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India's Number 1 Education App

MATHS

JEE (MAIN AND ADVANCED) MATHEMATICS

BINOMIAL THEOREM

EXAMPLE

1. Find the number of terms in the expansion of $(x + 2y)^{20} + (x - 2y)^{20}$

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2. The number of terms in the the expansion of $(x + a)^{20} + (x - a)^{20} + (x + ai)^{20} + (x - ai)^{20}$

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3. Find the coefficient of x^{15} in the expansion of $\left(2x^{12} - \frac{3}{x^3}\right)^5$

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4. How many middle terms are there in the expansion of $(2x + 3y)^{128}$

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5. How many middle terms $(x + 2z)^{4n+1}$ possesses ?

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6. The number of terms in $(2x + 3y + z - w)^{20}$ is

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7. The coefficient of $x^2y^3z^4$ in $(2x + y - z)^9$ is

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8. $n^2 - 1$ is divisible by 8, if n is

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9. Prove that : Find the 5^{th} term in the expansion of $(3x - 4y)^7$.

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10. Prove that : Find the 4^{th} term from the end in the expansion $(2a + 5b)^8$

.

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11. Find the number of terms in the expansion of $(4x - 7y)^{49} + (4x + 7y)^{49}$

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12. Find the coefficient of x^2 in the expansion of $\left(7x^3 - \frac{2}{x^2}\right)^9$.

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13. Find the middle terms are are there in the expansion of $(3a - 5b)^6$?

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14. Find the middle terms are in the expansion of $(2x + 3y)^7$

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15. Find the largest binomial coefficients in the expansion of $(1 + x)^{19}$

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16. Find the largest binomial coefficients in the expansion of $(1 + x)^{24}$

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17. Prove that : Find the numerically greatest term in the binomial expansion of $(1 - 5x)^{12}$ when $x = \frac{2}{3}$.

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18. Find the numerically greatest term in the expansion of $(3x - 5y)^{17}$ when $x = \frac{3}{4}, y = \frac{2}{7}$.

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19. Find the value of $\sum_{i=1}^{n}\sum_{i=1}^{n}\sum_{k=1}^{n} 1$

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20. Find the value of $\sum_{i=1}^{n}\sum_{i=1}^{n}\sum_{k=1}^{n} k$

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21. Find the value of $\sum_{1\le I,}\sum_{j\le n}(1)$.

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22. Find the value of $\sum_{1\le i\le}\sum_{j\le n}(1)$.

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23. Find the value of $\sum_{1 \le i}, \sum_{j \le n}(1)$.

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24. Find $\sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{k=1}^{n} (ijk)$

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25. $\sum_{1 \le i} \sum_{< j \le n} ij = \frac{\left(\sum_{i=1}^{n} i\right)^2 - \left(\sum_{i=1}^{n} i^2\right)}{2}$

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26. $\sum_{0 < i <} \sum_{j \le n} \left(C_i + C_j\right) = (n) \cdot 2^n$

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SOLVED EXAMPLES

1. Find the number of terms in the expansion of $(2a + 3b + c)^5$

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2. Find the coefficient of x^7 in $\left(1 + x^2\right)^4(1 + x)^7$

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3. Find the coefficient of

x^{-7} in $\left(\frac{2x^2}{3} - \frac{5}{4x^5}\right)^7$

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4. Prove that : Find the coefficient of x^6 in $\left(3 + 2x + x^2\right)^6$.

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5. Prove that : Find the coefficient of x^6 in $\left(3 + 2x + x^2\right)^6$.

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6. Find the coefficient of x^7 in $\left(1 - x - x^2 + x^3\right)^6$

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7. If the coefficients of $(2r + 4)^{th}$ term and $(3r + 4)^{th}$ term in the expansion of $(1 + x)^{21}$ are equal, find r.

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8. If the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ equals the coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$, then a and b satisfy the relation

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9. Find the term independent of x in the expansion of $\left(\sqrt{\frac{x}{7}} - \frac{\sqrt{5}}{x^2}\right)^{10}$

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10. the tern independent of x in $\left(1 + x + 2x^2\right)\left(3x^2/2 - 1/3x\right)^9$ is

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11. Find the middle term (s) in the expansion of

$\left(4a + \frac{3}{2}b\right)^{11}$

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12. Show that the middle term in the expansion of $(1 + x)^{2n}$ is $\frac{1.3.5...(2n - 1)}{n!}(2x)^n$.

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13. Find the numerically greatest terms in the expansion of $(3 + 2a)^{15}$ when $a = \frac{5}{2}$

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14. If $\left(3 + 7x - 9x^2\right)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$ prove the $a_0 + a_1 + a_2 + ... + a_{2n} = 1$

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15. If $\left(3 + 7x - 9x^2\right)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{2n} x^{2n}$ prove the $a_0 + a_2 + a_4 + \ldots\ldots = \frac{1 + (-13)^n}{2}$

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16. If $\left(1 + 3x - 2x^2\right)^{10} = a_0 + a_1 x + a_2 x^2. + \ldots + a_{20} x^{20}$ then prove that $a_0 + a_1 + a_2 + \ldots\ldots + a_{20} = 2^{10}$

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17. If $\left(1 + 3x - 2x^2\right)^{10} = a_0 + a_1 x + a_2 x^2. + \ldots + a_{20} x^{20}$ then prove that $a_0 - a_1 + + a_2 - a_3 + \ldots\ldots + a_{20} = 4^{10}$

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18. If $\left(1 + 3x - 2x^2\right)^{10} = a_0 + a_1 x + a_2 x^2 + \ldots . + a_{20} x^{20}$ thn prove that $a_0 + a_2 + a_4 + \ldots . + a_{20} = 2^9 + 2^{19}$

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19. If $\left(1 + 3x - 2x^2\right)^{10} = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{20}x^{20}$ thn prove that $a_1 + a_3 + a_5 + \ldots\ldots + a_{19} = 2^9 - 2^{19}$

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20. If the $2^{nd}, 3^{rd}$ and 4^{th} terms in the expansion of $(a + x)^n$ are respectively 240, 720, 1080, find a, x, n.

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21. If the coefficients of x^9, x^{10}, x^{11} in expansion of $(1 + x)^n$ are in A.P., the prove that $n^2 - 41n + 398 = 0$.

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22. If n is a positive integer, then prove that $81^n + 20n - 1$is divisible by 100`

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23. Using binomial theorem, prove that $5^{4n} + 52n - 1$ is divisible by 676 for all positive integers n.

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24. $3.\ C_0 + 7.\ C_1 + 11.\ C_2 + \ldots\ldots + (4n + 3).\ C_n =$

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25. With usual notations find the value of $3.\ C_1 + 5.\ C_2 + 7.\ C_3 + \ldots\ldots\ldots + n$ terms

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26. With usual notations prove that

$$C_1 + 2.\,C_2x + 3.\,C_3x^2 + \ldots\ldots + 2n.\,C_{2n}x^{2n-1} = 2n(1+x)^{2n-1}$$

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27. Prove that

$$\frac{^{20}C_1}{^{20}C_0} + 2.\frac{^{20}C_2}{^{20}C_1} + 3.\frac{^{20}C_3}{^{20}C_2} + \ldots\ldots + 20.\frac{^{20}C_{20}}{^{20}C_{19}} = 210$$

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28. If n is a positive integer and $C_4 = {}^nC_r$ then find the value of

$$\sum_{x=1}^{n} r^2\left(\frac{C_r}{C_{r-1}}\right)$$

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29. Prove that $\sum_{r=1}^{n+1} \frac{2^{r+1} C_{r-1}}{r(r+1)} = \frac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$

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30. Prove that $\sum_{r=0}^{n} r^2 . C_r = n(n+1).2^{n-2}$

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31. Prove that $\sum_{r=0}^{n} r^3 . C_r = n^2(n+3).2^{n-3}$

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32. If C_r denotes ${}^nC_r$ then show that

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3}x^2 + \ldots\ldots\ldots + C_n \cdot \frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

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33. If $C_0, C_1, C_2, \ldots\ldots . C_n$ are the coefficient in the expansion of $(1 + x)^n$

then show that

$$C_0C_r + C_1C_{r+1} + C_2C_{r+2} + \ldots\ldots\ldots . + C_{n-r}. C_n = \frac{(2n)!}{(n - r)!(n + r)!}$$

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34. With usual notations prove that

$$C_0 + 3. C_1 + 3^2. C_2 + \ldots\ldots\ldots . + 3^n. C_n = 4^n$$

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35. With usual notations prove that

$$\frac{C_1}{C_0} + 2. \frac{C_2}{C_1} + 3. \frac{C_3}{C_2} + \ldots\ldots + n. \frac{C_n}{C_{n-1}} = \frac{n(n + 1)}{2}$$

Hence prove that

$$\frac{15C_1}{15C_0} + 2. \frac{15C_2}{15C_1} + 3. \frac{15C_3}{15C_2} + \ldots\ldots . + 15. \frac{15C_{15}}{15C_{14}} = 120$$

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36. Prove that

$$\left(C_0 + C_1\right)\left(C_1 + C_2\right)\ldots.\left(C_{n-1} + C_n\right) = \frac{(n+1)^n}{n!}\left(C_1.C_2.C_3\ldots\ldots C_n\right)$$

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37. Prove that $\sum_{r=1}^{n} r^3\left(\frac{C_r}{\left(C_{r-1}\right)^2}\right) = \frac{n(n+1)^2(n+2)}{12}$

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38. Prove that following

$$C_0 + \frac{3}{2}.C_1 + \frac{9}{3}.C_2 + \frac{27}{4}.C_3 + \ldots\ldots + \frac{3^n}{n+1}.C_n = \frac{4^{n+1}-1}{3(n+1)}.$$

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39. Find the range of x for which the binomial expansion of the following are valid

$(3 - 4x)^{-5/2}$

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40. Find the range of x for which the binomial expansion of the following are valid

$(7 + 3x)^{-3}$

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41. Find the range of x for which the binomial expansion of the following are valid

$(3 + 4x)^{7}$

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42. Find the range of x for which the binomial expansion of the following are valid

$(9 + 5x)^{3/2}$

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43. Find the set of values of x for which the binomial expansions of the following are valid.

$(5 + x)^{3/2}$

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44. Find the set of values of x for which the binomial expansions of the following are valid.

$(2 + 3x)^{-2/3}$

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45. Write down the first three terms in the expansion of $(3 + x)^{-3/2}$

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46. Write down the first three terms in the expansion of $(4 - 5x)^{-1/2}$

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47. Write down and simplify 5th term in the expansion of $(6 + 7x)^{-2}$

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48. Write down and simplify 7th term in the expansion of $(2 + 3x)^{-5}$

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49. Prove that : Find the

10^{th} term of $\left(1 - \frac{3x}{4}\right)^{4/5}$

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50. Prove that : Find the

8^{th} term of $\left(1 - \frac{5x}{2}\right)^{-3/5}$

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51. 6th term of $\left(3 + \frac{2x}{3}\right)^{3/2}$

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52. Prove that : Write the general term in the expansion of

$(1 - 4x)^{-3}$

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53. Write the general term in the expansion of $\left(3 - \frac{5x}{4}\right)^{-1/2}$

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54. Find the coefficient of x^5 in $(3 - 4x)^{-1}$

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55. Find the coefficient of x^4 in $(8 - x)^{-1/3}$

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56. Find the coefficient of x^{12} in $\frac{(1 + 3x)}{(1 - 4x)^4}$

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57. If |x| is so small that x^4 and higher powers of x many be neglected , then find an approximate value of $\sqrt[4]{x^2 + 81} - \sqrt[4]{x^2 + 16}$

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58. If |x| is so small that x^2 and higher powers of x many be neglected , prove that $\frac{(1+7x)^{\frac{2}{3}}.(1-4x)^{-2}}{(4+7x)^{\frac{1}{2}}} = \frac{1}{2}\left(1+\frac{283}{24}x\right)$

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59. If |x| is so small that x^2 and higher powers of x many be neglected , prove that $\sqrt{9-2x}\left(3+\frac{4x}{5}\right)^{-1} = 1-\frac{17}{45}x$

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60. If x is so small that x^2 and higher powers of x many be neglected , show that $\frac{\left(1+\frac{3x}{2}\right)^{-4}(8+9x)^{1/3}}{(1+2x)^2} = 2-\frac{77x}{4}$

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61. If x is so small that x^2 and higher powers of x many be neglected , show that $\frac{\sqrt{4+x}+\sqrt[3]{8+x}}{(1+2x)+(1-2x)^{-1/3}} = 2 - \frac{5x}{2}$

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62. Find the value of $(1.02)^{3/2} - (0.98)^{3/2}$ correct to 6 decimals .

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63. Find the value of $\sqrt[5]{32.16}$ correct to 4 decimals places

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64. Find the value of $(627)^{1/4}$ correct to 4 decimals places

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65. Show that $+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{4.8.12}+\ldots\ldots=\sqrt{2}$

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66. Show that $1+\frac{1}{2.3}/5+\frac{1.3}{2.4}\left(\frac{3}{5}\right)^2+\frac{1.3.5}{2.4.6}\left(\frac{3}{5}\right)^3+\ldots\ldots=\sqrt{\left[\frac{5}{2}\right]}$

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67. Sho that $\frac{3}{6}+\frac{3.5}{6.9}+\frac{3.5.7}{6.9.12}+\ldots\ldots.\infty 3\sqrt{3}-4$

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68. Show that $1-\frac{3}{4}+\frac{3.5}{4.8}-\frac{3.5.7}{4.8.12}+\ldots\ldots.\sqrt{\left(\frac{8}{27}\right)}$

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69. Show that $\frac{4.6}{5.10} + \frac{4.6.8}{5.10.15} + \frac{4.6.8.10}{5.10.15.20} + \ldots\ldots = \frac{44}{45}$

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70. Show that $\frac{5}{6.12} + \frac{5.8}{6.12.18} + \frac{5.8.11}{6.12.18.24} + \ldots. = \frac{3\sqrt[3]{4} - 4}{6}$

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71. Show that $\frac{4}{12.18} - \frac{4.7}{12.18.24} + \frac{4.7.10}{12.18.24.30} - \ldots. = 3\sqrt[3]{\frac{9}{4}} - \frac{47}{12}$

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72. If $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \ldots.$ then prove that $9x^2 + 24x = 11$

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73. If $t = 1 - \frac{1}{8} + \frac{1.3}{8.16} - \frac{1.3.5}{8.16.24} + \ldots\ldots$ then prove that $5t^2 = 4$

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74. If n is a non zero rational number then show that

$$1 + \frac{n}{2} + \frac{n(n-1)}{2.4} + \frac{n(n-1)(n-2)}{2.4.6} + \ldots\ldots = 1 + \frac{n}{3} + \frac{n(n+1)}{3.6} + \frac{n(n+1)(n+2)}{3.6.9} +$$

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75. Prove that : Find the 7^{th} term in the expansion of $\left(\frac{4}{x^3} + \frac{x^2}{2}\right)^{14}$.

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76. Prove that : Find the 3^{rd} term from the end in the expansion of $\left(x^{-2/3} - \frac{3}{x^2}\right)^8$.

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77. Prove that : Find the coefficient of x^9 and x^{10} in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$.

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78. Prove that : Find the term independent of x (that is the constant term) in the expansion of $\left(\frac{\sqrt{x}}{3} + \frac{3}{2x^2}\right)^{10}$

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79. Prove that : If the coefficients of $(2r + 4)^{th}$ and $(r - 2)^{nd}$ terms in the expansion of $(1 + x)^{18}$ are equal, find r.

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80. Find the numerically greatest terms in the expansion of $(2x + 3x)^{10}$ when $x = \frac{11}{8}$

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81. Find the numerically greatest terms in the expansion of $(3x - 4y)^{14}$ when x = 8,y = 3

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82. Prove that : Suppose that n is a natural number and I, F are respectively the integral part and fractional part of $\left(7 + 4\sqrt{3}\right)^n$. Then show that

(i) I is an odd integer

(ii) $(I + F)(I - F) = 1$

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83. Prove that : Find the coefficient of x^6 in $\left(3 + 2x + x^2\right)^6$.

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84. The term independent of x in the expansion of $\left(1 + x + 2x^3\right)\left(3x^2/2 - 1/3x\right)^9$ is

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85. Find the number of terms free of radical sign in $\left(5^{1/2} + 7^{1/5}\right)^{220}$. Also find the number of irrational terms.

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86. Show that the coefficient of $x^k (0 \leq k \leq n)$ in the expansion of $1 + (1 + x) + (1 + x)^2 + \ldots + (1 + x)^n$ is ${}^{n+1}C_{K+1}$.

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87. For natural numbers m, n if $(1 - y)^m(1 + y)^n = 1 + a_1y + a_2y^2 + \ldots,$ and $a_1 = a_2 = 10,$ then (m, n) is

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88. Prove that : Find the coefficient of x^6 in $\left(3 + 2x + x^2\right)^6$.

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89. Find the number of distinct terms in the expansion of $\left(x + \frac{2}{x} + 1\right)^{20}$

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90. Find the coefficient of $a^5b^3c^4$ in the expansion of $\left(ab + \frac{bc}{2} - \frac{ca}{3}\right)^6$

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91. Prove that x^n - y^n is divisible by x - y for all positive integers n.

