MATHS

JEE (MAIN AND ADVANCED) MATHEMATICS

DEMOIVRE'S THEOREM

SOLVED EXAMPLES

1. Find product of values of $(1+2i)^{2/7}$

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2. Simplify $(1+\cos x+i\sin x)^n, n\varepsilon N.$

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3. Simplify $\frac{1+\cos x+i\sin x}{1+\cos x-i\sin x}$.

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4. If m, n integers and $x=\cos\alpha+i\sin\alpha, y=\cos\beta+i\sin\beta,$ then prove that $x^m y^n+\frac{1}{x^m y^n}=2\cos(m\alpha+n\beta)$ and $x^m y^n-\frac{1}{x^m y^n}=2i\sin(m\alpha+n\beta$.

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5. If $a=\cos 2\alpha+i\sin 2\alpha, b=\cos 2\beta+i\sin 2\beta, c=\cos 2\gamma+i\sin 2\gamma$ then prove that $\sqrt{abc}+\frac{1}{\sqrt{abc}}=2\cos(\alpha+\beta+\gamma)$

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6. If n is a positive integer prove that $(1+i)^{2n}+(1-i)^{2n}=2^{n+1}\cos\left(\frac{n\pi}{2}\right)$

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7. Find the value of $(-64)^{1/6}$.

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8. Find all the values of $(8i)^{1/3}$ and hence find sum and product of their values.

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9. Solve $x^9-x^5+x^4-1=0$

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10. Solve $(x-1)^n = x^n$, where n is a positive integer.

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11. Find the continued product of the four values of

$$\left(\frac{\cos(\pi)}{3} + i\frac{\sin(\pi)}{3}\right)^{3/4}$$

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12. Find the lengths of the sides of the triangle whose vertices are given by the equation $(z+\alpha\beta)^3 = \alpha^3$

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13. $\tan\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\} =$

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14. If $w(\neq 1)$ is a cube root of unity and $\left(1+w^2\right)^n=\left(1+w^4\right)^n$ then find the least positive integral value of n

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15. If $1, \omega, \omega^2$ are the cube roots of unity then prove the following

$i)\left(1-\omega+\omega^2\right)+\left(1+\omega-\omega^2\right)^5=32$

$ii)(x+y+z)\left(x+y\omega+z\omega^2\right)\left(x+y\omega^2+z\omega\right)=x^3+y^3+z^3-3xyz$

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ADDITIONAL SOLVED EXAMPLES

1. If $1, \alpha, \alpha^2, \ldots, \alpha^{n-1}$ are the n^{th} roots of unity show that $\sum_{r=1}^{n-1} r(\alpha_r+\alpha_{n-r})=-n.$

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2. If $x > 0$ the minimum value of x^x is

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3. In sin (A+iB)=x+iy show that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ and $\frac{x^2}{\cos h^2 B} + \frac{y^2}{\sin h^2 B} = 1$

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EXERCISE - 6 (VERY SHORT ANSWER QUESTIONS)

1. If $x = cis\theta$, then find the value of $\left[x^6 + \frac{1}{x^6}\right]$.

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2. If A,B,C are angles of a triangle such that $x = cisA, y = cisB, z = cisC$, then find the value of xyz .

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3. If $x + iy = cis, \alpha, cis\alpha$ then value of $x^2 + y^2$

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4. Simplify the following

$$\frac{cis5\alpha}{cis\alpha}$$

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5. Simplify the following

$$\frac{cis(-3\theta)cis2\theta}{cis4\theta(cis\theta)^6}$$

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6. $\frac{1 - \cos\theta + i\sin\theta}{1 + \cos\theta - i\sin\theta}$

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7. Simplify the following

$$\frac{(\cos\theta - i\sin\theta)^7}{(\sin 2\theta - i\cos 2\theta)^4}$$

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8. Simplify the following

$$\frac{(\sin\alpha + i\cos\alpha)^4}{(\sin 2\theta - i\cos 2\theta)^3}$$

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9. Simplify $\frac{(\cos\alpha + i\sin\alpha)^4}{(\sin\beta + i\cos\beta)^8}$

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10. If x=CiSθ, then find the value of (x^6+(1/x^6))

A. 2cos4θ

B. cos6θ

C. 2cos6θ

D. cos4θ

Answer: C

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11. If $x = cis\alpha, y = cis\beta$, then find

$$x^4y^3 + \frac{1}{x^4y^3}$$

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12. If $x = cis\alpha, y = cis\beta$, then find

$$xy + \frac{1}{xy}$$

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13. Find the values of the following

$$\left(1 + i\sqrt{3}\right)\frac{1}{3}$$

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14. Find the values of the following :

$$(1 - i)^8$$

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15. Find the values of the following

$$(1 + i)^{16}$$

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16. Find the values of the following :

$$\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5-\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$$

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17. Find the cube roots of 8 .

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EXERCISE - 6 (SHORT ANSWER QUESTIONS)

1. If n is an integer and $z=cis\theta$, $\left(\theta\neq(2n+1)\frac{\pi}{2}\right)$, then show that $\frac{z^{2n}-1}{z^{2n}+1}=i\tan n\theta\Big)$.

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2. $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{1/2}+\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^{11/2}$

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3. Show that one value of

$$\left(\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{\frac{8}{3}}=-1$$

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4. Find all the values of following .

$(-i)^{1/6}$

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5. Find all the values of following .

$(-16)^{1/4}$

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6. Find all the values of following .

$(-32)^{1/5}$

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7. Find all the values of the following

$\left(\sqrt{3}+i\right)^{1/4}$

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8. Find all the values of following .

$\left(1-I\sqrt{3}\right)^{1/3}$

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9. Find all the values of following .

$(1+i)^{2/3}$

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10. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that

$$\frac{1}{2+\omega} - \frac{1}{1+2\omega} = \frac{1}{1+\omega}.$$

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11. If $1, \omega, \omega^2$ are the cube roots of unity prove that

$$\left(1-\omega+\omega^2\right)^6 + \left(1-\omega^2+\omega\right)^6 = 128 = \left(1-\omega+\omega^2\right)^7 + \left(1+\omega-\omega^2\right)^7$$

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12. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that

$$(2-\omega)\left(2-\omega^2\right)\left(2-\omega^{10}\right)\left(2-\omega^{11}\right) = 49.$$

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13. If $1, \omega, \omega^2$ are the cube roots of unity prove that

$(a+b)\left(a\omega+b\omega^2\right)\left(a\omega^2+b\omega\right)=a^3+b^3$

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14. If $1, \omega, \omega^2$ are the cube roots of unity prove that

$x^2+4x+7=0 \quad \text{where} \quad x=\omega-\omega^2-2.$

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15. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$(a+b)^3+\left(a\omega+b\omega^2\right)^3+\left(a\omega^2+b\omega\right)^3$

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16. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$(a+2b)^2+\left(a\omega^2+2b\omega\right)^2+\left(a\omega+2b\omega^2\right)^2$

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17. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$\left(1-\omega+\omega^2\right)^3$

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18. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^8\right)$

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19. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$$\left[\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right]+\left[\frac{a+b\omega+c\omega}{b+c\omega+a\omega^2}\right]$$

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20. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$$(i+\omega)^3+\left(1+\omega^2\right)^3$$

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21. Prove that $-\omega, \text{ and } -\omega^2$ are the roots of $z6(2)-z+1=0$, where $\omega \text{ and } \omega^2$ are the complex cube roots of unity .

