $x=0$ and $x=1$.

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5. Show that $f(x)=\cos x$ is continuous for all values of x .

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6. If $f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x - 2)^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$

$f(x)$ is continuous for all real values of x , find the value of k .

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7. The function $f(x)$ is defined as follows : $\therefore f(x) = \begin{cases} 4x + a, & x < 1 \\ 6, & x = 1 \\ 3x - b, & x > 1 \end{cases}$

If $f(x)$ is continuous at $x=1$, find the value of a and b .

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8. A function $f(x)$ is defined as follows : $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Discuss its continuity at $x=0$

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9. Discuss of the continuity of the function $f(x) = \begin{cases} \frac{\sin x}{2}, & x < 0 \\ x + 2, & x \geq 0 \end{cases}$

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10. Prove that tangent function is continuous in its domain.

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11. Discuss the continuity of the function $f(x) = \begin{cases} 2x - 1, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases}$

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12. Discuss the differentiability of $f(x) = x^3$ at $x = 1$

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13. Show that the function $f(x) = \frac{x^3}{2}$ is not differentiable at $x=0$.

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14. Show that the function

$f(x) = \begin{cases} 1 + x, & x \leq 2, \\ 5 - x, & x > 2 \end{cases}$ is not differentiable at $x=2$

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15. Show that the function $f(x)=|x-2|$ is continuous but not differentiable at $x=2$.



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16. Differentiate $(px + q)^3$ with respect to x



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17. Differentiate $\tan px$ with respect to x



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18. Differentiate $\log \sin \frac{x}{3}$ with respect to 'x'



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19. Find the derivative of $\sin^2 x$ with respect to x

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20. Differentiate $\log(\sin x)$ with respect to x

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21. Find the derivative of $\tan x$ with respect to ' x '

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22. Differentiate $\log(\sec x + \tan x)$ with respect to x :

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23. Find the derivative of $\cos(\tan x^3)$

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24. Differentiate $\frac{1}{x\sqrt{1+x^2}}$ with respect to 'x'.

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25. If $y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}$ prove that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$.

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26. Differentiate $\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$

with respect to 'x'

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27. Find the derivative of $\cos^{-1} 2x$ with respect to 'x'.

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28. Find the derivative to $\sec(\tan^{-1} x)$ with respect to 'x'

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29. Find the derivative of $x \tan^{-1} x$ with respect to 'x'

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30. Find the derivative of $\cos^{-1}(\cot x)$ with respect to 'x'

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31. if $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ then prove that $(1-x)^2 \cdot \frac{d}{dx} = xy + 1$

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32. Find the derivative of $\tan^{-1}\left(\frac{1 - \cos x}{\sin x}\right)$ with respect to 'x'

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33. Find the derivative of $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ with respect to 'x'

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34. Differentiate $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$ with respect to 'x'

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35. If $y = \cot^{-1}\left(\frac{1 + \sqrt{1+x^2}}{x}\right)$ then find $\frac{dy}{dx}$

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36. Differentiate $\sin(-1) \left[\frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right]$ with respect to 'x'.

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37. If $y = \tan^{-1} \left[\frac{x}{1 + \sqrt{1-x^2}} \right]$ then find $\frac{dy}{dx}$

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38. If $y = \tan^{-1} \left(\frac{ax - b}{bx + a} \right)$ then prove that $d\frac{u}{dx} = \frac{1}{1+x^2}$.

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39. If $x^2 + y^2 = 25$ then find $\frac{dy}{dx}$.

A. $\frac{dy}{dx} = y$

B. $\frac{dy}{dx} = x$

$$C. \frac{dy}{dx} = -\frac{y}{x}$$

$$D. \frac{dy}{dx} = -\frac{x}{y}$$

Answer: D



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40. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$ differentiate both sides with respect to x

$$A. \frac{dy}{dx} = \frac{\sqrt{x}}{\sqrt{y}}$$

$$B. \frac{dy}{dx} = -\frac{\sqrt{x}}{\sqrt{y}}$$

$$C. \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$D. \frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x}}$$

Answer: C



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41. If $x^4 + y^4 - a^2xy = 0$ then find $\frac{dy}{dx}$

A. $\frac{dy}{dx} = \frac{a^2 - 4x^3}{4y^3 - a^2x}$

B. $\frac{dy}{dx} = \frac{a^2y - x^3}{4y^3 - a^2x}$

C. $\frac{dy}{dx} = \frac{a^2y - 4x^3}{y^3 - a^2x}$

D. $\frac{dy}{dx} = \frac{a^2y - 4x^3}{4y^3 - a^2x}$

Answer: D



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42. If $\cos(x+y) = y \sin x$ then find $\frac{dx}{dy}$



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43. If $y = x \sin y$ then prove that $x \frac{dy}{dx} = \frac{y}{1 - x \cos y}$



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44. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

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45. If $y = x^{\sin^{-1} x}$ then find $\frac{dy}{dx}$.

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46. Differentiate $x^{\sin x}$ with respect to 'x'.

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47. Find the derivative of $(1 + \cos x)^x$ with respect to x.

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48. Differentiate $x^x + (\sin x)^{\sin x}$ with respect to 'x'.

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49. Differentiate $\sqrt{\frac{1-x}{1+x}}$ with respect to x.

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50. Differentiate

$\sqrt{(x+1)(x+2)(x+3)}$ with respect to x.

A. $((x+1)(x+2)(x+3))$

$$\left(\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} \right).$$

B. $\frac{1}{4} \sqrt{(x+1)(x+2)(x+3)}$

$$\left(\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} \right).$$

C. $\frac{1}{\sqrt{(x+1)(x+2)(x+3)}}$

$$\left(\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} \right).$$

$$D. \frac{1}{2} \sqrt{(x+1)(x+2)(x+3)} \\ \left(\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} \right).$$

Answer: *D*

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51. If $x^a \cdot y^b = (x+y)^{a+b}$ then prove that $\frac{dy}{dx} = \frac{y}{x}$

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52. If $x^y + y^x = a^b$ then find $\frac{dy}{dx}$.

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53. If $y = (\sin x)^{\sin x^{\sin x \dots \infty}}$ then prove that $\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y \log(\sin x)}$

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54. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$ then prove that $(2y - 1) \frac{dy}{dx} = 1$.

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55. If $y = a^{x^{a^{x \dots \infty}}}$ then prove that $\frac{dy}{dx} = \frac{y^2 \log y}{x(1 - y \log x \log y)}$

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56.

$y = \sqrt{\sin x + \cos x + \sqrt{\sin x + \cos x + \sqrt{\sin x + \cos x + \dots \infty}}}$ then find

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57. If $x = at^2$ and $y = 2$ at then find $\frac{dy}{dx}$

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58. If $x = a(t + \sin t)$ and $y = a(1 - \cos t)$ then find $\frac{dy}{dx}$.

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59. If $x = a \cos^3 t$ and $y = a \sin^3 t$ then find $\frac{dy}{dx}$

- A. $\cot t$
- B. $-\tan t$
- C. $\tan t$
- D. $-\cot t$

Answer: B

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60. If $x = (a + b)\cos \theta - b \cos\left(\frac{a + b}{b}\right)\theta$ and $y = (a + b) \sin \theta - b \sin\left(\frac{a + b}{2b}\right)\theta$ then prove that

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61. If $x = \sin^{-1} \frac{2t}{1+t^2}$ and $y = \tan^{-1} \frac{2t}{1-t}$ then find $\frac{dy}{dx}$.

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62. Find the derivative of x^7 with respect to x^2 .

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63. Differentiate $\tan^{-1}(\tan x)$ with respect to $\sin x$.

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64. Differentiate $\tan^{-1} \left(\frac{2x}{1-x^2} \right)$ with respect to $\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$

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65. Find the 2nd derivative of $x^6 \cdot e^{6x}$ with respect to x .

