MATHS

BOOKS - NAGEEN MATHS (HINGLISH)

THREE-DIMENSIONAL GEOMETRY

Solved Example

1. A line makes 30°, 120° and 90° angles from the positive direction of x-axis, y-axis and z-axis respectively. Find its direction cosines.

A. $\frac{\sqrt{3}}{2}, 1, 0$

B. $\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0$

C. $\sqrt{3}, -1, 0$

D. $\frac{\sqrt{3}}{2}, \frac{1}{2}, 0$

Answer: B

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2. Find the direction ratios and direction cosines of a line joining the points $(3, -4, 6)$ and $(5, 2, 5)$.

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3. Show that the points $(2, 3, 4), (-1, -2, 1), (5, 8, 7)$ are collinear.

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4. A vector of magnitude 6 units makes equal makes angles from $OX-$, $OY-$ and OZ-axes. Find the vector.

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5. Find the equation of a line passing through the point $(2, -3, 5)$ and parallel to vector $\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$.

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6. Find the vector and cartesian equation of a line passes through the points $(1, 3, 2)$ and origin.

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7. Find the value of λ. If the points $A(-1, 3, 2), B(-4, 2, -2)$ and $C(5, \lambda, 10)$ are collinear.

A. 2

B. 3

C. 4

D. 5

Answer: D

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8. Find the angle between the lines $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(\hat{i} - \hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

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9. Show that the lines

$\frac{x-1}{1} = \frac{y}{-5} = \frac{z}{3}$ and $\frac{x+1}{7} = \frac{y}{2} = \frac{z-3}{1}$ are

A. perpendicular

B. parallel

C. perpendicular and parallel

D. none of these

Answer: A

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10. Show that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect. Also find the co-ordinates of their point of intersection.

A. $(5, 7, 6)$

B. $(5, -7, 5)$

C. $(5, -7, 6)$

D. $(-4, -7, 6)$

Answer: C

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11. Find the co-ordinates of a point where the line $\frac{x-1}{-2}=\frac{y-2}{3}=\frac{z+5}{-4}$, meets the plane $2x+4-z=3$.

A. $(-3, -1, -1)$

B. $(3, 1, 1)$

C. $(2, -3, -1)$

D. $(3, -1, -1)$

Answer: D

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12. Find the co-ordinates of the foot of perpendicular drawn from point $A(1, 6, 3)$ to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

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13. Find the shortest distance the lines $\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + \hat{j} + \hat{k}\right) - \mu\left(3\hat{i} - 5\hat{j}\right)$

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14. Find the shortest distance between the lines $\frac{x+2}{-4}=\frac{y}{1}=\frac{z-7}{1}$ and $\frac{x+3}{-4}=\frac{y-6}{3}=\frac{9}{2}$

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15. Find the shrotest distance between the lines $\vec{r}=\hat{i}+\hat{j}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu\left(2\hat{i}-\hat{j}+\hat{k}\right)$.

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16. Show that the points $A(0,4,3), B(-1,-5,-3), C(-2,-2,1)$ and $D(1,1,1)$ are coplanar. Also find the equation of the plane in which these points lie.

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17. Find the equation of the plane which cuts the intercepts of length $3, -4$ and 2units on the axes respectively.

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18. Convert the equation of plane $2x - 4y + 3z = 24$ into intercept from and find the intercepts cuts from the axes.

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19. A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the three coordinate axes is constant. Show that the plane passes through a fixed point.

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20. A plane cuts the co-ordinate axes at A, B and C respectively. If the centroid of ΔABC is $(2, -3, 4)$, find the equation of the plane.

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21. Find the vector and equation of a plane which is at a distance of 10 units from origin and normal vector from origin to this plane is $2\hat{i} - \hat{j} + 2\hat{k}$.

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22. The vector equation of a plane is $\vec{r} \cdot \left(3\hat{i} + 2\hat{j} - 6\hat{k}\right) = 56$. Convert it into normal form. Also find the length of perpendicular from origin and direction cosines of normal to the plane.

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23. The co-ordiantes of the foot of perpendicular from origin to a plane are $(1, 2, -3)$. Find the eqution of the plane.

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24. Find the normal form of the plane $2x + 3y - z = 5$. Also find the length of perpendicular from origin and d.c's of the normal to the plane.

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25. Find the vector and cartesian equation of plane which passes through the point $(1, 3, -2)$ and normal to the vector $\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$.

A. $2x + y + 3z = 11$

B. $2x + y - 3z = 11$

C. $2x - y - 3z = 11$

D. $2x - y + 3z = 11$

Answer: B

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26. Find the vector equation of the following plane in product form $\vec{r} = \left(\hat{i} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j}\right) + \mu\left(\hat{i} + 3\hat{j} - \hat{k}\right)$.

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27. Find the cartesian form of the equation of the plane.

$\vec{r} = (\lambda - \mu)\hat{i} + (1 - \mu)\hat{j} + (2\lambda + 3\mu)\hat{k}$

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28. Find the angle between the planes $3x + y + 2z = 1$ and $2x - y + z + 3 = 0$.

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29. Find the value of $'\lambda'$ if the planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)+3=0$ and $\vec{r}.\left(\lambda\hat{i}+2\hat{j}+7\hat{k}\right)=10$ are perpendicular.

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30. Find the equation of a plane passes through the point $(1,\ -2, 4)$ and parallel to the plane $3x-y+3+2z+5=0$.

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31. Find the equation of the plane which passes through the point $(2, 2, 2)$ and through the intersection of the planes $3x-y+2z=4$ and $x+y+z=2$.

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32. Find the vector equation of a plane passing through the intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=6$ and $\vec{r}.\left(2\hat{i}+3\hat{j}+4\hat{k}\right)-5=0$ and through the point $(2, 2, 1)$.

