MATHS

BOOKS - NAGEEN MATHS (HINGLISH)

RELATIONS AND FUNCTIONS

Solved Examples

1. If A = {1, 2, 3, 4}, define relations on A which have properties of being

(i) reflexive, transitive but not symmetric.

(ii) symmetric but neither reflexive nor transitive.

(iii) reflexive, symmetric and transitive.

2. A relation R is defined on the set of integers as follows :

$aRb \Leftrightarrow (a-b)$, is divisible by 6 where a, b, $\in$ I. prove that R is an equivalence relation.

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3. What is an equivalence relation?

Show that the relation of'similarity' on the set S of all triangle in a plane is an equivalence relation.

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4. Prove that the relation R on the set NxN defined by $(a, b)R(c, d)a+d=b+c$ for all $(a, b), (c, d) \in NxN$ is an equivalence relation.

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5. In the set of straight lines in a plane, for the relation 'perpendicular' check whether it is reflexive, symmetric and transitive.

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6. A relation R on the set of complex numbers is defined by $z_1 R z_2$ if and only if $\frac{z_1 - z_2}{z_1 + z_2}$ is real. Show that R is an equivalence relation.

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7. Let a relation R be defined by relation The $R = \{(4, 5), (1, 4), (4, 6), (7, 6), (3, 7)\}$ $R^{-1} o R$ is given by

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8. Which types of the following functions are ?

(i) {(a,1),(b,1),(c,1),(d,1),(e,1)}

(ii) {(3,2),(6,4)(9,2),(12,4)}

(iii) {(a,1),(b,2),(c,3),(d,4)}.

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9. prove that a function f = $\{(x, 2x+1) : x \in N\}$ defined on the set of natural numbers NxxN is one - one function.

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10. Prove that the function $f: N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

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11. The function $f: R \to R: f(x) = \sin x$ is

A. One One

B. Onto

C. Into

D. None Of these

Answer: C

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12. show that the function

$(i) f: N \rightarrow N: f(x) = x^2$ is one-one into

(ii) $f: Z \rightarrow Z: f(x) = x^2$ is many -one into .

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13. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are two mappings such that $f(x) = 2x$ and $g(x) = x^2 + 2$ then find fog and gog.

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14. If $f: R \rightarrow R, g: R \rightarrow R$ defined as $f(x) = \sin x$ and $g(x) = x^2$, then find the value of $(gof)(x)$ and $(fog)(x)$and also prove that $gof \neq fog$.

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15. If f and g two functions are defined as :

f= {(1,2),(3,6),(4,5)} and g = {(2,3),(6,7),(5,8)}, then find gof.

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16. A function $f: R \rightarrow R$ is defined as $f(x) = x^2 + 2$, then evaluate each of the following :

(i) $f^{-1}(-6)$ (ii) $f^{-1}(18)$

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17. Prove that the function f : $R \rightarrow R$ where R is the set of all real numbers, defined as f (x) = $3x + 4$ is one-one and onto . Also find the inverse function of f.

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18. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary operation on R. Further, show that division is a binary operation on the set R of nonzero real numbers.

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19. Show that subtraction and division are not binary operations on N.

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20. Let P be a set of all subset of set X. Prove that the functions defined by

$\cup : P \times P \to P, (A, B) \to A \cup B$ and $\cap : P \times P \to P, (A, B) \to A \cap B$

are binary on P.

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21. Show that the operation $\vee$ and $\wedge$ on R defined as $a \vee b =$ Maximum of a and b ; $a \wedge b =$ Minimum of a and b are binary operations of R.

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22. Show that $+ : R \times R \to R$and $\times : R \times R \to R$are commutative binary operations, but $: R \times R \to R$and $\div : R_{\cdot} \times R_{\cdot} \to R_{\cdot}$are not commutative.

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23. Prove that $*: R \times R \to R$ defined as $a * b = a + 2ab$ is not commutative .

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24. Prove that in the set of real numbers '+' and $' \times '$ are associative but '-' and $' \div '$ are not associative.