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22. If α, β are the roots of the equation $x^2 + x + 1 = 0$ then prove that $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = 0$.

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23. If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x-1)^3 + 8 = 0$ are

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24. Find the product of all the values of $(1+i)^{4/5}$.

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EXERCISE - 6 (LONG ANSWER QUESTIONS)

1. If n is a positive integer, show that

$$(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\left(\frac{n\pi}{4}\right).$$

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2. If n is a positive integer, show that

$$(P+iQ)^{1/n} + (P-iQ)^{1/n} = 2\left(P^2+Q^2\right)^{1/2n} \cos\left(\frac{1}{n}, \tan.\frac{Q}{P}\right).$$

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3. If n is an integer then show that

$$(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n(\theta/2)\cos\left(\frac{n\theta}{2}\right)$$

.

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4. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$ then for any $n \in N$ show that $\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$.

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5. If $\cos\alpha + \cos\beta + \cos\gamma = 0$ & $\sin\alpha + \sin\beta + \sin\gamma = 0$. Then Prove that

$\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

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6. If $\cos\alpha + \cos\beta + \cos\gamma = 0$ & $\sin\alpha + \sin\beta + \sin\gamma = 0$. Then Prove that

$\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

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7. If $\cos\alpha + \cos\beta + \cos\gamma = 0$

$= \sin\alpha + \sin\beta + \sin\gamma$ then show that

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

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8. If $\cos\alpha + \cos\beta + \cos\gamma = 0$

$= \sin\alpha + \sin\beta + \sin\gamma$ then show that

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

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9. If $\cos\alpha + \cos\beta + \cos\gamma = 0$

$= \sin\alpha + \sin\beta + \sin\gamma$ then show that

$\cos(\alpha + \beta) + \cos(\beta + \gamma) + \cos(\gamma + \alpha) = 0$

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10. If $\cos\alpha + \cos\beta + \cos\gamma = 0 \sin\alpha + \sin\beta + \sin\gamma$, Prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2} = \sin^2\alpha + \sin^2\beta + \sin^2\gamma.$$

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11. Solve the following equations.

$x^4 - 1 = 0$

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12. Solve the following equations.

$x^5 + 1 = 0$

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13. Solve the following equations.

$x^4 + 1 = 0$

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14. Solve the following equations

$x^{11} - x^7 + x^4 - 1 = 0$

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15. If $(1+x)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots + a_n x^n$, then show that

(i) $a_0 - a_2 + a_4 - a_6 + \ldots\ldots\ldots = 2^{n/2} \cos. \frac{n\pi}{4}$

(ii) $a_1 a_3 + a_5 - a_7 + \ldots\ldots\ldots = 2^{n/2} \sin. \frac{n\pi}{4}$

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16. If $(1+x)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots + a_n x^n$, then show that

(i) $a_0 - a_2 + a_4 - a_6 + \ldots\ldots\ldots = 2^{n/2} \cos. \frac{n\pi}{4}$

(ii) $a_1 a_3 + a_5 - a_7 + \ldots\ldots\ldots = 2^{n/2} \sin. \frac{n\pi}{4}$

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17. The common roots of the equation

$x^{12} - 1 = 0, x^4 + x^2 + 1 = 0$ are

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18. Find the number of 15^{th} roots of unity, which are also 25^{th} roots of unity .

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19. If $z^2 + z + 1 = 0$, where z is a complex number , prove that

$$\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right) + \left(z^3 + \frac{1}{z^3}\right)^2 + \left(z^4 + \frac{1}{z^4}\right)^2 + \left(z^5 + \frac{1}{z^5}\right)^2 +$$

.

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20. If $1, \alpha, \alpha^2, \alpha^3, \ldots, \alpha^{n-1}$ be the n^{th} roots of unity, then prove that $1^p + \alpha^p + \left(\alpha^2\right)^p + \left(\alpha^3\right)^p + \ldots + \left(\alpha^{n-1}\right)^p = \begin{cases} 0 & \text{if} \quad p \neq kn \\ n & \text{if} \quad p = kn \end{cases}$ where

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21. Prove the sum of $99t^{th}$ powers of the roots of the equation $x^7 - 1 = 0$ is zero and hence deduce the roots of $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$

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ADDITIONAL EXERCISE

1. Simplify

$$\frac{(\cos 2\theta + i \sin 2\theta)^6 (\cos 3\theta + i \sin 3\theta)^{-5}}{(\cos 5\theta + i \sin 5\theta)^{-8} (\cos 6\theta - i \sin 6\theta)^{-2}}$$

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2. If $2\cos\alpha = x + \frac{1}{x}$, find the value of x in terms of α.

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3. If x = cisA, y=cisB then find the value of cos(A+B) interms of x and y.

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4. If α is an n^{th} root of unity then prove that $1 + 2\alpha + 3\alpha^2 + 4\alpha^3 + \ldots + n\alpha^{n-1} = -\frac{n}{1-\alpha}$

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5. Find the value of $\sum_{k=1}^{10}\left\{\sin\left(\frac{2k\pi}{11}\right) - i\cos\left(\frac{2k\pi}{11}\right)\right\}$

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6. p,q,r are distinct cube roots of non-zero complex number z. Let a, b, c be complex numbers satisfying $ap + bq + cr \neq 0$. Then find the value of $\frac{(aq + br + cp)(ar + bp + cq)}{(ap + bq + cr)^2}$.

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7. If $(1 + x)^n = c_0 + c_1 x + c_2 x^2 + \ldots. + c_n x^n$, find the value of $c_0 - c_2 + c_4 - \ldots.$

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EXERCISE -I

1. The value of $(\cos 2\theta + i \sin 2\theta)^5 (\sin \theta + i \cos \theta)^4$ is

A. $\cos 10\theta + i \sin 10\theta$

B. $\cos 6\theta - i \sin 6\theta$

C. $\cos 6\theta + i \sin 6\theta$

D. $\cos 10\theta - i \sin 10\theta$

Answer: C

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2. $(1 + \cos\theta - i \sin\theta)^n$

A. $2^n \cos^n\left(\frac{\theta}{2}\right) cis \frac{n\theta}{2}$

B. $2^n \cos^n\left(\frac{\theta}{2}\right) cis\left(-\frac{n\theta}{2}\right)$

C. $2^n \sin^n\left(\frac{\theta}{2}\right) cis \frac{n\theta}{2}$

D. $2^n \sin^n\left(\frac{\theta}{2}\right) - cis \frac{n\theta}{2}$

Answer: B

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3. $(\sin x + i\cos x)^n$

A. $cisn\left(\frac{\pi}{2} + x\right)$

B. $cisn\left(\frac{\pi}{2} - x\right)$

C. $cisn\left(\frac{\pi}{3} + x\right)$

D. $cisn\left(\frac{\pi}{3} - x\right)$

Answer: B

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4. $(1 + \cos\theta + i\sin\theta)^n =$