A. $\vec{r}.\left(11\hat{i}+12\hat{j}+13\hat{k}\right)=59$

B. $\vec{r}.\left(11\hat{i}+12\hat{j}-13\hat{k}\right)=59$

C. $\vec{r}.\left(11\hat{i}-12\hat{j}+13\hat{k}\right)=59$

D. $\vec{r}.\left(11\hat{i}-12\hat{j}-13\hat{k}\right)=59$

Answer: A

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33. Find the angle between the line $\vec{r}=\left(2\hat{i}+\hat{j}-\hat{k}\right)+\lambda\left(2\hat{i}+2\hat{j}+\hat{k}\right)$ and the plane $\vec{r}.\left(6\hat{i}-3\hat{j}+2\hat{k}\right)+1=0.$

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34. If the line $\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + m\hat{j} - 2\hat{k}\right)$ is parallel to the plane $\vec{r}.\left(2\hat{i} + \hat{j} + m\hat{k}\right) = 1$ then find the value of m.

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35. Find the equation of a plane passes through the points $(0, 0, 0)$ and $(1, 3, 5)$ and parallel to the line $\frac{x}{-2} = \frac{y}{1} = \frac{z+3}{4}$.

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36. Find ten equation of the plane passing through the point $(0, 7, -7)$ and containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$.

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37. Find the perpendicular distance from the point $(1, -3, 4)$ to the plane $3x - 4y + 12z - 1 = 0$.

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38. Show that the point $(1, 2, 1)$ is equidistant from the planes $\vec{r}.\left(\hat{i}+2\hat{j}-2\hat{k}\right)=5$ and $\vec{r}.\left(2\hat{i}-2\hat{j}+\hat{k}\right)+3=0.$

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39. Show that the distance between planes $2x-2y+z+3=0$ and $4x-4y+2z+5=0 is \frac{1}{6}$

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40. Find the equations of het planes parallel to the plane $x+2y-2z+8=0$ which are at distance of 2 units from the point $(2, 1, 1)$.

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41. Find the co-ordinates of the foot of perpendicular and its perpendicular distance drawn from the point $(1, 3, 4)$ to the plane $2x - y + 2 + 3 = 0$. Also find the image of the point $(1, 3, 4)$ in the plane.

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42. Find the image of the point $(1, \ 2, \ 3)$ in the plane $\mathrm{x}+2\mathrm{y}+4\mathrm{z}=38$

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43. A variable plane is at a constant distance p from the origin and meets the coordinate axes in A, B, C . Show that the locus of the centroid of the tehrahedron $OABCisx^{-2} + y^{-2} + z^{-2} = 16p^{-2}$.

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44. Find the distance of the point $(2, 3, 4)$ from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane $3x + 2y + 2z - 5 = 0$.

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45. Show that the line

$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$

$\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$ are coplanar. Also find the equation of plane in which these lines lie.

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46. Prove that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} and \frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$ are coplanar . Aslo, find the plane containing these two lines.

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47. Find the equation of a plane passing through the parallel lines $\frac{x-3}{1}=\frac{y+2}{-4}=\frac{z}{5}$ and $\frac{z-4}{1}=\frac{y-3}{-4}=\frac{z-2}{7}$ are coplanar. Also find the equation of plane in which these lines lie.

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Exercise 11 A

1. If a line makes angles $90o,\quad 135o,\quad 45o$ with the x, y and z-axes respectively, find its direction cosines.

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2. Can a line make angle $45^\circ, 60^\circ, 120^\circ$ with $x-, y-$ and z-axes respectively.

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3. Find the direction cosines of that line whose direction ratios are as follows :

(i) $1, -2, 2$, (ii) $2, 6, 3$

(iii) $3, 1, -2$

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4. Find the direction cosines of the line joining the following points :

(i) $A(2, -1, 3), B(3, 1.1)$

(ii) $A(2, -1, 2), B(-4, 2, 0)$

(iii) $A(4, 3, -5), B(-2, 1, -8)$

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5. Show that the point $A(2, -3, -4), B(1, 2, 3), C(3, -8, -11)$ are collinear.

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6. Find the angle between those lines whose direction ratios are as follows :

(i) $(2, 3, 6)$ and $(1, 2, 2)$

(ii) $(4, -3, 5)$ and $(3, 4, 5)$

(iii) $(1, 2, 1)$ and $(4, -3, 2)$

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7. Find the angle between the following vectors :

(i) $\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

(ii) $\vec{a} = 6\hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} + 9\hat{k}$

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8. Show that the joint of the points (1,2,3), (4,5,7) is parallel to the join of the points (-4,3,-6),(2,9,2).

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9. If the co-ordinates of four points in space are $A(6, -6, 0), B(-1, -7, 6), C(3, -4, 4)$ and $D(2, -9, 2)$ then show that AB is perpendicular to CD.

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10. If $\vec{r}$ is a vector of magnitude 21 and has direction ratios $2, -3 and 6$, then find $\vec{r}$.

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11. Find the angles which the following vectors, makes form the co-ordinates axes :

(i) $2\hat{i} + \hat{j} + 3\hat{k}$, (ii) $3\hat{i} - 4\hat{j} + 5\hat{k}$

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12. Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$

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13. Prove that the lines whose directioncosines are given by the eqution $l + m + n = 0$ and $3lm - 5mn + 2nl = 0$ are mutually perpendicular.

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14. If the direction cosines of two lines are l_1, m_1, n_1 and l_2, m_2, n_2, then find the direction cosine of a line perpendicular to these lines.

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15. Find the angel between any two diagonals of a cube.

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16. Find the angle between two lines whose direction ratios are proportional to $1, 1, 2 and \left(\sqrt{3}-1\right), \left(-\sqrt{3}-1\right), 4$.