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25. Prove that $*: R \times R \to R$ defined as $a * b = a + 2ab$ is not associative

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26. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the operations $\div R \times R \to R$ and $\div : R_. \times R_. \to R_. \cdot$

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27. Show that a is the inverse of a for the addition operation + on R and $\frac{1}{a}$ is the inverse of $a \neq 0$for the multiplication operation $\times$ on R.

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28. Show that a is not the inverse of $a \in N$ for the addition operation + on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation $\times$ on N, for $a \neq 1$.

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Exercies 1 A

1. Give an example of a relation. Which is (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive. (iv) Reflexive and transitive but not symmetric. (v) Symm

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2. (i) If A= {x,y,z}, B=(1,2,3} and R= {(x,2),(y,3),(z,1),(z,2), then find R^{-1}.

(ii) If R is a relations such that R ={(4,5),(1,4),(4,6),(7,6),(3,7)}, then find $R^{-1}oR^{-1}$

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3. Prove that the relation R on Z defined by $(a,\ b) \in R \Leftrightarrow\ \ a-b$ is divisible by 5 is an equivalence relation on Z .

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4. Prove that the relation $R=\{(x,y): x, y \in N \text{ and } x-y \text{ is divisible by } 7\ \}$ defined on positive integers N is an equivlance relations.

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5. If A={a,b,c,d}, then on A .

(i) write the identity relation I_A .

(ii) write a reflexive relation which is not the identity relation.

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6. Show that the divisibility relation in the set of positive integers is reflexive and transitive, but not symmetric.

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7. Two points P and Q in a plane are related if $OP = OQ$, where is a fixed point. This relation is :

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8. Let N be the set of all natural numbers and let R be a relation on $N \times N$, defined by $(a, b)R(c, d)ad = bc$ for all $(a, b), (c, d) \in N \times N$. Show that R is an equivalence relation on $N \times N$. Also, find the equivalence class [(2,6)].

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9. (i) Show that in the set of positive integer, the relation ' is greater than ' is transitive but it is not reflexive or smmetric.

(ii) Let R be a relation on the set of natural numbers N defined as $_aR_b \Rightarrow$ a divides B where a,b $\in$ N. Is R symmetric ?

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10. Show that the relation is congruent to on the set of all triangles in a plane is an equivalence relation

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11. Show that the divisibility relation in the set of positive integers is reflexive and transitive, but not symmetric.

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12. Let R be a relation defined on the set of natural numbers N as $R = \{(x, y) : x, y \in N, 2x + y = 41\}$ Find the domain and range of R. Also, verify whether R is (i) reflexive, (ii) symmetric (iii) transitive.

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13. Let a relation R be defined by relation The $R = \{(4, 5), (1, 4), (4, 6), (7, 6), (3, 7)\}$ $R^{-1}oR$ is given by

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14. If $A = \{1, 2, 3, 5\}, B = \{2, 4, 6, 8\}$ and $C = \{4, 16, 36, 39\}$ are three sets and R is a relation from A to B and S from B to C defined as

${}_aR_b \Leftrightarrow b = 2a$ where $a \in A, b \in B$

${}_bS_c \Leftrightarrow c = b^2$ where $b \in B, c \in C$

$\therefore$ Find SoR.

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15. Show that the relation R in the set R of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$is neither reflexive nor symmetric nor transitive.

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16. If R and S are two equivalence relations on a set A; then $R \cap S$ is also an equivalence relation on R.

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Exercies 1 B

1. $f: R \to R$ is a function where f(x)= 2x-3 . Check whether f is noe -one ?

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2. On set $A = \{1, 2, 3\}$, relation R and S are given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ and $S = \{(1, 1), (2, 2), (3, 3), (1,$

Then

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3. Prove that $f: N \to N$ defined by $f(x) = 2x, x \in N$ is one-one and into.

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4. If R is the set of real numbers and a function $f: R \to R$ is defined as $f(x) = x^2, x \in R$, then prove that f is many-one into function.

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5. If Q is the set of rational numbers, then prove that a function $f: Q \to Q$ defined as $f(x) = 5x - 3, x \in Q$ is one -one and onto function.