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92. Find the remainder when 7^{103} is divided by 5.

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93. Find the remainder when $32^{32^{32}}$ is divided by 7.

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94. Find the (i) number of rational terms

(ii) Largest rational term in the expansion of $\left(2^{1/5}+\sqrt{3}\right)^{20}$

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95. If the coefficients of x^9, x^{10}, x^{11} in expansion of $(1+x)^n$ are in A.P., the prove that $n^2 - 41n + 398 = 0$.

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96. Prove that : Prove that

$2.C_0 + 7.C_1 + 12.C_2 + \ldots\ldots. + (5n+2)C_n = (5n+4)2_{n-1}$.

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97. Prove that : Prove that

$C_0 + 3.C_1 + 3_2.C_2 + \ldots\ldots + 3^n.C_n = 4^n$

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98. Prove that : Prove that

$$\frac{C_1}{C_0} + 2.\frac{C_2}{C_1} + 3.\frac{C_3}{C_2} + \ldots. + n.\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$$

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99. With usual notations prove that

$$C_1 + 2.\,C_2x + 3.\,C_3x^2 + \ldots\ldots + 2n.\,C_{2n}x^{2n-1} = 2n(1+x)^{2n-1}$$

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100. $$\frac{{}^{20}C_1}{{}^{20}C_0} + 2.\frac{{}^{20}C_2}{{}^{20}C_1} + 3.\frac{{}^{20}C_3}{{}^{20}C_2} + \ldots 20.\frac{{}^{20}C_{20}}{{}^{20}C_{19}} = 210$$

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101. If n is a positive integer and $C_r = {}^nC_r$ then find the vlaue of

$$\sum_{r=1}^{n} r^2\left(\frac{C_r}{C_{r-1}}\right).$$

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102. Prove that $5.C_0 + 5^2.\frac{C_1}{2} + 5^3.\frac{C_2}{3} + \ldots + 5^{n+1}.\frac{C_n}{n+1} = \frac{6^{n+1}-1}{n+1}$

Hence show that $5.C_0 + \frac{5^2}{2}.C_1 + \frac{5^3}{3}.C_2 + \ldots. + \frac{5^{11}}{11}.C_{10} = \frac{6^{11}-1}{11}$

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103. Prove that $1 - {}^nC_1\frac{1+x}{1+nx} + {}^nC_2\frac{1+2x}{(1+nx)^2}{}^nC_3\frac{1+3x}{(1+nx)^3} + \ldots. (n+1)$ terms = 0

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104. Show that $\sum_{k=0}^{n} C_k \cdot \sin(kx)\cos(n-k)x = 2^{n-1}\sin nx$ where $C_r = {}^nC_r$

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105. If n is a positive integer, prove that

$1 - 2n + \frac{2n(2n-1)}{2!} - \frac{2n(2n-1)(2n-2)}{3!} + \ldots + (-1)^{n-1}\frac{2n(2n-1)\ldots(n+2)}{(n-1)!} = (-1$

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106. If $\left(1+x+x^2\right)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$, then prove that $a_0 + a_1 + a_2 \ldots . + a_{2n} = 3^n$

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107. Find the set E of the vlaues of x for which the binomia expansions for the following are valid $(i)(3 - 4x)^{\frac{3}{4}}$ $(ii)(2 + 5x)^{\frac{-1}{2}}$ $(iii)(7 - 4x)^{-5}$

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108. Prove that : Find the

9^{th} term of $\left(2 + \frac{x}{3}\right)^{-5}$

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109. Find the 9^{th} term of $\left(1 - \frac{3x}{4}\right)^{\frac{4}{5}}$

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110. Prove that : Write the general term in the expansion of

$\left(3 + \frac{x}{2}\right)^{-2/3}$

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111. Prove that : Write the general term in the expansion of

$(2 - 3x)^{-1/3}$

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112. Prove that : Find the coefficient of x^{12} in $\frac{1 + 3x}{(1 - 4x)^4}$.

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113. Find the coefficient of x^6 in the expansion of $(1 - 3x)^{\frac{-2}{5}}$

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114. Prove that : Find the sum of the infinite series

$$1 + \frac{2}{3}.\frac{1}{2} + \frac{2.5}{3.6}\left(\frac{1}{2}\right)^2 + \frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^3 + ...\infty$$

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115. Prove that : Find the sum of the series

$$\frac{3.5}{5.10} + \frac{3.5.7}{5.10.15} + \frac{3.5.7.9}{5.10.15.20} + ... \infty$$

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116. If $x = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + ... \infty$ then find $3x^2 + 6x$.

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117. Find an approximate value of

(i) $\frac{1}{\sqrt[3]{999}}$ (ii) $(627)^{\frac{1}{4}}$ corrected to 5 decimal places

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118. Prove that : If $|x|$ is so small that x^3 and higher powers or x can be neglected, find approximate value of $\frac{(4 - 7x)^{1/2}}{(3 + 5x)^3}$.

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119. Prove that : Find an approximate value of $\sqrt[6]{63}$ correct to 4 decimal places.

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120. If |x| is so small that x^4 and higher powers of x many be neglected , then find an approximate value of $\sqrt[4]{x^2+81} - \sqrt[4]{x^2+16}$

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121. Prove that : Suppose that x and y are positive and x is very small when compared to y. Then find an approximate value of

$$\left(\frac{y}{y+x}\right)^{3/4} - \left(\frac{y}{y+x}\right)^{4/5}$$

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122. Find the coefficiant of x^6 in the expansion of $\left(1+x+x^2+x^3+x^4\right)^6$

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123. Sum the series $\frac{1}{3.6} + \frac{1.3}{3.6.9} + \frac{1.3.5}{3.6.9.12} + \ldots$

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124. If a, b and n are positive find the value of

$$1+\frac{na}{a+b}+\frac{n(n+1)}{2!}\left(\frac{a}{a+b}\right)^2+\ldots$$

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125. Prove that

$$5^n\left(1+\frac{n}{5}+\frac{n(n-1)}{5\cdot 10}+\frac{n(n-1)(n-2)}{5\cdot 10\cdot 15}+\ldots\infty\right)=3^n\left(1+\frac{n}{2}+\frac{n(n+1)}{2\cdot 4}+\frac{n(n+1)(}{2\cdot 4}\right.$$

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126. Show that the value of

$$\binom{2000}{2}+\binom{2000}{5}+\binom{2000}{8}+\ldots\binom{2000}{2000}=\frac{2^{2000}-1}{3}$$

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ADDITIONAL SOLVED EXAMPLES

1. Find the value of $\sum_{i=1}^{n}\sum_{i=1}^{n}\sum_{k=1}^{n} 1$

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2. Find the value of $\sum_{i=1}^{n}\sum_{i=1}^{n}\sum_{k=1}^{n} k$

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3. Find $\sum_{i=1}^{n}\sum_{i=1}^{n}\sum_{k=1}^{n} (ijk)$

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4. Prove that $\sum 1 \le i < \sum j \le n(i) = \frac{n\left(n^2 - 1\right)}{6}$

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5. $\sum 1 \le i \sum < j \le n ij = \frac{\left(\sum_{i=1}^{n} i\right)^2 - \left(\sum_{i=1}^{n} i^2\right)}{2}$

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6. Prove that $\sum 0 \le i \sum < j \le n \left(C_i + C_j\right) = n.2^n$

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7. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots . + C_n . x^n$ then find $C_0 - C_2 + C_4 - C_6 + \ldots\ldots$

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8. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots . + C_n . x^n$ then find $C_1 - C_3 + C_5 + \ldots\ldots$

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9. If $\left(1+x+x^2\right)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots\ldots . C_nx^n$ then find $C_0 + C_3 + C_6 + \ldots\ldots$

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10. If x is nearly equal to 1 then find the value of $\frac{mx^m - nx^n}{m - n}$

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EXERCISE - 3.1 (VERY SHORT ANSWER QUESTIONS)

1. Expand the following using binomial theorem

$(2a + 3b)^6$

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2. Expand the following using binomial theorem

$$\left(\frac{2}{3}x + \frac{7}{4}y\right)^5$$

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3. Expand the following using binomial theorem.

$$\left(\frac{2p}{5} - \frac{3q}{7}\right)^6$$

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4. Expand the following using binomial theorem

$$\left(\frac{2}{3}l - 3m\right)^6$$

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5. Find the number of terms in the expansion of

$(3p + 4q)^{14}$

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6. Find the number of terms in the expansion of

$\left(\frac{3a}{4} + \frac{b}{2}\right)^9$

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7. Find the number of terms in the expansion of $(4x - 7y)^{49} + (4x + 7y)^{49}$

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8. Find the number of terms in the expansion of $(x - 2y + 3z)^{10}$

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9. Find the number of terms in the expansion of

$(2x + 3y + z)^7$

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10. Write down and simplify

14th term in $(3 + x)^{15}$

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11. Write down and simplify

10^{th} term in $\left(\frac{3p}{4} - 5q\right)^{14}$

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12. Write down and simplify

r^{th} term in $\left(\frac{3a}{5} + \frac{5b}{7}\right)^8 \quad (1 \leq r \leq 9)$

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13. Write down and simplify

5th term in $(3x - 4y)^7$

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14. Write down and simplify

7th term in $\left(\frac{4}{x^3} + \frac{x^2}{2}\right)^{14}$

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15. Prove that : Find the 3^{rd} term from the end in the expansion of $\left(x^{-2/3} - \frac{3}{x^2}\right)^8$.

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16. Write down and simplify

Find the 4th term the end in $(2a + 5b)^8$

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17. Write down and simplify

Write down and simplify 6th term in $\left(\frac{2x}{3} + \frac{3y}{2}\right)^9$

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18. Find the coefficient of

x^{-6} in $\left(3x - \frac{4}{x}\right)^{10}$

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19. Find the coefficient of x^{11} in $\left(2x^2 + \frac{3}{x^3}\right)^{13}$

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20. Find the coefficient of x^2 in $\left(7x^3 - \frac{2}{x^2}\right)^9$

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21. Find the coefficient of x^{-8} in $\left(2x^4 - \frac{1}{3x^2}\right)^5$

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22. Find the coefficient of x^3 in $\left(\sqrt{x^5} + \frac{3}{\sqrt{x^3}}\right)^6$

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23. Find the coefficient of x^9 and x^{10} in $\left(2x^2 - \frac{1}{x}\right)^{20}$

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24. Find the independent of x in $\left(\frac{x^{1/2}}{3} - \frac{4}{x^2}\right)^{10}$

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25. Find the independent of x in $\left(\frac{3}{\sqrt[3]{x}}+5\sqrt{x}\right)^{25}$

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26. Find the independent of x in $\left(4x^3+\frac{7}{x^2}\right)^{14}$

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27. Find the term independent of x in the expansion of

$\left(\frac{2x^2}{5}+\frac{15}{4x}\right)^9$

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28. Find the independent of x in $\left(3x^2+\frac{5}{x^3}\right)^{12}$

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29. Find the independent of x in $\left(2x^2 - \frac{3}{x}\right)^9$

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30. Find the independent of x in $\left(\frac{\sqrt{x}}{3} - \frac{4}{x\sqrt{x}}\right)^{12}$

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31. Prove that : Find the term independent of x (that is the constant term) in the expansion of $\left(\frac{\sqrt{x}}{3} + \frac{3}{2x^2}\right)^{10}$

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32. Find the term independent of x in

$$(1 + 3x)^n \left(1 + \frac{1}{3x}\right)^n.$$

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33. Find the middle term (s) in the expansion of $\left(\frac{3x}{7} - 2y\right)^{10}$

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34. Find the middle term (s) in the expansion of $\left(4x^2 + 5x^3\right)^{17}$

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35. Find the middle term (s) in the expansion of $\left(\frac{3}{p^3} + 5p^4\right)^{20}$

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36. Find the middle term (s) in the expansion of $\left(3x^2 + \frac{5}{x^3}\right)^{12}$

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37. Find the middle terms are in the expansion of $(2x + 3y)^7$

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38. If the k^{th} term is the middle term in $\left(x^2 - \frac{1}{2x}\right)^{20}$ find T_k and T_{k+3}

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39. Find the largest binomial coefficients in the expansion of $(1 + x)^{19}$

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40. Find the largest binomial coefficients in the expansion of $(1+x)^{24}$

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41. If ${}^{22}C_r$ is the largest binomial coefficient in the expansion of $(1+x)^{22}$, find the value of ${}^{13}C_r$.

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42. Find the number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$.

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43. Find the sum of last 20 coefficients in the expansions of $(1+x)^{39}$.

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44. If A and B are coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then find the value of $\frac{A}{B}$.

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EXERCISE - 3.1 (SHORT ANSWER QUESTIONS)

1. Prove that : If the coefficients of $(2r+4)^{th}$ and $(r-2)^{nd}$ terms in the expansion of $(1+x)^{18}$ are equal, find r.

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2. Find the sum of the coefficients of x^{32} and x^{-18} in the expansion of $\left(2x^3 - \frac{3}{x^2}\right)^{14}$.

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3. If the coefficient of x^{11} and x^{12} in $\left(2+\frac{8x}{3}\right)^n$ are equal find n

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4. If the coefficient of x^{10} in $\left(ax^2+\frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-10} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ then show that ab = -1

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5. Find the numerically greatest terms in the expansion of $(2x+3x)^{10}$ when $x=\frac{11}{8}$

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6. Find the numerically greatest terms in the expansion of $(3x-4y)^{14}$ when x = 8,y = 3

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7. Find the numerically greatest terms in the expansion of $\left(3+\frac{2x}{5}\right)^{12}$ when x = 3/4`

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8. Find the numerically greatest terms in the expansion of $(3y+7x)^{10}$ when $x=\frac{1}{3}, y=\frac{1}{2}$

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9. Find the numerically greatest term (s) in the expansion of $(4+3x)^{15}$ when $x=\frac{7}{2}$

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10. Find the numerically greatest term (s) in the expansion of

$(3x + 5y)^{12}$ when $x = \frac{1}{2}, y = \frac{4}{3}$

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11. Find the numerically greatest term (s) in the expansion of

$(4a - 6b)^{13}$ when $a = 3, b = 5$

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12. Find the numerically greatest term (s) in the expansion of

$(3 + 7x)^{n}$ when $x = \frac{4}{5}, n = 15$

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13. Find the numerically greatest terms in the expansion of $(1 - 5x)^{12}$ when $x = \frac{2}{3}$

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14. If $\left(1+x+x^2\right)^n = a_0 + a_1x + a_2x^2 + \ldots. + a_{2n}x^{2n}$ then prove that $a_0 + a_1 + a_2 + \ldots\ldots a_{2n} = 3^n$

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15. If $\left(1+x+x^2\right)^n = a_0 + a_1x + a_2x^2 + \ldots. + a_{2n}x^{2n}$ then prove that $a_0 + a_2 + a_4 + \ldots\ldots + a_{2n} = \frac{3^n+1}{2}$

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16. If $\left(1+x+x^2\right)^n = a_0 = a_1x + a_2x^2 + \ldots. + a_{2n}x^{2n}$ then prove that $a_1 + a_3 + a_5 + \ldots. a_{2n-1} = \frac{3^n-1}{2}$

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17. If $\left(1+x+x^2\right)^n = a_0 + a_1 x + a_2 x^2 + \ldots. + a_{2n} x^{2n}$, then prove that

$a_0 + a_3 + a_6 + a_9 + \ldots\ldots = 3^{n-1}$

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18. If $\left(1+x+x^2+x^3\right)^7 = b_0 + b_1 x + b^2 x^2 + \ldots. b_{21} x^{21}$, then find the value of

$b_0 + b_2 + b_4 + \ldots\ldots + b_{20}$

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19. If $\left(1+x+x^2+x^3\right)^7 = b_0 + b_1 x + b^2 x^2 + \ldots. b_{21} x^{21}$, then find the value of

$b_1 + b_3 + b_5 + \ldots\ldots + b_{21}$

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20. Find the remainder when 2^{2013} in divided by 17.

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21. Find the number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$.