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17. Find the angles of a triangle whose verties are $A(3, 2, 1), B(35, 2)$ and $C(5, -2, 3)$.

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18. If a line makes angles $90o, \quad 135o, \quad 45o$ with the x, y and z-axes respectively, find its direction cosines.

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19. Find the direction cosines of a line which makes equal angles with the coordinate axes.

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20. If a line has the direction ratios $18, 12, 4$, then what are its direction cosines?

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21. Show that the points $(2, 3, 4), (-1, -2, 1), (5, 8, 7)$ are collinear.

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22. Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, 4), (1, 1, 2)$and $(5, 5, 2)$.

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Exercise 11 B

1. Find the vector equation of a lin e passes through the point whose position vector is $\left(2\hat{i}-\hat{j}-\hat{k}\right)$ and parallel to vector $\hat{i}+5\hat{k}$.

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2. Find the vector equation of a line passes through the point $\hat{i}+3\hat{j}+\hat{k}$ and parallel to vector $3\hat{i}-2\hat{j}+k$.

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3. Find the equation of a line passes through the point $(2, 3, 4)$ and whose direction ratios are $3, -1, -2$.

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4. Find the equation of a line parallel to the line $\frac{x-5}{3}=\frac{y+1}{-2}=\frac{z}{1}$ and passes through the point $(0, -1, 2)$.

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5. The cartesian equation of a line is $\frac{x+2}{1} = \frac{y+3}{-2} = \frac{z}{3}$, find its vector equation.

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6. Find the vector equation of the line through $A(3, 4, -7) and B(1, -1, 6)$. Find also, its Cartesian equations.

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7. Find the equation of a line passes through the points whose position vectors are $\left(\hat{i} + 4\hat{j} + \hat{k}\right)$ and $\left(2\hat{i} - \hat{j} + 5\hat{k}\right)$.

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8. Prove that the points $A(2, 0, -3), B(1, -2, -5)$ and $C(3, 2, -1)$ are collinear.

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9. Prove that the points $A(9, -1, 4), B(-1, -3, 2)$ and $C(4, -2, 3)$ are collinear.

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10. Prove that the point $A(1, 2, 3), B(-2, 3, 5)$ and $C(7, 0, -1)$ are collinear.

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11. Find the values of λ ad μ if the points $A(-1, 4, -2), B(\lambda, \mu, 1)$ and $C(0, 2, -1)$ are collinear.

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12. Find the equation of a line passes through the point $\hat{i} + \hat{j} + 5\hat{k}$ and parallel to line joining the points $(2, -4, 1)$ and $(0, 1, 3)$.

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13. The cartesian equation of a line is $6x + 1 = 3y - 2 = 3 - 2x$. Find its direction ratios.

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14. Find the angle between the following pairs of lines

(i) $\vec{i} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 5\hat{j} + \hat{k} + \mu\left(3\hat{i} + 2\hat{j} + 6j\hat{k}\right)$

(ii) $\vec{r} = \hat{i} + \hat{j} + \lambda\left(\hat{i} + 2\hat{j} + \hat{k}\right)$ and $\vec{r} = \mu\left(3\hat{i} + 6\hat{j} + 4\hat{k}\right)$

(iii) $\frac{x-1}{2} = \frac{y-2}{3} - \frac{z-1}{-3}$ and $\frac{x+3}{-1} = \frac{y-5}{8} = \frac{z-1}{4}$

(iv) $\frac{5-x}{-2}=\frac{y+3}{-2}=\frac{z-5}{1}$ and $\frac{x+1}{2}=\frac{2y-3}{4}=\frac{z-2}{1}$

(v) $\frac{x+3}{1}=\frac{y-1}{2}, z=3$ and $\frac{x-1}{-2}=\frac{y+3}{3}=\frac{z+5}{4}$

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15. (i) Show that the line $\frac{x+3}{2}=\frac{y+1}{-1}=\frac{z+3}{3}$ and $\frac{x}{5}=\frac{y-5}{1}=\frac{z-3}{-3}$ are perpendicular.

(ii) Show that the lines $\frac{x-4}{-2}=\frac{y+3}{4}=\frac{z+1}{1}$ are mutually perpendicular.

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16. Find the values of λ if the following of lines perpendicular :

$\frac{1-x}{3}=\frac{7y-14}{3\lambda}=\frac{z+1}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y}{1}=\frac{1-z}{5}$

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17. Show that the following pairs of lines intersect. Also find their point of intersection :

(i) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$

(ii) $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$

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18. Show that the lines

$\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-1}{1}$ and $\frac{x-1}{1-}=\frac{y-1}{-1}=\frac{z-1}{2}$ do not intersect.

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19. Show that the lines $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda\left(3\hat{i}-\hat{j}\right)$ and $\vec{r}=4\hat{i}+\hat{k}+\mu\left(2\hat{i}+3\hat{k}\right)$ intersect. Also find the co[-ordinates of their point of intersection.

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20. Find the co-ordinates of that point at which the lines joining the points $(1, 1, 2)$ and $(3, 5, -1)$ meets the xy-plane.

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21. Find the co-ordinates of that point at which the line joining the points $(-2, 1, 4)$ and $(2, 0, 3)$ meets the $yz-$ plane.

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22. Find the co-ordinates of a point at which the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z+5}{6}$, meets the plane $3x - y + z = 3$.

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23. Find the co-ordinates of a point at which the line $\frac{x+1}{2} = \frac{y-1}{-2} = \frac{z+5}{6}$, meets the plane $x - 2y + 3z = 8$.

