MATHS

JEE (MAIN AND ADVANCED) MATHEMATICS

MEAN VALUE THEOREMS

Example

1. Verify Rolle's theorem for the function sinx-sin2x on $[0, \pi]$

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2. Let f(x) = (x - 1)(x - 2)(x -3). Prove that there is more than one ‘c’ in (1,3) such that f ’(c) = 0

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3. Show that there is no real number K, which the equation $x^2 = x^2 - 3x + x \in [0, 1]$ = 0 has two distinct roots in [0,1]

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4. It is given that Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax$ on [1,3] with C = $2 + \frac{1}{\sqrt{3}}$. Find the values a and b

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5. Verify Rolle's theorem for the functions $f(x) = x(x+3)e^{-x/2}$ on [-3,0]

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6. Verify whether Rolle's theorem is applicable to f(x) = tanx on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$?

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7. Verify Rolle's theorem for the function $f(x) = (x - a)m(x - b)^n$ on [a, b] where m and n are positive integers.

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8. Find "c" of the Rolle's theorem for $f(x) = \frac{\log(x^2 + ab)}{a + bx}$ in [a, b] where a, b $>$ 0.

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9. If 2a + 3b + 6c = 0, show that there exists at least one root of the equation $ax^2 + bx + c = 0$ in the interval (0,1).

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10. The value of 'a' for which $x^3 - 3x + a = 0$ has two distinct roots in [0,1] is given by

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11. Find “θ ” of the Rolle’s theorem for the function

$f(x) = x^2 - 5x + 7$ in [2,3],

at least one root

atmost one root

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12. Find c of the LMVT for $f(x) = x(x-1)(x-2)$ in $\left[0, \frac{1}{2}\right]$

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13. Show that the square roots of two successive natural numbers greater than N^2 differ by less than $\frac{1}{2N}$

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Exercise

1. $f(x) = x^2 - 1$ in [-1,1]

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2. f(x) = $x^2 - 5x + 6$ in [-3, 8]

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3. $f(x) = x^2 + 4 \in [-3, 3]$

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4. $f(x) = (x^2 - 1)(x - 2) \in [-1, 2]$

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5. $f(x) - \log(x^2 + 2) - \log 3 \in [-1, 1]$

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6. The constant c of Rolle's theorem for the function $f(x) = 2x^3 + x^2 - 4x - 2$ in $[-\sqrt{2}, \sqrt{2}]$ is

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7. Find c of the R olle's theorem for the functions

$$f(x) = \frac{\log(x^2 + 3)}{4x}$$ in [1,3]

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8. The value of 'c' in Lagrange's mean value theorem

for f(x) = (x - a)m $(x - b)^m [a, b]^n$ is [a,b] is

A. $\frac{mb + na}{m + n}$

B. $\frac{ma + nb}{m + n}$

C. $\frac{a + b}{m + n}$

D. $\frac{a+b}{2}$

Answer: A

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9. If $f(x) = x^3 + bx^2 + ax$ satisfies the conditions of Rolle's theorem in [1, 3] with $c = 2 + \frac{1}{\sqrt{3}}$ then (a, b) is equal to

A. $(11, 6)$

B. $(11, -6)$

C. $(-6, 11)$

D. $(6, 11)$

Answer: B

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10. If a + b + c = 0, then the equation $3ax^2 + 2bx + c = 0$ has, in the interval (0, 1)

A. at least one root

B. atmost one root

C. no root

D. exactly one root

Answer: A

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11. If $27a+9b+3c+d=0$, then the equation $4ax^3+3bx^2+2cx+d=0$ has atleast one real root lying between

A. 0 and 1

B. 1 and 3

C. 0 and 3

D. 0 and 2

Answer: C

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12. Rolle's theorem cannot be applicable for

A. $f(x) = \cos x - 1 \in [0, 2\pi]$

B. $f(x) = x(x-2)^2 \in [0, 2]$

C. $f(x) = 3 + (x-1)^{2/3} \in [0, 3]$

D. $f(x) = \sin^2 x \in [0, \pi]$

Answer: C

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13. The value of 'c' in Lagrange's thorem for $f(x) = lx^2 + mx + n[l \neq 0]$ on [a,b] is

A. $a/2$

B. $b/2$

C. $\frac{a-b}{2}$

D. $\frac{a+b}{2}$

Answer: D

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14. If f'(x) = $\frac{1}{1+x^2}$ for all x and f(0) = 0, then:

A. $f(2) \leq 0.4$

B. $f(2) > 2$

C. $0.4 < f(2) < 2$

D. $f(2) = 2$

Answer: C

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15. Let f be a function which is continuous and differentiable for all real x. If f(2) = -4 and f'(x) $\geq$ 6 for all $x \in [2, 4]$, then

A. $f(4) < 8$

B. $f(4) \geq 8$

C. $f(4) > 12$

D. $f(4) > 8$

Answer: B

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16. In [0,1], Lagrange's mean value theorem is not applicable to

(a) $f(x) = \begin{cases} \frac{1}{2} - x & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2 & x \geq \frac{1}{2} \end{cases}$

(b) $f(x) = \begin{cases} \frac{\sin}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$f(x) = x|x|$

(d) f(x)=|x|

A. a

B. b

C. c

D. d

Answer: A

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17. Let f(x) and g(x) be differentiable functions for $0 \leq x \leq 1$ such that f(0) = 2, g(0) = 0, f(1) = 6`. Let there exist a real number c in (0,1) such that f ’(c) = 2 g ’(c), then g(1) =

A. 1

B. 2

C. -2

D. -1

Answer: B

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18. The value of 'a' for which $x^3 - 3x + a = 0$ has two distinct roots in [0,1] is given by

A. not value of k

B. at least one value of k

C. infinitely many values of k

D. exactly one value of k

Answer: A

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19. Let $R \rightarrow R$ be a continuous function define by

f(x) $= \dfrac{1}{e^x + 2e}$

statement 1 : $f(c) = \dfrac{1}{3}$ for some c $\in$ R

statement 2 $0 < f(x) \le \dfrac{1}{2\sqrt{2}}$ for all x $\in$ R

A. statement -1 is true statement 2 is true

statement 2 is not the correct explanation for

statement -1

B. statement -1 is true statement -2 is false

C. statement -1 is false statement 2 is true

D. statement -1 is true statement -2 is true

statement -2 is the correct explantion for statement -1

Answer: D

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20. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]

A. lies between 1 and 2

B. lies between 2 and 3

C. lies between -1 and 3

D. does not exist

Answer: D

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21. If f and 'g' are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in [0,1]$:

A. $2f(c) = g(c)$

B. $2f(c)=3g(c)$

C. $f(c)=g(c)$

D. f(c) =2g (C)

Answer: D

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22. If the Rolle's theorem holds for the function $f(x) = 2x^3 + ax^2 + bx$ in the interval [-1,1] for the point $c = \frac{1}{2}$, then the value of 2a + b is

A. 1

B. -1

C. 2

D. -2

Answer: B

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23. Let f(1)= -2 and f'(x) $\geq$ 4.2 for 1 $\leq$ x $\leq$ 6 .

The possible value of /(6) lies in the interval:

A. $[15, 19]$

B. $(-\infty, 12)$

C. $[12, 15)$

D. $[19, \infty]$

Answer: D

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24. A value of c for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is

A. $\log_e 3$

B. $2 \log_3 e$

C. $1/2 \log_e 3$

D. $\log_3 e$

Answer: B

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25. If 2a + 3b + 6c = 0, then atleast one root of the equation $ax^2 + bx + c = 0$ lies in the interval

A. 0,1

B. 1,2

C. 2,3

D. 1,3

Answer: A

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26. If the equation $a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1x = 0(a_1 \neq 0, n \geq 2)$ has a positive root x = α then the equation $na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots\ldots + a_1 = 0$ has a positive root, which is

A. equal to α

B. $\geq \alpha$

C. $< \alpha$

D. $> \alpha$

Answer: C

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27. Let f be differentiable for all x. If f(1) = -2 and f'(x) $\geq$ 2 for all $x \in [1, 6]$, then

A. $f(6) < 8$

B. $f(6) \geq 8$

C. $f(6) \geq 5$

D. $f(6) \leq 5$

Answer: B

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28. Consider the functions, $f(x) = |x - 2| + |x - 5|$, $x \in R$ Statement-1 : $f'(4) = 0$ Statement-2 : f is continuous in [2, 5], differentiable in (2, 5) and F(2) =F(5)

A. S-1 is false S-2 is true

B. S-1 is true S-2 is true S-2 is a correct explanation for S-1

C. s-1 is true S-2 is true S-2 is a correct explantion for S-1

D. S-1 is true S-2 is false

Answer: B

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