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EXERCISE - 3.1 (LONG ANSWER QUESTIONS)

1. If 36, 84, 126 are three successive binomial coefficients in the expansion of $(1+x)^n$, find n.

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2. If the coefficients of 4 consecutive terms in the expansion of $(1+x)^n$ are a_1, a_2, a_3, a_4 respectively, then show that

$$\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$$

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3. If the coefficient of $r^{th}, (r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1+x)^n$ are in A.P then show that $n^2-(4r+1)n+4r^2-2=0$

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4. If P and Q are the sum of odd terms and the sum of even terms respectively in the expansion of $(x+a)^n$ then prove that

$$P^2-Q^2=\left(x^2-a^2\right)^n$$

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5. If P and Q are the sum of odd sum terms and the sum of even terms respectively in the expansion of $(x + a)^n$ then prove that $4PQ = (x + a)^{2n} - (x - a)^{2n}$

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6. Using binomial theorem prove that $50^n - 49n - 1$ is divisible by 49^2, $\forall n \in N$

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7. If I, n are positive integers , $0 < f < 1$ and if $\left(7 + 4\sqrt{3}\right)^n = I + f$, then show that I is an odd integer

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8. If I, n are positive integers , $0 < f < 1$ and if $\left(7 + 4\sqrt{3}\right)^n = I + f$, then show that $(I + f)(1 - f) = 1$

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9. If R, n are positive integers, n is odd,

$0 < F < 1$ and if $\left(5\sqrt{5} + 11\right)^n = R + F,$ then prove that

R is an even integer and

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10. If R, n are positive integers, n is odd,

$0 < F < 1$ and if $\left(5\sqrt{5} + 11\right)^n = R + F,$ then prove that

$(R + F).F = 4^n$

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EXERCISE - 3.2 (SHORT ANSWER QUESTIONS)

1. Prove that following

i) $2.C_0 + 5.C_1 + 8.C_2 + \ldots . + (3n + 2)C_n = (3n + 4).2^{n-1}$

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2. With usual notations prove that

$2.C_0 + 7.C_1 + 12.C_2 + \ldots\ldots . + (5n + 2).C_n = (5n + 4).2^{n-1}$

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3. $C_0 + 4.C_1 + 7.C_2 + \ldots\ldots(n + 1)$ terms =

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4. Prove that $C_0 + 2.C_1 + 4.C_2 + 8.C_3 + \ldots\ldots + 2^n.C_n = 3^n$

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5. Prove that $3.C_0 + 6.C_1 + 12.C_2 + \ldots\ldots\ldots + 3.2^n.C_n = 3^{n+1}$

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EXERCISE - 3.2 (LONG ANSWER QUESTIONS)

1. With usual notation prove that

$$2^2.C_0 + 3^2.C_1 + 4^2.C_2 + \ldots\ldots + (n+2)^2.C_n = \left(n^2 + 9n + 16\right)2^{n-2}$$

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2. Prove that

$$\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \frac{C_7}{8} + \ldots\ldots = \frac{2^n - 1}{n+1}$$

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3. Prove that : If n is a positive integer, then prove that

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}.$$

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4. Prove that

$$C_0 . C_3 + C_1 . C_4 + C_2 . C_5 + \ldots . + C_{n-3} . C_n = {}^{2n}C_{n+3}$$

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5. Prove that

$$\left({}^{2n}C_0\right)^2 - \left({}^{2n}C_1\right)^2 + \left({}^{2n}C_2\right) - \left({}^{2n}C_3\right)^2 + \ldots\ldots + \left({}^{2n}C_{2n}\right)^2 = (-1)^n {}^{2n}C_n .$$

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EXERCISE - 3.3 (VERY SHORT ANSWER QUESTIONS)

1. Find the range of x for which the binomial expansions of the following are valid .

$(2 + 5x)^{-1/2}$

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2. Find the range of x for which the binomial expansions of the following are valid .

$(7 - 4x)^{-5}$

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3. Find the set of values of x for which the binomial expansions of the following are valid.

$(2 + 3x)^{-2/3}$

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4. Find the range of x for which the binomial expansions of the following are valid .

$(x + 5)^{3/2}$

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5. Find the range of x for which the binomial expansions of the following are valid .

$(7 + 3x)^{-5}$

Watch Video Solution

6. Find the range of x for which the binomial expansions of the following are valid .

$\left(4 - \frac{x}{3}\right)^{-1/2}$

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7. Find the range of x for which the binomial expansions of the following are valid .

$(a + bx)^r$

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8. Write down the first three terms is the following expansions

$\left(1 + \frac{x}{2}\right)^{-5}$

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9. Write down the first three terms is the following expansions

$(3 + 4x)^{-2/3}$

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10. Write down the first three terms is the following expansions

$(3 + 5x)^{-\frac{7}{3}}$

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11. Write down the first three terms is the following expansions

$(1 + 4x)^{-4}$

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12. Write down the first three terms is the following expansions

$(8 - 5x)^{2/3}$

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13. Find the

6^{th} term of $\left(1 + \frac{x}{2}\right)^{-5}$

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14. Find the

7^{th} term of $\left(1 - \frac{x^2}{3}\right)^{-4}$

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15. Find the

10^{th} term of $(3 - 4x)^{-2/3}$.

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16. Find the

5^{th} term of $\left(7 + \frac{8y}{3}\right)^{7/4}$.

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17. Find the general term $(r+1)^{th}$ term in the expansion of

$$\left(1+\frac{4x}{5}\right)^{5/2}$$

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18. Find the general term in the expansion of $\left(3+\frac{x}{2}\right)^{-2/3}$

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19. Find the general term in the expansion of $\left(1-\frac{5x}{3}\right)^{-3}$

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20. Find the general term in the expansion of $(4+5x)^{-3/2}$

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21. Find the coefficient of x^6 in $(1 - 3x)^{-2/5}$

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22. Find the coefficient of x^4 in $(1 - 4x)^{-3/5}$

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EXERCISE - 3.3 (LONG ANSWER QUESTIONS)

1. Find the coefficient of x^{10} in the expansion of $\frac{1 + 2x}{(1 - 2x)^2}$.

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2. Find the coefficient of x^5 in $\frac{(1 - 3x)^2}{(3 - x)^{3/2}}$.

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3. Find the coefficient of x^8 in $\frac{(1+x)^2}{\left(1-\frac{2}{3}x\right)^3}$.

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4. Find the coefficient of x^7 in $\frac{(2+3x)^3}{(1-3x)^4}$.

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5. Find the coefficient of x^3 in $\frac{(1-5x)^3\left(1+3x^2\right)^{3/2}}{(3+4x)^{1/3}}$

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6. If |x| is so small that x^2 and higher powers of x may be neglected show that

$$\left(\sqrt{4-x}\right)\left(3-\frac{x}{2}\right)^{-1}=\frac{2}{3}\left(1+\frac{x}{24}\right)$$

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7. If |x| is so small that x^2 and higher powers of x may be neglected show that

$$\frac{(4+3x)^{1/2}}{(3-2x)^2} = \frac{2}{9} + \frac{41}{108}x$$

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8. If |x| is so small that x^2 and higher powers of x may be neglected show that

$$\frac{\left(1-\frac{2}{3}x\right)^{3/2}.(32+5x)^{1/5}}{(3-x)^3} = \frac{2}{27}\left(1+\frac{x}{32}\right)$$

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9. If |x| is so small that x^2 and higher powers of x may be neglected show that

$$\frac{(8+3x)^{2/3}}{(2+3x)\sqrt{4-5x}} = 1 - \frac{5x}{8}$$

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10. If |x| is so small that x^2 and higher powers of x may be neglected show that

$$\frac{(4-7x)^{1/2}}{(3+5x)^3} = \frac{2}{27}\left(1 - \frac{47}{8}x\right)$$

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11. BY neglecting x^4 and higher powers of x show that $\sqrt[3]{x^2+64} - \sqrt[3]{x^2+27} = 1 - \frac{7}{432}x^2$

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12. Suppose p, q are positive and p is very small when compared to q. Then find an approximate value of

$\left(\frac{q}{q+p}\right)^{1/2} + \left(\frac{q}{q-p}\right)^{1/2}$.

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13. Suppose s and t are positive and t is very small when compared to s.

Then find an approximate value of

$\left(\frac{s}{s+t}\right)^{1/3} - \left(\frac{s}{s-t}\right)^{1/3}$.

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14. Find the value of the $\sqrt{199}$ correct to 4 decimal places

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15. Find the value of the $\sqrt[5]{242}$ correct to 4 decimal places

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16. Find an approximate value of the following corrected to 4 decimal places.

$\sqrt[7]{127}$

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17. Find the value of the $\frac{1}{\sqrt[3]{999}}$ correct to 4 decimal places

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18. Find an approximate value of the following corrected to 4 decimal places.

$\sqrt[3]{1002} - \sqrt[3]{998}$

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19. Expand $3\sqrt{3}$ in increasing power of $\frac{2}{3}$

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20. Expand $5\sqrt{5}$ in increasing power of 4/5`

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21. Show that $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\ldots..=\sqrt{3}$

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22. Show that $1+\frac{2}{3.1}/2+\frac{2.5}{3.6}\left(\frac{1}{2}\right)^2+\frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^3+\ldots..=\sqrt[3]{4}$

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23. Find the sum of the infinite series

$$\frac{7}{5}\left(1+\frac{1}{10^2}+\frac{1.3}{1.2}.\frac{1}{10^4}+\frac{1.3.5}{1.2.3}.\frac{1}{10^6}+\ldots.\right)$$

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24. Show that $\frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \ldots\ldots = \sqrt{8} - 1$

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25. Show that $\frac{3.5}{5.10} + \frac{3.5.7}{5.10.15} + \frac{3.5.7.9}{5.10.15.20} + \ldots . \infty = \frac{5\sqrt{5}}{3\sqrt{3}} - \frac{8}{5}$

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26. Show that $\frac{3}{4.8} - \frac{3.5}{4.8.12} + \frac{3.5.7}{4.8.12.16} - \ldots\ldots = \sqrt{\frac{2}{3}} - \frac{3}{4}$

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27. $\frac{1}{4} - \frac{5}{4.8} + \frac{5.7}{4.8.12} - \ldots\ldots =$

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28. If $x = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + \ldots. \infty$ then find $3x^2 + 6x$.

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29. If $x = \frac{5}{(2!).3} + \frac{5.7}{(3!).3^2} + \frac{5.7.9}{(4!).3^3} + \ldots.$

then find the value of $x^2 + 4x$.

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30. If $t = \frac{4}{5} + \frac{4.6}{5.10} + \frac{4.6.8}{5.10.15} + \ldots\ldots\infty$ then prove that 9t = 16

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ADDITIONAL EXERCISE

1. The term independent of x $(x > 0, x \neq 1)$ in the expansion of $\left[\frac{(x+1)}{\left(x^{2/3}-x^{1/3}+1\right)}-\frac{(x-1)}{\left(x-\sqrt{x}\right)}\right]^{10}$ is

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2. Find coeff. of x^{25} in the expansion of $\sum_{k=0}^{50}(-1)^{k}{}^{50}C_{k}(2x-3)^{50-k}(2-x)^{k}$.

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3. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}}+x^{2}\log_{10}x\right)^{8}$ is 5600, then the value of $x =$

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4. Find the coefficient of x^{6} in the expansion of $\left(1+x^{2}-x^{3}\right)^{8}$

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5. Find the number of distinct terms in the expansion

$(x + 2y + 3z + w)^{20}$

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6. Find the number of distinct terms in the expansion

$(x + y + z)^{10} + (x + y - z)^{10}$

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7. Find the coefficient of x^{10} in the expansion of

$(1 + 2x)^{21} + (1 + 2x)^{22} + \ldots\ldots + (1 + 2x)^{30}$

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8. The expansion $\left[x+\left(x^3-1\right)^{1/2}\right]^5+\left[x-\left(x^3-1\right)^{1/2}\right]^5$ is a polynomial of degree

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9. If the middle term of the expansion of $(1+x)^{2n}$ is the greatest term then prove that x lies between $\frac{n}{n+1}$ and $\frac{n+1}{n}$

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10. Show that the greatest coefficient in the expansion of $(x+y+z+w)^{15}$ is $\frac{15!}{3!(4!)^3}$

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11. Find the sum of the coefficients of integral powers of x in $\left(1+3\sqrt{x}\right)^{20}$

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12. If n is integer greater than 1 show that

$$a - {}^{n}C_1(a-1) + {}^{n}C_2(a-2) + {}^{n}C_3(a-3)\ldots\ldots + (-1)^{n}(a-n) = 0$$

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13. Prove that $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \frac{C_6}{7} + \ldots\ldots = \frac{2^n}{n+1}$

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14. Prove that

$$\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots\ldots = \frac{2^{n-1}}{n!}$$

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15. Show that $\sum_{r=0}^{50} \left({}^{100}C_r\right).\left({}^{200}C_{150+r}\right) = {}^{300}C_{50}$

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16. Show that $C_0 + \left(C_0 + C_1\right) + \left(C_0 + C_1 + C_2\right) + \ldots\ldots + \left(C_0 + C_1 + \ldots + C_n\right) = (n + 2).2^{n-1}$

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17. If x > 0 , write the first negative term in the expansion of $(1 + 2x)^{23/2}$

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18. Find the coefficient of x^n in the expansion of $\left(1 + 2x + 3x^2 + 4x^3 + \ldots.\right)^2$

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19. Find the coefficient of x^6 in $\left(1+x+x^2+x^3+\ldots\ldots\infty\right)\left(1+x^2+x^4+x^6+\ldots\ldots\infty\right)$

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20. Find the coefficient of x^5 in $\left(1+x+x^2+x^3\right)^5\left(1+x+x^2+x^3+x^4\right)^{-5}$

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21. Find the coefficient of x^7 in the expansion of $\left(1+2x+3x^2+4x^3+5x^4+6x^5+7x^6+8x^7\right)^{10}$

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22. If x is small and if the expansion of $a+\frac{b}{1+2x}+\frac{c}{1-3x^2}$ is $1+x+2x^2+\ldots\ldots\infty$ find (a,b,c)

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23. If x nearly equal to 1 show that $\frac{px^p - qx^q}{p - q} = x^{p+q}$ (nearly)

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24. If x nearly equal to 1 show that $\frac{px^q - qx^p}{x^q - x^p} = \frac{1}{1 - x}$ (nearly)

Watch Video Solution

25. $1 + n.\frac{2n}{1+n} + \frac{n(n+1)}{1.2}\left(\frac{2n}{1+n}\right)^2 + \ldots..$

Watch Video Solution

EXERCISE - I

1. Ratio of middle term in $\left(px^3 + \frac{q}{x^2}\right)^{15}$ is

A. $p : qx^2$

B. $px^5 : q$

C. $px^2 : q$

D. $p : qx^5$

Answer: B

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2. The numerically greatest term in the expansion $(2x - 3y)^{12}$ when x = 1 and y = 5/2 is the

A. 11th term

B. 10th term

C. 9th term

D. 6th term

Answer: A

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3. No. of term in $\left(1 + 5\sqrt{2}x\right)^9 + \left(1 - 5\sqrt{2}x\right)^9$ if x > 0 is

A. 3

B. 5

C. 4

D. 6

Answer: B

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4. No. of term in $\left(1 + 3x + 3x^2 + x^3\right)^6$ is

A. 17

B. 19

C. 21

D. 16

Answer: B

Watch Video Solution

5. The number of distinct terms in $(a + b + c + d + e)^3$ is

A. 35

B. 38

C. 42

D. 45

Answer: A

Watch Video Solution

6. If a and b are respective coefficients of x^m and x^n in the expansion of $(1+x)^{m+n}$ then

A. a + b = m+ n

B. a = 2b

C. a = b

D. b = 2a

Answer: C

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7. If the coefficient of $(3r)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1+x)^{2n}$ are equal then n =

A. 2r

B. 3r

C. 3r + 1

D. 2r + 1

Answer: A

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8. If the coefficients ${}^nC_4$, ${}^nC_5$, ${}^nC_6$ of $(1 + x)^n$ are in A.P. then n is equal to

A. 12

B. 11

C. 7

D. 8

Answer: C

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9. $C_0 + 2.C_1 + 4.C_2 + \ldots\ldots + C_n.2^n = 243$, then n =

A. 5

B. 10

C. 15

D. 20

Answer: A

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10. $\frac{1}{2}.{}^nC_0 + {}^nC_1 + 2.{}^nC_2 + 2^2.{}^nC_3 + \ldots\ldots + 2^{n-1}.{}^nC_n =$

A. $\frac{3^{-n}}{2}$

B. $\frac{4^{2n}}{2}$

C. $\frac{4^n}{2}$

D. $\frac{3^n}{2}$

Answer: D

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11. ${}^{(2n+1)}C_0 + {}^{(2n+1)}C_1 + {}^{(2n+1)}C_2 + \ldots\ldots + {}^{(2n+1)}C_n =$

A. 2^n

B. 2^{-n}

C. 2^{2n}

D. 3^{2n}

Answer: C

Watch Video Solution

12. ${}^{(2n+1)}C_0 - {}^{(2n+1)}C_1 + {}^{(2n+1)}C_2 - \ldots\ldots + {}^{(2n+1)}C_{2n} =$

A. 0

B. 1

C. -1

D. 2

Answer: B

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13. $\frac{\left(1 + {}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + \ldots\ldots + {}^{n}C_n\right)^2}{1 + {}^{2n}C_1 + {}^{2n}C_2 + {}^{2n}C_3 + \ldots\ldots\ldots + {}^{2n}C_{2n}} =$

A. 1

B. -1

C. 2

D. 3

Answer: A

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14. ${}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \ldots\ldots\ldots + {}^{21}C_{10} =$

A. 2^{10}

B. 2^{20}

C. 2^{21}

D. 2^{19}

Answer: B

Watch Video Solution

15. $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots\ldots + C_{25}^2 =$

A. ${}^{49}C_{50}$

B. ${}^{49}C_{25}$

C. ${}^{50}C_{25}$

D. ${}^{39}C_{40}$

Answer: C

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16. Prove that

$$\left({}^{2n}C_0\right)^2 - \left({}^{2n}C_1\right)^2 + \left({}^{2n}C_2\right) - \left({}^{2n}C_3\right)^2 + \ldots\ldots + \left({}^{2n}C_{2n}\right)^2 = (-1)^n {}^{2n}C_n.$$