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24. Find the co-ordiantes of the foot of perpendicular drawn from the point $(1, 2, 3)$ to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

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25. Find the length and the foot of the perpendicular drawn from the point $(2, -1, 5)$ to the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{x+8}{11}$

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26. Find the co-ordinates of the foot of perpendicular and length of perpendicular drawn from point $\left(\hat{i} + 6\hat{j} + 3\hat{k}\right)$ to the line $\vec{r} = \hat{j} + 2\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$.

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27. Find the image of the point $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

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28. Find the image of the point $(0, 2, 3)$ in the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

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29. Find the image of the point $\left(3\hat{i} - \hat{j} + 11\hat{k}\right)$ in the line $\vec{r} = 2\hat{j} + 3\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$.

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30. Find the shortest distance between the following lines :

(i) $\vec{r} = 4\hat{i} - \hat{j} + \lambda\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and

$\vec{r} = \hat{i} - \hat{j} + 2\hat{k} + \mu\left(2\hat{i} + 4\hat{j} - 5\hat{k}\right)$

(ii) $\vec{r} = -\hat{i} + \hat{j} - \hat{k} + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$ and

$\vec{r} = \hat{i} - \hat{j} + 2\hat{k} + \mu\left(- \hat{i} + 2\hat{j} + \hat{k}\right)$

(iii) $\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$ and

$\frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2}$

(iv) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$

(v) $\vec{r} = \vec{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and

$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(- \hat{i} + \hat{j} - \hat{k}\right)$

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31. Find the co-ordinates of the point at a distance of $\sqrt{5}$units from the point $(1, 2, 3)$ on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$.

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32. Find the co-ordinates of the point at a distance of $\sqrt{14}$ from the mid-point of AB on the line joining the point $A(1, 2, 3)$ and $B(3, 6, 9)$.

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Exercise 11 C

1. Find the equations of the plane passing through the following points :

(i) $A(2, 1, 0), B(3, -2, -2), C(3, 1, 7)$

(ii) $A(1, 1, 1), B(1, -1, 2), C(-2, -2, 2)$

(iii) $A(0, -1, 0), B(2, 1, -1), C(1, 1, 1)$

(iv) $A(1, -2, 5), B(0, -5, -1), C(-3, 5, 0)$

(v) $(4, -1, -1), B(2, 0, 2), C(3, -1, 2)$

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2. Show that the points $A(-1, 4, -3), B(-3, 2, 1), C(3, 2, -5)$ and $D(-3, 8, -5)$ are coplanar. Also find the equation of the plane passing through these points.

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3. Show that the point $A(4, -1, 2), B(-3, 5, 1), C(2, 3, 4)$ and $D(1, 6, 6)$ are coplanar. Also find the equation of the plane passing through these points.

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4. Find the equation of a plane which cuts the interscepts $4, 3$ and -2 units on x, y and z-axes respectively.

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5. Find the equation of a plane passes through the point $(1, 2, 3)$ and cuts equal intercepts on the co-ordinate axes.

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6. Find the equation of a plane passes through the point $(4, 4, 1)$ and the ratio of intercepts cuts on axes from this plane is $2:1:1$.

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7. A plane meets the coordinate axes at A, $BandC$ respectively such that the centroid of triangle ABC is $(1, -2, 3)$. Find the equation of the plane.

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Exercise 11 D

1. Find th equation of the plane which is at distance of 8 units from origin and the perpendicular vector from origin to this plane is $\left(2\hat{i} + \hat{j} - 2\hat{k}\right)$.

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2. Find the equation of the plane which is at a distance of $\sqrt{29}$ units from origin and the perpendicular vector from oiring to this plane is $\left(4\hat{i}-2\hat{j}+3\hat{k}\right)$.

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3. The vector equation of a plane is $\vec{r}.\left(\hat{i}+2\hat{j}+2\hat{k}\right)=12$. Convert it into normal form. Find the d.c.s' of the perpendicular vector drawn from origin to this plane and length of perpendicular.

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4. The vector equation of a plane is $\vec{r}.\left(6\hat{i}-3\hat{j}-\hat{k}\right)+2=0$. Convert it into normal form. Also find the length of perpendicular from origin and the d.c.'s of this perpendicular vector.

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5. Find the equation of a plane which is a distance of 2 units from origin and the d.r's of perpendicular vectors are $2, -1, 2$.

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6. Find the angle between the planes $\vec{r}.\left(\hat{i}+\hat{j}-2\hat{k}\right)=3$ and $\vec{e}r.\left(2\hat{i}-2h*j+\hat{k}\right)=22$

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7. Find the vector equation of the following planes whose Cartesian equations are $x+2y+3z+5=0$

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8. The co-ordiantes of the foot of perpendicular from origin to a plane are $(3, -2, 1)$. Find the equation of the plane.

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9. Find the normal form of the plane $x + 2y - 2z + 6 = 0$. Also find the length of perpendicular from origin to this plane and the d.c.'s of the normal.

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10. Find the d.c.'s of the normal and length of perpendicular from origin to the plane $x = 2$.

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11. In each of the following cases,determine the direction cosines of the normal to the plane ned its distance from the origin: $x + y + z = 1$

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12. Find the coordinates of the foot of perpendicular drawn from origin to the planes: $2x - 3y + 4z - 6 = 0$

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13. Find the coordinates of the foot of perpendicular drawn from origin to the planes: $x + y + z = 1$

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14. Find the vector and Cartesian equation of the plane that passes through the point (1,4,6) and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$.

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15. Find the vector and cartesian equation of a plane which passes through the point $(2, -1, 3)$ and perpendicular to a line whose d.r.'s are

$1, -3, 5.$

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16. Find the Cartesian equtionof the followig plane: `vecr=(lamd-2mu)hati+(3-mu)hatj+(2lamda+mu)hatk.

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17. Convert the equation of the plane $\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(-\hat{i} + \hat{j} + 2\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + \hat{k}\right)$ into scalar product form.