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66. Find the 2nd derivative of $\sin(\cos x)$ with respect to x .

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67. If $y = \sec x + \tan x$ then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$.

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68. If $y = e^{ax} \sin bx$ then prove that $\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0$.

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69. If $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ then find $\frac{d^2y}{dx^2}$

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70. If $y = A \sin^2 x + B \cos^2 x$ then prove that

$$\sin 2x \cdot y_2 - 2 \cos 2x \cdot y_1 = 0$$

where $y_1 = \frac{dy}{dx}$, $y_2 = \frac{d^2y}{dx^2}$.

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71. Verify Rolles theorem for function $f(x) = x^2 - 4x + 3$ on $[1, 3]$

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72. Verify Rolle's theorem for the function $f(x) = x^2$ in the interval $[-1, 1]$.

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73. Verify Rolle's theorem for the function $f(x) = x^3 + 3x^2 - 24x - 80$ in the interval $[-4, 5]$.

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74. Verify Rolle's theorem for $f(x) = \sqrt{1 - x^2}$ in the interval $[-1, 1]$.

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75. Verify Rolle's theorem for the function $f(x) = \cos 2x$ in the interval $[0, \pi]$.

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76. Verify Rolle's theorem for the function $f(x) = \{\log(x^2 + 2) - \log 3\}$ in the interval $[-1, 1]$.

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77. Discuss the applicability the Rolle's theorem for the function $f(x) = x^2$ in the interval $[2,3]$.

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78. Discuss the applicability of the Rolle's theorem for the function $f(x) = |x|$ in the interval $[-1, 1]$

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79. Verify Lagrange's Mean value theorem for the function $f(x) = x^2 - 1$ in the interval $[3,5]$.

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80. $f(x) = \frac{1}{4x - 1}$ in $[1, 4]$

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81. Using Lagrange's Mean Value theorem , find the co-ordinates of a point on the curve $y = x^2$ at which the tangent drawn is parallel to the line joining the points (1,1) and (3,9).

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Exercies 5 A

1. Prove that the function $f(x) = 2x^2 - 3x + 2$ is continuous at at $x=1$.

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2. The fuction $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x - 3 & x < 2 \\ x - 1 & x \geq 2 \end{cases}$$

Prove that $f(x)$ is continuous x at =2.

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3. Discuss the continuity of the function $f(x) = \begin{cases} 1 + x^2 & x \leq 1 \\ 1 - x & x > 1 \end{cases}$ at $x=1$.

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4. If $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$ then show that $f(x)$ is

continuous at $x=3$.

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5. Discuss the continuity of the function

$f(x) = \begin{cases} x, & x \geq 0 \\ 2, & x < 0 \end{cases}$ at $x=0$

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6. The function $f(x)$ is defined in the interval $[0,1]$ as follows:

$$f(x) = \begin{cases} 0, & x = 0 \\ \frac{1}{2} - x, & 0 < x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ \frac{2}{3} - x, & \frac{1}{2} < x < 1 \\ 1, & x = 1 \end{cases}$$

Discuss the continuity of the function at $x = \frac{1}{2}$



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7. Discuss the continuity of the function

$$= f(x) = \begin{cases} (3-x), & x \leq 0 \\ x, & x > 0 \end{cases} \text{ at } x=0.$$



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8. Show that the function $f(x) = \begin{cases} x - 4, & x \leq 5 \\ 5x - 24, & x > 5 \end{cases}$ is continuous at $x=5$.



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9. Discuss the continuity of the following function at $x=1$ and 2

$$f(x) = \begin{cases} 5x - 4, & x \leq 1 \\ 4x^2 - 3x, & 1 < x < 2 \\ 3x + 4, & x \geq 2 \end{cases}$$



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10. If $f(x) = \begin{cases} x^2 + 1, & x \neq 1 \\ 3, & x = 1 \end{cases}$, then check whether the function $f(x)$ is continuous or discontinuous at $x=1$



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11. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \text{ at } x=0.$$



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12. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \text{ at } x=0$$

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13. (i) Discuss the continuity of the function $f(x) = \begin{cases} |x - a|, & x \neq a \\ 0, & x = a \end{cases}$ at $x = a$ (ii) Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{|x-3|}{x-3}, & x \neq 3 \\ 0, & x = 3 \end{cases} \text{ at } x = 3$$

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14. Show that $f(x) = \sin x$ is continuous for all values of x .

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15. Prove that $f(x) = \begin{cases} \frac{\sin x}{x}; & x \neq 0 \\ 1; & x = 0. \end{cases}$ is continuous at $x = 0$.

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16. If $f(x) = \begin{cases} \frac{\sin 3x}{\sin 5x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then discuss its continuity at $x=0$.

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17. Show that the function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, is discontinuous at $x=0$.

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18. Discuss the continuity of $f(x) = \begin{cases} \frac{\sin^2 2x}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, at $x=0$.

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19. Discuss the continuity of $f(x) = \begin{cases} \cos \frac{1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$,

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20. Discuss the continuity of $f(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$,

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21. Discuss the continuity of $f(x) = \begin{cases} x \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$,

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22. Discuss the continuity of $f(x) = \begin{cases} \frac{\sin^2 x}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$,

at $x=0$

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23. If the function $f(x) = \begin{cases} \frac{3x^3 - 2x^2 - 1}{x - 1}, & x \neq 1 \\ K, & x = 1 \end{cases}$,

is continuous at $x=1$, find the value of k .



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24. For what value of k , the function

$$f(x) = \begin{cases} Kx^2, & x \leq 2 \\ 5, & x > 2 \end{cases}$$

is continuous at $x=2$.



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25. For what value of k , the function

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ k, & x = 2 \end{cases}$$

is continuous at $x = 2$.



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26. For what value of k , the function

$$f(x) = \begin{cases} 2x + 1, & x > 2 \\ k, & x = 2, \\ 3x - 1, & x < 2 \end{cases}$$

is continuous at $x=2$.



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27. If the function $f(x)$ given by $f(x) = \begin{cases} 3ax + b & \text{if } x > 1 \\ 11 & \text{if } x = 1 \\ 5ax - 2b & \text{if } x < 1 \end{cases}$ is

continuous at $x = 1$ then find the values of a and b .



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28. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{1 + \cos x}{\tan^2 x}, & x \neq \pi \\ \frac{1}{2}, & x = \pi \end{cases},$$

at $x = \pi$.



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29. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x < 0 \\ x + 1, & x \geq 0 \end{cases},$$

at $x=0$.



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30. Prove that the function $f(x) = 2x - |x|$ is continuous at $x=0$.



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Exercis 5 B

1. Prove that the function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x < 0 \\ x + 3, & x \geq 0 \end{cases}$ is everywhere continuous .



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2. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x < 0 \\ 2x + 3, & x \geq 0 \end{cases}$$

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3. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$$

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4. Discuss the continuity of the function

$$f(x) = \begin{cases} 3x + 5, & x \geq 2 \\ 6x - 1, & x < 2 \end{cases}$$

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5. Prove that $\cot x$ is continuous in its domain

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6. Discuss the continuity of the function $f(x) = \begin{cases} 4x - 2, & x \leq 2 \\ 3x, & x > 2 \end{cases}$

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7. If the function $f(x) = \begin{cases} 5, & x \leq 2 \\ ax + b, & 2 < x \leq 10 \\ 21, & x > 10 \end{cases}$

continuous, find the values of a and b

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Exercises 5 C

1. Show that the function $f(x) = x^2$ is continuous and differentiable at $x=2$.

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2. Show that the function $f(x) = \begin{cases} 5 - x, & x \geq 2 \\ x + 1, & x < 2 \end{cases}$

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3. Show that the function $f(x) = \begin{cases} 1 - x, & x < 1 \\ x^2 - 1, & x \geq 1 \end{cases}$ is continuous but not differentiable at $x=1$

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4. Show that the function $f(x) = \begin{cases} 3 + x, & x \geq 0 \\ 3 - x, & x < 0 \end{cases}$ is not differentiable at $x=0$

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5. show that the function is always differentiable .