A. $(-1)^n . {}^{2n}C_n$

B. $(-1)^{2n} . {}^{2n}C_n$

C. $(-1)^n . {}^{3n}C_n$

D. $(-1)^n . {}^{n}C_n$

Answer: A

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17. $C_0C_2 + C_1C_3 + C_2C_4 + \ldots\ldots . + C_{n-2}C_n =$

A. ${}^{2n}C_{n-2}$

B. ${}^{2n}C_{n}$

C. ${}^{2n}C_{n-1}\Big)$

D. ${}^{2n}C_{2n-2}$

Answer: A

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18. If the coefficients of x^2 and x^4 in the expansion of $\left(x^{\frac{1}{3}}+\frac{2}{x^{\frac{1}{3}}}\right)^{18}, (x<0)$ are m and n respectively then m/n is equal to

A. 27

B. 182

C. $\frac{5}{4}$

D. $\frac{4}{5}$

Answer: B

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19. Sum of the coefficients of $\left(1+\frac{x}{3}+\frac{2y}{3}\right)^{12}$

A. 2^6

B. 2^8

C. 2^{12}

D. 10^2

Answer: C

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20. Sum of coefficients of x^{2r}, $r = 1, 2, 3....$ in $(1+x)^n$ is

A. $\left(2n^{n-1} - 1\right)$

B. $\left(2^{n-1}+1\right)$

C. $\left(2^{n-2}+1\right)$

D. $\left(2n^{n-2}-1\right)$

Answer: A

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21. Sum of coefficients of terms of even powers of x in $\left(1-x+x^2-x^3\right)^7$ is

A. -2^{17}

B. 2^{17}

C. 2^{13}

D. -2^{13}

Answer: C

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22. Sum of coefficients of terms of odd powers of x in $\left(1 - x + x^2 - x^3\right)^9$ is

A. -2^{17}

B. 2^{17}

C. 2^{13}

D. 2^{12}

Answer: A

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23. The range of x of which the expansion $\left(9 + 25x^2\right)^{-6/5}$ is valid is

A. $(-3/5, 3/5)$

B. $[-3/5, 3/5]$

C. $(-\infty, 3/5)$

D. $(-\infty, -3/5) \cup (3/5, \infty)$

Answer: A

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24. If the expansion $(4a - 8x)^{1/2}$ were to possible then

A. $2 < \left|\frac{a}{x}\right|$

B. $2 > \left|\frac{a}{x}\right|$

C. $2 < \left|\frac{x}{a}\right|$

D. $2 > \left|\frac{x}{a}\right|$

Answer: A

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25. For $|x| < \frac{1}{2}$, the value of the fourth term of $(1 - 2x)^{-3/4}$ is

A. $-\frac{77}{16}x^3$

B. $\frac{16}{77}x^3$

C. $\frac{77}{16}x^3$

D. $-\frac{16}{77}x^3$

Answer: C

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26. The coefficient of x^7 in

$\left(1 + 2x + 3x^2 + 4x^3 + \ldots\ldots \text{to } \infty\right)$

A. 0

B. 6

C. 8

D. 36

Answer: C

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27. $1+\frac{1}{10^2}+\frac{1.3}{1.2}.\frac{1}{10^4}+\frac{1.3.5}{1.2.3}.\frac{1}{10^6}+\ldots\ldots\infty=$

A. $\frac{\sqrt{7}}{2}$

B. $\frac{5\sqrt{2}}{7}$

C. $\left(\frac{5}{7}\right)^{1/2}$

D. $\frac{5\sqrt{2}}{3}$

Answer: B

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EXERCISE - II

1. If r^{th} term is middle term in $\left(x^2-\frac{1}{2x}\right)^{20}$ then $(r+3)^{th}$ term is

A. $\frac{{}^{20}C_7 x}{2^{13}}$

B. $-\left(\frac{{}^{20}C_5 x}{4^{13}}\right)$

C. $-\left(\frac{{}^{20}C_7 x}{2^{13}}\right)$

D. $-\left(\frac{{}^{20}C_{14} x}{4^{13}}\right)$

Answer: C

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2. If 'a' is the coefficient of the middle term in the expansion of $(1+x)^{2n}$ and b,c are the coefficients of two middle terms in the expansion of $(1+x)^{2n-1}$ then

A. a + b = c

B. `a = b+ c

C. b= c+a

D. b + c = 2a

Answer: B

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3. The coefficient of the middle term in $(1 + \alpha x)^4$ and $(1 - \alpha x)^6$ is same then α =

A. $-5/3$

B. $3/5$

C. $-3/10$

D. $10/3$

Answer: C

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4. The numerical value of middle terms in $\left(1 - \frac{1}{x}\right)^n (1 - x)^n$ is

A. ${}^{2n}C_n$

B. ${}^{n}C_n$

C. $-\left({}^{2n}C_n\right)$

D. $-\left({}^{n}C_n\right)$

Answer: A

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5. If the middle term of $(1 + x)^{2n}$ is $\frac{1.3.5.... (2n - 1)k}{n!}$ then $k =$

A. $(3x)^{n+1}$

B. $(2x)^{n+1}$

C. $(2x)^n$

D. $(3x)^n$

Answer: C

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6. The coefficient of the middle term in the expansion of $(1+x)^{40}$ is

A. $\frac{1.3.5 - - - - 39}{20!}.2^{20}$

B. $\frac{1.3.5 - - - 39}{20!}$

C. $\frac{40!}{20!}$

D. $40!2^{20}$

Answer: A

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7. If the third term in the expansion of $\left(x + x\log_{10}x\right)^5$ is 10^6 then x is

A. 1

B. 100

C. $\frac{1}{\sqrt{10}}$

D. 10

Answer: D

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8. In the expansion of $(1 + x)^n$ if the 2nd and 3rd terms are respectively a,b then x =

A. $\frac{a^2 - 2b}{a}$

B. $\frac{a^2}{a^2 - 2b}$

C. $\frac{a}{a^2 - 2b}$

D. $\frac{a^2 - 2a}{a}$

Answer: D

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9. $\frac{18^3 + 7^3 + 3.18.7.25}{3^6 + 6.243.2 + 15.81.4 + 20.27.8 + 15.9.16 + 6.3.32 + 64} =$

A. 4

B. 3

C. 2

D. 1

Answer: D

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10. The expansion $\left[x + \left(x^3 - 1\right)^{1/2}\right]^5 + \left[x - \left(x^3 - 1\right)^{1/2}\right]^5$ is a polynomial of degree

A. 7

B. 4

C. 5

D. 6

Answer: A

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11. If the ratio of the 7th term from the beginning to the 7th term from the end in the expansion of $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)^x$ is $\frac{1}{6}$, then x is

A. 3

B. 6

C. 12

D. 9

Answer: D

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12. If the sum of odd terms and the sum of even terms in $(x + a)^n$ are p and q respectively then $p^2 - q^2 =$

A. $\left(x^2 + a^2\right)^n$

B. $\left(x^2 - a^2\right)^n$

C. $\left(a^2 - x^2\right)^n$

D. $x^2 - a^2$

Answer: B

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13. In the expansion of $\left(\sqrt[5]{3} + \sqrt[7]{2}\right)^{24}$, then rational term is

A. T_{14}

B. T_{16}

C. T_{15}

D. T_7

Answer: C

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14. The sum of the rational terms in expansion of $\left(\sqrt{2}+3^{1/5}\right)^{10}$ is

A. 41

B. 42

C. 32

D. 38

Answer: A

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15. The number of rational terms in the binomial expansion of $\left(\sqrt[4]{5}+\sqrt[5]{4}\right)^{100}$ is

A. 50

B. 5

C. 6

D. 51

Answer: C

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16. The number of irrational terms in the expansion of $\left(\sqrt[5]{3}+\sqrt[3]{7}\right)^{36}$ is

A. 30

B. 34

C. 31

D. 29

Answer: B

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17. The number integral terms in the expansion of $\left(\sqrt{3}+\sqrt[8]{5}\right)^{256}$ is

A. 35

B. 32

C. 33

D. 34

Answer: C

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18. The sum of the coefficient of x^{32} and x^{-17} in $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is

A. 1

B. -1

C. 2

D. 0

Answer: D

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19. If the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ equals the coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$, then a and b satisfy the relation

A. ab = 1

B. a/b = 1

C. a + b = 1

D. a - b = 1

Answer: A

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20. The term independent of x in $\left(2x^{1/2} - 3x^{-1/3}\right)^{20}$

A. ${}^{20}C_7 2^8 3^{12}$

B. ${}^{20}C_8 2^7 3^{13}$

C. ${}^{20}C_8 C^8 3^{12}$

D. ${}^{20}C_7 2^7 3^{13}$

Answer: C

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21. If the absolute term (independent of x) in the expansion of $\left(\sqrt{x} - k/x^2\right)^{10}$ is 405 then $k =$

A. $\pm 3^{1/4}$

B. $\pm 4^{1/3}$

C. ± 2

D. ± 3

Answer: D

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22. If the 5th term is the term independent of x in the expansion of $\left(x^{2/3} + \frac{1}{x}\right)^n$ then n =

A. 10

B. 8

C. 7

D. 12

Answer: A

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23. If a term independent of x is to exist in the expansion of $\left(x+\frac{1}{x^2}\right)^n$ then n must be

A. a multiple of 2

B. a multiple of 3

C. a multiple of 5

D. a multiple of 7

Answer: B

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24. The term independent of x in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is

A. $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots\ldots + C_n^2$

B. ${}^{2n}C_n$

C. $\frac{1.3.5\ldots..(2n-1)}{n!}2^n$

D. All of the above

Answer: B

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25. The sum of the coefficient in the expansion of $\left(1+x+x^2\right)^n$ is

A. 2

B. 2^n

C. 3^n

D. 4^n

Answer: C

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26. The sum of the binomial coefficients in the expansion of $\left(\frac{2x}{3}+\frac{3}{2x^2}\right)^n$ is 64 then the term independent of x is

A. $20/3$

B. $3/20$

C. $10/3$

D. $3/10$

Answer: A

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27. The two successive terms in the expansion $(1 + x)^{24}$ whose coeff's are in the ratio $4 : 1$ are

A. 18th , 19th

B. 19th, 20th

C. 20th, 21th

D. 21th, 22nd

Answer: C

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28. If two consecutive terms in the expansion of $(x + a)^n$ are equal to where n is a positive integer then $\frac{(n + 1)a}{x + a}$ is

A. Negative integer

B. rational number

C. a real number

D. a positive integer

Answer: D

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29. The coefficient of x^{53} in $\sum_{r=0}^{100} {}^{100}C_r(x-3)^{100-r}.2^r$ is

A. ${}^{100}C_{47}$

B. ${}^{100}C_{53}$

C. $-{}^{100}C_{53}$

D. ${}^{100}C_{100}$

Answer: C

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30. Let $n \in N$. If $(1+x)^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ and $a_{n-3}, a_{n-2}, a_{n-1}$ are in A.P then

Statement - I : a_1, a_2, a_3 are in A.P.

Statement -II : n = 7

The true statements are :

A. only I

B. only II

C. both I, II

D. neither I nor II

Answer: C

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31. If the coefficient of rth term and $(r+1)th$ term in the expansion of $(1+x)^{20}$ are in the ration $1:2$, then $r =$

A. 6

B. 7

C. 8

D. 9

Answer: B

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32. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the binomial expansion of $(1+y)^m$ are in A.P. then m and r satisfy the equation

A. $m^2 - m(4r-1) + 4r^2 + 2 = 0$

B. $m^2 - m(4r+1) + 4r^2 - 2 = 0$

C. $m^2 - m(4r+1) + \left(4r^2 + 2\right) = 0$

D. $m^2 - m(4r-1) + 4r^2 - 2 = 0$

Answer: B

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33. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are 45, 120 and 210 then the value of n is

A. 8

B. 12

C. 10

D. 14

Answer: C

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34. The coefficient of x^n in expansion of $(1 + x)(1 - x)^n$ is

A. $(n - 1)$

B. $(- 1)^n(1 - n)$

C. $(-1)^{n-1}(n-1)^2$

D. $(-1)^{n-1}n$

Answer: B

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35. The coefficient of x^{11} in the expansion of $\left(1-2x+3x^2\right)(1+x)^{11}$ is

A. 164

B. 144

C. 116

D. 261

Answer: B

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36. The coefficient of x^5 in the expansion of $\left(x^2 - x - 2\right)^5$ is

A. -83

B. -82

C. -81

D. 0

Answer: C

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37. The coefficient of x^4 in $\left(1 + x + x^2 + x^3\right)^{11}$ is

A. -144

B. 144

C. -128

D. -142

Answer: A

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38. Coefficient of x^5 in $(1+x)^{21}+(1+x)^{22}+\ldots\ldots+(1+x)^{30}$ is

A. ${}^{51}C_5$

B. ${}^{9}C_5$

C. ${}^{31}C_6$ - ${}^{21}C_6$

D. ${}^{30}C_5+{}^{20}C_5$

Answer: C

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39. Coefficient of $a^8b^6c^4$ in $(a+b+c)^{18}$ is

A. $\frac{18!}{4!10!5!}$

B. $\frac{18!}{3!8!8!}$

C. $\frac{18!}{2!7!9!}$

D. $\frac{18!}{8!6!4!}$

Answer: D

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40. The coefficient of x^9 in $(x - 1)(x - 4)(x - 9)\ldots\ldots (x - 100)$ is

A. -235

B. 235

C. 385

D. -385

Answer: D

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41. The term in $(x + y)^{50}$ which is greatest in absolute value if $|x| = \sqrt{3}|y|$ is

A. T_{17}

B. T_{19}

C. T_{20}

D. T_{21}

Answer: B

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42. The greatest integer which divides the number $101^{100} - 1$ is

A. 10^2

B. 10^3

C. 10^4

D. 10^5

Answer: C

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43. $9^7 + 7^9$ is divisible by

A. 6

B. 24

C. 64

D. 72

Answer: C

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44. Larger of $99^{50} + 100^{50}$ and 101^{50}

A. 101^{50}

B. $99^{50} + 100^{50}$

C. Both are equal

D. can not be decided

Answer: A

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45. The remainder left out when 8^{2n} - $6(2)^{2n+1}$ is divided by 9 is

A. 2

B. 7

C. 8

D. 0

Answer: A

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46. Integral part of 7 + 4sqrt3)^n*is*(n in N)`

A. an even number

B. an odd number

C. an even or an odd number depending upon the value of n

D. nothing can be said

Answer: B

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47. Integral part of $\left(7+5\sqrt{2}\right)^{2n+1}$ is $(n \in N)$

A. an even number

B. an odd number

C. an even or an odd number depending upon the value of n

D. nothing can be said

Answer: A

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48. If $\left(6\sqrt{6}+14\right)^{2n+1}=R$ and $F=[R]$, where [R] denotes the greatest integer less than or equal to R thwn $RF=$

A. 20^n

B. 20^{2n}

C. 20^{2n+1}

D. 1

Answer: C

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49. The integral part of $\left(\sqrt{2}+1\right)^6$ is

A. 198

B. 196

C. 197

D. 199

Answer: C

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50. ${}^{n}C_{r+1} + 2{}^{n}C_{r} + {}^{n}C_{r-1} =$

A. ${}^{(n+1)}C_{r+1}$

B. ${}^{n+2}C_{r}$

C. ${}^{n+2}C_{r+1}$

D. ${}^{n+1}C_{r}$

Answer: C

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51. If C_k is the coefficient of x^k in the expansion of $(1+x)^{2005}$ and if a,d are real numbers then $\sum_{k=0}^{2005}(a+kd).C_k$ =

A. $(2a+2005d)2^{2004}$

B. $(2a+2005d)2^{2005}$

C. $(2a+2004d)2^{2005}$

D. $(2a+2004d)2^{2005}$

Answer: A

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52. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots {}^{20}C_{10}$ is

A. ${}^{20}C_{10}$

B. $-\left({}^{20}C_{10}\right)$

C. $\frac{1}{2}\cdot\left({}^{20}C_{10}\right)$

D. 0

Answer: C

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53. If $S_n = \sum_{r=0}^{n} \frac{1}{{}^{n}C_r}$ and $T_n = \sum_{r=0}^{n} \frac{r}{{}^{n}C_r}$ then $\frac{t_n}{s_n}$ =

A. $\frac{1}{4}n$

B. $\frac{1}{3}n$

C. $\frac{1}{3}n$

D. n

Answer: C

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54. $C_0 - \left[C_1 - 2.C_2 + 3.C_3 - \ldots\ldots. + (-1)^{n-1}.n.C_n\right] =$

A. 0

B. 1

C. -1

D. 2

Answer: B

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55. $C_0 + 4.C_1 + 7.C_2 + \ldots\ldots(n+1)$ terms =

A. $(3n+2).2^{n-1}$

B. $(2n+2).2^{n-1}$

C. $(2n+2).3^{n-1}$

D. $(2n-2).3^{n+1}$

Answer: A

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56. $2.C_2 + 6.C_3 + 12.C_4 + \ldots . + n(n-1).C_n =$

A. $n(n-1).2^{n-1}$

B. $2n(n-1).2^{n-2}$

C. $n(n-1).2^{n-2}$

D. $2n(n+1).2^{n-1}$

Answer: C

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57. Sum of last 8 coefficients in $(1+x)^{16}$ is

