MATHS

BOOKS - NAGEEN MATHS (HINGLISH)

LINEAR PROGRAMMING

Solved Example Type

1. Solve the Following Linear Programming Problem graphically : Maximise $Z = 3x + 4y$

subject to the constraints :

$x + y \leq 4, x \geq 0, y \geq 0.$

A. 16

B. 18

C. 20

D. 14

Answer: A

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2. Solve the following linear programming problem graphically:

Minimize : $z = 100x + 50y$

Subject to constraints : $x + 2y \geq 10$

$3x + 4y \geq 24$

$x \geq 0$

$y \geq 0$

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3. Solve the following linear programming problem graphically.

Minimize : $z = 2x + 3y - 1$

Subject to:

$x - y \geq 0$

$-x + 2y \geq 2$

$x \geq 3$

$y \leq 4$

$y \geq 0$

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4. Solve the following linear programming problem graphically:

Minimize: $z = x + 3y$

Subject to:

$$x + y \leq 8$$

$$3x + 5y \geq 15$$

$$x \geq 0$$

$$y \geq 0$$

5. Solve the following linear programming problem graphically:

Maximize : $z = 5x + 3y$

Subject to:

$$x + 3y \leq 5$$

$$x + y \leq 3$$

$$x \geq 0$$

$$y \geq 0$$

6. A man wishes to mix two types of food X and Y in such a way that the vitamins contents of the mixture contain at least B units of vitamin A and 11 units of vitamin B. Food X costs Rs. 50 per kg and food Y costs Rs. 60 per kg. Fod X contains 2 units per kg of vitamin A and 5 kg per unit of vitamin B while food Y contains 5 units

per kg of vitamin A and 2 units per kg of vitamin B. Determine the minimum cost of the mixture.

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7. (Allocation problem) A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be

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8. A factory makes tennis rackets and cricket bats. A tennis racket takes 1. 5 hours of machine time and 3 hours of craftmans time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftmans time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsmans time. If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find the number of tennis rackets and crickets bats that the factory must manufacture to earn the

maximum profit. Make it as an L.P.P. and solve graphically.

9. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs.17.50 per package on nuts and Rs.7 per package of bolts. How many packages of each should be produced each day

so as to maximize his profits if he operates his machines for at the most 12 hours a day? Form the linear programming problem and solve it graphically.

10. There are factories located one at place P and other at place Q from which a certin commodity is to be delivered to each of three depot situated at A,B and C. The weekly requirement of the depots are respectively 5,5 and 4 units of the commodity while the

production capacity of the factories P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below.

From/To	Cost (in ₹)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. Also find the minimum transportation cost.

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1. Solve the following linear programming problem graphically:

Maximize : $z = 60x + 15y$

Subject to : $x + y \leq 50$

$3x + y \leq 90$

$x \geq 0$

$y \geq 0$

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2. Solve the following linear programming problem graphically:

Maximize : $z = 3x + 5y$

Subject to: $x + y \geq 2$

$x + 3y \geq 3$

$x \geq 0$

$y \geq 0$

3. Solve the following linear programming problem graphically:

Maximize : $z = 30x + 25y$

Subject to: $x + h \leq 6$

$3x + 2y \leq 15$

$x \geq 0$

$y \geq 0$

4. Solve the following linear programming problem graphically:

Maximize : $z = 200x + 500y$

Subject to: $x + 2y \geq 10$

$3x + 4y \leq 24$

$$x \geq 0$$

$$y \geq 0$$

5. Solve the following linear programming problem graphically:

Maximize : $z = x + 9y$

Subject to: $x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0$

$y \geq 0$

6. Solve the following linear programming problem graphically:

Maximize : $z = x + 2y$

Subject to: $2x + y \geq 3$

$x + 2y \geq 6$

$x \geq 0$

$y \geq 0$

Show that z is minimum at two points.

7. Solve the following linear programming problem graphically:

Maximize :$z = x + 2y$

Subject to: $x - y \leq 2$

$x + y \leq 4$

$x \geq 0$

$y \geq 0$

8. Solve the following linear programming problem graphically:

Maximize : $z = 4x + 3y$

Subject to: $2x + y \geq 40$

$x + 2y \geq 50$

$x + y \geq 35$

$x \geq 0$

$y \geq 0$

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Exercise 12 B

1. One kind of cake requires $200g$ of flour and $25g$ of fat and another kind of cake requires

$100g$ of flour and $50g$ of fat. Find the maximum number of cakes which can be made from $5kg$ of flour and $1kg$ of fat assuming that there is no shortage of other ingredients used in making the cakes.

2. A woman wishes to mix two types of foods in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and at least 10 units of vitamin B. Food I contains 2 units/kg of vitamin A and 1 unit /kg of vitamin B.