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5. $\left(1 + i\sqrt{3}\right)^9 =$

A. -2^6

B. -2^8

C. -2^9

D. -2^{12}

Answer: C

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6. A value of n such that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^n=1$ is

A. 12

B. 3

C. 2

D. 1

Answer: A

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7. $\left(\cos 15^0 + i\sin 15^0\right)\left(\cos 45^0 + i\sin 45^0\right) =$

A. $\frac{1}{2}\left(1+\sqrt{3}i\right)$

B. $\frac{1}{\sqrt{2}}\left(1-\sqrt{3}i\right)$

C. $\frac{1}{\sqrt{3}}\left(1+\sqrt{i}\right)$

D. $\frac{1}{\sqrt{3}}\left(1-\sqrt{3}i\right)$

Answer: A

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8. If $x+\frac{1}{x}=2\cos\alpha, y+\frac{1}{y}=2\cos\beta,$ then $xy+\frac{1}{xy}=$

A. $\cos(\pm\alpha\pm\beta)$

B. $2\cos(\pm\alpha\pm\beta)$

C. $2\sin(\pm\alpha\pm\beta)$

D. $\sin(\pm\alpha\pm\beta)$

Answer: B

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9. If $x + (1/x) = 2\cos(\pi/10),$ then $x^5 + \left(1/x^5\right) =$

A. 10

B. 2

C. 0

D. 32

Answer: C

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10. If $x - \frac{1}{x} = 2i\sin\theta,$ then $x^4 - \frac{1}{x^4} =$

A. $2i\sin 4\theta$

B. $2\cos 5\theta$

C. $2i\cos^5\theta$

D. $2\cos^5\theta$

Answer: A

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EXERCISE -II

1. $\left[\sqrt{2}(\cos 56^\circ 15 + i\sin 56^\circ 15')\right]^8$

A. 1

B. i

C. 16

D. 16i

Answer: D

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2. $\frac{(\cos\alpha + i\sin\alpha)^5}{(\sin\beta + i\cos\beta)^3} =$

A. $\sin(5\alpha + 3\alpha) + i\cos(5\alpha + 3\beta)$

B. $-\sin(5\alpha + 3\beta) + i\cos(5\alpha + 3\beta)$

C. $\sin(3\alpha + 5\beta) + i\cos(3\alpha + 5\beta)$

D. $-\sin(3\alpha + 5\beta) + i\cos(3\alpha + 5\beta)$

Answer: B

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3. $\frac{(\cos 3\theta - i\sin 3\theta)^4}{(\sin 2\theta + i\cos 2\theta)^5} =$

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4. $\left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n$

A. $cis \quad n\theta$

B. $-cis \quad n\theta$

C. $cis \quad \theta$

D. $-cis \quad \theta$

Answer: A

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5. If $\left(\frac{1+\cos\theta+i\sin\theta}{\sin\theta+i+i\cos\theta}\right)^4 = \cos n\theta + i\sin n\theta$ then n=

A. 4

B. 8

C. 6

D. 2

Answer: A

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6. $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6+\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6=$

A. 2

B. -2

C. 1

D. 0

Answer: A

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7.

$[(\cos\alpha-\cos\beta)+i(\sin\alpha-\sin\beta)]^n+[(\cos\alpha-\cos\beta)-i(\sin\alpha-\sin\beta)]^r$

A. $2^{n+1}.\sin^n\left(\frac{\alpha-\beta}{2}\right)\cos\left[\left(\frac{\pi}{2}+\frac{\alpha+\beta}{2}\right)\right]$

B. $2^{n-1}.\sin^n\left(\frac{\alpha-\beta}{2}\right)\cos\left[\left(\frac{\pi}{2}+\frac{\alpha+\beta}{2}\right)\right]$

C. $2^{n+1}.\sin^n\left(\frac{\alpha+\beta}{2}\right)\cos\left[\left(\frac{\pi}{2}+\frac{\alpha-\beta}{2}\right)\right]$

D. $2^{n-1}.\sin^n\left(\frac{\alpha+\beta}{2}\right)\cos\left[\left(\frac{\pi}{2}+\frac{\alpha-\beta}{2}\right)\right]$

Answer: A

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8. If $\left(\sqrt{3}+i\right)^{100}=2^{99}(a+ib)$ then b=

A. 1

B. $\sqrt{3}$

C. $\sqrt{2}$

D. 2

Answer: B

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9. If $z = \cos\theta + i\sin\theta$, then $\dfrac{z^{2n}-1}{z^{2n}+1}$

A. $\cos n\theta$

B. $\sin n\theta$

C. $-i\sin \quad n \quad \theta$

D. $i\tan \quad n \quad \theta$

Answer: D

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10. If $x = \cos\alpha + i\sin\alpha, y = \cos\beta + i\sin\beta,$ then $\dfrac{x-y}{x+y} =$

A. $i.\cos\left(\dfrac{\alpha-\beta}{2}\right)$

B. $i.\sin\left(\dfrac{\alpha-\beta}{2}\right)$

C. $i.\cot\left(\dfrac{\alpha-\beta}{2}\right)$

D. $i.\tan\left(\frac{\alpha-\beta}{2}\right)$

Answer: D

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11. If $x=\cos\alpha+i\sin\alpha, y=\cos\beta+i\sin\beta,$ then $\frac{x}{y}-\frac{y}{x}=$

A. $2i\sin(\alpha+\beta)$

B. $2i\sin(\alpha-\beta)$

C. $2i\cos(\alpha+\beta)$

D. $2i\cos(\alpha-\beta)$

Answer: B

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12. If $\theta = \frac{\pi}{6}$, then the 10th term of the series $1 + (\cos\theta + i\sin\theta) + (\cos\theta + i\sin\theta)^2 + ...$ Is

A. i

B. -1

C. 1

D. $-i$

Answer: D

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13. If n is a positive integer and $(1+i)^{2n} + (1-i)^{2n} = k\cos(n\pi/2)$ then the value of k is

A. 2^n

B. 2^{n-1}

C. 2^{n+1}

D. 1

Answer: C

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14. If n is positive integer and

$\left(1+i\sqrt{3}\right)^n+\left(1-i\sqrt{3}\right)^n=2^{n+1}\cos\theta$, then the value of θ is

A. $n\pi/3$

B. $n\pi/2$

C. $n\pi/4$

D. $n\pi/6$

Answer: A

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15. If α and β are the roots of $x^2 - 2x + 4 = 0$ then $\alpha^n + \beta^n$

A. $2^n \cos(n\pi/2)$

B. $2^{n+1} \cos(n\pi/2)$

C. $2^{n-1} \sin(n\pi/6)$

D. $2^{n+1} \sin(n\pi/3)$

Answer: D

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16. Show that one value of

$$\left(\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{\frac{8}{3}} = -1$$

A. 1+i

B. 1-i

C. 1

D. -1

Answer: D

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17. If n is a positive integer, then $(a+ib)^{m/n}+(a-ib)^{m/n}$