A. $\left[2^{15} - \frac{1}{2}.{}^{16}C_8\right]$

B. $\left[2^{15}+\frac{1}{2}.{}^{6}C_{2}\right]$

C. $\left[2^{15}-\frac{1}{2}.{}^{6}C_{2}\right]$

D. $\left[2^{15}-\frac{1}{4}.{}^{6}C_{2}\right]$

Answer: A

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58. $C_0+\frac{C_1}{2}(4)+\frac{C_2}{3}(16)+\ldots\ldots\ldots\ldots+\frac{C_n}{n+1}\left(2^{2n}\right)$

A. $\frac{5^{n+1}+1}{n-1}$

B. $\frac{5^{n+1}-1}{4(n+1)}$

C. $\frac{5^{n+1}+1}{4(n+1)}$

D. $\frac{5^{n+1}+1}{4(n-1)}$

Answer: B

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59. $(1+x)^{15} = a_0 + a_1 x + \ldots . + a_{15} x^{15} \Rightarrow \sum_{r=1}^{15} r \frac{a_r}{a_{r-1}} =$

A. 110

B. 115

C. 120

D. 135

Answer: C

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60. The value of $\sum_{r=1}^{15} r^2 \left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}} \right)$ of is equal to

A. 1240

B. 560

C. 1085

D. 680

Answer: D

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61. If a_k is the coefficient of x^k in the expansion of $\left(1+x+x^2\right)^n$ for k = 0,1,2,.....,2n then $a_1 + 2a_2 + 3a_3 + \ldots\ldots\ldots + 2n.\, a_{2n} =$

A. $-a_0$

B. 3^n

C. $n.3^n$

D. $-n.3^n$

Answer: C

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62. Find the sum of the following

$$\frac{{}^{15}C_1}{{}^{15}C_0} + 2\frac{{}^{15}C_2}{{}^{15}C_1} + 3\frac{{}^{15}C_3}{{}^{15}C_2} + \ldots\ldots + 15\frac{{}^{15}C_{15}}{{}^{15}C_{14}}$$

A. 2^{15}

B. 240

C. 120

D. 136

Answer: C

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63. If $a_0, a_1, a_2 \ldots\ldots a_n$ are binomial coefficients then

$$\left(1 + \frac{a_1}{a_0}\right)\left(1 + \frac{a_2}{a_1}\right)\ldots\ldots\ldots\ldots\ldots\left(1 + \frac{a_n}{a_{n-1}}\right) =$$

A. $\frac{(n-1)^n}{n!}$

B. $\frac{(n+1)^n}{n!}$

C. $\frac{(n+1)^{2n}}{n!}$

D. $\frac{(n-1)^{n}}{2n!}$

Answer: B

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64. $\frac{(C_0+C_1)(C_1+C_2)(C_2+C_3).....(C_{n-1}+C_n)}{C_0C_1C_2...C_n}$

A. $\frac{(n+1)^{n}}{n!}$

B. $\frac{n+1}{n!}$

C. $\frac{(n+1)^{n-1}}{n!}$

D. $\frac{(n-1)^{n}}{n!}$

Answer: A

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65. If $(1+x)\left(1+x+x^2\right)\left(1+x+x^2+x^3\right).....\left(1+x+x^2+.....+x^{n-1}\right) =$ $a_0+a_1x+a_2x^2+.....+a_mx^m$ then $a_0+a_1+....+a_m =$

A. n!

B. 2n!

C. 3n!

D. 4n!

Answer: A

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66. If $\left(1-x+x^2\right)^n = a_0+a_1x+a_2x^2+....+a_{2n}x^{2n}$ then $a_0+a_2+a_4+.....+a_{2n} =$

A. $\frac{3^n+1}{2}$

B. $\frac{3^n-1}{2}$

C. $\frac{3^{2n}-1}{2}$

D. $\frac{3^n - 1}{4}$

Answer: A

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67. If $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots. =$

A. $\frac{2^{n-1}}{n!} \forall n \in N$

B. $\frac{2^{n-1}}{2n!} \forall n \in N$

C. $\frac{2^{n-1}}{n!} \forall n \in N$

D. $\frac{3^{2n-1}}{n!} n \in N$

Answer: A

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68. $C_0 + C_1 + 2.C_2(3) + 3.C_3\left(3^2\right) + 4.C_4\left(3^3\right) + \ldots\ldots + n.C_n 3^{n-1} =$

A. $n.4^{n-1} + 1$

B. $2n.\ 4^{n-1} + 1$

C. $n.4^{n-1} - 1$

D. $n.4^{n+1} - 1$

Answer: A

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69. $\left(1 + x + x^2\right)^8 = a_0 + a_1 x + \ldots\ldots + a_{16} x^{16}$ then

$a_0 - a_2 + a_4 - a_6 + \ldots\ldots . + a_{16} =$

A. 1

B. 2

C. 3

D. 4

Answer: A

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70. If a_r is the coefficient x^r in the expansion of $\left(1+x+x^2\right)^n$ then $a_1 - 2a_2 + 3a_3 - \ldots\ldots - 2na_{2n} =$

A. 0

B. n

C. $-n$

D. 2n

Answer: C

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71. If $\left(1+2x+3x^2\right)^{10} = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{20}x^{20}$ then $\frac{a_2}{a_1} =$

A. 10.5

B. 21

C. 10

D. 5.5

Answer: A

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72. ${}^{15}C_2 + 2.\ {}^{15}C_3 + 3.\ {}^{15}C_4 + \ldots . + 14.\ {}^{15}C_{15} =$

A. $13.2^{14} - 1$

B. $13.2^{14} + 1$

C. $12^{14} + 1$

D. $12^{14} - 1$

Answer: B

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73. $^{20}C_{10}.\ ^{15}C_0 + {}^{20}C_9.\ ^{15}C_1 + {}^{20}C_8.\ ^{15}C_2 + \ldots . + {}^{20}C_0.\ ^{15}C_{10} =$

A. $^{20}C_{10}$

B. $^{25}C_{10}$

C. $^{35}C_{10}$

D. $^{40}C_{10}$

Answer: C

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74.

$^{10}C_1.\ ^9C_5 + {}^{10}C_2.\ ^9C_4 + {}^{10}C_3.\ ^9C_3 + {}^{10}C_4.\ ^9C_2 + {}^{10}C_5.\ ^9C_1 + {}^{10}C_6 = {}^{19}C_6 + x$

then x =

A. -84

B. 84

C. 81

D. -81

Answer: A

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75. $\sum_{r=1}^{10} r. \frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} =$

A. 5(2n - 9)

B. 5(n-9)

C. 2(n-9)

D. 2(5n-9)

Answer: A

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76. $\sum_{r=0}^{n-1} \frac{C_r}{C_r + C_{r+1}} =$

A. $\frac{n}{2}$

B. $\frac{n}{3}$

C. $\frac{n}{4}$

D. $\frac{2n}{3}$

Answer: A

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77. The sum of the coefficients in the binomial expansion of $\left(\frac{1}{x} + 2x\right)^n$ is equal to 6561. The constant term in the expansion is

A. ${}^8C_4$

B. $16\left({}^8C_4\right)$

C. ${}^6C_{4.2}^4$

D. $8\left({}^{8}C_{4}\right)$

Answer: B

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78. If $\frac{1}{(1-2x)(1+3x)}$ is to tbe expanded as a power series of x, then

A. $|x| < 1/2$

B. $|x| < 1/6$

C. $-1/3 < x < 1/2$

D. $|x| < 1/3$

Answer: D

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79. $1 + {}^{2}C_{1}x + {}^{3}C_{2} + {}^{4}C_{3}x^{3} + \ldots..$ to ∞ terms can be summed up if

A. $x < 1$

B. $x > -1$

C. $-1 < x < 1$

D. $-\infty < x < \infty$

Answer: C

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80. If S_n denotes the sum of first n natural number then $S_1 + S_2x + S_3x^2 + \ldots\ldots + S_nx^{n-1} + \ldots\ldots\infty$ terms =

A. $(1 - x)^{-1}$

B. $(1 - x)^{-2}$

C. $(1 - x)^{-3}$

D. $(1 - x)^{-4}$

Answer: C

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81. ${}^4C_1 + {}^5C_2 . \left(\frac{1}{2}\right) + {}^6C_3 . \left(\frac{1}{2}\right)^2 + \ldots . .$ to ∞ terms :

A. 30

B. 40

C. 900

D. 15

Answer: A

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82. If $|x| < 1$, then the coefficient of x^n in expansion of $\left(1 + x + x^2 + x^3 + \ldots\ldots\right)^2$ is

A. n

B. n-1

C. n+2

D. n+1

Answer: D

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83. If the expansion in powers of x of the function $\frac{1}{(1 - ax)(1 - bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_n$ then coefficient of x^n is

A. $\frac{a^n - b^n}{b - a}$

B. $\frac{a^{n+1} - b^{n+1}}{b - a}$

C. $\frac{b^{n+1} - a^{n+1}}{b - a}$

D. $\frac{b^n - a^n}{b - a}$

Answer: C

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84. The coefficient of x^{24} in the expansion of $\left(1+x^2\right)^{12}\left(1+x^{12}\right)\left(1+x^{24}\right)$ is

A. $12C_6$

B. $12C_6 + 2$

C. $12C_6 + 4$

D. $12C_6 + 6$

Answer: B

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85. If $0 < x < 1$, the first negative term in the expansion of $(1+x)^{27/5}$ is

A. 7th term

B. 5th term

C. 8th term

D. 6th term

Answer: C

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86. Find the coefficient of x^{10} in the expansion of $\frac{1+2x}{(1-2x)^2}$.

A. $r.2^r$

B. $(2r-1)2^r$

C. $r.2^{2r+1}$

D. $(2r+1)2^r$

Answer: D

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87. The first negative term in the expansion of $(1+x)^{3/4}$ is

A. T_2

B. T_4

C. T_3

D. T_7

Answer: C

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88. If T_{r+1} is the first negative term in the expansion of $(1+x)^{7/2}$ then r =

A. 5

B. 6

C. 7

D. 4

Answer: A

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89. If x^3, x^4, x^5 .....can be neglected then $\sqrt{x^2+16} - \sqrt{x^2+9} =$

A. $1 - \frac{x^2}{4}$

B. $1 - \frac{x^2}{8}$

C. $1 - \frac{x^2}{12}$

D. $1 - \frac{x^2}{24}$

Answer: D

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90. If 'c' is small in comparison with l then $\left(\frac{l}{l+c}\right)^{1/2} + \left(\frac{l}{l-c}\right)^{1/2} =$

A. $2 + \frac{3c}{4l}$

B. $2 + \frac{3c^2}{4l^2}$

C. $l + \frac{3c^2}{4l^2}$

D. $l + \frac{3c}{4l}$

Answer: B

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91. If x is numerically so small so that x^2 and higher powers of x can be neglected , then $\left(1 + \frac{2x}{3}\right)^{\frac{3}{2}}, (32 + 5x)^{-\frac{1}{5}}$ is approximately equal to :

A. $\frac{32 + 31x}{64}$

B. $\frac{32 + 32x}{64}$

C. $\frac{31 + 32x}{64}$

D. $\frac{1 - 2x}{64}$

Answer: A

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92. If x is small so that x^2 and higher powers can be neglected, then the approximately value for $\frac{(1-2x)^{-1}(1-3x)^{-2}}{(1-4x)^{-3}}$ is

A. 1-2x

B. 1-3x

C. 1-4x

D. 1-5x

Answer: C

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93. $1+\frac{2}{4}+\frac{2.5}{4.8}+\frac{2.5.8}{4.8.12}+\frac{2.5.8.11}{4.8.12.16}+\ldots\ldots=$

A. $4^{-2/3}$

B. $\sqrt[5]{16}$

C. $\sqrt[3]{4}$

D. $4^{2/3}$

Answer: C

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94. $\frac{3}{4.8} + \frac{3.5}{4.8.12} + \frac{3.5.7}{4.8.12.16} + \ldots. =$

A. $\sqrt{\frac{3}{2}} - \frac{3}{4}$

B. $\sqrt{\frac{2}{3}} - \frac{3}{4}$

C. $\sqrt{\frac{3}{2}} - \frac{1}{4}$

D. $\sqrt{\frac{2}{3}} - \frac{1}{4}$

Answer: A

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95. Observe the following statements :

Statement -I : In the expansion of $(1+x)^{50}$, the sum of the coefficients of odd powers of x is 2^{50} .

Statement -II : The coefficient of x^4 in the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is equal to $\frac{504}{259}$.

Then the true statements are :

A. only I

B. only II

C. both I and II

D. neither I nor II

Answer: D

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96. The following coefficients which are in increasing order are :

A. ${}^{25}C_{11}, {}^{25}C_{12}, {}^{25}C_{13}, {}^{25}C_{14}$

B. ${}^{25}C_{12}, {}^{25}C_{13}, {}^{25}C_{14}, {}^{25}C_{15}$

C. ${}^{25}C_{9}, {}^{25}C_{10}, {}^{25}C_{11}, {}^{25}C_{12}$

D. ${}^{25}C_{13}, {}^{25}C_{14}, {}^{25}C_{15}, {}^{24}C_{16}$

Answer: D

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97. Observe the following statements :

Statement - I : $\frac{1}{2}.\ {}^{10}C_0 - {}^{10}C_1 + 2.\ {}^{10}C_2 - 2^2.\ {}^{10}C_3 + \ldots\ldots + 2^9.\ {}^{10}C_{10} = -\frac{1}{2}$

Statement - II : ${}^{20}C_1 - 2\left({}^{20}C_2\right) + 3.\ \left({}^{20}C_3\right) - \ldots. - 20.\ \left({}^{20}C_{20}\right) = 0$

Then the false statements are :

A. only I

B. only II

C. both I and II

D. neither I nor II

Answer: C

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98. The number of terms in the expansions of $\left(1+3x+3x^2+x^3\right)^6, (x+y-z)^{16}, \left(x^2-2+\frac{1}{x^2}\right)^{25}$ are n_1, n_2, n_3 then

A. $n_1 < n_2 < n_3$

B. $n_1 < n_3 < n_2$

C. $n_2 < n_1 < n_3$

D. $n_2 < n_3 < n_1$

Answer: B

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99. $(1+x)^n = \sum_{r=0}^{n} C_r x^r$ then match the following

List-I	List-II
I. $1 + 2x + 3x^2 + 4x^3 + \ldots\ldots\infty$	a) $(1-x)^{-3}$
II. $1 - x + x^2 - x^3 + \ldots + (-1)^r x^r + \ldots\infty$	b) $(1-x)^{-2}$
III. $S_1 + S_2 x + S_3 x^2 + \ldots\ldots + S_{r+1} x^r + \ldots\ldots\infty$	c) $(1+x)^{-1}$

Where $S_r = 1 + 2 + 3 + \ldots\ldots\ldots + r$

	I	II	III		I	II	III		I	II	III		I	II	III
1)	b	a	c	2)	a	c	b	3)	c	a	b	4)	b	c	a

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100. $(1+x)^n = \sum_{r=0}^{n} C_r x^r$ then match the following

List-I	List-II
I. $1 + 2x + 3x^2 + 4x^3 + \ldots\ldots\infty$	a) $(1-x)^{-3}$
II. $1 - x + x^2 - x^3 + \ldots + (-1)^r x^r + \ldots\infty$	b) $(1-x)^{-2}$
III. $S_1 + S_2 x + S_3 x^2 + \ldots\ldots + S_{r+1} x^r + \ldots\ldots\infty$	c) $(1+x)^{-1}$

Where $S_r = 1 + 2 + 3 + \ldots\ldots\ldots + r$

	I	II	III		I	II	III		I	II	III		I	II	III
1)	b	a	c	2)	a	c	b	3)	c	a	b	4)	b	c	a

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101. Assertion (A): In the expansion of $(1+x)^n$, three consecutive terms are 5, 10,10 then n = 5

Reason (R): If the coefficient of r^{th}, $(r+1)^{th}$, $(r+2)^{th}$ terms of $(1+x)^n$ are in A.P. then $(n-2r)^2 = n+2$

A. A and R are true , R is correct explanation of A

B. A and R are true, R is not the correct explanation of A

C. A is true , R is false

D. A is false R is true

Answer: D

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102. Assertion (A) : In the $(1+x)^{50}$, the sum of the coefficients of odd powers of x is 2^{49}

Reason (R) : The sum of coefficients of odd powers of x in $(1+x)^n$ is 2^{n-1}

A. A and R are true , R is correct explanation of A

B. A and R are true, R is not the correct explanation of A

C. A is true , R is false

D. A is false R is true

Answer: A

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103. If $ab \neq 0$ and the sum of the coefficient of x^7 and x^4 in the expansion of $\left(\frac{x^2}{a} - \frac{b}{x}\right)^{11}$ zero, then

A. a = b

B. a + b = 0

C. ab = -1

D. ab = 1

Answer: D

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104. Coefficient of $x^3y^4z^2$ in $(2x - 3y + 4z)^9$ is