Food II contains 1 unit/kg of vitamin A and 2 unit/kg of vitamin B. The cost of food I is Rs. 50/kg and of food II is Rs. 70/kg. Find the minimum cost of such a mixture.

3. A woman wishes to mix two types of food in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food I costs $Rs.\,60/kg$ and food of vitamin B. Food II cost Rs. 80/kg. Food I contains 3 units/kg of vitamin A and 5

units/kg of vitamin B while food II contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Find the minimum cost of the mixture.

4. Two tailors A and B earns 15 and 20 per dayrespectively. A can stitch 6 shirts and 4 paints while Bcan stitch 10 shirts and 4 paints per day. To minimise the cost to stitch 60 shirts and 32 paints, how manydays should they work?

5. (Manufacturing problem) A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and

6. A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost Rs 25000 and Rs

40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the nu

7. A company manufacturers two types of products P and Q. The product P requires 5 minutes each for cutting and 10 minutes each for assembling. The product Q requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes availabe for cutting and 4 hours for assembling.

The profit is 50 paise each for type P and 60 paise each for type Q. How many products of each type should the company manufacture in order to maximize the profit?

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8. A dietician wishes to mix together two kinds of food X and Y m such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below:

One kg of foo

A. $Rs.\ 115$

B. $Rs.\ 116$

C. $Rs.\ 114$

D. $Rs.\ 112$

Answer: D

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9. There are two types of fertilisers F_1 and $F_2.F_1$ consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10%

phosphoric acid. After testing he soil conditions a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs. 6/kg and F_2 costs Rs. 5/kg, determine how much of each type of fertiliser should be used to that nutrient requirements are met at a minimum cost. What is the minimum cost?

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10. A manufacturer makes two types of toys A and B. Three machines are needed for this

purpose and the tune (in minutes) required for each toy on the machines is given below: Each machine is available for a maximum of 6 hours per day. If the profit

11. Two godowns A and B have gram capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D. E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to

12. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distances (in km) between the

13. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of 70 paise and screws B at a profit of Rs 1.

Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Formulate the above LPP and solve it graphically and determine the maximum profit.

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14. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls

per week and the demand for dolls of type B is

at most half of that for do

15. An aeroplane can carry a maximum of 200

passengers. A profit of Rs. 400 is made on each

first class ticket and a profit of Rs. 300 is made

on each second class ticket. The airline reserves

at least 20 seats for first class. However, at least

four times as many passengers prefer to travel

by second class then by first class. Determine

how many tickets of each type must be sold to

maximise profit for the airline. Form an LPP and solve it graphically.

16. A company produces two types of leather belts A and B on which the profits are Rs. 40 and Rs. 30 per belt respectively. Each belt of type A requires twice as much time as required by a belt to type B.If all belts were of type B the company could produce 1000 belts per day. But the supply of leather is sufficient only for 800 belts per day. Belt A requires a fancy buckle and

only 400 fancy buckles are available per day. For belt of type B only 700 buckles are available per day. How should the company manufacture the two types of belts in order to have maximum overall profit?

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17. A manufacturer has three machines A,B and C installed in his factory. Machines A and B are capable of being operated for atmost 12 hours whereas machine C must be operated for atleast 5 hours a day. He produces only two

items X and Y each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of X and Y on three machines are given in the following table:

Item	Number of hours required on machines		
	A	B	C
X	1	2	1
Y	2	1	1.25

He makes a profit of Rs. 600 and Rs. 400 on each item of X and Y respectively. How many of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produced?

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1. Maximise $Z = 3x + 4y$

Subject to the constraints

$x + y \leq 4, x \geq 0, y \geq 0$

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2. Minimise $Z = -3x + 4y$

Subject   to

$x + 2y \leq 8, 3x + 2y \leq 12, x \geq 0, y \geq 0.$

3. Maximise $Z = 5x + 3y$

Subject   to

$3x + 5y \leq 15, 5x + 2y \leq 10, x \geq 0, y \geq 0.$

4. Minimise $Z = 3x + 5y$

Such that $x + 3y \geq 3, x + y \geq 2, x, y \geq 0.$

5. Maximise $Z = 3x + 2y$

Subject to $x + 2y \leq 10,\ 3x + y \leq 15,\ x, y \geq 0$

6. Minimise $Z = x + 2y$

Subject to $2x + y \geq 3,\ x + 2y \geq 6,\ x, y \geq 0.$

Show that the minimum of Z occurs at more than two points.