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18. Find the vector equation of the plane passing thrugh the points (2,5,-3),(-2,-3,5),(5,3,-3).

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19. Find the equation of the plane passing through $A(2, 2, -1), B(3, 4, 2) and C(7, 0, 6)$. Also find a unit vector perpendicular to this plane.

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20. Find the cartesian equation of plane passing through the points $(1, 1, 1), (1, -1, 1)$ and $(-7, -3, -5)$.

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21. Find the angle between the folowing planes :-

(i) $\vec{r}.\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=5$ and $\vec{r}.\left(-\hat{i}+\hat{j}\right)=1$

(ii) $\vec{r}.\left(2\hat{i}+3\hat{j}-6\hat{k}\right)=1$ and

$\vec{r}.\left(\hat{i}-2\hat{j}+2\hat{k}\right)+3=0$

(iii) $x+y-2z=3$ and $2x-2y+z+1=0$

(iv) $x+y-z=8$ and $-x+2y+z-1=0$

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22. Find the value of $'\lambda'$ if the following planes are perpendicular.

(i) $2x - 4y + 3z + 1 = 0$ and $x + 2y + \lambda z = 3$

(ii) $\vec{r}.\left(3\hat{i} - 6\hat{j} - 2\hat{k}\right) = 1$ and

$\vec{r}.\left(2\hat{i} + \hat{j} - \lambda\hat{i}\right) = 2$

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23. Find the equation of the plane passes through the point $(2, 3, 5)$ and parallel to the plane $x - 3y + z = 8$.

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24. Find the equation of the plane passes through the point $(1, -3, 1)$ and parallel to the plane $2x + 3y + z = 1$.

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25. Find the equation of the plane passes through the point $(2, 1, -2)$ and parallel to the plane $\vec{r}.\left(3\hat{i}+\hat{j}-\hat{k}\right)=0$.

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26. Find the equation of the plane through the points (2,2,1) and (9,3,6) and $perpendicar \rightarrow thepla \neq$ 2x+6y+6z=1`

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27. Find the equation of a plane passes through the point $(0, 0, 0)$ and perpendicular to each to the planes $x+2y-z=1$ and $3x-4y+z=5$.

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28. Find the equation of the plane passing through the point $(1, 3, 2)$and perpendicular to each of the planes $x+2y+3z=5$and

$3x + 3y + z = 0.$

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29. Find the equation of a plane passing through the intersection of the planes $\vec{r}.\left(\hat{i} + 3\hat{j} - \hat{k}\right) = 5$ and $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 3$ and passes through the point $(2, 1, -2)$.

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30. Find the equation of a plane containing the line of intersection of the planes $x + y + z - 6 = 0 and 2x + 3y + 4z + 5 = 0$ passing through $(1, 1, 1)$.

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31. Find the equation of the plane passing through the intersection of the planes $2x - 3y + z - 4 = 0 and x - y + z + 1 = 0$ and perpendicular to

the plane $x + 2y - 3z + 6 = 0$.

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32. Find the equation of a plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i} - 7\hat{j} + 4\hat{k}\right) = 3$ and $\vec{r}.\left(3\hat{i} - 5\hat{j} + 4\hat{k}\right) + 11 - 0$ and passes through the point $\left(-2\hat{i} + \hat{j} + 3\hat{k}\right)$.

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33. Find the equation of a plane passing through the intersection of the planes $x - 3y + 2z - 5 - 0$ and $2x + y + 3z - 1 = 0$ and passes through the point $(1, -2, 3)$.

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34. Prove that the equaton of a plane through point $(2, -4, 5)$ and the line o lintersection of the planes $\vec{r}.\left(2\hat{i} + 3\hat{j} - \hat{k}\right) = 1$ and

$\vec{r}.\left(3\hat{i}+\hat{j}-2\hat{k}\right)=2$ is $\vec{r}.\left(2\hat{i}+8\hat{j}+7\hat{k}\right)=7.$

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35. Find the vector equation to the plane through the point $(2, 1, -1)$ passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right)=0ad\vec{r}.\left(\hat{j}+2\hat{k}\right)=0$

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Exercise 11 E

1. Find the angle between the following lines and the planes :

(i) line $\vec{r}=\left(\hat{i}+2\hat{j}-\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and planes $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)=4.$

(ii) line $\vec{r}=\left(2\hat{i}+3\hat{j}+9\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ and plane $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=5.$

(iii) line $\frac{x+1}{3}=\frac{y}{2}=\frac{z}{4}$ and plane $2x+y-3z=5$.

(iv) line $\frac{x-3}{9}=\frac{y+4}{6}=\frac{z+2}{2}$ and plane $3x-y+z=0$.

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2. Find the value of $'m'$ for which the line $\vec{r}=\left(\hat{i}+2\hat{j}-\hat{k}\right)+\lambda\left(2\hat{i}+\hat{j}+\hat{k}\right)$ is parallel to the plane $\vec{r}.\left(3\hat{i}-2\hat{j}+m\hat{k}\right)=5$.

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3. Find the value of 'm' for which the line $\vec{r}=\hat{i}+\lambda\left(2\hat{i}-m\hat{j}-3\hat{k}\right)$ is parallel to the plane $\vec{r}.\left(m\hat{i}+3\hat{j}+\hat{k}\right)=1$.

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4. Find the Cartesian equation of the plane passing through the points $A\ (0,0,0)$ and $b(3,\ -1,2)$ and parallel to the line

$$\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$$

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5. Find the equation of the plane passes through the point $(2, 3, -4)$ and $(1, -1, 3)$ and parallel to x-axis.