A. $\frac{9!}{4!4!}2^3 3^4 4^2$

B. $\frac{-9!}{3!2!4!}2^3 3^4 4^2$

C. $\frac{9!}{4!4!}2^3 3^4 4^2$

D. $\frac{9!}{3!4!2!}2^3 3^4 4^2$

Answer: D

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105. Coefficient of x^{18} in $\left(x^2 + 1\right)\left(x^2 + 4\right)\left(x^2 + 9\right)\ldots.\left(x^2 + 100\right)$ is

A. -385

B. 385

C. 285

D. -285

Answer: B

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106. If the middle term in the expansion of $(1+x)^{2n}$ is the greatest term, then x lies in the interval

A. $n-1 < x < n$

B. $\frac{n}{n+1} < x < \frac{n+1}{n}$

C. $n < x < n+1$

D. $\frac{n+1}{n} < x < \frac{n}{n+1}$

Answer: B

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107. If $2^{2006} - 2006$ divided by 7, the remainder is

A. 0

B. 1

C. 2

D. 4

Answer: A

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108. The greatest coefficient in $\left(\frac{x^{3/2}y}{2}+\frac{2}{xy^{3/2}}\right)^{12}$ is

A. $12\left(2^{11}\right)$

B. $12\left(2^{10}\right)$

C. $12\left(2^{22}\right)$

D. $33\left(2^{9}\right)$

Answer: D

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109. The term independent of x $(x > 0, x \neq 1)$ in the expansion of $\left[\frac{(x+1)}{\left(x^{2/3} - x^{1/3} + 1\right)} - \frac{(x-1)}{\left(x - \sqrt{x}\right)}\right]^{10}$ is

A. 105

B. 210

C. 315

D. 420

Answer: B

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PRACTICE EXERCISE

1. The middle term in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is

A. ${}^{2n}C_n$

B. ${}^{2n}C_n x$

C. $\frac{{}^{2n}C_n}{x}$

D. ${}^{2n}C_{n-1}x$

Answer: A

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2. If the first three terms in the binomial expansion of $(1+bx)^n$ in ascending powers of x are 1,6x and $16x^2$ respectively then b + n =

A. A) $\frac{28}{3}$

B. B) $\frac{15}{2}$

C. C) $\frac{29}{3}$

D. D) $\frac{17}{3}$

Answer: C

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3. If the first three terms in the expansion of $(1 - ax)^n$ where n is a positive integer are 1,-4x and $7x^2$ respectively then a =

A. $1/5$

B. $1/4$

C. $1/3$

D. $1/2$

Answer: D

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4. In the expansion of $\left(\frac{x^2}{2} - \frac{2}{x^2}\right)^8$, the third term from the last is

A. $\frac{448}{x^6}$

B. $\frac{428}{x^8}$

C. $\frac{324}{x^2}$

D. $\frac{448}{x^8}$

Answer: D

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5. The third term in the expansion of $\left(\frac{1}{x} + x^{\log_{10} x}\right)^5$ is 10^6 then x =

A. 10^{-1}

B. 10

C. 10^2

D. 10^3

Answer: B

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6. The numerically greatest term in the expansion $(5x - 6y)^{14}$ when x = 2/5, y = 1/2 is

A. ${}^{14}C_6 2^8 3^6$

B. ${}^{14}C_7 2^6 3^8$

C. ${}^{14}C_6 2^6 3^8$

D. ${}^{14}C_7 2^8 3^6$

Answer: C

View Text Solution

7. The no. of terms in $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$

A. 8

B. 6

C. 7

D. 4

Answer: D

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8. No. of distinct terms in $(x+y-z)^{16}$ is

A. 154

B. 126

C. 133

D. 153

Answer: D

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9. No. of distinct terms in $(a + b + c + d)^n, n \in N$ is

A. ${}^{(n+3)}C_2$

B. ${}^{(n+3)}C_3$

C. ${}^{(n+2)}C_3$

D. ${}^{(n+4)}C_3$

Answer: B

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10. If the coefficient of $(2r + 4)^{th}$ term and $(r - 2)^{th}$ term in the expansion of $(1 + x)^{18}$ are equal then find r.

A. 9

B. 4

C. 6

D. 3

Answer: C

View Text Solution

11. The number of integral terms in the expansion of $\left(\sqrt{2}+\sqrt[4]{3}\right)^{100}$ is

A. 75

B. 25

C. 26

D. 101

Answer: C

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12. The term independent of x in the expansion of $\left(x^2 - \frac{1}{x}\right)^6$ is

A. -12

B. 15

C. 24

D. -15

Answer: B

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13. If 5th term of the expansion $\left(\sqrt[3]{x} - \frac{1}{x}\right)^n$ is independent of x then n =

A. 16

B. 12

C. 8

D. 4

Answer: A

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14. Term independent of x in $(1 + 4x)^p \left(1 + \frac{1}{4x}\right)^q$ is :

A. ${}^{(p+q)}C_{4p}$

B. ${}^{(p+q)}C_{p}$

C. ${}^{(p+q)}C_{2p}$

D. ${}^{(p+q)}C_{3p}$

Answer: B

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15. Term independent of x in $\left(x - \frac{1}{x}\right)^4\left(x + \frac{1}{x}\right)^3$ is

A. 1

B. 2

C. 0

D. 4

Answer: C

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16. The sum of the binomial coefficients of the 3rd, 4th terms from the beginning and from the end of $(a + x)^n$ is 440 then n =

A. 10

B. 11

C. 12

D. 13

Answer: B

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17. If the sum of odd terms and the sum of even terms in the expansion of $(x + a)^n$ are p and q respectively then $p^2 + q^2 =$

A. $\frac{(x + a)^{2n} - (x - a)^{2n}}{2}$

B. $(x + a)^{2n} - (x - a)^{2n}$

C. $\frac{(x + a)^{2n} + (x - a)^{2n}}{2}$

D. $(x + a)^{2n} + (x - a)^{2n}$

Answer: C

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18. If the sum of odd terms and the sum of even terms in $(x + a)^n$ are p and q respectively then 4pq =

A. $(x + a)^{2n} - (x - a)^{2n}$

B. $\left(x^2 - a^2\right)^{2n} + (x + a)^{2n}$

C. $\left(x^2 - a^2\right)^{n} - (x - a)^{2n}$

D. $\left(x^2 + a^2\right)^{n} + (x - a)^{2n}$

Answer: A

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19. If A and B are coefficients of x^n in the expansion of $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ respectively, then find the value of $\frac{A}{B}$.

A. 1

B. 2

C. 3

D. 4

Answer: B

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20. The coefficient of x^{-17} in $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is

A. -1365

B. 1365

C. 465

D. -465

Answer: A

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21. The coefficient of x in $\left(x^2 + \frac{a}{x}\right)^5$ is 270 then a =

A. 3

B. 4

C. 5

D. 2

Answer: A

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22. If the 21st and 22nd terms in the expansion $(1 + x)^{44}$ are equal then x =

A. 7

B. 8

C. 7/8

D. 8/7

Answer: C

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23. If the coefficients of 2nd, 3rd, 4th terms of $(1 + x)^n$ are in A.P. then $n =$

A. 4

B. 5

C. 7

D. 6

Answer: C

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24. If the coefficients of 2nd , 3rd and 4th terms of the expansion of $(1 + x)^{2n}$ are in A.P. then the value of $2n^2 - 9n + 7$ is

A. 0

B. 5

C. 2

D. 6

Answer: A

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25. If the coefficients of 3rd, 4th , 5th terms in $(1+x)^{2n}$ are A.P. then $4n^2 - 26n + 40 =$

A. 3

B. 8

C. 9

D. 6

Answer: D

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26. If the coefficients of r,(r +1),(r+2) terms in $(1 + x)^{14}$ are in A.P. then r =

A. 3,2

B. 5,9

C. 2,4

D. 5,3

Answer: B

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27. If three successive coefficients in $(1 + x)^n$ are 6,15,20 then n =

A. 5

B. 7

C. 6

D. 9

Answer: C

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28. If a_1, a_2, a_3, a_4 are the coefficients of 2nd, 3rd, 4th and 5th terms of $(1+x)^n$ respectively then $\frac{a_1}{a_1+a_2}, \frac{a_2}{a_2+a_3}, \frac{a_3}{a_3+a_4}$ are in

A. A.P.

B. G.P

C. H.P

D. A.G.P

Answer: A

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29. If a,b,c,d are conseentive binomial coefficients of $(1+x)^n$ then $\frac{a+b}{a}, \frac{b+c}{b}, \frac{c+d}{c}$ are is

A. A.P.

B. G.P

C. H.P

D. A.G.P

Answer: C

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30. Coefficient of x^5 in $\left(1+x+x^2+x^3\right)^{10}$ is

A. 1910

B. 1902

C. 1819

D. 1932

Answer: B

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31. The coefficient of x^4 in the expansion of $\left(1 + x - 2x^2\right)^7$ is

A. -81

B. -91

C. 81

D. 91

Answer: B

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32. Binomial coefficients which are in decreasing order are

A. ${}^{15}C_5, {}^{15}C_6, {}^{15}C_7$

B. ${}^{15}C_{10}, {}^{15}C_9, {}^{15}C_8$

C. ${}^{15}C_6, {}^{15}C_7, {}^{15}C_8$

D. $615C_7, {}^{15}C_6, {}^{15}C_5$

Answer: D

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33. Coefficient of x^3 in $1 + (1 + x) + (1 + x)^2 + \ldots . + (1 + x)^n$ is

A. ${}^nC_4$

B. ${}^{(n+1)}C_4$

C. ${}^{(n+2)}C_4$

D. ${}^{(n+1)}C_2$

Answer: B

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34. Sum of the coefficients of $\left(1 + 2x - 4x^2\right)^{2003}$

A. 1

B. 2

C. 3

D. - 1

Answer: D

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35. Sum of coefficients of terms of even powers of x in $\left(1 + x + x^2 + x^3\right)^5$ is

A. 512

B. 516

C. 612

D. 234

Answer: A

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36. Sum of coefficients of terms of odd powers of in $\left(1 + x - x^2 - x^3\right)^8$ is

A. 0

B. 1

C. 2

D. -1

Answer: A

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37. If $\left(1 + x - 2x^2\right)^8 = 1 - a_1x + a_2x^2 + \ldots\ldots + a_{16}x^{16}$, then $a_2 + a_4 + a_6 + \ldots\ldots + a_{16} =$

A. 120

B. 123

C. 127

D. 231

Answer: C

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38. If $\left(1 + x - 2x^2\right)^8 = 1 + a_1x + a_2x^2 + \ldots\ldots + a_{16}x^{16}$, then $a_1 + a_3 + a_5 + \ldots\ldots + a_{15} =$

A. 2^7

B. -2^7

C. 3^2

D. 4^6

Answer: B

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39. $(1+x)^{25} = \sum_{r=0}^{25} C_r x^r$ then $C_1 - C_3 + C_5 - C_7 + \ldots\ldots - C_{25} =$

A. 2^{10}

B. -2^{10}

C. 2^{12}

D. -2^{12}

Answer: C

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40. The sum of the coefficients in the expansion of $\left(1 + x - 3x^2\right)^{171}$ is

A. 0

B. 1

C. -1

D. 2

Answer: C

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41. If the sum of all the binomial coefficients in $(x+y)^n$ is 512 , then the greatest binomial coefficient is

A. ${}^{10}C_5$

B. ${}^{9}C_4$ or ${}^{9}C_5$

C. ${}^{11}C_5$ or ${}^{11}C_6$

D. ${}^{12}C_6$

Answer: B

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42. The coefficient of x^{-n} in $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is

A. 0

B. 1

C. 2^n

D. ${}^{2n}C_n$

Answer: B

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43. If n is a positive integer then $2^{4n} - 15n - 1$ is divisible by

A. 64

B. 196

C. 225

D. 256

Answer: C

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44. Smaller of $\left(19^{10} + 20^{10}\right)$ and 21^{10} is

A. $19^{10} + 20^{10}$

B. 21^{10}

C. Both are equal

D. can not be decided

Answer: A

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45. $9^{11} + 11^9$ is divisible by

A. 7

B. 8

C. 9

D. 10

Answer: D

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46. Integral part of $\left(8 + 3\sqrt{7}\right)^n$ is

A. an even number

B. an odd number

C. an even or an odd number depending upon the value of n

D. nothing can be said

Answer: B

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47. If $n = \left(\sqrt{2} + 1\right)^6$. Then the integer just greater than n is

A. 199

B. 198

C. 197

D. 196

Answer: B

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48. $1.\ C_0 + 3.\ C_1 + 3^2.\ C_2 + \ldots. + 3^n.\ C_n =$

A. 4^n

B. 3^n

C. 5^n

D. 2^n

Answer: A

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49. $C_0 + \frac{C_1}{2} + \frac{C_2}{2^2} + \frac{C_3}{2^3} + \ldots . + \frac{C_n}{2^n} =$

A. $(1/2)^n$

B. $(3/2)^n$

C. $(3/2)^{2n}$

D. $(3/2)^{-n}$

Answer: B

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50. ${}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \ldots\ldots + {}^{11}C_5 =$

A. 2^7

B. 2^8

C. 2^9

D. 2^{10}

Answer: D

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51. $C_0^2 + C_1^2 + C_2^2 - \ldots\ldots\ldots - C_{15}^2 =$

A. 1

B. 2

C. 3

D. 0

Answer: D

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52. $\frac{1}{2}.\ {}^{10}C_0 - {}^{10}C_1 + 2.\ {}^{10}C_2 - 2^2.\ {}^{10}C_3 + \ldots . + 2^9.\ {}^{10}C_{10} =$

A. $1/2$

B. $1/4$

C. $3/2$

D. $1/3$

Answer: A

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53. ${}^{2n+1}C_0^2 - {}^{2n+1}C_1^2 + {}^{2n+1}C_2^2 - \ldots\ldots - {}^{2n+1}C_{2n+1}^2 =$

A. 0

B. $^{(2n+1)}C_n$

C. $-\left(^{2n+1}C_n\right)$

D. $-\frac{1}{2}\left(^{2n}C_n\right)$

Answer: A

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54. Prove that following

i) $2.C_0 + 5.C_1 + 8.C_2 + \ldots . + (3n+2)C_n = (3n+4).2^{n-1}$

A. $(4-3n).2^{2n-1}$

B. $(4-3n).2^{n+1}$

C. $(4+3n).2^{n-1}$

D. $(2-3n).2^{n+1}$

Answer: C

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55. ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \ldots\ldots + {}^{20}C_{10} =$

A. $\left[2^{19} - \frac{1}{2}.\ {}^{18}C_{10}\right]$

B. $\left[2^{19} + \frac{1}{2}.\ {}^{20}C_{10}\right]$

C. $2^{19} - \frac{1}{4}.\ {}^{18}C_{10}\right]$

D. $\left[2^{19} - \frac{1}{4}.\ {}^{6}C_{3}\right]$

Answer: B

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56. $C_1 + 2.\ C_2 + 3.\ C_3 + \ldots\ldots + n.\ C_n =$

A. 2^n

B. $n.2^n$

C. $n.2^{n-1}$

D. $n.2^{n+1}$

Answer: C

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57. If a_r is the coefficient of x^r in the expansion of $(1+x)^n$ then $\frac{a_1}{a_0} + 2.\frac{a_2}{a_1} + 3.\frac{a_3}{a_2} + \ldots . + n.\frac{a_n}{a_{n-1}} =$

A. $\frac{n(n+1)}{2}$

B. $\frac{n(n+3)}{2}$

C. $\frac{n(n-1)}{2}$

D. n^2

Answer: A

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58. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_nx^n$, then $C_0 + 2.C_1 + 3.C_2 + \ldots + (n+1).C_n =$

A. $2^n + n.2^{n-1}$

B. $2^{n-1} + n.2^n$

C. $2^n + (n+1)2^{n-1}$

D. 2^{n+1}

Answer: A

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59. $1.\ {}^{20}C_1 - 2.\ {}^{20}C_2 + 3.\ {}^{20}C_3 - \ldots - 20.\ {}^{20}C_{20} =$

A. 1

B. 2

C. -1

D. 0

Answer: D

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60.