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6. Show that the function $f(x) = \begin{cases} x^2 + 2, & x \geq 1 \\ 2x + 1, & x < 1 \end{cases}$ is

always differentiable at $x=1$

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7. Show that the function $f(x) = |x - 3|$, $x \in \mathbb{R}$, is continuous but not differentiable at $x = 3$.

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8. Show that the function

$f(x) = \begin{cases} x \frac{\sin 1}{x}, & \text{when } x \neq 0 \\ \text{when } x = 0 \end{cases}$ is continuous but

not differentiable at $x = 0$.

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9. If $f(x) = \begin{cases} 1 + \sin x, & 0 \leq x < \pi/2 \\ 1, & x < 0 \end{cases}$ then show that $f'(0) = |x|$ does not exist.

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10. If $f(x) = |x|$ then show that $f'(3) = 1$

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11. If $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$ is differentiable for all values of x , then find the values of 'a' and 'b'.

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12. If $f(x) = \begin{cases} x^2, & x \leq 1 \\ x^2 - x + 1, & x > 1 \end{cases}$ then show that $f(x)$ is not differentiable at $x=1$.

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13. Prove that the function $f(x) = \begin{cases} \frac{x}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is not differentiable

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14. Prove that the function

if $f(x) = \begin{cases} 6x - 5, & x \leq 3 \\ 2x^2 - 6x + 13, & x > 3 \end{cases}$ is differentiable at $x=3$.

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Exercies 5 D

1. $\sin 5x$

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2. $\tan 3x$

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3. $\cos(x^4)$

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4. $\tan(x^2)$

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5. $\sin^4 x$

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6. $\cot^2 x$

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7. e^{5x}



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8. $e^{\frac{x}{a}}$



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9. e^{x^2+1}



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10. $\log(x^2 + 3)$



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11. \sin°



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12. $\cos x^\circ$



Watch Video Solution

13. $\log \cos x$



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14. $\log \tan x$



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15. $\sqrt{\sin x}$



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Watch Video Solution

16. $\sqrt{\sec x}$

 Watch Video Solution

17. $\tan \sqrt{x}$

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18. $\operatorname{cosec} \sqrt{x}$

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19. $e^{\sin x}$

 Watch Video Solution

20. $e^{\tan x}$

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21. cose^x

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22. $\tan(e^x + 5)$

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23. $\frac{1}{\sqrt{x+1} + \sqrt{x}}$

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24. $e^{mx} \cdot \cos nx$



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25. $e^{-2x} \sin 4x$

 Watch Video Solution

26. $\sqrt{ax^2 + bx + c}$

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27. $\sqrt{\log x}$

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28. $\frac{e^x + e^{-x}}{e^x - e^{-x}}$

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29. $\log\left(x + \frac{1}{x}\right)$



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30. $\log s\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$



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31. $\sin^3(ax + b)$



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32. $\log(\sin x^2)$



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33. $\log(\sec 2x + \tan 2x)$

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34. $\log \{\log (\cos x)\}$

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35. Differentiate $\log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$ with respect to x :

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36. Differentiate $\log \sqrt{\frac{1 + \sin x}{1 - \sin x}}$ with respect to x :

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37. $\frac{x}{\sqrt{1-x^2}}$

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38. $y = \frac{a^2 + x^2}{\sqrt{a^2 - x^2}}$ then $dy/dx =$

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39. $e^{(x^2) / (1+x^2)}$

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40. $\log(x + \sqrt{x^2 - 1})$

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41. $\log \sqrt{\frac{1+x^2}{1-x^2}}$

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42. $\tan^2\left(\frac{\pi x^2}{2}\right)$

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43. $\sqrt{\frac{1 - \tan x}{1 + \tan x}}$

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44. $e^x \cdot \log(\sin 2x)$

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45. If $y = \sqrt{\frac{1-x}{1+x}}$ then $\frac{dy}{dx}$ equals

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46. find the derivative of the function w.r.t $x \sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$

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47. $\sin\left(\frac{1 + x^2}{1 - x^2}\right)$

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48. If $x \in (\pi, 2\pi)$, prove that

$$\frac{(\sqrt{1 + \cos x}) + (\sqrt{1 - \cos x})}{(\sqrt{1 + \cos x}) - \sqrt{1 - \cos x}} = \cot\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

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49. If $y = \sin x \cdot \cos(2x)$ then prove that $\frac{dy}{dx} = y[\cot x - 2 \tan 2x]$

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50. If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$. Prove that $\frac{dy}{dx} = \frac{x-1}{2x(x+1)}$

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51. if $y = \sqrt{\frac{1 - \cos x}{2}}$, then prove that $\frac{dy}{dx} = \frac{1}{2} \cos \frac{x}{2}$.

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Exercises 5 E

1. $\sin^{-1} 3x$

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2. $\cot^{-1} \sqrt{x}$

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3. $\sin^{-1}(ax)$

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4. $\cos^{-1}\left(\frac{x}{a}\right)$

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5. (i) $\tan^{-1}\sqrt{x}$ (ii) $\tan^{-1}(2x + 1)$

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6. $\log(\sin^{-1} x)$

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7. $\cos ec^{-1}3x$



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8. $\sec^{-1}\left(\frac{x}{a}\right)$



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9. $\cos^{-1}\left(\frac{1-x}{1+x}\right)$



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10. $\sin^{-1}\frac{x}{1+x}$



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11. $\sin(\tan^{-1} 2x)$



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12. $\tan^{-1}(\cos \sqrt{x})$

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13. $2 \tan^{-1} x - \log(1 + x^2)$

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14. $\sqrt{\cot^{-1} \sqrt{x}}$

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15. $e^{ax} \cdot \sin^{-1} bx$

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16. $e^{ax} \cdot \sin^{-1} bx$



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17. (i) $b \tan^{-1} \left(\frac{x}{a} + \tan^{-1} \frac{x}{a} \right)$

(ii) $(\sin^{-1} x)^2 - (\cos^{-1} x)^2$



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18. $\cos \left(a \sin^{-1} \frac{1}{x} \right)$



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19. If $y = x \sin^{-1} x + \sqrt{1 - x^2}$ then prove that $\frac{dy}{dx} = \sin^{-1} x$.



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1. Differentiate $\tan^{-1}\left\{\sqrt{\frac{1+\cos x}{1-\cos x}}\right\}$, $x \neq 0$

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2. $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$

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3. $\cos^{-1}\left(\frac{\sqrt{1 + \cos x}}{2}\right)$

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4. $\tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$

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$$5. \tan^{-1} \left(\sqrt{\frac{1 - \cos 3x}{1 + \cos x}} \right)$$

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$$6. \sec^{-1} \left(\frac{1 + \tan^2 x}{1 - \tan^2 x} \right)$$

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$$7. \operatorname{cosec}^{-1} \left(\frac{1 + \tan^2 x}{2 \tan x} \right)$$

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$$8. \sin^{-1}(\cos x) + \tan^{-1}(\cot x)$$

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$$9. \sin^{-1}(1 - 2x^2)$$



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10. The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. \cos^{-1} is



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11. $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$



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12. Differentiate $\cos^{-1}(4x^3 - 3x)$ with respect to x , if $x \in \left(\frac{1}{2}, 1\right)$



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13. Differentiate $\sin^{-1}(3x - 4x^3)$ with respect to x , if $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$



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14. $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

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15. $\cos^{-1}\left(\frac{1-x^{2n}}{1+x^{2n}}\right)$

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16. $\cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