A. $2\left(a^n+b^2\right)^{m/2n}\cos\left[\frac{m}{n}\frac{\tan^{-1}b}{a}\right]$

B. $2\left(a^n+b^2\right)^{m/2n}\sin\left[\frac{m}{n}\frac{\tan^{-1}b}{a}\right]$

C. $2\left(a^n+b^2\right)^{m/2n}\cot\left[\frac{m}{n}\frac{\tan^{-1}b}{a}\right]$

D. $2\left(a^n+b^2\right)^{m/2n}\tan\left[\frac{m}{n}\frac{\tan^{-1}b}{a}\right]$

Answer: A

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18. If $p = \cos 2\alpha + i \sin 2\alpha, q = \cos 2\beta + i \sin 2\beta$ then $\sqrt{p/q} - \sqrt{q/p} =$

A. $2i \sin(\alpha + \beta)$

B. $2i \sin(\alpha - \beta)$

C. $2i \cos(\alpha - \beta)$

D. $2i \cos(\alpha + \beta)$

Answer: B

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19. The values of $\left(1 - \sqrt{3}i\right)^{1/3}$ are

A. $2^{1/3} cis\left[\frac{2k\pi + \pi/3}{3}\right], k = 0, 1, 2$

B. $2^{1/5} cis\left[\frac{2k\pi + \pi/6}{3}\right], k = 0, 1, 2$

C. $2^{1/3} cis\left[\frac{2k\pi - \pi/3}{3}\right], k = 0, 1, 2$

D. $2^{1/5} cis\left[\frac{2k\pi - \pi/6}{3}\right], k = 0, 1, 2$

Answer: C

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20. The product of the four values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{3/4}$ is

A. -1

B. 1

C. i

D. $-i$

Answer: B

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21. Two of the three values of $(-1)^{1/3}$ are $\cos(\pi/3)+i\sin(\pi/3), \cos(5\pi/3)+i\sin(5\pi/3)$.The third value is

A. $\cos(\pi/3) - i\sin(\pi/3)$

B. $\cos(5\pi/3) - i\sin(5\pi/3)$

C. -1

D. 1

Answer: C

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22. One value of $(1+i)^{1/2}$ is $2^{1/4}e^{i\pi/8}$.The other value is

A. $2^{1/4}e^{i(\pi/8)}$

B. $2^{1/4}r^{i(5\pi/8)}$

C. $2^{1/4}e^{-i(5\pi/8)}$

D. $2^{1/4}.\ e^{i(9\pi/8)}$

Answer: D

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23. If $z + \frac{1}{z} = 1$ and z be any complex number,then the value of $z^{99} + \frac{1}{z^{99}}$ is

A. 1

B. -1

C. 2

D. -2

Answer: D

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24. If $\left(\sqrt{3} - i\right)^n = 2^n$, $b\varepsilon 1$,the set of integers,then n is a multiple of

A. 6

B. 10

C. 9

D. 12

Answer: D

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25. If A+B+C=180° and cis A=x,cis B=y,cis C=z, them xyz=

A. -1

B. 0

C. 1

D. 2

Answer: A

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26. If $x_n = \cos\left(\frac{\pi}{2^n}\right) + i\sin\left(\frac{\pi}{2^n}\right)$, then x1 x2 x3.....infinity

A. -1

B. 1

C. $1/\sqrt{2}$

D. $i/\sqrt{2}$

Answer: A

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27. If $z_k = \cos\left(\frac{k\pi}{10}\right) + i\sin\left(\frac{k\pi}{10}\right)$, then $z_1 z_2 z_3 z_4$ is equal to

A. -1

B. 1

C. -2

D. 2

Answer: A

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28. The general value of θ which satisfies the equation $(\cos\theta + i\sin\theta)(\cos 3\theta + i\sin 3\theta)\ldots[(\cos(2r-1)\theta + i\sin(2r-1)\theta)] = 1$ is

A. $\frac{n\pi}{r^2}$

B. $(n-1)\frac{\pi}{r^2}$

C. $(2n+1)\frac{\pi}{r^2}$

D. $\frac{2r\pi}{n^2}$

Answer: D

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29. $\sum_{k=1}^{6}\left[\sin\left(\frac{2k\pi}{7}\right) - i\cos\left(\frac{2k\pi}{7}\right)\right] =$

A. -1

B. 0

C. $-i$

D. i

Answer: D

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30. The value of $\prod_{k=1}^{6}\left((\sin)\frac{2\pi k}{7}-(i\cos)\frac{2\pi k}{7}\right)$ is

A. -1

B. 0

C. $-i$

D. i

Answer: A

Watch Video Solution

31. The value of

$$1+\sum_{k=0}^{14}\left\{(\cos)\frac{(2k+1)\pi}{15}+(i\sin)\frac{(2k+1)\pi}{15}\right\} \text{ is}$$

A. 0

B. 1

C. -1

D. i

Answer: B

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32. If α and β are the roots of the equation $x^2-2x+4=0$, then $\alpha^9+\beta^9=$

A. -2^8

B. 2^9

C. -2^{10}

D. 2^{10}

Answer: C

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33. If $\cos\alpha + \cos\beta + \cos\gamma = 0$ & $\sin\alpha + \sin\beta + \sin\gamma = 0$. Then Prove that

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0 = \sin 2\alpha + \sin 2\beta + \sin 2\gamma$

A. 2

B. 1

C. $\frac{1}{2}$

D. 0

Answer: D

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34. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$

A. 0

B. $\frac{1}{2}$

C. $\frac{3}{2}$

D. 2

Answer: C

Watch Video Solution

35. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\cos(2\alpha - \beta - \gamma) + \cos(2\beta - \gamma - \alpha) + \cos(2\gamma - \alpha - \beta) =$

A. 0

B. 1

C. 2

D. 3

Answer: D

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36. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $x^{\sin(2\alpha-\beta-\gamma)} \cdot x^{\sin(2\beta-\gamma-\alpha)} \cdot x^{\sin(2\gamma-\alpha-\beta)} =$

A. 0

B. 1

C. x

D. x^3

Answer: B

View Text Solution

37. If $\cos\alpha + coa\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\cos(\alpha+\beta) + \cos(\beta+\gamma) + \cos(\gamma+\alpha) =$

A. 0

B. 1

C. $\cos 2(\alpha+\beta+\gamma)$

D. -1

Answer: A

Watch Video Solution

38. If A+B+C=π and $\cos A + \cos B + \cos C = 0 = \sin A + \sin B + \sin C$ then $\cos 3A + \cos 3B + \cos 3C =$

A. 3

B. -3

C. 0

D. 1

Answer: B

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39. If $x^6 + 1 = 0$, then x =

A. $cis\left(\frac{(2k+1)\pi}{6}\right), k = 0, 1, 2, 3, 4, 5$

B. $cis\left(\frac{(2k-1)\pi}{6}\right), k = 0, 1, 2, 3, 4, 5$

C. $cis\left(\frac{(3k+1)\pi}{6}\right), k = 0, 1, 2, 3, 4, 5$

D. $cis\left(\frac{(3k-1)\pi}{6}\right), k = 0, 1, 2, 3, 4, 5$

Answer: A

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40. find the roots $x^{11} - x^7 + x^4 - 1 = 0$

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41. The number of common roots of 15th and of 25th roots of unity are

A. 1

B. 5

C. 6

D. 10

Answer: B

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42. The common roots of the equation