$\left(1 + x + x^2 + \ldots\ldots + x^p\right)^n = a_0 + a_1 x + a_2 x^2 + \ldots. + a_{np} x^{np} \Rightarrow a_1 + 2a_2 + 3a_3 +$

A. $\frac{np(p+1)^n}{2}$

B. $\frac{np(p+1)^n}{4}$

C. $\frac{np(p-1)^n}{4}$

D. $\frac{np(p-1)^{2n}}{4}$

Answer: A

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61. $C_1 + 4.\ C_2 + 7.\ C_3 + \ldots\ldots. + (3n-2).\ C_n =$

A. $(3n + 4)2^{n-1} - 2$

B. $(2n - 4)2^{n-1} - 2$

C. $(3n - 4)2^{n-1} - 2$

D. $(3n - 4)2^{n-1} + 2$

Answer: D

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62. $\sum_{r=1}^{n} (-1)^{r-1} {}^{n}C_{r}(a - r) =$

A. a

B. $-a$

C. 2a

D. 3a

Answer: A

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63. $2.\ C_0 + \frac{2^2}{2}.C_1 + \frac{2^3}{3}.C_2 + \ldots\ldots + \frac{2^{11}}{11}.C_{10} =$

A. $\frac{3^{11} - 1}{11}$

B. $\frac{3^9 - 1}{9}$

C. $\frac{3^{11} + 1}{11}$

D. $\frac{3^{12} - 1}{12}$

Answer: A

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64. ${}^{11}C_0^2 - {}^{11}C_1^2 + {}^{11}C_2^2 - {}^{11}C_3^2 + \ldots\ldots - {}^{11}C_{11}^2 =$

A. 0

B. ${}^{22}C_{11}$

C. $(-1)^{11}.{}^{22}C_{11}$

D. 1

Answer: A

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65. $\frac{C_0}{1} + \frac{C_2}{3} + \frac{C_4}{5} + \ldots\ldots + \frac{C_{16}}{17} =$

A. $\frac{2^{15}}{14}$

B. $\frac{2^{16}}{17}$

C. $\frac{2^{15}}{16}$

D. $\frac{2^{20}}{22}$

Answer: B

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66. $\frac{C_1}{2} + \frac{C_3}{4} + \ldots\ldots + \frac{C_{15}}{16} =$

A. $\frac{2^{15} - 1}{16}$

B. $\frac{2^{15} - 1}{16}$

C. $\frac{2^{14} - 1}{16}$

D. $\frac{2^{40} + 1}{16}$

Answer: A

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67. The coefficient of x^3 in $(1 - 4x)^{1/2}$ is

A. -4

B. 2

C. -8

D. 12

Answer: A

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68. The range of x for which the expansion of $\left(1 - \frac{3}{x}\right)^{-3/4}$ is valid is

A. $|x| < 1$

B. $|x| < 3/4$

C. $|x| < -3/4$

D. $|x| > 3$

Answer: D

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69. The range of x of which the expansion of $\left(2 - 3x^2\right)^{-\frac{11}{2}}$ is valid is

A. $\left(-\sqrt{\frac{2}{3}}, \frac{2}{3}\right)$

B. $\left(-\frac{2}{3}, \sqrt{\frac{2}{3}}\right)$

C. $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$

D. $\left(-\infty, \sqrt{\frac{2}{3}}\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$

Answer: C

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70. The fifth term of $\left(1 - \frac{2x}{3}\right)^{3/4}$ is

A. $\frac{-5x^4}{1152}$

B. $\frac{5x^4}{1152}$

C. $-\frac{5x^4}{1052}$

D. $\frac{5x^4}{1052}$

Answer: A

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71. For $|x| < 1$, the $(r+1)^{th}$ term in the expansion of $\sqrt{1-x}$ is

A. $\frac{1.3.5\ldots\ldots.(2r-3)}{r!}\left(\frac{x}{2}\right)^r$

B. $-\frac{1.3.5\ldots\ldots.(2r-3)}{r!}\left(\frac{x}{2}\right)^r$

C. $-\frac{1.3.5\ldots\ldots.(2r-3)}{r!}(x)^r$

D. $\frac{1.3.5\ldots\ldots.(2r-3)}{r!}(x)^r$

Answer: B

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72. The general term of $(2a-3b)^{-1/2}$ is

A. $\frac{1.3.5\ldots\ldots(2r-5)}{r!}\frac{1}{\sqrt{2a}}\left(\frac{3b}{4a}\right)^r$

B. $\frac{1.3.5\ldots\ldots(2r-3)}{r!}\frac{1}{\sqrt{2a}}\left(\frac{3b}{4a}\right)^r$

C. $\frac{1.3.5\ldots\ldots(2r-1)}{r!}\frac{1}{\sqrt{2a}}\left(\frac{3b}{4a}\right)^r$

D. $\frac{1.3.5.....(2r-3)}{r!}\frac{1}{\sqrt{a}}\left(\frac{3b}{4a}\right)^r$

Answer: C

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73. In the expansion of $\left(1+x+x^2+x^3+....\infty\right)^3$, coefficient of x^3 is

A. 4

B. 6

C. 8

D. 10

Answer: D

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74. In the expansion of $\left(1-2x+3x^2-4x^3+....\text{to}\right)^4$ coefficient of x^2 is

A. 72

B. 36

C. -36

D. -72

Answer: B

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75. The coefficient of x^{24} in $\left(1 + 3x + 6x^2 + 10x^3 + \ldots\ldots\infty\right)^{2/3}$ is

A. 25

B. 125

C. 50

D. 300

Answer: A

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76. The coefficient of x^{10} in $\frac{1 - 2x + 3x^2}{1 - x}$ is

A. 1

B. 2

C. 3

D. -2

Answer: B

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77. The coefficient of x^9 in the expansion of $\frac{1 - 5x}{1 + x}$ is

A. -6

B. 9

C. -9

D. 6

Answer: A

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78. The coefficient of x^4 in the expansion of $\frac{(1 - 3x)^2}{(1 - 2x)}$ is

A. 1

B. 2

C. 3

D. 4

Answer: D

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79. For $|x| < 1$, the coefficient of x^r in the expansion of $\frac{(1 + x)^2}{(1 - x)^3}$ is

A. $2r^2 + 2r + 1$

B. $2r^2 + 2r - 1$

C. $2r^2 - 2r + 1$

D. $r^2 + 2r - 3$

Answer: A

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80. If $x = 1 + 3a + 6a^2 + 10a^3 + \ldots\ldots$ to ∞ terms $|a| < 1, y = 1 + 4a + 10a^2 + 20a^3 + \ldots.$ to ∞ terms, $|a| < 1$, then x:y

A. $(1 - a) : 1$

B. $1 : (1 - a)$

C. $(1 + a) : 1$

D. $1 : (1 + a)$

Answer: A

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81. Prove that : If $|x|$ is so small that x^2 and higher powers of x may be neglected, then find an approximate value of

A. $1 + \frac{11x}{12}$

B. $2 + \frac{35x}{6}$

C. $1 - \frac{5x}{12}$

D. $1 + \frac{5x}{12}$

Answer: B

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82. If |x| is so small that all terms containing x^2 and higher powers of x can be neglected , then the approximate value of $\frac{(3 - 5x)^{1/2}}{(5 - 3x)^2}$, where $x = \frac{1}{\sqrt{363}}$, is

A. $\frac{\sqrt{3}}{25}$

B. $\frac{1 + 30\sqrt{3}}{75}$

C. $\frac{1 - 30\sqrt{3}}{75}$

D. $\frac{1 + 30\sqrt{3}}{750}$

Answer: D

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83. If $x = \frac{1 \cdot 3}{3 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{3 \cdot 6 \cdot 9} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{3 \cdot 6 \cdot 9 \cdot 12} + \ldots$ to infinite terms, then $9x^2 + 24x =$

A. 11

B. 21

C. 31

D. 41

Answer: A

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84. $1 - \frac{1}{5} + \frac{1.4}{5.10} - \frac{1.5.7}{5.10.15} + \ldots . =$

A. $\frac{1}{3}\sqrt[3]{5}$

B. $\frac{1}{2}\sqrt[3]{5}$

C. $\frac{1}{2}\sqrt[3]{4}$

D. $\sqrt[3]{5}$

Answer: B

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85. $1 + \frac{1}{2}.\frac{3}{5} + \frac{1.3}{2.4}.\frac{9}{25} + \frac{1.3.5}{2.4.6}.\frac{27}{125} + \ldots\ldots\infty =$

A. $\sqrt{\frac{5}{2}}$

B. $\sqrt{\frac{3}{5}}$

C. $\sqrt{\frac{2}{5}}$

D. $\sqrt{\frac{5}{3}}$

Answer: A

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86. $1+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{4.8.12}+\ldots\ldots=$

A. $\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: A

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87. If $x = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + \ldots. \infty$ then find $3x^2 + 6x$.

A. 0

B. 1

C. 2

D. -1

Answer: C

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88. $2 + \frac{5}{2!3} + \frac{5.7}{3!3^2} + \frac{5.7.9}{4!3^2} + \ldots . \infty =$

A. $\sqrt{3}$

B. $\sqrt[3]{3}$

C. $3\sqrt{3}$

D. $2\sqrt{3}$

Answer: C

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89. Prove that : Find the sum of the infinite series

$$1+\frac{2}{3}.\frac{1}{2}+\frac{2.5}{3.6}\left(\frac{1}{2}\right)^2+\frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^3+.....\infty$$

A. $\sqrt[3]{3}$

B. $\sqrt[4]{4}$

C. $\sqrt[4]{8}$

D. $\sqrt[3]{4}$

Answer: D

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90. Sho that $\frac{3}{6}+\frac{3.5}{6.9}+\frac{3.5.7}{6.9.12}+\ldots\ldots.\infty 3\sqrt{3}-4$

A. $2\sqrt{3} - 4$

B. $3\sqrt{3} - 2$

C. $3\sqrt{3} - 4$

D. $2\sqrt{3} + 4$

Answer: C

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91. $1 + \frac{1}{3} + \frac{1.3}{1.2} \cdot \frac{1}{3^2} + \frac{1.3.5.}{1.2.3} \cdot \frac{1}{3^3} + \ldots\ldots\infty =$

A. $\sqrt{2}$

B. $\sqrt{3}$

C. $\sqrt{5}$

D. $\frac{1}{2\sqrt{3}}$

Answer: B

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92. Observe the following statements :

Statement - I: The total number of terms in the expansion of $(x + y)^{100} + (x - y)^{100}$ after simplification is 51

Statement - II : If ${}^{43}C_{r-6} = {}^{43}C_{r+1}$ then r = 12

Statement - III : The coefficient of x^n in $(1 - x)^{-2}$ is (n+1) .

Then the true statements are :

A. only I, II

B. only II, III

C. only III, I

D. all the three

Answer: C

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93. Let l,m,n are the coefficients of x^5 in

$$\left(1 + 2x + 3x^2 + \ldots.\right)^{-3/2}, \left(1 + x + x^2 + x^3 + \ldots.\right)^2, (1 + x)^5$$

Respectively then :

A. $l < m < n$

B. $m < n < l$

C. $n < l < m$

D. $l < n < m$

Answer: D

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94. Match the following

List - I (Expansion)	List - II (No. of terms)
I. $(p+q)^{50}+(p-q)^{50}$	a) 23
II. $(p+q)^{50}-(p-q)^{50}$	b) 24
III. $(p+q)^{47}+(p-q)^{47}$	c) 25
IV. $(p+q)^{45}-(p-q)^{45}$	d) 26

The correct match is

	I	II	III	IV		I	II	III	IV
1)	a	b	c	d	2)	d	c	b	a
3)	a	c	d	b	4)	d	a	b	c

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95. Match the following question

COLUMN -I	COLUMN -II
A) $R\text{-}CONH_2$	p) most reactive towards acyl substitution
B) R - COOR'	q) reduces $HgCl_2$
C) HCOOH	r) high boiling point
D) RCOCl	s) fruit flavour

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EXERCISE - 1.1 (Level - 1)

1. Expand the following using binomial theorem.

$(4x + 5y)^7$

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2. Expand the following using binomial theorem.

$\left(\frac{2}{3}x + \frac{7}{4}y\right)^5$

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3. Expand the following using binomial theorem.

$\left(\frac{2p}{5} - \frac{3q}{7}\right)^6$

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4. Write down and simplify 6^{th} term in $\left(\frac{2x}{3} + \frac{3y}{2}\right)^9$

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5. Write down and simplify

7^{th} term in$(3x - 4y)^{10}$

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6. Write down and simplify

10^{th} term in $\left(\frac{3p}{4} - 5q\right)^{14}$

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7. Find the number of terms in the expansion of

$\left(\frac{3a}{4} + \frac{b}{2}\right)^9$

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8. Find the number of terms in the expansion of

$(3p + 4q)^{14}$

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9. Find the number of terms in the expansion of

$(x + y + z)^{20}$

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10. Find the number of terms in the expansion of $(x - 2y + 3z)^{10}$

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11. Find the number of terms in the expansion of

$(2x + 3y + z)^7$

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12. Find the middle term(s) in the expansion of $n \in N$

$\left(\frac{1}{2}x - 3y\right)^{20}$

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13. Find the middle term(s) in the expansion of $n \in N$

$\left(x\sqrt{x} - \frac{2}{x}\right)^{15}$

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14. Find the middle term(s) in the expansion of $n \in N$

$$\left(p^2 - 2q\right)^{2n-1}$$

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15. Find the middle term(s) in the expansion of $n \in N$

$$\left(a^3 + \frac{2}{b}\right)^{4n}$$

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16. Show that the sum of the coefficients of x^{-6} and x^{-1} in $\left(5x^2 - \frac{7}{x^3}\right)^{12}$ is positive.

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17. Show that the sum of the coefficients of x^{25} and x^{-10} of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is zero.

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18. Find the numerically greatest term of

$(3 + 7x)^{15}, x = \frac{4}{5}$

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19. Find the numerically greatest term of

$(2x - 3y)^{12}, x = 1, y = \frac{5}{3}$

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20. Find the numerically greatest term of

$(5x - 6y)^{14}, x = \frac{2}{5}, y = \frac{1}{2}$

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21. If the coefficients of x^9, x^{10}, x^{11} in expansion of $(1 + x)^n$ are in A.P., the prove that $n^2 - 41n + 398 = 0$.

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22. If the $2^{nd}, 3^{rd}$ and 4^{th} terms in the expansion of $(a + x)^n$ are respectively 240, 720, 1080, find a, x, n.

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23. If $\left(1 + x + 2x^2 + 4x^3\right)^{10} = a_0 + a_1x + a_2x^2 + \ldots. + a_{30}x^{30}$.

Find the value of

$a_0 + a_1 + a_2 + \ldots + a_{30}$

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24. If $\left(1 + x + 2x^2 + 4x^3\right)^{10} = a_0 + a_1 x + a_2 x^2 + \ldots + a_{30} x^{30}$.

Find the value of

$a_0 - a_1 + a_2 - a_3 \ldots + a_{30}$

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25. If $\left(1 + x + 2x^2 + 4x^3\right)^{10} = a_0 + a_1 x + a_2 x^2 + \ldots + a_{30} x^{30}$.

Find the value of

$a_0 + a_2 + a_4 + \ldots + a_{30}$

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26. If $\left(1 + x + 2x^2 + 4x^3\right)^{10} = a_0 + a_1 x + a_2 x^2 + \ldots + a_{30} x^{30}$.

Find the value of

$a_1 + a_3 + a_5 + \ldots + a_{29}$

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27. If $\left(1 + x + x^2\right)^n = b_0 + b_1 x + b_2 x^2 + \ldots + b_{2n} x^{2n}$ then prove that

$b_0 + b_1 + b_2 + \ldots + b_{2n} = 3^n$

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28. If $\left(1 + x + x^2\right)^n = b_0 + b_1 x + b_2 x^2 + \ldots + b_{2n} x^{2n}$ then prove that

$b_0 - b_1 + b_2 - \ldots + b_{2n} = 1$

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29. If $\left(1 + x + x^2\right)^n = b_0 + b_1 x + b_2 x^2 + \ldots + b_{2n} x^{2n}$ then prove that

$b_0 + b_2 + b_4 + b_6 + \ldots + b_{2n} = \frac{3^n + 1}{2}$

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30. If $\left(1+x+x^2\right)^n = b_0 + b_1x + b_2x^2 + \ldots. + b_{2n}x^{2n}$ then prove that

$b_1 + b_3 + b_5 + \ldots.. + b_{2n-1} = \frac{3^n - 1}{2}$

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31. If $\left(1+x+x^2\right)^n = b_0 + b_1x + b_2x^2 + \ldots. + b_{2n}x^{2n}$ then prove that

$b_0 + b_3 + b_6 + b_9 + \ldots.. = 3^{n-1}$

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32. If $\left(1+x+x^2\right)^n = b_0 + b_1x + b_2x^2 + \ldots. + b_{2n}x^{2n}$ then prove that

If $n = 25, b_0 - b_2 + b_4 - b_6 + \ldots. - b_{2n} = 0$

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33. If $\left(1+x+x^2\right)^n = b_0 + b_1x + b_2x^2 + \ldots. + b_{2n}x^{2n}$ then prove that

If $n = 37, b_1 - b_3 + b_5 - b_7 + \ldots b_{2n-1} = 1$

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34. Prove that $\forall n \in N$

$3^{3n} - 26n - 1$ is divisible by 676.

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35. Prove that $\forall n \in N$

$6^n - 5n$ leaves remainder 1 when divided by 25.

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36. Prove that $\forall n \in N$

$9^{n+1} - 8n - 9$ is divisible by 64

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37. Prove that $\forall n \in N$

$5^{4n} + 52n - 1$ is divisible by 676.

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38. If the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ equals the coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$, then a and b satisfy the relation

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39. Show that the number of terms with integral values in the expansion of $\left(3^{1/3} + 7^{1/2}\right)^{400}$ is 67.

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40. Find the coefficient of x^7 in the expansion of $\left(1 - x - x^2 + x^3\right)^6$.

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41. Find the coefficient of x^3 in the expansion of $\left(2 - x + 5x^2\right)^6$.

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42. If the coefficients of r^{th}, $(r+1)^{\text{th}}$ and $(r+2)^{\text{nd}}$ terms in the expansion of $(1+x)^n$ are in A.P. then show that $n^2 - (4r+1)n + 4r^2 - 2 = 0$.