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17. Differentiate $\tan^{-1}\left(\frac{a+x}{1-ax}\right)$ with respect to x

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18. $\cos^{-1}\left(\frac{x - x^1}{x + x^{-1}}\right)$

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19. $\cot^{-1}\left(\sqrt{1 + x^2} - x\right)$

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20. Differentiate $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), -1/\sqrt{3}$

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21. $\tan^{-1}\left(\frac{3 - 5x}{1 + 15x}\right)$

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22. $\tan^{-1}\left(\frac{5x}{1 - 6x^2}\right)$



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$$23. \tan^{-1}\left(\frac{2x}{1-15x^2}\right)$$



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$$24. \text{Differentiate } \tan^{-1}\left(\frac{4\sqrt{x}}{1-4x}\right)$$



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$$25. \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$$



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$$26. \sin^{-1}\left(\frac{x-1}{x+1}\right) + \sec^{-1}\left(\frac{x+1}{x-1}\right)$$



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$$27. \tan^{-1} \left(\frac{\sqrt{1+x^2} - 1}{x} \right)$$

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$$28. \text{ If } y = \tan^{-1} \left\{ \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right\}, \quad -1$$

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29. Differentiate the following functions w.r.t.x

$$\sin \left[2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right]$$

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$$30. \tan^{-1} \sqrt{\frac{1+\sin x}{1-\sin x}}$$

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31. If $y = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)$ then prove

that $\frac{dy}{dx} = 2\sqrt{a^2 - x^2}$

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Exercies 5 G

1. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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2. If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$

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3. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

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4. If $x^{2/3} + y^{2/3} = a^{2/3}$, find $\frac{dy}{dx}$

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5. $5x^2 + 5y^2 - 7y + 3x = 2 = 0$

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6. $\sin(xy) + \frac{x}{y} = x^2 - y$

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7. $x^2 + y^2 = \log(xy)$

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8. $x \sin 2y = y \cos 2x$



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9. $y \sec x - y^2 \cos x + 2x = 0$



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10. $e^x \log y = \sin^{-1} x + \sin^{-1} y$



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11. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$, then prove that

$$\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$$



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12. If $e^x + e^y = e^{x+y}$ then prove that $\frac{dy}{dx} = -e^{y-x}$

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Exercies 5 H

1. $y = x^{\sin 2x}$

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2. $y = (\log x)^x$

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3. $y = (\log_e x)^{\sin x}$

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4. $y = x^{\tan x}$

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5. $y = (\sin x)^{\log x}$

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6. $y = (1 + x)^x$

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7. Find $\frac{dy}{dx}$ when $(\tan x)^y = (\tan y)^x$

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8. $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$



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$$9. y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$$



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$$10. y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$$



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$$11. y = x^{\log x} + (\log x)^x$$



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$$12. y = e^{\sin x} + (\tan x)^x$$



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$$13. (\cos x)^y = (\sin y)^x$$

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$$14. y = (\tan x)^{\log x} + (\cos x)^{\sin x}$$

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$$15. y = x^{\sin x} + a^{\sin x}$$

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$$16. (i) y = \frac{(x-a)(x-b)}{\sqrt{x-c}} \quad (ii) y = \sqrt{\frac{(x-a)(x-b)}{(x-c)(x-d)}}$$

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$$17. y = \sin x \cdots \in 2x \cdots \in 4x \cdots \in 8x$$



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$$18. y = \sqrt{\frac{x^2 + x + 1}{x^2 - x + 1}}$$



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$$19. y = \frac{(x + 1)^2 \cdot \sqrt{x - 1}}{(x + 3)^3 e^x}$$



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$$20. y = (x + 1)^2 (x + 2)^3 (x + 3)^4$$



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$$21. y = \tan x \tan 2x \tan 3x \tan 4x$$



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22. (i) $x^y \cdot y^x = 1$ (ii) $y = e^{x^x}$

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23. (i) $y = e^x \sin^3 x \cos^4 x$ (ii) $y = x \cdot e^{x \sin x}$

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24. If $y = \sqrt{x + y}$ then prove that $\frac{dy}{dx} = \frac{1}{2y - 1}$

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25. If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$

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26. If $x(x) + y^y = 1$ then find $\frac{dy}{dx}$.



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Exercis 5 I

1.

If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$ then prove that $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$



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2.

If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \infty}}}$ then prove that $\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$



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3.

If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots \infty}}}$ then prove that $\frac{dy}{dx} = \frac{\sec^2}{2y}$

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4.

If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \infty}}}$ then prove that $\frac{dy}{dx} = \frac{1}{x(2y)}$

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5. If $y = x^{x^{x \dots \infty}}$ then prove that $x \frac{dy}{dx} = \frac{y^2}{1 - y \log x}$

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6. If $y = (\cos x)^{(\cos x)^{(\cos x) \dots \infty}}$ then prove that $\frac{dy}{dx} = \frac{y^2 \tan x}{y \log(\cos x) - 1}$.

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7. If $y = e^{x+e^x+e^{e^x+\dots\infty}}$, prove that $\frac{dy}{dx} = \frac{y}{1-y}$

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8. If $y = x + \frac{1}{x + \frac{1}{x + \dots\infty}}$ then prove that $x \frac{dy}{dx} = y/(2y-x)$.

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9. $y = x^2 + \frac{1}{x^2 + \frac{1}{x^2 + \dots\infty}}$ then prove that $x \frac{dy}{dx} = \frac{2xy^2}{1+y^2}$.

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10. If $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}\dots\infty}}$ then prove that $\frac{dy}{dx} = \frac{y^2}{x(2-y \log x)}$.

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1. $x = a \cos t, y = b \sin t$

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2. $x = a \left(\cos t + \log \tan \left(\frac{t}{2} \right) \right), y = a \sin t$ then find $\frac{dy}{dx}$

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3. $x = a \tan \theta, y = b \sec \theta$

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4. $x = 2 \cos^2 t, y = 6 \sin^2 t$

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$$5. x = \sqrt{\sin 2t}, y = \sqrt{\cos 2t}$$



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$$6. x = a(t - \sin t), y = a(1 - \cos t)$$



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$$7. \text{ If } x = \frac{3at}{1+t^3}, y = \frac{3at^2}{1+t^3}, \text{ then } \frac{dy}{dx} \text{ is}$$



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$$8. x = 2 \cos t - \cos 2t, y = 2 \sin t - \sin 2t \text{ Find } \frac{dy}{dx}$$



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$$9. x = a \sin 2t(1 + \cos 2t), y = b \cos 2t(1 - \cos 2t) \text{ find } \frac{dy}{dx}$$



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10. Find $\frac{dy}{dx}$, when $x = \frac{\cos^{-1} 1}{\sqrt{1+t^2}}$ and $y = \frac{\sin^{-1} t}{\sqrt{1+t^2}}$, $t \in \mathbb{R}$



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11. $x = 3 \sin t - 2 \sin^3 t$, $y = 3 \cos t - 2 \cos^3 t$ then find $\frac{dy}{dx}$



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12. $x = e^t(\sin t + \cos t)$, $y = e^t(\sin t - \cos t)$



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13. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, $a \neq 0$



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14. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{dy}{dx}$.

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Exercises 5 K

1. Differentiate x^8 with respect to x^4

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2. Differentiate e^x with respect to \sqrt{x}

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3. Differentiate $x \sin^{-1} x$ with respect to $\cos^{-1} x$.

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4. Differentiate $(\log x)$ with respect to $\tan x$.

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5. Differentiate $\log x$ with respect to $\sin x$

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6. Differentiate $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to $\sin^{-1}x$.

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7. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}x, x \neq 0$.

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8. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, if $x \in (-1, 1)$

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9. Differentiate $\sec^{-1}\frac{1}{2x^2-1}$ with respect to $\sqrt{1-x^2}$

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10. Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}(2x^2-1)$.