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6. If R is the set of real numbers then prove that a function $f: R \to R$ defined as $f(x) = \frac{1}{x}, x \neq 0, x \in R,$ is one-one onto.

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7. Prove that the function $f: R^{+} \to R$ which is defined as $f(x) = \log_e x$ is one - one .

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8. If R is the set of real numbers prove that a function $f: R \to R$, $f(x) = e^x, x \in R$ is one to one mapping.

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9. A function $f: R \to R$ is defined as $f(x) = 4x - 1, x \in R,$ then prove that f is one - one.

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10. Let $A = R - \{3\}$ and $B = R - [1]$. Consider the function $f: A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is one-one and onto and hence find f^{-1}

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11. Let the function $f: R \to R$ be defined by $f(x) = \cos x, \forall x \in R$.

Show that f is neither one-one nor onto.

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12. If a function $f: R \to R$ is defined as $f(x) = x^3 + 1$, then f is

A. Onto

B. Into

C. cant say

D. Not a function

Answer: A

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13. Function $f : R \to R$ and $g : R \to R$ are defined as $f(x) = \sin x$ and $g(x) = e^x$.

Find (gof)(x) and (fog)(x).

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14. If $f : R \to R$ and $g : R \to R$ be two functions defined as f(x)=2x+1 and $g(x) = x^2 - 2$ respectively , then find (gof) (x) and (fog) (x) and show that (fog) (x) $\neq$ (gof) (x).

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15. If f and g are two functions from R to R which are defined as $f(x) = x^2 + x + 1$ and $g(x) = 2x - 1$ for each $x \in R$, then show that (fog) (x) $\neq$ (gof) (x).

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16. If $f: R \rightarrow R$ and $g: R \rightarrow R$ be two functions defined as $f(x) = x^2$ and $g(x) = 5x$ where $x \in R$, then prove that (fog)(2) $\neq$ (gof) (2).

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17. If $f: R \rightarrow R$ defined as $f(x) = 3x + 7$, then find $f^{-1}(-2)$

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18. If Q is the set of rational numbers and a function $f: Q \rightarrow Q$ is defined as $f(x) = 5x - 4, x \in Q$, then show that f is one-one and onto. Also define f^{-1}.

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Exercies 1 C

1. In the given sets, working operations $*$ is defined, check whether $*$ is binary or not ? Justify your answer.

(i) In Z^+, $a * b = a - b$

(ii) In Z^+, $a * b = ab$

(iii) In R, $a * b = ab^2$

(iv) In Z^+, $a * b = |a - b|$

(v) In Z^+, $a \cdot b = a$

(vi) In Z^+, $a * b = a - 3b$

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2. In the given sets the binary operation * is defined. Check the commutativity and associativity is each case for *:

(i) In Z, a*b=a-b

(ii) In Q, $a * b = 1 + ab$

(iii) In Q, $a * b = \frac{ab}{2}$

(iv) In Z^+, $a * b = 2^{ab}$

(v) In Z^+, $a * b = a^b$

(vi) In $R - \{-1\}$, $a * b = \frac{a}{b+1}$

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3. Construction a composition table for binary operation $\wedge$ defined as $a \wedge b$= minimum of {a,b} in the set {1,2,3,4,5} and

(i) evaluate $(2 \wedge 3) \wedge 4 \text{ and } 2 \wedge (3 \wedge 4)$

(ii) is $\wedge$ commutative ?

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4. Consider the infimum binary operation $\wedge$ on the set $S = \{1, 2, 3, 4, 5\}$ defined by $a \wedge b =$ Minimum of $a \, and \, b$. Write the composition table of the operation $\wedge \cdot$

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5. Let $' \cdot '$ be a binary operation on N given by $a \cdot b = LCMa, b$ for all $a, b \in N$. Find $5 \cdot 7, 20 \cdot 16$ (ii) Is * commutative? Is * associative? Find the identity element in N Which element of N are invertible? Find them.

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6. Let $*$ be a binary operation on the set Q of rational numbers as follows:

(i) $a * b = a - b$

(ii) $a * b = a^2 + b^2$

(iii) $a * b = a + ab$

(iv) $a * b = (a - b)^2$

(v) $a \cdot b = \frac{ab}{4}$

(vi) $a * b = ab^2$.