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6. Find the equation of a line passing through the point (1,2,3) and perpendicular to the plane $\vec{r}.\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=1.$

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7. The equation of the line passing through $(1, 2, 3)$ and parallel to the planes $x-y+2z=5$ and $3x+y+z=6$ is.

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8. Find the perpendicular distance from the point $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$ to the plane $\vec{r}.\left(3\hat{i}-4\hat{j}+12\hat{k}\right)=1.$

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9. Find the perpendicular distance from the point $\left(2\hat{i}+\hat{j}-\hat{k}\right)$ to the plane $\vec{r}.\left(i-2\hat{j}+4\hat{k}\right)=3.$

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10. Find the distance of the point $(2,1,0)$ from the plane $2x+y+2z+5=0.$

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11. Find the distance of each of the following points from the corresponding given plane: $(2,3,-5), x+2y-2z=9$

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12. Ifthe points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane, $3x + 4y - 12z + 13 = 0$, then λ satisfiesthe equation

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13. Find the distance between the planes $2x - y + 2z = 4$ and $6x - 3y + 6z = 2$.

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14. Find the distance between the parallel planes

$\vec{r} \cdot \left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = 5$ and

$\vec{r} \cdot \left(6\hat{i} - 9\hat{j} + 18\hat{k}\right) + 20 = 0.$

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15. Find the equation of the plane parallel to the plane x - 2y + 2z - 3 = 0, which is at a unit distance from (1,2,3)

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16. Find the length of the foot of the perpendicular from the point (1,1,2) to the plane $\vec{r}.\left(2\hat{i}-2\hat{j}+4\hat{k}\right)+5=0$

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17. Find the co-ordinates of the foot of perpendicular and the length of perpendicular drawn from the point $(2,3,7)$ to the plane $3x-y-z=7$.

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18. Find the image of the point $(1,3,4)$ in the plane $2x-y+z+3=0$.

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19. Find the image of point $(0, 0, 0)$ in the plnae $3x + 4 - 6z + 1 = 0$.

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20. A variable plane is at a constant distance p from the origin and meets the coordinate axes in A, B, C . Show that the locus of the centroid of the tehrahedron $OABCisx^{-2} + y^{-2} + z^{-2} = 16p^{-2}$.

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21. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$, is

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22. Find the distance of the point (0.-3. -2) from the plane $x + 2y - z = 1$ measured parallel to $\frac{x+1}{2} = \frac{y+1}{2} = \frac{z}{3}$

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23. Find the equation of the plane passing through the intersection of the planes $2x - 3y + z - 4 = 0 and x - y + z + 1 = 0$ and perpendicular to the plane $x + 2y - 3z + 6 = 0$.

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24. Find the equation of the plane through the intersection of the planes $3x - 4y + 5z = 10 and 2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

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25. Find the equation of a line passing through the point $\left(2\hat{i}-3\hat{j}-5\hat{k}\right)$ and perpendicular to the plane $\vec{r}.\left(6\hat{i}-3\hat{j}+5\hat{k}\right)+2=0$. Also find the point of intersection of this line and the plane.

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26. Find the equation of the plane which is at a distance of 5 units fom the origin and perpendiculat to $2\hat{i}-3\hat{j}+6\hat{k}$

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Exercise 11 F

1. Show that the lines $\frac{x-3}{2}=\frac{y+1}{-3}=\frac{z+2}{1}dn\frac{x-7}{-3}=\frac{y}{1}=\frac{z+7}{2}$ are coplanar. Also find the equation of the plane containing them.

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2. Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar.

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3. Find the vector equation of the plane in which the lines $\vec{r}=\hat{i}+\hat{j}+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$ and $\vec{r}=\left(\hat{i}+\hat{j}\right)+\mu\left(-\hat{i}+\hat{j}-2\hat{k}\right)$ lie.

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4. Show that the lines

$\vec{r}=\left(2\hat{j}-3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and

$\vec{r}=\left(2\hat{i}+6\hat{j}+3\hat{k}\right)+\mu\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$

are coplanar. Also the find the equation of the plane passing through these lines.

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5. The equation of the plane which contains two parallel lines $\frac{x+1}{3} = \frac{y-2}{2} = \frac{z}{1}$ and $\frac{x-3}{3} = \frac{y+4}{2} = \frac{z-1}{1}$ is

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Exercise 11 G

1. Show that the line through the points $(1, 1, 2), (3, 4, 2)$ is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

A. 30°

B. 60°

C. 90°

D. 180°

Answer: C

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2. Find the shortest distance between the following pair of line: $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu\left(2\hat{i} + 3\hat{j}\right.$

A. $\frac{4}{\sqrt{19}}$

B. $\frac{3}{\sqrt{19}}$

C. $\frac{2}{\sqrt{19}}$

D. $\frac{1}{\sqrt{19}}$

Answer: B

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3. Find the distance of the plane $2x - 2y + 4z = 6$ from the origin.

A. $\frac{8}{\sqrt{29}}$

B. $\frac{6}{\sqrt{29}}$

C. $\frac{4}{\sqrt{29}}$

D. $\frac{2}{\sqrt{29}}$

Answer: A

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4. A plane meets the coordinate axes at $A, BandC$ respectively such that the centroid of triangle ABC is $(1, -2, 3)$. Find the equation of the plane.

A. $\alpha x + \beta y + \gamma z = 1$

B. $\alpha^2 x + \beta^2 y + \gamma^2 z = 3$

C. $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

D. $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3.$

Answer: D

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5. Find the euqationof the plane which cuts intercepts 2,3,-4 on the axes.