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11. Differentiate $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ with respect to $\cos^{-1}x^2$

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12. Differentiate $\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\}$ with respect to $\cos^{-1} x^2$

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Exercis 5 L

1. Find the 2nd derivative if $x^3 \log x$ with respect to x .

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2. If $y = \tan^{-1} x^3$ then find $\frac{d^2y}{dx^2}$.

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3. Find the 2nd derivative of e^{ax+b} with respect to x .

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4. If $y = x + \cot x$ then prove that $\sin^2 x \frac{d^2 y}{dx^2} - 2y \cos x = 0$.

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5. If $y = \log(\sin x)$, prove that $\frac{d^3 y}{dx^3} = 2 \cos x \csc^3 x$.

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6. If $y = A \cos nx + B \sin nx$, show that $\frac{d^2 y}{dx^2} + n^2 y = 0$

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7. (i) If $y = a \sin(\log x)$ then prove that $x^2 \cdot \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$. (ii) If $y = a \cos(\log_e x) + b \sin(\log_e x)$, then prove that $x^2 \cdot y_2 + x \cdot y_1 + y = 0$.

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8. If $y = (\sin^{-1} x)^2$ then prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$.

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9. If $y = \sin(\sin x)$ then prove that

$$\frac{d^2y}{dx^2} + \tan x \cdot \frac{dy}{dx} + y \cos^2 x = 0$$

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10. IF $y = e^{\tan^{-1} x}$ then prove that :

$$(1 - x^2) \frac{d^2y}{dx^2} + (2x - 1) \frac{dy}{dx} = 0.$$

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11. If $y^3 - 3ax^2 + x^3 = 0$, then prove that $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0$

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12. If $y = (\tan^{-1} x^2)$, show that $(x^2 + 1)^2 \frac{d^2 y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2$.

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13. If $y = e^{\tan x}$ then prove that:

$$\cos^2 x \frac{d^2 y}{dx^2} - (1 + \sin 2x) \frac{dy}{dx} = 0$$

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14. If $y = Ae^{-kt} \cos(pt + c)$, then prove that $\frac{d^2 y}{dt^2} + 2k \frac{dy}{dt} + n^2 y = 0$,

where $n^2 = p^2 + k^2$

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15. If $x = at^2$, $y = 2$ at then find $\frac{d^2 y}{dx^2}$.

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16. If $x = a(t - \sin t)$, $y = a(1 - \cos t)$ then find $\frac{d^2y}{dx^2}$.

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17. If

$x = \sin t$ and $y = \sin mt$ then prove that $\sqrt{1-x^2} \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y$

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18. If $y = (\sin^{-1} x)^2 + (\cos^{-1} x)^2$, then prove that

$$(1-x^2)y_2 - xy_1 - 4 = 0.$$

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1. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = x^2 \text{ in the interval } [-2, 2].$$

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2. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = x^2 - 5x - 6 \text{ in the interval } [-1, 6].$$

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3. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = x^2 - 6x + 8 \text{ in the interval } [2, 4].$$

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4. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = x(x - 4)^2 \text{ in the interval } [0, 4].$$





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5. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = x^3 - 7x^2 + 16x - 12 \text{ in the interval } [2,3].$$



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6. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = (x - 2)(x - 4)(x - 6), \text{ in the interval } [2,6].$$



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7. Verify Rolle's theorem for the following functions in the given intervals.

(i) $f(x) = (x - 2)(x - 3)^2$ in the interval $[2,3]$.

(ii) $f(x) = x^3(x - 1)^2$ in the interval $[0,1]$.



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8. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = (x - 2)^4(x - 3)^3 \text{ in the interval } [2,3].$$

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9. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) \sin x \text{ in the interval } [0, \pi].$$

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10. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = \sin 3x \text{ in the interval } [0, \pi].$$

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11. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = \sin x + \cos x \text{ in the interval } [0, \pi/2].$$



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12. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = \sin^2 x \text{ in the interval } [0, \pi].$$



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13. Verify Rolle's theorem for the following functions in the given intervals.

$$f(x) = e^x \cos x \text{ in the interval } [-\pi/2, \pi/2].$$



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14. Discuss the applicability of the Rolle's theorem for the following functions in the given intervals.

(i) $f(x) = (x - 2)(2x - 1)$ in the interval $[1, 2]$.

(ii) $f(x) = \tan x$ in the interval $[0, \pi]$.

(iii) $f(x) = \sin \frac{1}{x}$ in the interval $[-2,2]$.

(iv) $f(x) = |x|$ in the interval $[-2,2]$.

(v) $f(x) = x^{1/3}$ in the interval $[-2,2]$.

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15. Verify Rolles theorem for the function $f(x) = \log \left\{ \frac{x^2 + ab}{x(a+b)} \right\}$ on $[a, b]$, where $a < b$

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16. Using Rolle's theorem , find a point on the curve $y = x^2, x \in [-1, 1]$ at which the tangent is parallel to X-axis.

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17. Using Rolle's theroem, find the point on the curve $y = x(x - 4), x \in [0, 4]$, where the tangent is parallel to X-axis.



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18. Verify Rolle's theorem for the function

$$f(x) = 2x^3 + x^2 - 4x - 2 \text{ in the interval } \left[-\frac{1}{2}, \sqrt{2} \right].$$



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19. It is given that for the function f given by $f(x) = x^3 + bx^2 + ax$, $x \in [1, 3]$. Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b .



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Exercies 5 N

1. Verify Lagrange's Mean Value theorem for the following functions in the given intervals

$f(x) = x^2 + x - 2$ in the interval $[0,4]$.

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2. Verify Lagrange's Mean Value theorem for the following functions in the given intervals

$f(x) = 1 + 2x - x^2$ in the interval $[0,1]$.

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3. Verify Lagrange's Mean Value theorem for the following functions in the given intervals

$f(x) = x(x + 2)^2$ in the interval $[0,2]$

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4. Verify Lagrange's Mean Value theorem for the following functions in the given intervals

$f(x) = (x - 2)(x - 3)(x - 4)$, in the interval $[2,5]$ /

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5. Verify Lagrange's Mean Value theorem for the following functions in the given intervals

$f(x) = e^x$ in the interval $[0,1]$.

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6. Verify Lagrange's Mean Value theorem for the following functions in the given intervals

(i) $f(x) = \sin x$ in the interval $[\pi/2, 5\pi/2]$.

(ii) $f(x) = \log_e x$ in the interval $[1,e]$.

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7. Verify Lagrange's Mean Value theorem for the following functions in the given intervals

$$f(x) = \sqrt{x^2 - 4} \text{ in the interval } [2,4]$$



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8. Verify Lagrange's Mean Value theorem for the following functions in the given intervals

$$f(x) = \tan^{-1} x \text{ in the interval } [0,1]$$



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9. Using Lagrange's theorem , find the value of c for the following functions :

(i) $x^3 - 3x^2 + 2x$ in the interval $[0,1/2]$.

(ii) $f(x) = 2x^2 - 10x + 1$ in the interval $[2,7]$.

(iii) $f(x) = (x-4)(x-6)$ in the interval $[4,10]$.

(iv) $f(x) = \sqrt{x - 1}$ in the interval $[1,3]$.

(v) $f(x) = 2x^2 + 3x + 4$ in the interval $[1,2]$.

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10. Using Lagrange's mean value theorem, find a point on the curve $y = \sqrt{x - 2}$ defined on the interval $[2,3]$, where the tangent is parallel to the chord joining the end points of the curve.

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11. Using Lagrange's Mean Value theorem, find the co-ordinates of a point on the curve $y = x^3$ at which the tangent drawn is parallel to the chord joining the points $(1,1)$ and $(3,27)$.