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43. If a_1, a_2, a_3, a_4 are the coefficients of the $2^{nd}, 3^{rd}, 4^{th}$ and 5^{th} terms respectively in the binomial expansion of $(1+x)^n$ where n is a positive integer prove that $\frac{a_1}{a_1 + a_2}, \frac{a_2}{a_2 + a_3}, \frac{a_3}{a_3 + a_4}$ are in arithmetic progression.

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44. If P and Q are the sum of odd terms and the sum of even terms respectively in the expansion of $(x + a)^n$ then prove that

$P^2 - Q^2 = \left(x^2 - a^2\right)^n$

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45. If P and Q are the sum of odd terms and the sum of even terms respectively in the expansion of $(x + a)^n$ then prove that

$4PQ = (x + a)^{2n} - (x - a)^{2n}$

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46. Simplify the following

$\left(\sqrt{3} + \sqrt{2}\right)^4 - \left(\sqrt{3} - \sqrt{2}\right)^4$

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47. Simplify the following

$$\left(\sqrt{2}+1\right)^5+\left(\sqrt{2}-1\right)^5$$

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48. Simplify the following

$$\left(\sqrt{3}+1\right)^5-\left(\sqrt{3}-1\right)^5$$

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49. Simplify the following

$$\left(\sqrt{x+1}+\sqrt{x-1}\right)^6+\left(\sqrt{x+1}-\sqrt{x-1}\right)^6$$

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50. Using binomial theorem, prove that $50^n - 49n - 1$ is divisible by 49^2 for all positive integers n.

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51. Using binomial theorem, prove that $5^{4n} + 52n - 1$ is divisible by 676 for all positive integers n.

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52. Find the remainder when 2^{2013} in divided by 17.

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EXERCISE - 1.2 (Level - 1)

1. Prove the following:

$C_0 + 2 \cdot C_1 \_ + 3 \cdot C_2 + \ldots (n+1) \cdot C_n = (n+2).2^{n-1}$

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2. Prove the following:

$2 \cdot C_0 + 3 \cdot C_1 + 4 \cdot C_2 + \ldots + (n+2) \cdot C_n = (n+4).2^{n-1}$

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3. Prove that following

i) $2.C_0 + 5.C_1 + 8.C_2 + \ldots . + (3n+2)C_n = (3n+4).2^{n-1}$

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4. Prove the following:

$3 \cdot C_1 + 7 \cdot C_2 + 11 \cdot C_3 + \ldots (4n-1) \cdot C_n = 1 + (2n-1)2^n$

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5. Prove the following:

$$\sum_{r=0}^{n} (-1)^r (3r+5).C_r = 0$$

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6. Prove that following

$$C_0 + \frac{3}{2}.C_1 + \frac{9}{3}.C_2 + \frac{27}{4}.C_3 + \ldots\ldots + \frac{3^n}{n+1}.C_n = \frac{4^{n+1}-1}{3(n+1)}.$$

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7. Prove the following:

$$1.2.C_1 + 2.3C_2 + 3.4.C_3 + \ldots. n(n+1).C_n = n.(n+3).2^{n-2}$$

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8. Prove that : Prove that

$$\frac{C_1}{C_0} + 2.\frac{C_2}{C_1} + 3.\frac{C_3}{C_2} + \ldots. + n.\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$$

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9. Prove the following:

$$C_0 + \frac{C_2}{3} + \frac{C_4}{5} + \ldots = \frac{2^n}{n+1}$$

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10. Prove that following

$$\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \frac{C_7}{8} + \ldots\ldots = \frac{2^n - 1}{n+1}$$

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11. Prove the following:

$$C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \ldots + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$$

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12. Prove that

$$\left({}^{2n}C_0\right)^2 - \left({}^{2n}C_1\right)^2 + \left({}^{2n}C_2\right) - \left({}^{2n}C_3\right)^2 + \ldots\ldots + \left({}^{2n}C_{2n}\right)^2 = (-1)^n {}^{2n}C_n.$$

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13. Prove that

$$\left(C_0 + C_1\right)\left(C_1 + C_2\right)\left(C_2 + C_3\right)\ldots\ldots\left(C_{n-1} + C_n\right) = \frac{(n+1)^n}{n!} \cdot C_0 \cdot C_1 \cdot C_2 \ldots\ldots C_n.$$

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14. Prove that

$$\left(C_0 + C_1 + C_2 + \ldots\ldots + C_n\right)^2 = 1 + {}^{2n}C_1 + {}^{2n}C_2 + \ldots\ldots + {}^{2n}C_{2n}$$

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15. Use the identity $(1+x)^m(1+x)^n = (1+x)^{m+n}$ to prove Vandermonde's theorem,

$^{m}C_{r} + {}^{m}C_{r-1} \cdot {}^{n}C_{1} + {}^{m}C_{r-2} \cdot {}^{n}C_{2} + \ldots + {}^{n}C_{r} = {}^{(m+n)}C_{r}$

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16. Find the sum of the series $C_1 + 2^2 \cdot C_2 x + 3^2 \cdot C_3 x^2 + 4^2 \cdot C_4 x^3 + \ldots + n^2 \cdot C_n \cdot x^{n-1}$ and deduce $(n > 2)$ the value of $C_1 - 2^2 \cdot C_2 + 3^2 - C_3 - \ldots + (-1)^{n-1} \cdot n^2 \cdot C_n$

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17. Show that

$$\frac{2^2 \cdot C_0}{1 \cdot 2} + \frac{2^3 \cdot C_1}{2 \cdot 3} + \frac{2^4 \cdot C_2}{3 \cdot 4} + \ldots + \frac{2^{n+2} \cdot C_n}{(n+1)(n+2)} = \frac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$$

Hence deduce that $\frac{C_0}{1.2} - \frac{C_1}{2.3} + \frac{C_2}{3.4} - \ldots = \frac{1}{n+2}$

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18. Show that $^{n}C_0 + {}^{(n+1)}C_1 + {}^{(n+2)}C_2 + \ldots + {}^{(n+k)}C_k = {}^{(n+k+1)}C_k$

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19. Show that ${}^{30}C_0 + {}^{30}C_1 + {}^{30}C_2 + \ldots + {}^{30}C_{14} = 2^{29} - \frac{1}{2}\left({}^{30}C_{15}\right)$

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EXERCISE - 1.3 (Level - 1)

1. Find the range of x for which the following expansions are valid .

$$\left(2 - \frac{3x}{4}\right)^{-15/4}$$

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2. Find the range of x for which the following expansions are valid .

$$\left(3 + \frac{5x^2}{3}\right)^{-3/7}$$

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3. Find the range of x for which the binomial expansions of the following are valid .

$$\left(4 - \frac{x}{3}\right)^{-1/2}$$

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4. Write the first three terms of the expansion of

$$\left(1 + \frac{x^2}{2}\right)^{-5}$$

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5. Write the first three terms of the expansion of

$$(2 + x)^{-1/2}$$

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6. Write down the first three terms is the following expansions

$(8 - 5x)^{2/3}$

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7. Find 4^{th} term of $(8 - x)^{1/3}$

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8. Find 6^{th} term of $\left(3 - 4x^2\right)^{-1}$

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9. Prove that : Find the

8^{th} term of $\left(1 - \frac{5x}{2}\right)^{-3/5}$

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10. Find the Coefficient of x^4 in $(8 - x)^{1/3}$

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11. Find the coefficient of x^6 in $(1 - 3x)^{-2/5}$

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12. Find the Coefficient of x^{10} in $\frac{1 + 2x}{(1 - 2x)^2}$

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13. Find the Coefficient of x^n in $\left(\frac{1 + x}{1 - x}\right)^2$

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14. Find the general term in the expansion of

$\left(4 - 7x^2\right)^{-2/5}$

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15. Prove that : Write the general term in the expansion of

$(2 - 3x)^{-1/3}$

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16. Find the coefficient of x^4 in $\frac{(2 + 3x)^3}{(1 - 3x)^4}$

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17. Find the coefficient of x^n in $\frac{(1 + 2x)^3}{(1 - x)^2}$

Watch Video Solution

18. Find the coefficient of x^{10} in the expansion of $\frac{1+2x}{(1-2x)^2}$.

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19. Find the coefficient of x^4 in the expansion of $(1-4x)^{-3/5}$.

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20. Find the coefficient of x^5 in $\frac{(1-3x)^2}{(3-x)^{3/2}}$.

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21. Find the coefficient of x^8 in $\frac{(1+x)^2}{\left(1-\frac{2}{3}x\right)^3}$.

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22. Find the coefficient of x^7 in $\frac{(2+3x)^3}{(1-3x)^4}$.

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23. Find the coefficient of x^3 in the expansion of $\frac{\left(1+3x^2\right)^{\frac{3}{2}}}{(3+4x)^{\frac{1}{3}}}$

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24. If $|x|$ is so small that x^2 and higher powers of x may be neglected then find the approx-imate values of the following

$$\frac{\sqrt{4+x}+\sqrt[3]{8+x}}{(1+2x)+(1-2x)^{-1/3}}$$

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25. By neglecting x^4 and higher powers of x, find an approximate value of $\sqrt[3]{x^2+64}-\sqrt[3]{x^2+27}$.

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26. Expand : $3\sqrt{3}$ in ascending powers of $\frac{1}{3}$

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27. Prove that : Expand $5\sqrt{5}$ in increasing powers of $\frac{4}{5}$.

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28. If b is small compared to 'a' show that $\frac{c}{(a-b)^2} - \frac{c}{(a+b)^2} = \frac{4bc}{a^3}$ approximately.

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29. If 'c' is small in comparison with l then $\left(\frac{l}{l+c}\right)^{1/2} + \left(\frac{l}{l-c}\right)^{1/2} =$

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30. Find the values of the following correct to five decimals.

$$\frac{1}{\sqrt[3]{128}}$$

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31. Find the values of the following correct to five decimals.

$$\sqrt{3.96}$$

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32. Find the values of the following correct to five decimals.

$$\sqrt{1.01} - \sqrt{0.99}$$

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33. Find the sum of the infinite series

$$1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\ldots\ldots$$

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34. Find the sum of the infinite series

$$1-\frac{4}{5}+\frac{4.7}{5.10}-\frac{4.7.10}{5.10.15}+\ldots\ldots\ldots$$

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35. Find the sum of the infinite series

$$\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+\ldots\ldots$$

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36. Find the sum of the infinite series

$$\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}-\ldots\ldots$$

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37. If $x = \frac{3}{16}(3) + \frac{3.7}{16.32}(3)^2 + \frac{3.7.11}{16.32.48}(3)^3 + \ldots$ then show that $x^2 + 2x = 7$

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38. If $t = \frac{4}{5} + \frac{4.6}{5.10} + \frac{4.6.8}{5.10.15} + \ldots\ldots\ldots\infty$ then prove that $9t = 16$.

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39. If $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \ldots.$ then prove that $9x^2 + 24x = 11$.

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40. If $x = \frac{5}{(2!).3} + \frac{5.7}{(3!).3^2} + \frac{5.7.9}{(4!).3^3} + \ldots.$

then find the value of $x^2 + 4x$.

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41. Find the sum of the infinite series

$$\frac{7}{5}\left(1+\frac{1}{10^2}+\frac{1.3}{1.2}.\frac{1}{10^4}+\frac{1.3.5}{1.2.3}.\frac{1}{10^6}+....\right)$$

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42. Show that

$$1+\frac{x}{2}+\frac{x(x-1)}{2.4}+\frac{x(x-1)(x-2)}{2.4.6}+....$$
$$=1+\frac{x}{3}+\frac{x(x+1)}{3.6}+\frac{x(x+1)(x+2)}{3.6.9}+....$$

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EXERCISE - 1.4 (Level-2)

1. Find the coefficient of term independent of x in the expansion of

$$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$$

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2. Find coeff. of x^{25} in the expansion of $\sum_{k=0}^{50}(-1)^{k50}C_k(2x-3)^{50-k}(2-x)^k$.

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3. Find the coefficient of x^6 in the expansion of $\left(1+x^2-x^3\right)^8$

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4. Find the coefficient of x^{10} in the expansion of $\left(1+x^2-x^3\right)^8$

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5. Find the coefficent of x^6 in $\left(1 + 3x + 9x^2\right)^{10}$

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6. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}} + x^2 \log_{10} x\right)^8$ is 5600, then the value of $x =$

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7. Find the range of values of the term independent of x in $\left(x \sin^{-1} p + \frac{\cos^{-1} p}{x}\right)^{10}$ where $p \in [-1, 1]$.

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8. The remainder when 5^{99} is divided by 13 is

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9. Find the remainder when 3^{37} is divided with 80 ?

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10. Find the remainder when 2^{60} is divided with 7.

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11. Find the number of distinct terms in the expansion

$(x + 2y + 3z + w)^{20}$

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12. Find the number of distinct terms in the following expansions.

$$\left(x^2 + 1 + \frac{1}{x^2}\right)^{40}$$

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13. Find the number of distinct terms in the following expansions.

$(p + q)^{70} + (p - q)^{70} + (p + qi)^{70} + (p - qi)^{70}$ where $i = \sqrt{-1}$

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14. Find the number of distinct terms in the expansion

$(x + y + z)^{10} + (x + y - z)^{10}$

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15. Prove that, $C_0 \cdot {}^{2n}C_n - C_1 \cdot {}^{(2n-2)}C_n + C_2 \cdot {}^{(2n-4)}C_n - \ldots . = 2^n$

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16. If $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$ and if $\sum_{r=0}^{n} \frac{1}{{}^nC_r} = \lambda$, then show that

$$\sum 0 \le i \le n \sum 0 \le j \le n \left(\frac{i}{{}^nC_i} + \frac{j}{{}^nC_j} \right) = n(n+1)\lambda$$

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17. Prove that $C_0 - \frac{1}{3} \cdot C_1 + \frac{1}{5} \cdot C_2 - \ldots + (-1)^n \cdot \frac{1}{2n+1} C_n = \frac{2^{2n}(n!)^2}{(2n+1)!}$

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18. If a, b, c, d be $3^{rd}, 4^{th}, 5^{th}$ and 6^{th} terms of the expansion of $(p+q)^n$, where n is a positive integer prove that $\frac{b^2 - ac}{c^2 - b} = \frac{5a}{3c}$.

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19. Show that

$$\sum_{r=0}^{n}(-1)^r.\,{}^nC_r\left\{\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\ldots.\text{ upto m terms}\right\}=\frac{2^{mn}-1}{2^{mn}\left(2^n-1\right)}$$

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20. If $\sum_{r=0}^{2n}a_r(x-2)^r=\sum_{r=0}^{2n}b_r(x-3)^r$ and $a_k=1$ for all $k\geq n$ then show that $b_n={}^{(2n+1)}C_{(n+1)}$.

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21. Show that the coefficient of x^ny^n in the expansion of $\{(1+x)(1+y)(x+y)\}^n$ is $C_0^3+C_1^3+C_2^3+\ldots C_n^3$.

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22. If $\frac{1}{1!(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+\ldots.=$

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23. If $\sum_{r=0}^{n}\left(\frac{r+2}{r+1}\right)\cdot {}^{n}C_{r}=\frac{255}{6}$. Find n.

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24. Show that $1\sum_{r=0}^{50}\left({}^{100}C_{r}\right)\cdot\left({}^{200}C_{150+r}\right)={}^{300}C_{50}$

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25. Prove that $\sum_{k=1}^{n-r}{}^{n-k}C_{r}={}^{n}C_{r+1}$.

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26. Show that

$$C_0+\left(C_0+C_1\right)+\left(C_0+C_1+C_2\right)+\ldots+\left(C_0+C_1+\ldots+C_n\right)=(n+2).2^{n-1}$$

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27. If x > 0 , write the first negative term in the expansion of $(1 + 2x)^{23/2}$

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28. If x is small and the expansion of $\frac{a}{(1 - x)^2} + \frac{b}{(2 + 3x)^2}$ is $1 + x + \ldots\infty$ find (a, b) and coefficient of term containing x^n.

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29. If x is small and if the expansion of $a + \frac{b}{1 + 2x} + \frac{c}{1 - 3x^2}$ is $1 + x + 2x^2 + \ldots\ldots\infty$ find (a,b,c)

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30. If $\frac{1 + 2x}{1 - x - x^2} = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$ then find $\left(a_0, a_1, a_2, a_3\right)$.

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31. If x is nearly equal to 1 show that

(a) $\frac{px^p - qx^q}{p - q} = x^{p+q}$ (nearly)

(b) $\frac{px^q - qx^p}{x^q - x^p} = \frac{1}{1 - x}$ (nearly) (Hint : Take $x = 1 + \delta x$ and proceed)

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32. If $|x| < 1$ then $1 + n\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1=x}\right)^2 + \ldots\ldots . \infty =$

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33. Prove the following

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}(1) = n^4$$

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34. Find the value of $\sum_{i=1}^{n}\sum_{i=1}^{n}\sum_{k=1}^{n} k$

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35. Prove the following

$$\sum 1 \le i \sum < j \le n (i + j) = \frac{n\left(n^2 - 1\right)}{2}$$

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36. Prove the following

$$\sum_{r=0}^{n}\sum_{s=0}^{n}\left(C_r \cdot C_s\right) = 4^n$$

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