A. $6x + 4y - 3z = 12$

B. $6x - 4y + 3z = 12$

C. $2x - 3y + 4z = 12$

D. None of these

Answer: B

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6. Find the perpendicular distasnce of the point (1,0,0) from the lines (x-1)/2=(y+1)/(-3)=(z+10)/8`

A. $\sqrt{6}$ unit

B. $2\sqrt{6}$ unit

C. $3\sqrt{6}$ unit

D. None of these

Answer: B

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7. Find the image of the point $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

A. $(1, 0, 7)$

B. $(0, 1, 7)$

C. $(7, -1, 0)$

D. $(-7, -1, 0)$

Answer: A

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8. Find the image of the point $(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$.

A. $(3, 5, -2)$

B. $(2, 3, -5)$

C. $(-3, 5, 2)$

D. $(-2, 3, 5)$

Answer: C

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9. The distance between the planes $3x + 5y + z = 8$ and $3x + 5y + z + 27 = 0$ is :

A. $\frac{8}{\sqrt{35}}$

B. $\frac{27}{\sqrt{35}}$

C. $\sqrt{35}$

D. $2\sqrt{35}$

Answer: C

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10. Find the equation of the plane passing through the points $(1, -1, 2)$ and $(2, -2, 2)$ and which is perpendicular to the plane $x - 2y + 2z = 9$

A. $2x + 2y + z = 2$

B. $x + y - 2z + 1 = 0$

C. $x + y - 2z = 4$

D. None of these

Answer: A

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Exercise 11 H

1. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is

A. $x - 2y + 2z - 3 = 0$

B. $x - 2y + 2z + 1 = 0$

C. $zx - 2y + 2z - 1 = 0$

D. $x - 2y + 2z + 5 = 0$

Answer: A

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2. If $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = 1$ and $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9$ then $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right| = ?$

A. 1

B. 2

C. 3

D. 4

Answer: C

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3. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to

A. -5

B. 5

C. 2

D. -2

Answer: A

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4. The vector parallel to the line of intersection of the planes $\vec{r}.\left(3\hat{i}-\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2$ is :

A. $2\hat{i}+7\hat{j}+13\hat{k}$

B. $-2\hat{j}+7\hat{k}+13\hat{k}$

C. $2\hat{i}-7\hat{j}+13\hat{k}$

D. $-2\hat{i}-7\hat{j}+13\hat{k}$

Answer: B

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5. Equation of a plane passing through the intersection of the planes $\vec{r}.\left(3\hat{i}-\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2$ and passing through the point $\left(\hat{i}+2\hat{j}-\hat{k}\right)$ is :

A. $\vec{r}.\left(2\hat{i}-7\hat{j}-13\hat{k}\right)=1$

B. $\vec{r}.\left(2\hat{i}+7\hat{j}+13\hat{k}\right)=1$

C. $\vec{r}.\left(2\hat{i}-7\hat{j}-13\hat{k}\right)=4$

D. None of these

Answer: A

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6. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the coordinate axes at A,B and C respectively. Find the equation of the sphere OABC.

A. $x^2+y^2+z^2+az+by+cz=0$

B. $x^2+y^2+z^2+2az+2by+2cz=0$

C. $x^2+y^2+z^2-ax-by-cz=0$

D. None of these

Answer: C

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7. Equation of the line passing through $(1,1,1)$ and parallel to the plane $2x+3y+z+5=0$ is

A. $\frac{x-1}{-1}=\frac{y+2}{1}=\frac{z-3}{-1}$

B. $\frac{x-1}{3}=\frac{y+2}{2}=\frac{z-3}{-1}$

C. $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-3}{3}$

D. None of above

Answer: A

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8. The perpendicular distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda\left(\hat{i}-\hat{j}+4\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}+5\hat{j}+\hat{k}\right)=5$ is :

A. $\frac{10}{3}$

B. $\frac{5}{3\sqrt{3}}$

C. $\frac{10}{3\sqrt{3}}$

D. $\frac{5}{3}$

Answer: C

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9. Find the vector equation to the plane through the point $(2, 1, -1)$ passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right)=0$ and $\vec{r}.\left(\hat{j}+2\hat{k}\right)=0$

A. $\vec{r}.\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$

B. $\vec{r}.\left(\hat{i}+9\hat{j}+11\hat{k}\right)=6$

C. $\vec{r}.\left(\hat{i}-9\hat{j}-11\hat{k}\right)=0$

D. None of the above

Answer: A

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10. A line makes angles $\alpha, \beta, \gamma$and $\delta$with the diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{5}{3}$

Answer: C

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Exercise 11 2

1. Show that the three lines with direction cosines $\frac{12}{13}, -\frac{3}{13}, -\frac{4}{13}, \frac{4}{13}, \frac{12}{13}, \frac{3}{13}, \frac{3}{13}, -\frac{4}{13}, \frac{12}{13}$ are mutually perpendicular.

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2. Show that the line thorugh the points (1,-1,2) and (3,4,-2)` is perpendicular to the line through the points (0,3,2)andf (3,5,6).

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3. Show that the line through the points $(4, 7, 8), (2, 3, 4)$is parallel to the line through the points $(1, 2, 1),(1, 2, 5)$.

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4. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

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5. Find the equation of the line in Cartesiasn form that passes through the point with positoin vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$.

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6. Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

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7. The cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its vector form.

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8. Find the vector and the cartesian equations of the lines that passes through the origin and $(5, 2, 3)$.

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9. Find the vector and the cartesian equations of the line that passes through the points$(3, 2, 5), (3, 2, 6)$.

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10. Find the angle between the following pairs of lines.

(i) $\hat{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\overrightarrow{r} = 7\hat{i} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

(ii) $\overrightarrow{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\overrightarrow{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

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11. Find the angle between the following pair of lines: (i) $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$ (ii) $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$

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12. Find the values of p so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles.

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13. Show that the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are perpendicular to each other.