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1. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \text{ at } x=0$$

- A. $x > 0$
- B. $x < 0$
- C. 0
- D. None of these

Answer: C



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2. Prove that the function defined by $f(x) = \tan x$ is a continuous function.

- A. \mathbb{R}
- B. $\mathbb{R} - \{n\pi, n \in \mathbb{Z}\}$
- C. $\mathbb{R} - \left\{ \frac{n\pi}{2}, n \in \mathbb{Z} \right\}$

D. None of these

Answer: D

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3. The function $f(x) = \begin{cases} 2ax, & x \geq 3 \\ 3x + 1, & x < 3 \end{cases}$ continuous at $x=3$, then $a = ?$

A. $\frac{5}{3}$

B. $\frac{5}{2}$

C. 5

D. None of these

Answer: A

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4. the function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$ then find the value of k

A. 1

B. 2

C. 3

D. None of these

Answer: B



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5. The function $f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$

A. continuous at $x=1$

B. discontinuous at $x= 1$

C. continuous at $x= 2$

D. None of these

Answer: A



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6. The function $f(x) = 2x - |x|$

A. discontinuous at $x=0$

B. continuous at $x=0$

C. discontinuous at $x=1$

D. None of these

Answer: B



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7. The value of 'k' for which $f(x) = \begin{cases} kx^2, & x \geq 2 \\ 12, & x < 2 \end{cases}$

continuous at $x=2$ is :

A. 1

B. 2

C. 3

D. 4

Answer: C



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8. The value of k for which $f(x) = \begin{cases} \frac{1 - \cos 2x}{x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$

continuous at $x=0$, is :

A. 1

B. 2

C. 3

D. 4

Answer: B



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9. If the function $f(x) = \begin{cases} 3ax + b, & x > 1 \\ 11, & x = 1 \\ 5ax - 2b, & x < 1 \end{cases}$

continuous at $x=1$ then $(a, b) = ?$

A. (3,2)

B. (2,3)

C. (1,4)

D. (4,1)

Answer: A



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10. The value of 'a' for which $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

is continuous at $x=0$, is

A. ± 1

B. ± 2

C. 0

D. ± 3

Answer: A



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11. If $y = \sin^{-1} \frac{1}{\sqrt{1+x^2}}$ then $\frac{dy}{dx}$ at $x = 0$ is :

A. 1

B. 3

C. -1

D. None of these

Answer: C

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12. If $y = \frac{x}{x+5}$ then $x \frac{dy}{dx} = ?$

A. $y(1-y)$

B. $y(1-y)$

C. $(1-y)$

D. $(1+y)$

Answer: A

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13. If $x^y = e^{x-y}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{\log x}$

B. $\frac{1}{(\log ex)^2}$

C. $\frac{\log x}{(\log ex)^2}$

D. $\frac{1}{(\log ex)^2}$

Answer: C



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14. If $y = \tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{\sqrt{a^2 - x^2}}$

B. $\frac{1}{\sqrt{x^2 - a^2}}$

C. $\frac{1}{\sqrt{a^2 + x^2}}$

D. $\frac{1}{\sqrt{1 + x^2}}$

Answer: A



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15. If $y = \tan^{-1}\left(\frac{1-x}{1+x}\right) + \cot^{-1}\left(\frac{1-x}{1+x}\right)$ then $\frac{dy}{dx}$?

A. -1

B. 1

C. 0

D. None of these

Answer: C



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16. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx} = -\frac{2}{1+x^2}$ (b) $\frac{2}{1+x^2}$ (c) $\frac{1}{2-x^2}$ (d) $\frac{2}{2-x^2}$

A. $\frac{2}{1+x^2}$

B. $\frac{-2}{1+x^2}$

C. $\frac{-1}{1+x^2}$

D. None of these

Answer: B

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17. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \infty}}}$, then $\frac{dy}{dx}$ is $\frac{x}{2y-1}$ (b)

$\frac{x}{2y+1}$ $\frac{1}{x(2y-1)}$ (d) $\frac{1}{x(1-2y)}$

A. $\frac{x}{2x-1}$

B. $\frac{y}{2x-1}$

C. $\frac{1}{x}(2y-1)$

D. $\frac{1}{y(2x-1)}$

Answer: C

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18. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, if $x \neq \pm 1$

A. 1

B. 2

C. -1

D. 2

Answer: A



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19. If $f(x) = x^2 + 7x + 10$ then $f'(2) = ?$

A. -4

B. $-\frac{5}{2}$

C. -11

D. 11

Answer: D



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20. At which point the slope to tangent is zero for the curvey

$$y = x^2 - 6x + 8?$$

A. (3,1)

B. (3,-1)

C. (-3 ,1)

D. (-3 ,-1)

Answer: B



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$$1. f(x) = \begin{cases} \frac{|x^2 - x|}{x^2 - x} & x \neq 0, 1 \\ 1, & x = 0 \\ -1, & x = 1 \end{cases} \text{ is continuous for all :}$$

A. x

B. x except at $x = 0$

C. x except at $x = 1$

D. x except at $x = 0$ and $x = 1$

Answer: D



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$$2. \text{ Let } f(x) = \begin{cases} \frac{x - 4}{|x - 4|} + a, & x < 4 \\ \frac{x - 4}{|x - 4|} + b, & x > 4 \end{cases} \text{ Then } f(x)$$

is continuous at $x = 4$ when a. $a = 0, b = 0$ b. $a = 1, b = 1$ c.

$a = -1, b = 1$ d. $a = -1, b = -1$

A. (0, 0)

B. (1, 1)

C. (-1, 1)

D. (1, -1)

Answer: D



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3. The points of discontinuity of the function

$$f(x) = \begin{cases} 3x + 1, & 0 \leq x < 2 \\ 4x - 1, & 2 < x \leq 6 \\ 5x + 2, & 6 < x \leq 10 \end{cases} \text{ are:}$$

A. 6

B. 2

C. 6, 2

D. 6, 2, 0

Answer: A



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4. If $f(x) = x \frac{\sin(1)}{2}$, $x \neq 0$ then the value of the ltbRgt function at $x = 0$

so that the function is continuous at $x = 0$, is :

A. 1

B. 0

C. -1

D. None of these

Answer: B



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5. The value of $f(0)$, so that the function

$f(x) = \frac{(27 - 2x)^2 - 3}{9 - 3(243 + 5x)^{1/5} - 2}$ ($x \neq 0$) is continuous, is given $\frac{2}{3}$ (b) 6

(c) 2 (d) 4

A. 2

B. 4

C. 6

D. None of these

Answer: A



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6. If $f(x) = |x| + |x - 1|$, then :

A. $f(x)$ is continuous at $x = 0$ and at $x = 1$

B. $f(x)$ is continuous at $x = -0$ and discontinuous at a $x = 1$

C. $f(x)$ is discontinuous at $x = 0$ and continuous at $x = 1$

D. None of these

Answer: A



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$$7. \text{ If } f(x) = \begin{cases} \frac{\sqrt{1+\lambda x} - \sqrt{1-\lambda x}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{x-2}, & -0 \leq x < 1 \end{cases} \text{ is}$$

continuous at $x = 0$ then $\lambda = ?$

A. -1

B. 1

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

Answer: C



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$$8. \text{ If } f(x) \begin{cases} \frac{1 - \cos 8x}{x^2}, & x \geq 0 \\ \lambda, & x < 0 \end{cases} \text{ is continuous at}$$

$x = 0$ then $\lambda = ?$

A. 32

B. 16

C. 64

D. 8

Answer: A



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9. $f(x) = [x]$ (greatest integer function) is continuous at $x = 0$ then $f(0) = ?$

A. 0

B. -1

C. 1

D. $1/2$

Answer: D



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10. If $f(x) = \frac{2x + 3 \sin x}{3x + 2 \sin x}$, $x \neq 0$ is continuous at $x = 0$ then $f(0) = ?$