Find which of the binary operations are commutative and which are associative.

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7. Let $A = N \times N$ and $\cdot$ be the binary operation on A defined by $(a, b) \cdot (c, d) = (a + c, b + d)$. Show that $\cdot$ is commutative and associative. Find the identity element for $\cdot$ on A, if any.

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8. State whether the following statements are true or false. Justify. (i) For an arbitrary binary operation $\cdot$ on a set $N, a \cdot a = a \forall a \in N$. (ii) If $\cdot$ is a commutative binary operation on N, then `a" "*" "(b" "*" "c)"

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9. If * is a binary operation in N defined as a*b $= a^3 + b^3$, then which of the following is true :

(i) * is associative as well as commutative.

(ii) * is commutative but not associative

(iii) * is associative but not commutative

(iv) * is neither associative not commutative.

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Exercies 1 D

1. Relation " parallel" in the set of all straight lines in a plane is :

A. only reflexive

B. only symmetric

C. only transitive

D. equivalence relation

Answer: D

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2. In the set of straight lines in a plane, for the relation 'perpendicular' check whether it is reflexive, symmetric and transitive.

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: B

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3. Relation " similar" in triangles in a plane is :

A. reflexive, symmetric , transitive

B. reflexive, transitive but not symmetric

C. symmetric , transitive but not reflexive

D. none of the above

Answer: A

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4. If $f : N \to N$ defined as $f(x) = x^2 \, \forall x \in N,$ then f is :

A. many-one

B. one-one

C. onto

D. none of these

Answer: B

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5. The function f is defined as :

$$f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$

The range of f is :

A. {1,0}

B. {0,-1}

C. {1,-1}

D. {1,0,-1}

Answer: D

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6. If $f: R \to A,$ where A =[-1,1] , is defined as $f(x) = \cos x,$ thn find f is

A. into

B. one-one

C. onto

D. none of these

Answer: C

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7. Let $f: N \to N$ be defined as $f(n) = \frac{n+1}{2}$ if n is odd and $f(n) = \frac{n}{2}$ if n is even for all n $\in$ N State whether the function f is bijective. Justify your answer

A. one-one into

B. one-one onto

C. many-one into

D. many-one onto

Answer: B

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8. If $f: R \to R$ is defined as f(x)=2x+5 and it is invertible , then $f^{-1}(x)$ is

A. $\frac{x-5}{2}$

B. $\frac{x-2}{5}$

C. $\frac{x+5}{2}$

D. none of these

Answer: A

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9. Is $\cdot$ defined on the set $\{1, 2, 3, 4, 5\} by a \cdot b = L\dot{C}\dot{M}$.of a and b a binary operation? Justify your answer.

A. 6

B. 24

C. 36

D. none of these

Answer: C

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10. On the set Z of integers, if the binary operation * is defined by $a \cdot b = a + b + 2,$ then find the identity element.

A. commutative

B. associative

C. commutative and associative

D. none of above

Answer: D

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Exercies 1 E

1. A relation $R = \{(x, y) : x, y \in A \text{ and } x < y\}$ is defined on set A={1,2,3,4,5}. The relation R is :

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: C

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2. If R and S are two non-empty relations on set A, then incorrect statement is :

A. R and S are reflexive , then $R \cap S$ is also reflexive .

B. R and S are symmetric , then $R \cup S$ is also symmetric

C. R and S are transitive , then $R \cap S$ is also transitive .

D. R and S are transitive , then $R \cup S$ is also transitive.

Answer: B

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3. If $f(x) = \frac{x-1}{x+1}$ then $f(2x)$ is equal to

A. $\frac{1+f(x)}{3+f(x)}$

B. $\frac{1+3f(x)}{3+f(x)}$

C. $\frac{3+f(x)}{1+f(x)}$

D. $\frac{1+3f(x)}{3-f(x)}$

Answer: B

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4. If n(A)=3 and n(B)=4 , then no. of of one-one function from A to B is :

A. 12

B. 24

C. 36

D. none of these

Answer: B

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5. Let $f(x) = \dfrac{ax+b}{cx+d}$. Then the $fof(x) = x$, provided that : $(a \neq 0, b \neq 0, c \neq 0, d \neq 0)$

A. a=b=c=d=1

B. a=b=1

C. a=d

D. a=-d

Answer: D

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6. Let $f:[-1,\infty] \rightarrow [-1,)$ is given by $f(x)=(x+1)^2-1, x \geq -1$. Show that f is invertible. Also, find the set $S=\{x: f(x)=f^{-1}(x)\}$.