$x^{12} - 1 = 0, x^4 + x^2 + 1 = 0$ are

A. ω, ω^2

B. $-\omega, \ -\omega^2$

C. $\pm\omega,\ \pm\omega^2$

D. $\pm\omega^2,\ \pm\omega^5$

Answer: C

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43. The product of all nth roots of unity $(n > 1)$ is

A. 0

B. $(-1)^{n-1}$

C. -1

D. 1

Answer: B

View Text Solution

44. The product of the distinct $(2n)^{th}$ roots $1+i\sqrt{3}$ is

A. 0

B. $-1-i\sqrt{3}$

C. $1+i\sqrt{3}$

D. $-1+i\sqrt{3}$

Answer: B

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45. If α is a non real root of $x^6=1$ then $\dfrac{\alpha^5+\alpha^3+\alpha+1}{\alpha^2+1}=$

A. α^2

B. $-\alpha^2$

C. α

D. 0

Answer: B

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46. If $1, \alpha_1, \alpha_2, \ldots . \alpha_{n-1}$ are the n^{th} roots of unity then $(1-\alpha_1)(1-\alpha_2)\ldots . (1-\alpha_{n-1}) =$

A. n-1

B. n

C. -1

D. 1

Answer: B

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47. If $1, \alpha_1, \alpha_2, \ldots . \alpha_{n-1}$ are the n^{th} roots of unity and n is an odd natural number then

$(1+\alpha_1)(1+\alpha_2).....(1+\alpha_{n-1}) =$

A. 1

B. -1

C. 0

D. -2

Answer: A

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48. If α is an n^{th} root of unity then prove that $1+2\alpha+3\alpha^2+4\alpha^3+....+n\alpha^{n-1} = -\frac{n}{1-\alpha}$

A. $\frac{n}{1-\alpha}$

B. $\frac{-n}{1-\alpha}$

C. $\frac{-n}{(1-\alpha)^2}$

D. $\frac{n}{(1-\alpha)^2}$

Answer: B

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49. Statement-I :

$$\frac{(\cos\theta + i\sin\theta)^5(\cos 3\theta - i\sin 3\theta)^6}{(\cos 2\theta + i\sin 2\theta)^3(\cos 4\theta - i\sin 4\theta)^5} = 1$$

Statement-II :

$$\frac{(\cos 2\theta + i\sin 2\theta)^3(\cos 3\theta - i\sin 3\theta)^4}{(\cos 3\theta + i\sin 3\theta)^2(\cos 4\theta + i\sin 4\theta)^{-3}} = 1$$

A. Only I is true

B. Only II is true

C. Both I and II are true

D. Neither I nor II are true

Answer: B

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50. Statement-I :

If $x_n = \cos(\pi/2^n) + i\sin(\pi/2^n)$ then $x_1.\, x_2.\, x_3.\ldots. = -1$

Statement-II : $x_n = \cos(\pi/4^n) + i\sin(\pi/4^n)$ then

$x_1.\, x_2.\, x_3.\ldots.\infty = -1$

A. Only I is true

B. Only II is true

C. Both I and II are true

D. Neither I nor II are true

Answer: A

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51. Statement-I : If $e^{i\theta} = \cos\theta + i\sin\theta$ then for the $\Delta ABC e^{iA} e^{iB} e^{iC} = -1$

Statement-II : If $\left(\sqrt{3}+1\right)^{100} = 2^{99}(a+ib)$ then $b = 2\sqrt{3}$

A. Only I is true

B. Only II is true

C. Both I and II are true

D. Neither I nor II are true

Answer: A

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52. The ascending order of the values of

$z_1 = (1+i)^4, z_2 = \left(\sqrt{3}+i\right)^{12}, z_3 = \left(1+i\sqrt{3}\right)^9 + \left(1-i\sqrt{3}\right)^9$

A. z_1, z_2, z_3

B. z_2, z_3, z_1

C. z_3, z_1, z_2

D. z_1, z_3, z_2

Answer: C

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53. The descending order of the values of

$$z_1 = \left(\frac{\sqrt{3}}{2}i + \frac{1}{2}\right)^6, z_2 = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^4, z_3 = \left(\frac{\sqrt{3}}{4} + \frac{i}{4}\right)^6$$

A. z_1, z_2, z_3

B. z_3, z_2, z_1

C. z_3, z_1, z_2

D. z_3, z_1, z_2

Answer: D

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54. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma,$ then match the following.

I) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma =$	a) 0
II) $\sin 2\alpha + \sin 2\beta + \sin 2\gamma =$	b) 3
III) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$	c) $3/2$
IV) $\cos(2\alpha - \beta - \gamma) + \cos(2\beta - \gamma - \alpha) + \cos(2\gamma - \alpha - \beta) =$	d) $3\,\mathrm{co}$

A. A) d,a,c,b

B. B) b,c,a,d

C. C) d,a,b,c

D. D) d,c,b,a

Answer: A

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55. Match the following.

I) $(i)^i =$	a) $i\pi/2$
II) $\log_e i =$	b) $i\pi/2 + \log \pi/2$
III) $\log(\log i) =$	c) $\sqrt{2}$
IV $\sqrt{i} + \sqrt{-i} =$	$e^{-\pi/2}$

A. d,a,c,d

B. b,c,a,d

C. d,a,b,c

D. d,c,b,a

Answer: C

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56. Assertion (A) : $\left(\sqrt{3}+i\right)^6+\left(\sqrt{3}-i\right)^6=-128$

Reason (R) : If n is a positive integer then

$\left(\sqrt{3}+i\right)^n+\left(\sqrt{3}-i\right)^n=2^{n+1}\frac{\cos(n\pi)}{6}$

A. Both A and R are true R is correct explanation to A

B. Both A and R are true but R is not correct explanation to A

C. A is true R is false

D. A is false R is true

Answer: A

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57. If A+B+C=180° and cis A=x,cis B=y,cis C=z, them xyz=

A. Both A and R are true R is correct explanation to A

B. Both A and R are true but R is not correct explanation to A

C. A is true R is false

D. A is false R is true

Answer: A

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58. p,q,r are distinct cube roots of non-zero complex number z. Let a, b, c be complex numbers satisfying $ap + bq + cr \neq 0$. Then find the value of $\frac{(aq + br + cp)(ar + bp + cq)}{(ap + bq + cr)^2}$.