A. -1

B. 0

C. 1

D. None of these

Answer: C



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11. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then $f(x)$ is differentiable on

A. $[-1, 1]$

B. $\mathbb{R} - \{-1, 1\}$

C. $\mathbb{R} - (-1, 1)$

D. None of these

Answer: B



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12. If $\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\} = \alpha$, then prove that $x^2 = \sin 2\alpha$

A. $\frac{-x}{\sqrt{1-x^4}}$

B. $\frac{x}{\sqrt{1-x^4}}$

C. $\frac{-x}{\sqrt{1-x^2}}$

D. $\frac{x}{\sqrt{1-x^2}}$

Answer: A



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13. $\frac{d}{dx} \left[\log \left\{ e^x \left(\frac{x-2}{x+2} \right)^{\frac{3}{4}} \right\} \right] =$

A. $((x^2 - 1))$

B. 1

C. $\frac{x^2 + 1}{x^2 - 4}$

D. $e^x \cdot \frac{x^2 - 1}{x^2 - 4}$

Answer: A



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14. If $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ then $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = ?$

A. $\tan^2 \theta$

B. $\sec^2 \theta$

C. $\sec \theta$

D. $|\sec \theta|$

Answer: D



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15. If $f(x) = \tan^{-1} \sqrt{\frac{1 + \sin x}{1 - \sin x}}$, $0 \leq x \leq \frac{\pi}{2}$ then $f' \left(\frac{\pi}{6} \right) = ?$

A. $-\frac{1}{4}$

B. $-\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

Answer: D



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16. The differential coefficient of

$$\cot^{-1}(\sqrt{\cos 2x}) \text{ at } x = \frac{\pi}{6} \text{ is}$$

A. $\sqrt{\frac{2}{3}}$

B. $\sqrt{\frac{1}{3}}$

C. $\sqrt{3}$

D. $\sqrt{6}$

Answer: A



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17. If $y^2 = ax^2 + bx + c$ where a, b c are constants then

$$y^3 \frac{d^2y}{dx^2} = ?$$

- A. A constant
- B. A function of x
- C. A function of y
- D. None of these

Answer: A



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18. if $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then prove that $\frac{dy}{dx} = \frac{1}{x^3y}$

A. 0

B. 1

C. -1

D. None of these

Answer: B



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19. If $y^{\frac{1}{m}} = x + \sqrt{1 + x^2}$ then $(1 + x^2)y_2 + xy_1 = ?$

A. my

B. m^2y

C. m^2y^2

D. None of these

Answer: B

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20. If $y = \sin^2 \alpha + \cos^2(\alpha + \beta) + 2 \sin \alpha \sin \beta \cos(\alpha + \beta)$

then $\frac{d^3y}{da^3} = ?$

A. $\frac{\sin^3(\alpha + \beta)}{\cos \alpha}$

B. $\sin(\alpha + \beta)$

C. 0

D. 1

Answer: C

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1. Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$, at $x = -3$ and at $x = 5$.



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2. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$



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3. Examine the following functions for continuity. (a) $f(x) = x - 5$ (b)

$$f(x) = \frac{1}{x - 5} \quad \text{(c) } f(x) = \frac{x^2 - 25}{x + 5} \quad \text{(d) } f(x) = |x - 5|$$



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4. Prove that the function $f(x) = x^n$ is continuous at $x = n$, where n is a positive integer.



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5. Is the function f defined by

$$f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 5 & \text{if } x > 1 \end{cases}$$

continuous at $x = 0$? At $x = 2$?



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6. $f(x) = 2x + 3$ if $x \leq 2$ and $f(x) = 2x - 3$ if $x > 2$



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$$7. f(x) = \begin{cases} |x| + 3 & \text{if } x \leq -3 \\ -2x & \text{if } -3 < x < 3 \\ 6x + 2 & \text{if } x \geq 3 \end{cases}$$



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$$8. \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

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$$9. \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

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$$10. f(x) \begin{cases} x + 1, & \text{if } x \geq 1 \\ x^2 + 1, & \text{if } x < 1 \end{cases}$$

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$$11. f(x) \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$$

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$$12. f(x) \begin{cases} 10^3 - 1, & \text{if } x \leq 1 \\ x^2, & \text{if } x > 1 \end{cases}$$

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13. Is the function defined by

$$f(x) = \begin{cases} x + 5, & \text{if } x \leq 1 \\ x - 5, & \text{if } x > 1 \end{cases} \text{ a continuous function?}$$

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14. Discuss the continuity of the function f , where f is defined by

$$f(x) = \begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$$

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15. $f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases}$

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$$16. f(x) \begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$$



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17. Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases} \text{ is continuous at } x = 3.$$



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18. For what value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases} \text{ continuous at}$$

$x = 0$? What about continuity at $x = 1$?



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19. Show that the function defined by $g(x) = x - [x]$ is discontinuous at

all integral points which $[x]$ denotes the greatest integer function.



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20. Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$?



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21. Discuss the continuity of the following functions: (a)

$f(x) = s \in x + \cos x$ (b) $f(x) = s \in x \cos x$ (c) $f(x) = s \in x \cos x$



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22. Discuss the continuity of the cosine, cosecant, secant and cotangent functions.



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23. Find all points of discontinuity of f , where

$$f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x + 1, & \text{if } x \geq 0 \end{cases}$$

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24. Determine if f defined by $f(x) = \begin{cases} x^2 \frac{\sin 1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

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25. Examine the continuity of f , where f is defined by

$$f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$$

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$$26. f(x) = \begin{cases} \frac{k \cos x}{(\pi - 2x)}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$$

$$\text{at } x = \frac{\pi}{2}.$$

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27. $f(x) = \begin{cases} kx^2, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$ at $x = 2$

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28. Find the value of k for which $f(x) = \begin{cases} kx + 1 & \text{when } x \leq \pi \\ \cos x & \text{when } x > \pi \end{cases}$

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29. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \begin{cases} kx + 1, & \text{if } x \leq 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$ at $x = 5$

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30. Find the values of a and b such that the function defined by $f(x) = \begin{cases} 5, & \text{if } x < 2 \\ 2a + b, & \text{if } x = 2 \end{cases}$

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31. Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function.

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32. Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

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33. Examine that $\sin |x|$ is a continuous function.

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34. Find all the points of discontinuity of f defined by

$$f(x) = \frac{|x|}{|x + 1|}.$$

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Exercies 5 2

1. $\sin(x^2 + 5)$

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2. $\cos(\sin x)$

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3. $\sin(ax + b)$





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4. differentiate $\sec(\tan(\sqrt{x}))$



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5. Differentiate : $\frac{\sin(ax + b)}{\cos(cx + d)}$



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6. Differentiate $\cos x^3 \cdot \sin^2(x^5)$



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7. Find $\frac{dy}{dx}$ for $y=2\sqrt{\cot(x^2)}$



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8. If $y = f(a)$ is a differentiable function of a and $a = g(x)$ is a differentiable function of x then $y = f(g(x))$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{da} \times \frac{da}{dx}$. This rule is also known as CHAIN RULE. Based on the above information find the derivative of functions with respect to x in the following questions.

$$\cos(\sqrt{x})$$

- A. $\frac{\cos(\sqrt{x})}{2\sqrt{x}}$
- B. $-\frac{\cos(\sqrt{x})}{2\sqrt{x}}$
- C. $\frac{\sin(\sqrt{x})}{2\sqrt{x}}$
- D. $-\frac{\sin(\sqrt{x})}{2\sqrt{x}}$

Answer: D

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9. Prove that the function f given by $f(x) = |x - 1|$, $x \in \mathbb{R}$ is not differentiable at $x = 1$

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10. Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 3$ is not differentiable at $x = 1$ and $x = 2$.