A. $\left\{0, -1, \dfrac{-3 \pm \sqrt{3}}{2}\right\}$

B. {0,1,-1}

C. {0,-1}

D. {}

Answer: C

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7. If n(A)=10, then no of different functions from A to A is :

A. $\underline{|10}$

B. 10^{10}

C. 2^{10}

D. $2^{10} - 1$

Answer: B

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8. If $g(f(x)) = |\sin x|$ and $f(g(x)) = \left(\sin(\sqrt{x})\right)^2$ then

A. $f(x) = \sin^2 x, \, g(x) = \sqrt{x}$

B. $f(x) = \sin x g(x) = |x|$

C. $f(x) = x^2, \, g(x) = \sin\sqrt{x}$

D. f(x) and g(x) cannot be determined

Answer: A

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9. If the function $f: A \rightarrow B$ is one-one onto and $g: B \rightarrow A$, is the inverse of f, then fog =?

A. f

B. g

C. I_A

D. I_B

Answer: D

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10. If $f(x)=\left(ax^2+b\right)^3$, then find the function g such that $f(g(x))=g(f(x))$

A. $\left(\frac{x^{1/3}-b}{a}\right)^{1/2}$

B. $\frac{1}{\left(ax^2+b\right)^3}$

C. $\dfrac{1}{(ax^2+b)^{1/3}}$

D. none of these

Answer: A

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Exercise 1 1

1. Determine whether each of the following relations are reflexive, symmetric and transitive :

(i) Relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as

$$R = \{(x, y) : 3x - y = 0\}$$

(ii) Relation R in the set N of natural numbers defined as

$$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$R = \{(x, y) : y$ is divisible by x}

(iv) Relation R in the set Z of all integers defined as

$R = \{(x, y) : x - y$ is an integer $\}$

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x, y) : \text{ and } y$ work at the same place $\}$

(b)$R = \{(x, y) : x \text{ and } y$ live in the same locality $\}$

(c) $R = \{(x, y) : x$ is exactly 7 cm taller than y$\}$

(d) $R = \{(x, y) : x$ is wife of $y\}$

(e) $R = \{(x, y) : x$ is father of $y\}$

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2. Give an example of a relation. Which is (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric. (iii) Reflexive and symmetric but not transitive. (iv) Reflexive and transitive but not symmetric. (v) Symm

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3. Show that the relation R on the set A of points in a plane, given by $R = \{(P, Q):$ Distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further show that the set of all points related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

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4. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1$ is similar to $T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides $3, 4, 5,$ T_2 with sides $5, 12, 13$ and T_3 with sides $6, 8, 10$. Which triangles among T_1, T_2 and T_3 are related ?

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5. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1(\text{ and } P)_2 havesamenumberofsides\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle

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6. Let L be the set of all lines in $XY = plane$ and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4.$

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7. Let R be the relation in the set $\{1, \quad 2, \quad 3, \quad 4\}$ given by $R = \{(1, \quad 2), \quad (2, \quad 2), \quad (1, \quad 1), \quad (4, \quad 4), \quad (1, \quad 3), \quad (3,$
. Choose the correct answer. (A) R is reflexive and symmetric but not transitive. (B) R is re

A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

Answer: B

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8. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer.

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer: C

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Exercise 1 2

1. Show that the function $f: R_0 \rightarrow R_0$, defined as $f(x) = \frac{1}{x}$, is one-one onto, where R_0 is the set of all non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0 ?