A. 1

B. -1

C. i

D. $-i$

Answer: A

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PRACTICE EXERCISE

1. $\left(\sqrt{3}+i\right)^{12}=$

A. 2^6

B. 2^8

C. 2^9

D. 2^{12}

Answer: D

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2. $(1+\cos\theta-i\sin\theta)^4=$

A. $2^4 \cos^4\left(\frac{\theta}{2}\right) cis2\theta$

B. $2^4 \cos^4\left(\frac{\theta}{2}\right) cis(-2\theta)$

C. $2^4 \sin^4\left(\frac{\theta}{2}\right) cis2\theta$

D. $2^4 \sin^4\left(\frac{\theta}{2}\right) cis(-2\theta)$

Answer: B

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3. $(1-\cos\theta+i\sin\theta)^6=$

A. $2^6 \frac{\sin^6(\theta)}{2}(i\cos 3\theta+\sin 3\theta)$

B. $2^6 \frac{\sin^6(\theta)}{2}(\sin 3\theta-i\cos 3\theta)$

C. $2^6 \frac{\sin^6(\theta)}{2}(\cot 3\theta+i\sin 3\theta)$

D. $2^6 \frac{\sin^6(\theta)}{2}(-\cot 3\theta+i\sin 3\theta)$

Answer: B

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4. $(1+i)^5+(1-i)^5$

A. -8

B. 81

C. 8

D. 32

Answer: A

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5. If $(\sin\alpha+i\cos\alpha)^4(\cos 2\alpha-i\sin 2\alpha)^{-2}=\cos\theta+i\sin\theta,\quad \text{then}\quad \theta=$

A. 0

B. 4α

C. 6α

D. 5α

Answer: A

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6. The values of A and θ if $(1+\sin\alpha+i\cos\alpha)^8 = A(\cos\theta+i\sin\theta)$ are respectively

A. A) $2^8\cos^8\left(\frac{\pi}{4}-\frac{\alpha}{2}\right),\ -2\alpha$

B. B) $2^8\cos 3\alpha,\ -3\alpha$

C. C) $2^8\cos^8\left(\frac{\pi}{4}-\frac{\alpha}{2}\right),\ -4\alpha$

D. D) $2^8\cos\alpha,\ -\alpha$

Answer: C

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7. $(1-\cos\theta+i\sin\theta)^8=$

A. $256\sin^8(\theta/2)[\cos 4\theta+i\sin 4\theta]$

B. $256\cos^8(\theta/2)[\cos 4\theta-i\sin 4\theta]$

C. $256\sin^8(\theta/2)[\sin 4\theta-i\cos 4\theta]$

D. $256\sin^8(\theta/2)[\cos 4\theta-i\sin 4\theta]$

Answer: D

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8. $\dfrac{\cos 75^0+i\sin 75^0}{\cos 30^0+i\sin 30^0}=$

A. $\dfrac{1}{\sqrt{2}}(1+i)$

B. $\dfrac{1}{\sqrt{2}}(1-i)$

C. $\dfrac{1}{\sqrt{3}}(1+i)$

D. $\dfrac{1}{\sqrt{3}}(1-i)$

Answer: A

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9. If $x + \frac{1}{x} = 2\cos\theta$ then $x^{10} + \frac{1}{x^{10}} =$

A. $2^{10}\cos 10\theta$

B. $2\cos 10\theta$

C. $2^{10}\cos^{10}\theta$

D. $2\cos^{10}\theta$

Answer: B

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10. If x satisfies the equation $x^2 - 2x\cos\theta + 1 = 2\cos\theta$,then the value of $x^n + 1/x^n$ is

A. $2^n \cos n\theta$

B. $2^n \cos^n \theta$

C. $2 \cos n\theta$

D. $2 \cos^n \theta$

Answer: C

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11. $\frac{(\cos\theta + i\sin\theta)^5 (\cos 3\theta - i\sin 3\theta)^6}{(\cos 2\theta + i\sin 2\theta)^3 (\cos 4\theta - i\sin 4\theta)^5} =$

A. 1

B. $cis(\theta)$

C. i

D. -1

Answer: B

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12. $\left(\frac{\sin\pi/8+i\cos\pi/8}{\sin\pi/8-i\cos\pi/8}\right)^8$

A. -1

B. 0

C. 1

D. 2i

Answer: C

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13. $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n$

A. $\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$

B. $\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}+n\theta\right)$

C. $\cos\left(\frac{n\pi}{2}+n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$

D. $cis(n\pi - \theta)$

Answer: A

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14. If $\left(\frac{\sqrt{3}/2 + (1/2)i}{\sqrt{3}/2 - (1/2)i}\right)^{120} = p + iq$,then

A. $p = \cos 20^\circ, q = \sin 20^\circ$

B. $p = -1, q = 0$

C. $p = 0, q = 1$

D. $p = \cos 40^\circ, q = \sin 40^\circ$

Answer: D

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15. If n is a positive integer, then

$(1+i)^n + (1-i)^n =$

A. $2^{(n/2)+1}.\dfrac{\cos(n\pi)}{3}$

B. $2^{(n/2)+1}.\dfrac{\cos(n\pi)}{4}$

C. $2^{(n/2)+1}.\dfrac{\cos(n\pi)}{5}$

D. $2^{(n/2)+1}.\dfrac{\cos(n\pi)}{6}$

Answer: B

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16. Assertion (A) : $\left(\sqrt{3}+i\right)^6 + \left(\sqrt{3}-i\right)^6 = -128$

Reason (R) : If n is a positive integer then

$\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = 2^{n+1}\dfrac{\cos(n\pi)}{6}$

A. $2^{n+1}\dfrac{\cos(n\pi)}{2}$

B. $2^{n+1}\dfrac{\cos(n\pi)}{3}$

C. $2^{n+1}\frac{\cos(n\pi)}{5}$

D. $2^{n+1}\frac{\cos(n\pi)}{6}$

Answer: D

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17. $\left(\sqrt{3}+i\right)^7+\left(\sqrt{3}-i\right)^7=$

A. $128\sqrt{3}$

B. $256\sqrt{3}$

C. $-128\sqrt{3}$

D. $-256\sqrt{3}$

Answer: C

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18. $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{1/2}+\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^{11/2}$

A. 1

B. -1

C. 0

D. 2

Answer: C

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19. One value of $\left(1+i\sqrt{3}\right)^{3/4}+\left(1-i\sqrt{3}\right)^{3/4}$ is equal to

A. $(16)^{1/4}$

B. $(64)^{1/4}$

C. $(32)^{1/4}$

D. $(48)^{1/4}$

Answer: C

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20. One of the value of $\frac{[\cos(\pi/6) - i\sin(\pi/6)]^{11/2}}{[\cos(\pi/6) + i\sin(\pi/6)]^{1/2}}$ is

A. 1

B. 0

C. -1

D. $1/2$

Answer: C

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21. If $x = \cos\alpha + i\sin\alpha, y = \cos\beta + i\sin\beta,$ then $\frac{x}{y} + \frac{y}{x}$

A. $2\sin(\alpha + \beta)$

B. $2\sin(\alpha - \beta)$

C. $2\cos(\alpha + \beta)$

D. $2\cos(\alpha - \beta)$

Answer: D

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22. If $x = cis\alpha, y = cis\beta$ then $x^3y^4 - \frac{1}{x^3y^4} =$

A. $2i\cos(3\alpha + 4\beta)$

B. $2i\cos(3\alpha - 4\beta)$

C. $2i\sin(3\alpha + 4\beta)$

D. $2i\sin(3\alpha - 4\beta)$

Answer: C

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23. If a,b and c are in G.P then $\frac{b-a}{b-c}+\frac{b+a}{b+c}$

A. $2\sin(3\alpha+\beta-\gamma)$

B. $2\cos(3\alpha+\beta-\gamma)$

C. $\cos(3\alpha+\beta-\gamma)$

D. $\sin(3\alpha+\beta-\gamma)$

Answer: B

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24. $cis\frac{\pi}{10}.cis\frac{2\pi}{10}.cis\frac{3\pi}{10}.cis\frac{4\pi}{10}=$