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Exercies 5 3

1. Find $\frac{dy}{dx}$ in the following

$$2x + 3y = \sin x$$

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2. $2x + 3y = \sin y$

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3. $ax + by^2 = \cos y$

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4. $xy + y^2 = \tan x + y$

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5. $x^2 + xy + y^2 = 100$

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6. $x^3 + x^2y + xy^2 + y^3 = 81$

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7. $\sin^2 y + \cos xy = k$



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8. $\sin^2 x + \cos^2 y = 1$

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9. Graph of $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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10. $y = \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right], -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

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11. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) 0 < x < 1$

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$$12. y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

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$$13. y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$$

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$$14. y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

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$$15. y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$$

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1. $\frac{e^x}{\sin x}$

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2. Evaluate: $\int e^{\sin^{-1}((-1)x)} dx$.

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3. $= e^{x^3}$

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4. Find derivative of $\sin(\tan^{-1} e^{-x})$ w.r.t. to x

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5. Differentiate the following w.r.t. x : $\log(\cos e^x)$



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6. Differentiate the following w.r.t. x : $e^x + e^{x^2} + \dots + e^{x^5}$



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7. Differentiate the following w.r.t. x : $\sqrt{e^{\sqrt{x}}}$, $x > 0$



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8. $\log(\log x)$, $x > 1$



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9. Differentiate the following w.r.t. x : $\frac{\cos x}{\log x}$, $x > 0$



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10. Find derivative of $\cos(\log x + e^x)$, $x > 0$ w.r.t. to x

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Exercies 5 5

1. $\cos x \cdot \cos 2x \cdot \cos 3x$, find $d y / d x$

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2. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x

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3. Differentiate $(\log x)^{\cos x}$ with respect to x .

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4. Find the derivative of $x^x - 2^{\sin x}$ w.r.t. x



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5. Find the derivative of $(x + 3)^2(x + 4)^3(x + 5)^4$ w.r.t. x



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6. Differentiate the functions given w.r.t. x : $\left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)}$



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7. Differentiate the following w.r.t. x : $(\log x)^x + x^{\log x}$



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8. Differentiate the function $(\sin x)^x + \sin^1 x$ with respect to x .



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9. $x^{\sin x} + (\sin x)^{\cos x}$



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10. Differentiate $x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$ with respect to x :



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11. Differentiate the following w.r.t. x : $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$



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12. $x^y + y^x = 1$



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13. $y^x = x^y$ then find dy/dx



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14. If $(\cos x)^y = (\cos y)^x$ find $\frac{dy}{dx}$.



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15. If $xy = e^{(x-y)}$, then find $\frac{dy}{dx}$.



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16. Find the derivative of the function given by

$f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and, hence, find $f'(1)$.



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17. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below: (i) by using product rule (ii) by expanding the product to obtain a single polynomial. (iii) by logarithmic differentiation. Do they all give the same answer?

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18. If u , v and w are functions of x , then show that $\frac{d}{dx}(uvw) = \frac{du}{dx}vw + u\frac{dv}{dx}w + uv\frac{dw}{dx}$ in two ways - first by repeated application of product rule, second by logarithmic differentiation.

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Exercies 5 6

1. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = 2at^2$, $y = at^4$

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2. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx} \cdot x = a \cos \theta, y = b \cos \theta$

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3. $x = \sin t, y = \cos 2t$

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4. $x = 4t, y = \frac{4}{t}$

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5. $x = \cos \theta - \cos 2\theta, y = \sin \theta - \sin 2\theta$

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6. If $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$ find $\frac{d^2y}{dx^2}$

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7. If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ show that $\frac{dy}{dx} = 0$ at $t = \frac{\pi}{6}$

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8. In the curve $x = a \left(\cos t + \log \tan \left(\frac{t}{2} \right) \right)$, $y = a \sin t$. Show that the portion of the tangent between the point of contact and the x-axis is of constant length.

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9. If $x = a \sec \theta$, $y = b \tan \theta$ then $\frac{dy}{dx} = ?$

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10. if $x = a(\cos \theta + \theta \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$ then $\frac{d^2y}{dx^2}$

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11. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$ then show that, $\frac{dy}{dx} = -\frac{y}{x}$.

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Exercies 5 7

1. $x^2 + 3x + 2$

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2. x^{20}

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3. $x \cdot \cos x$

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4. $\log x$

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5. Find derivative of $x^3 \log x$ w.r.t. to x

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6. $e^x \sin 5x$

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7. $e^{6x} \cos 3x$



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8. $\tan^{-1} x$



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9. differentiate: $\log(\log x)$



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10. the derivative of $\sin(\log x)$ is



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11. If $y = 5 \cos x - 3 \sin x$, prove that $\frac{d^2 y}{dx^2} + y = 0$.



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12. If $y = \cos^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.



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13. If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that $x^2 y_2 + x y_1 + y = 0$



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14. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m + n) \frac{dy}{dx} + mny = 0$.



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15. If $y = 500 e^{7x} + 600 e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49 y$.



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16. If $e^y(x + 1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

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17. If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$

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Exercises 5 8

1. Verify Rolles theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$.

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2. Examine if Rolles theorem is applicable to any of the following functions. Can you say something about the converse of Rolles theorem

from these example?(i) $f(x) = [x]$ for $x \in [5, 9]$ (ii) $f(x) = [x]$ for $x \in [-2, 2]$ (iii) $f(x) = x^2$

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3. If $f: [-5, 5] \rightarrow \mathbb{R}$ is differentiable function and if $f'(x)$ does not vanish anywhere, then prove that $f(-5) \neq f(5)$.

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4. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$, where $a = 1$ and $b = 4$.

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5. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval $[a, b]$, where $a = 1$ and $b = 3$. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

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6. Examine the applicability of Mean Value Theorem for all three functions given in the above exercise 2.



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Miscellaneous Exercise

1. Differentiate w.r.t. x the function. $(3x^2 - 9x + 5)^9$



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2. Differentiate w.r.t. x the functions $\sin^3 x + \cos^6 x$



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3. Find $\frac{dy}{dx}$ if $y = (5x)^{3 \cos 2x}$

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4. $\sin^{-1}(x\sqrt{x}), 0 \leq x \leq 1$

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5. Differentiate w.r.t. x the function $(\cos^{-1}x/2)/(\sqrt{2x+7}), -2$

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6. $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

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7. $(\log x)^{\log x}, x > 1$

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8. Differentiate w.r.t. x the function $\cos(a \cos x + b \sin x)$, for some constant a and b .

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9. $(\sin x - \cos x)^{(\sin x - \cos x)}$, $\frac{\pi}{4} < x < \frac{3\pi}{4}$

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10. Differentiate w.r.t. x the function $x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and $x > 0$.

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11. Differentiate $x^x + (2 - 3)^x + (x - 3)^x + 2$ with respect to x :

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12. Find $\frac{dy}{dx}$, if $y=12(1-\cos t)$, $x=10(t-\sin t)$, $t=\pi/2$

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13. If $y = \sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}$ then find $\frac{dy}{dx}$

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14. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

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15. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that

$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b .

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16. If $\cos y = x \cos(a + y)$, with $\cos a \neq \pm 1$, prove that

$$\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}.$$

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17. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.

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18. If $f(x) = |x|^3$, show that $f(x)$ exists for all real x and find it.

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19. Using mathematical induction prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all positive integers n .

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20. Using the fact that $s \in (A + B) = s \in A \cos B + \cos A s \in B$ and the differentiation, obtain the sum formula for cosines.

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21. Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer.

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22. If $y = |f(x)g(x)h(x)lmnabc|$, prove that

$$\frac{dy}{dx} = |f'(x)g'(x)h'(x)lmnabc|$$

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23. If $y = e^a \cos^{(-1)x}$, $-1 \leq x < 1$, show that

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2y = 0$$



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