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2. Check the injectivity and surjectivity of the following functions:(i) $f: N \rightarrow N$given by $f(x) = x^2$(ii) $f: Z \rightarrow Z$given by $f(x) = x^2$(iii) $f: R \rightarrow R$given by $f(x) = x^2$(iv) $f: N \rightarrow N$given by $f(x) = x^3$(v) `f : Z - >

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3. Prove that the Greatest Integer Function $f: R \rightarrow R$, given by $f(x) = [x]$, is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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4. Show that the Modulus Function $f: R \rightarrow R$, given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$is x, if x is positive or 0 and $|x|$is x, if x is negative.

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5. Show that the Signum function $f: R \rightarrow R$, given by $f(x) = \{1, \quad \text{if} \quad x > 00, \quad \text{if} \quad x = 0 - 1, \quad \text{if} \quad x < 0$ is neither one-one nor onto.

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6. Let $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$and let $f = \{(1, 4), (2, 5), (3, 6)\}$be a function from A to B. Show that f is one-one.

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7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.(i) $f: R \to R$, defined by $f(x) = 34x$(ii) $f: R \to R$, defined by $f(x) = 1 + x^2$

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8. Let A and B be sets. Show that $f : A \times B$,$B \times A$ such that $f (a, b) = (b, a)$ is bijective function.

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9. Let $f: N \to N$ be defined as $f(n) = \frac{n+1}{2}$ if n is odd and $f(n) = \frac{n}{2}$ if n is even for all n $\in$ N State whether the function f is bijective. Justify your answer

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10. Let $A = R - \{3\}$and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

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11. Let $f: R \to R$ be defined as $f(x) = x^4$. Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D

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12. Let $f: R \to R$be defined as $f(x) = 3x$. Choose the correct answer

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: A

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Exercise 1 3

1. Let $f:\{1, 3, 4\} \to \{1, 2, 5\}$and $g:\{1, 2, 5\} \to \{1, 3\}$be given by $f = \{(1, 2), (3, 5), (4, 1)\}$and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

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2. Let f, g and h be functions from R to R. Show that

$(f+g)oh = foh + goh(fg)oh = (foh)(goh)$

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3. Find and , if (i) () $f(x) = \quad |x|$ and $g(x) = |5x - 2|$ (ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

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4. If $f(x) = \frac{4x+3}{6x-4}, \ x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. What is the inverse of f?

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5. State with reason whether following functions have inverse

(i) $f: \{1, 2, 3, 4\} \to \{10\}$ with $f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

(ii) $g: \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$ with $g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii) $h: \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$ with $h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

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6. Show that $f: [-1, 1] \rightarrow R$, given by $f(x) = \frac{x}{(x+2)}$ is one-one. Find the inverse of the function $f: [-1, 1] \rightarrow$ Range f.

(Hint: For $y \in$ Range f, $y = f(x) = \frac{x}{x+2}$, for some x in $[-1, 1]$, i.e., $x = \frac{2y}{(1-y)}$)

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7. Consider $f: R \rightarrow R$given by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f.

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8. Consider $f: R_{\pm} > [4, \quad \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of given f by $f^{-1}(y) = \sqrt{y - 4}$ where R_+ is the set of all non-negative real numbers.

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9. Consider $f: R \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y + 6} - 1}{3}\right)$.

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10. Let $f: X \to Y$be an invertible function. Show that f has unique inverse. (Hint: suppose $g_1($ and $g)_2$are two inverses of f. Then for all $y \in Y$, $fog_1(y) = I_Y(y) = fog_2(y)$Use one oneness of f).

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11. Consider $f:\{1, 2, 3\} \to \{a, b, c\}$given by $f(1) = a,\ f(2) = b$ and $f(3) = c$. Find $f^{-1}$and show that $\left(f^{-1}\right)^{-1} = f$.

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12. Let $f: X \to Y$be an invertible function. Show that the inverse of f^{-1} is f, i.e., $\left(f^{-1}\right)^{-1} = f$.