A. 1

B. -1

C. 0

D. 2

Answer: B

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25. If $x_n = \cos\left(\frac{\pi}{2^n}\right) + i\sin\left(\frac{\pi}{2^n}\right)$, then x1 x2 x3.....infinity

A. $x_1 + x_2 + \ldots = 1$

B. $x_1 - x_2 + x_3 \ldots = 0$

C. $x_1 x_2 x_3 \ldots = 1$

D. $x_1 . x_2 . x_3 \ldots = -1$

Answer: D

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26. If $x_n = \cos\left(\frac{\pi}{4^n}\right) + i\sin\left(\frac{\pi}{4^n}\right)$, then $x_1, x_2, x_3 \ldots . \infty$ is equal to

A. $\frac{1 + i\sqrt{3}}{2}$

B. $\frac{-1+i\sqrt{3}}{2}$

C. $\frac{1-i\sqrt{3}}{2}$

D. $\frac{-1-i\sqrt{3}}{2}$

Answer: A

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27. The continued product of the four values of $\left(1-i\sqrt{3}\right)^{3/4}$ is

A. $2^{1/4}$

B. $2^{3/4}$

C. 2^{3}

D. $2^{1/3}$

Answer: C

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28. The continued product of all values of $\sqrt[5]{4}$ is

A. A) $\frac{1}{2}$

B. B) $-\frac{1}{2}$

C. C) 4

D. D) -4

Answer: C

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29. $\sum_{r=1}^{8}[\sin(2\pi r/9)+i\cos(2\pi r/9)]$=

A. -1

B. 1

C. i

D. $-i$

Answer: D

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30. The values of

$$\sum_{m=1}^{10} [\sin(2\pi m/11) - i\cos(2\pi m/11)] \text{ is}$$

A. i

B. $-i$

C. 1

D. -1

Answer: A

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31. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma =$

A. $3\sin(\alpha+\beta+\gamma)$

B. $2\sin(\alpha+\beta+\gamma)$

C. $\sin(\alpha+\beta+\gamma)$

D. $\cos(\alpha+\beta+\gamma)$

Answer: A

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32. If $\cos\alpha+\cos\beta+\cos\gamma=0$ & $\sin\alpha+\sin\beta+\sin\gamma=0$. Then Prove that

$\sin 2\alpha+\sin 2\beta+\sin 2\gamma=0$

A. 2

B. 1

C. $1/2$

D. 0

Answer: D

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33. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

A. 0

B. $\frac{1}{2}$

C. $\frac{3}{2}$

D. 2

Answer: C

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34. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\sin(2\alpha - \beta - \gamma) + \sin(2\beta - \gamma - \alpha) + \sin(2\gamma - \alpha - \beta) =$

A. 0

B. 1

C. 2

D. 3

Answer: A

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35. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma =$

A. 0

B. 1

C. $\cos 2(\alpha + \beta + \gamma)$

D. -1

Answer: A

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36. If $\cos\alpha + coa\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\cos(\alpha+\beta) + \cos(\beta+\gamma) + \cos(\gamma+\alpha) =$

A. 0

B. 1

C. $\sin 2(\alpha+\beta+\gamma)$

D. 2

Answer: A

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37. If $a = cis\alpha, b = cis\beta, c = cis\gamma$, and $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} - 1 = 0$ then $\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) =$

A. 0

B. 1

C. -1

D. 2

Answer: B

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38. $If A + B + C = \pi$ and

$\cos A + \cos B + \cos C = 0 = \sin A + \sin B + \sin C$,

then $\sin 3A + \sin 3B + \sin 3C =$

A. 0

B. 1

C. 2

D. 3

Answer: A

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39. The cube roots of 1+I are

A. A) $(i-1)/\sqrt[3]{2}, \{(i-1)/\sqrt[3]{2}\}\omega, \{(i-1)/\sqrt[3]{2}\}\omega^2$

B. B) $(i-1)/\sqrt{2}, \{(i-1)/\sqrt{2}\}\omega, \{(i-1)/\sqrt[3]{2}\}\omega^2$

C. C) $(i-1)/\sqrt{3}, \{(i-1)/\sqrt{3}\}\omega, \{(i-1)/\sqrt[3]{3}\}\omega^2$

D. D) $(i+1)/\sqrt[3]{2}, \{(i+1)/\sqrt[3]{2}\}\omega, \{(i+1)/\sqrt[3]{2}\}\omega^2$

Answer: A

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40. The values of $\left(\sqrt{3}+i\right)^{1/4}$ are

A. $2^{1/4}cis\left[\frac{2k\pi+\pi/6}{4}\right], k=0,1,2,3$

B. $2^{1/4}cis\left[\frac{2k\pi+\pi/3}{4}\right], k=0,1,2,3$

C. $2^{1/4}cis\left[\frac{2k\pi-\pi/6}{4}\right], k=0,1,2,3$

D. $2^{1/4}cis\left[\frac{2k\pi - \pi/3}{4}\right], k = 0, 1, 2, 3$

Answer: A

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41. The values of $(1+i)^{2/3}$ are

A. $2^{1/3}cis\left[(6k+1)\frac{\pi}{6}\right], k = 0, 1, 2$

B. $2^{1/3}cis\left[(7k+1)\frac{\pi}{6}\right], k = 0, 1, 2$

C. $2^{1/3}cis\left[(4k+1)\frac{\pi}{6}\right], k = 0, 1, 2$

D. $2^{1/3}cis\left[(5k+1)\frac{\pi}{6}\right], k = 0, 1, 2$

Answer: C

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42. All the values of $(-i)^{1/3}$ are given by

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43. If $x^4 - 1 = 0$ then x=

A. $\frac{1 \pm i}{\sqrt{2}}, \frac{-1 \pm i}{\sqrt{2}}$

B. $\frac{1 \pm i}{2}, \frac{-1 \pm i}{\sqrt{2}}$

C. $\frac{1 \pm i}{\sqrt{2}}, \frac{-1 \pm i}{2}$

D. $\pm 1, \ \pm i$

Answer: A

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44. Solve $x^9 - x^5 + x^4 - 1 = 0$

A. $cis\frac{\pi}{5}, cis\frac{3\pi}{5}, cis\pi, cis\frac{7\pi}{5}, cis\frac{9\pi}{5}, \ \pm 1, \ \pm i$

B. $cis\frac{\pi}{3}, cis\frac{2\pi}{3}, cis\pi, cis\frac{7\pi}{3}, cis\frac{9\pi}{3}, \ \pm 1$

C. $cis\frac{\pi}{2}, cis\frac{3\pi}{2}, cis\pi, cis\frac{7\pi}{2}, cis\frac{9\pi}{2}, \ \pm 1$

D. $1, i, cis\frac{\pi}{2}, cis\frac{3\pi}{2}, cis\pi, cis\frac{7\pi}{2}$

Answer: A

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45. $1, \alpha_1, \alpha_2, \alpha_3 \ldots . \alpha_{n-1}$ are the n^{th} roots of unity and n is an even natural number ,then $(1+\alpha_1)(1+\alpha_2)(1+\alpha_3) \ldots . (1+\alpha_{n-1}) =$