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14. Find the shortest distance between the following pair of line: $\vec{r}=\hat{i}+2\hat{j}+\hat{k}+lamd\left(\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu\left(2\hat{i}+\hat{j}+\right.$

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15. Find the shortest distance between the following lines: $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}; \frac{3-x}{-1}=\frac{y-5}{-2}=\frac{z-7}{1}$

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16. Find the shortest distance between the following pair of line: $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and $\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu\left(2\hat{i}+3\hat{j}\right.$

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17. Find the shortest distance between the following pair of line: $\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2$

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Exercise 11 3

1. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.(a) $z = 2$ (b) $x + y + z = 1$ (c) $2x + 3yz = 5$(d) $5y + 8 = 0$

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2. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$

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3. Find the Cartesian equations of the following planes whose vector equations are: $\vec{r}.\left[(s-2t)\hat{i} + 93 - t\right)\hat{j} + (2s+t)\hat{k}\Big] = 15$

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4. In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.(a) $2x + 3y + 4z12 = 0$ (b) $3y + 4z6 = 0$(c) $x + y + z = 1$ (d) $5y + 8 = 0$

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5. Find the vector and Cartesian equation of the plane that passes through the point (1,4,6) and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$.

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6. Find the equations of the planes that passes through three points. (a) $(b)(c)((d)(e)1, \quad 1, \quad 1(f)), \quad ((g)(h)6, \quad 4, \quad 5(i)), \quad ((j)(k) \quad 4$ (n) (o) `(p) (q)((r) (s)1," "1," "0(t))," "((

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7. Find the intercepts cut off by the plane $2x + yz = 5$.

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8. Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

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9. Find the equation of the plane through the intersection of the planes $3x \quad y \quad + \quad 2z \quad 4 \quad = \quad 0$ and $x \quad + \quad y \quad + \quad z \quad 2 \quad = \quad 0$ and the point (2, 2, 1).

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10. Find the vector of the plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 7, \vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) = 9$ and the point (2,1,3)`.

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11. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$

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12. Find the direction cosines of the unit vector perpendcular to the plane $\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{i} - 3\hat{j} + 5\hat{k}\right) = 3$

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13. In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them. (a) $7x + 5y + 6z + 30 = 0$ and $3x \quad y \quad 10z + 4 = 0$ (b) `2x" "+"

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14. In the following cases, find the distance of each of the given points from the corresponding given plane.

	Point	Plane
(a)	(0, 0, 0)	$3x - 4y + 12z = 3$
(b)	(3,–2, 1)	$2x - y + 2z + 3 = 0$
(c)	(2, 3, –5)	$x + 2y - 2z = 9$
(d)	(–6, 0, 0)	$2x - 3y + 6z - 2 = 0$

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Miscellaneous Exercise

1. Show that the line joining the origin to the point $(2, 1, 1)$ is perpendicular to the line determined by the points $(3, 5, -1)$ and $(4, 3, -1)$.

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2. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$.

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3. Find the angle between the lines whose direction ratios are a, b, c and bc, ca, ab.

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4. Find the equation of a line parallel to x axis and passing through the origin.

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5. If the coordinates of the points $A, B, C, Dbe91, 2, 3), (4, 5, 7), (-4, 3, -6)$ and $(2, 9, 2)$ respectively then find the angle between AB and CD.

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6. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k.

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7. Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\rightarrow r\hat{i} + 2\dot{\hat{j}} - 5\hat{k} + 9 = 0$.

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8. Find the equation of the plane passing through (a,b,c) and paralle toteh plne $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=2.$

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9. Find the shortest distance between lines

$\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$ and

$\vec{r}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right).$

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10. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4,1) crosses the YZ-plane.

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11. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX-plane.

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12. find the coordinates of point where the line through (3,-4,-5) and (2,-3,1) crosses the plane $2x + y + z = 7$.

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13. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$

A. $7x - 8y - 3z + 25 = 0.$

B. $7x - 8y + 3z - 25 = 0.$

C. $7x + 8y + 3z + 25 = 0.$

D. $7x - 8y + 3z + 25 = 0.$

Answer: D

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14. If the points $(1, \quad 1, \quad p) \quad and \quad (\quad 3, \quad 0, \quad 1)$ be equidistant from the plane $\rightarrow r3\hat{i} + 4\hat{j} - 12\hat{k} + 13 = 0$, then find the value of p.

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15. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ and $\vec{r}.\left(2\hat{i} + 3\hat{j} - \hat{k}\right) + 4 = 0$ and parallel to x-axis.

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16. If O be the origin and the coordinates of P be (1,2,-3) then find the equation of of the plane passing through P and perpendicular to OP.

A. $x + 2y - 3z - 14 = 0$

B. $x + 2y - 3z + 14 = 0$

C. $2x + 2y - 3z - 28 = 0$

D. none of these

Answer: A

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17. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0, \vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$

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18. Find the distance of the point $(-1, -5, -10)$ from the point of the intersection of the line $\vec{r} = 2\hat{i} - 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$.

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19. Find the vector equation o the line passing through (1,2,3) and parallel to the planes $\vec{r}.\left(\hat{i} + \hat{j} + 2\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6$

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20. Find the vector equation of the line passing through the point $(1, 2, 4)$ and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

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21. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

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22. Find the distance between the planes $2x+3y+4z=4$ and $4+6y+8z=12$.

A. $\frac{4}{\sqrt{29}}$ units

B. 4 units

C. 8 units

D. $\frac{2}{\sqrt{29}}$ units

Answer: D

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23. The planes: $2xy + 4z = 5 and 5x2.\ 5y + 10z = 6$are(A) Perpendicular (B) Parallel(C) intersect y-axis (D) passes through $\left(0, 0, \frac{5}{4}\right)$

A. perpendicular

B. parallel

C. intersect y-axis

D. passes through $\left(0, 0, \frac{5}{4}\right)$

Answer: B

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