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13. If $f: R \to R$ be given by $f(x) = \left(3 - x^3\right)^{1/3}$, then $fof(x)$is

A. $x^{1/3}$

B. x^3

C. x

D. $\left(3 - x^3\right)$

Answer: C

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14. Let $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R$be a function as $f(x) = \frac{4x}{3x+4}$. The inverse of f is map, $g: Ran \geq f \rightarrow R - \left\{-\frac{4}{3}\right\}$given by.(a) $g(y) = \frac{3y}{3-4y}$ (b) $g(y) = \frac{4y}{4-3y}$(c) $g(y) = \frac{4y}{3-4y}$ (d) $g(y) = \frac{3y}{4-3y}$

A. $g(y) = \frac{3y}{3-4y}$

B. $g(y) = \frac{4y}{4-3y}$

C. $g(y) = \frac{4y}{4-3y}$

D. $g(y) = \frac{4y}{3-4y}$

Answer: b

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Exercise 1 4

1. Determine whether or not each of the definition of given below gives a binary operation. In the event that is not a binary operation, given justificantion for this.

(i) On Z^+, define * by a * b = a- b

(ii) On Z^+, define * by a * b = ab

(iii) On R,define * by a * $b - ab^2$

(iv) On Z^+, define * by a *b $= |a - b|$

(v) On Z^+, define * by a* b = a

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2. For each operation * defined below, determine whether * is binary, commutative or associative.

(i) On Z, define a*b = a-b

(ii) On Q, define a*b = ab + 1

(iii) On Q, define a*b $= \frac{ab}{2}$

(iv) On Z^+, define a*b $= \frac{a}{b+1}$

(v) On Z^+, define a*b =a^b

(vi) On R- {-1}, define a*b $= \frac{a}{b+1}$

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3. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a \wedge b = \min\{a, b\}$. Write the operation table of the operation.

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4. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table.

(i) Compute (2*3)*4 and 2*(3*4)

(ii) Is * commutative ?

(iii) Compute (2*3) *(4*5)

(Hint : uset the following table)

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5. Let $\cdot$ 'be the binary operation on the set $\{1, 2, 3, 4, 5\}$defined by $a \cdot 'b = H\dot{C}\dot{F}$.of a and b. Is the operation $\cdot$ 'same as the operation $\cdot$ defined in Exercise 4 above? Justify your answer.

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6. Let $\cdot$ be the binary operation on N given by $a \cdot b = L\dot{C}\dot{M}$.of a and b. Find (i) $5 \cdot 7, 20 \cdot 16$(ii) Is $\cdot$ commutative? (iii) Is $\cdot$ associative? (iv) Find the identity of $\cdot$ in N (v) Which elements of N are invert

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7. If $*$ defined on the set $\{1, 2, 3, 4, 5\}$ by $a * b = LCM$of a and b a binary operation? Justify your answer.

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8. Let * be the binary operation on N defined by $a * b = HCF$ of a and b. Does there exist identity for this binary operation on N ?

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9. Let $\cdot$ be a binary operation on the set Q of rational numbers as follows: (i) $a \cdot b = a - b$ (ii) $a \cdot b = a^2 + b^2$ (iii) $a \cdot b = a + ab$ (iv) $a \cdot b = (a - b)^2$ (v) $a \cdot b = \frac{ab}{4}$ (vi) $a \cdot b = ab^2$. Find wh

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10. Find the which of the operations given above has identity ?

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11. Let $A = N \times N$ and $\cdot$ be the binary operation on A defined by $(a, b) \cdot (c, d) = (a + c, b + d)$. Show that $\cdot$ is commutative and associative. Find the identity element for $\cdot$ on A, if any.

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12. State whether the following statements are true or false. Justify. (i) For an arbitrary binary operation $\cdot$ on a set N, $a \cdot a = a \forall a \in N$. (ii) If $\cdot$ is a commutative binary operation on N, then `a" "*" "(b" "*" "c)"

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13. Consider a binary operation $\cdot$ on N defined as $a \cdot b = a^3 + b^3$. Choose the correct answer. (A) Is $\cdot$ both associative and commutative? (B) Is $\cdot$ commutative but not associative? (C) Is $\cdot$ associative but not commutative? (D) Is

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Miscellaneous Exercise

1. Let $f: R \rightarrow R$be defined as $f(x) = 10x + 7$. Find the function $g: R \rightarrow R$such that$gof = fog = 1_R$

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2. Let $f: W \rightarrow W$be defined as $f(n) = n - 1$, if is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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3. If $f: R \rightarrow R$is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

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4. Show that the function $f: R \rightarrow \{x \in R: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one- one and onto function .