A. 1

B. 0

C. -1

D. 2

Answer: B

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46. If $1, \alpha, \alpha^2, \ldots\ldots, \alpha^{n-1}$ are the n^{th} roots of unity, then the value of $(3-\alpha)(3-\alpha^2)\ldots\ldots(3-\alpha^{n-1})$ is

A. n

B. 0

C. $(3^n-1)/2$

D. $(3^n+1)/2$

Answer: C

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47. A : $(1+i)^6+(1-i)^6=0$

R : If n is a positive integer then $(1+i)^n+(1-i)^n=2^{(n/2)+1}.\cos\frac{n\pi}{4}$

A. Both A and R are true R is correct explanation to A

B. Both A and R are true but R is not correct explanation to A

C. A is true R is false

D. A is false R is true

Answer: A

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48. Assertion (A): If A, B, C are the angles of a triangle such that $\cos A + \cos B + \cos C = 0 = \sin A + \sin B + \sin C$ then $\cos 3A + \cos 3B + \cos 3C = -3$

Reason (R) : If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\gamma + \alpha + \beta)$

A. Both A and R are true R is correct explanation to A

B. Both A and R are true but R is not correct explanation to A

C. A is true R is false

D. A is false R is true

Answer: A

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49. IF ω represents a complex cube root of unity, then $\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\ldots+\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$

A. Both A and R are true R is correct explanation to A

B. Both A and R are true but R is not correct explanation to A

C. A is true R is false

D. A is false R is true

Answer: A

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50. Assertion (A) : If $\alpha=\cos\left(\frac{2\pi}{7}\right)+i\sin\left(\frac{2\pi}{7}\right), p=\alpha+\alpha^2+\alpha^4, q=\alpha^3+\alpha^5+\alpha^6$ then the equation whose roots are p and q is $x^2+x+2=0$

Reason (R) : If α is a roots of $z^7=1$ then $1+\alpha+\alpha^2+\ldots\ldots+\alpha^6=0$

A. Both A and R are true R is correct explanation to A

B. Both A and R are true but R is not correct explanation to A

C. A is true R is false

D. A is false R is true

Answer: A

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51. If $z = \frac{\sqrt{3} - i}{2}$ then $\left(i^{101} + z^{101}\right)^{103} =$

A. A) z

B. B) $-z$

C. C) $\bar{z}$

D. D) iz

Answer: A

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52. The roots of $(z-1)^n = i(z+1)^n$ are plotted on the argand plane they are

A. on x = 0

B. concyclic

C. on a parabola

D. on y = 0

Answer: A

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53. If $'\alpha'$ be the non-real n^{th} roots of unity, then $1 + 3\alpha + 5\alpha^2 + \ldots\ldots(2n-1)\alpha^{n-1}$ is equal to

A. $\frac{2n}{1-\alpha}$

B. $\frac{n}{1-\alpha}$

C. $\frac{n}{2(1-\alpha)}$

D. $\frac{2n}{\alpha-1}$

Answer: D

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54. If $\cos\alpha + 2\cos\beta + 3\cos\gamma = \sin\alpha + 2\sin\beta + 3\sin\gamma = 0$ then $\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma =$

A. $18\cos(\alpha+\beta+\gamma)$

B. $18\sin(\alpha+\beta+\gamma)$

C. $\sin(\alpha+\beta+\gamma)$

D. $\cos(\alpha+\beta+\gamma)$

Answer: B

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55. If $\sin(A+iB)=x+iy$ then

$$\left(\frac{x^2}{\sin^2 A}-\frac{y^2}{\cos^2 A}\right)+\left(\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}\right)=$$

A. 1

B. 2

C. 3

D. 4

Answer: B

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56. $I: \sum_{k=1}^{6}\left[\sin\left(\frac{2k\pi}{7}\right)-i\cos\left(\frac{2k\pi}{7}\right)\right]=i$

$II: \sum_{r=1}^{8}\left[\sin\left(\frac{2\pi r}{9}\right)+i\cos\left(\frac{2\pi r}{9}\right)\right]=i$

Which of the above statements are true

A. A) only I

B. B) only II

C. C) Both I and II

D. D) neither I nor II

Answer: B

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57. Match the following.

$Column - I$	$Column - II$
$I \ : \ x = \cos\theta + i\sin\theta \ \text{ then } \ x^n - \frac{1}{x^n}$	a) $2i\sin(\alpha - \beta)$
$II \ : \text{If } \ z = \cos\theta + \sin\theta \ \text{ then } \ \frac{z^n - 1}{z^{2n} + 1}$	b) $2\cos(\alpha - \beta)$
$III \ : \ x = cis\alpha, y = cis\beta \ \text{ then } \ \frac{x}{y} + \frac{y}{x} =$	c) $i\tan n\theta$
$IV \ : \ x = cis\alpha, y = cis\beta \ \text{ then } \ \frac{x}{y} - \frac{y}{x} =$	d) $2i\sin n\theta$

A. a,b,c,d

B. b,c,a,d

C. d,a,b,c

D. d,c,b,a

Answer: D

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58. The ascending order of the values of

$z_1(1+i)^4+(1-i)^4, z_2=\left(\sqrt{3}+i\right)^{12}+\left(\sqrt{3}-i\right)^{12}, z_3=\left(1+i\sqrt{3}\right)^9+($

A. z_1, z_2, z_3

B. z_2, z_3, z_1

C. z_3, z_1, z_2

D. z_1, z_3, z_2

Answer: C

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59. I : Value of $i^i=e^{-\frac{\pi}{2}}$

II : $z=i\log\left(2+\sqrt{3}\right)$ then sinz=2

which of above statement is true

A. only I

B. only II

C. Both I and II

D. neither I nor II

Answer: A

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60. Assertion (A) : $(\cos\theta + i\sin\theta)^{10} + (\cos\theta - i\sin\theta)^{10} = 2\cos 10\theta$

Reason (R) : $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$

A. A) Both A and R are true R is correct explanation to A

B. B) Both A and R are true but R is not correct explanation to A

C. C) A is true R is false

D. D) A is false R is true

Answer: B

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61. The modulus argument form of 1+iTanθ when $\theta \varepsilon \left(\frac{\pi}{2}, \pi \right)$ is

A. $\sec\theta CIs\theta$

B. $-\sec\theta Cis(\pi+\theta)$

C. $-\sec\theta Cis(\theta-\pi)$

D. none of these

Answer: C

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62. If $Z = \left(i^i\right)^i$ where $i = \sqrt{-1}$ then

A. A) $|Z| = e^{-\frac{\pi}{2}}$

B. B) |Z|=1

C. C) Re(Z)=0

D. D) $Re(Z) = e^{-\frac{\pi}{2}}$

Answer: B

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63. If α is a non-real and $\alpha = (1)^{1/5}$ then the value of $2^{|1+\alpha+\alpha^2+\alpha^{-2}+\alpha^{-1}|}$ is equal to

A. 1

B. 2

C. 3

D. 4

Answer: D

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64. If $(x-1)^4 - 16 = 0$,then the sum of non-real complex values of x is

A. 2

B. 0

C. 4

D. 3

Answer: A

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