7. Minimise and Maximise $Z = 5x + 10y$

Subject   to

$x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, y \geq 0.$

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8. Minimise and Maximise $Z = x + 2y$

Subject   to

$x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y \geq 0$

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9. Maximise $Z = -x + 2y$

Subject to the constraints

$x \geq 3,\ x + 3y \geq 5,\ x + 2y \geq 6,\ y \geq 0$

10. Maximise $Z = x + y$

Subject to

$x - y \leq -1,\ -x + y \leq 0,\ x, y \geq 0.$

1. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at leat 8 units of vitamin A and 11 units of vitamin B. Food P cost Rs. 60/kg and Food Q costs Rs. 80/kg. Food P contains 3 units/kg of vitamin A and 5 units/kg of Vitamin B while food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

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2. One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which an be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

3. A factory makes tennis rackets and cricket bats. A tennis rackets takes 1.5 hour of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of cratman's time. In a daythe factory has the availability of not more than 42 hours of machine time and 24 hours of craftman's time.

(i) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If profit on a racket and on a bat is Rs. 20 and

Rs. 10 respectively, find the maximum profit of the factory when it works at full capacity.

4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should be produced each day

so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

5. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package on screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacturer a

package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs. 7 and screws B at a profit of Rs. 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day order to maximise his profit? Determine the maximum profit.

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6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/ cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lam. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hour and the grinding/ cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs. 5 and that from a shade isRs. 3.

Assuming that the manufacturer can sell all the lamps and shades that the produces how should be scehdule his daily production in order to maximise his profit?

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7. A company manufacturers two types of novelty suouvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours

20 minutes available for cutting and 4 hours for assembling. The profit is Rs. 5 each for type A and Rs. 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?

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8. A merchant plans to sell two types of personal computers -a desktop model and a portable model that will cost Rs. 25000 and Rs. 40000 respectively. He estimates that the total monthly demand of computers will not exceed

250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and if his profit on the desktop model is Rs. 4500 and on portable model is Rs. 5000.

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9. A diet is to contain at leat 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 cost Rs. 4 per unit and F_2 costs Rs. 6 per unit. One unit of food F_1

contains 3 uinits of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

10. There are two types of fertilisers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric

acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing he soil conditions a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs. 6/kg and F_2 costs Rs. 5/kg, determine how much of each type of fertiliser should be used to that nutrient requirements are met at a minimum cost. What is the minimum cost?

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11. The corner points of the feasible region determined by the following system of linear inequalities:

$2x + y \geq 10, \; x + 3y \leq 15, \; x, \; y \geq 0$ are

$(0, 0), \; (5, 0), \; (3, 4)$ and $(0, 5)$. Let $Z = px + qy$, where $p, \; q \geq 0$. Condition on p and q so that the maximum of Z occurs at both $(3, 4)$ and $(0, 5)$ is:

(a) $p = q$ (b) $p = 2q$ (c) $p = eq$ (d) $q = 3p$

1. (Diet problem) A dietician has to develop a special diet using two foods P and Q. Each packet (containing $30g$) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calciums atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food

should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A? How many packets of each food should be used to maximise the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

2. A farmer mixes two brands P and Q of cattle feed. Brand P costing Rs. 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q

costing Rs. 200 per bag contains 1.5 units of nutritonal element A, 11.25 units of element B and 3 units of element C. The minimum requirements of nutrients A,B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

3. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

1 kg of food X costs Rs. 16 and 1 kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet?

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4. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each purpose and the time (in minutes) required for each toy on the machines is given below:

Types of toys	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs. 7.50 and that on each toy of type B is Rs. 5, show that 15 toys of type A and 30 of type B

should be manufactured in a day to get maximum profit.

5. An aeroplane can carry a maximum of two passengers. A profit of Rs. 1000 is made on each executive clas ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However at least 4 times as many passengers prefer to travel by economy clas than by the executive class. Determine how many tickets of

each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

6. Two godowns A and B have grain capacity of 100 quintals and 50 qunitals respectively. They supply to 3 ration shops, D,E and F whose requirements are 60,50 and 40 quintals repectively.The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in ₹)		
From/To	A	B
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

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7. An oil company has two depots A and B with capacities of $7000L$ and $4000L$ respectively. The company is to supply oil to three petrol pumps, D,E and F whose requirements are $4500L$, $3000L$

and $3500L$ respectively. The distances (in km) between the depots and the petrol pumpa is given in the following table:

From/To	Distance (in km) A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Rs. 1 per km, how shold the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

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8. A fruit grower canuse two types of fertilizer in his garden brand P and brand Q. The amounts (in kg) of nitrogen phosphoric acid, potash, and chlorine a bag of each brand are given in the stable. Tests indicate that the garden needs at least 240kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the

garden?

	kg per bag Brand P	Brand Q
Nitro en	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

9. A toy company manufactures two types of dolls A and B. Market research and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further

the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs. 12 and Rs. 16 per doll respectively on dolls A and B how many of each should be produced weekly in order to maximise the profit?