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5. Show that the function $f: R \rightarrow R$given by $f(x) = x^3$is injective.

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6. Give examples of two functions $f: \quad N \rightarrow Z \quad and g: \quad Z \rightarrow Z$ such that o f is injective but is not injective. (Hint: Consider $f(x) \quad = \quad x \quad and g(x) \quad = \quad |x|$)

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7. Given examples of two functions $f: \quad N \rightarrow N \quad andg: \quad N \rightarrow N$ such that of is onto but f is not onto. (Hint: Consider $f(x) \quad = \quad x \quad andg(x) \quad = \quad |x|$).

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8. Given a non-empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows: For subsets A, B in P(X), ARB if and only if A B. Is R an equivalence relation on P(X)? Justify you answer

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9. Given a non-empty set X, consider the binary operation $\cdot : P(X) \times P(X) \rightarrow P(X)$given by $A \cdot B = A \cap B \forall A, B \in P(X)$is the power set of X. Show that X is the identity element for this operation and X is the only invertible element i

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10. Find the number of all onto functions from the set $\{1, 2, 3, , n)$to itself.

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11. Let $S = \{a, b, c\} and T = \{1, 2, 3\}$. Find $F^{-1}$of the following functions F from S to T, if it exists.(i $)F = \{(a, 3), (b, 2), (c, 1)\}$(ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

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12. Consider the binary operations $\cdot : R \times R \rightarrow R$ and $o: R \times R \rightarrow R$ defined as $a \cdot b = |a - b|$ and $aob = a$ for all $a, b \in R$. Show that $\cdot$ is commutative but not associative, o is associative but not commutative. Further, show that $\cdot$ is distributive over o . Dose o distribute over $\cdot$? Justify your answer.

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13. Given a non -empty set X, let $\cdot : \quad P(X) \quad \times \quad P(X) \rightarrow P(X)$ be defined as A * B = (A B) ∪ (B A), ∀ A, B ∈ P(X) $A \quad \cdot \quad B \quad = \quad (A \quad - \quad B) \cup (B \quad - \quad A), \forall A, \quad B \in P(X)$.

Show that the empty set φ is the identity for the

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14. Define a binary operation * on the set $A = \{0, 1, 2, 3, 4, 5\}$ as $a \cdot b = a + b$ (mod 6). Show that zero is the identity for this operation and each element a of the set is invertible with $6 - a$ being the inverse of a. OR A binary operation * on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as $a \cdot b = \{a + b, \quad \text{if} \quad a + b < 6a + b - 6, \quad \text{if} \quad a + b \geq 6$ Show that zero is the identity for this operation and each element a of set is invertible with $6 - a,$ being the inverse of a.

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15. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \rightarrow B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \left(\frac{1}{2}\right)\right| - 1, x \in A$. Are f and g equal? Justify your answer.

(Hint: One may note that two functio

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16. Let $A = \{1, 2, 3\}$ Then number of relations containing $(1, 2) and (1, 3)$ which are reflexive and symmetric but not transitive is

A. 1

B. 2

C. 3

D. 4

Answer: A

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17. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is (A) 1 (B) 2 (C) 3 (D) 4

A. 1

B. 2

C. 3

D. 4

Answer:

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18. Let $f: R \rightarrow R$be the Signum Function defined as $f(x) = \{1, x > 00, x = 0 - 1, x < 1$ and $g: R \rightarrow R$be the Greatest Integer Function given by $g(x) = [x]$, where [x] is greatest integer less than or equal to x. Then does fo

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19. Number of binary operations on the set {a, b} are (A) 10 (B) 16 (C) 20 (D) 8

A. 10

B. 16

C. 20

D. 8

Answer:

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