



MATHS

JEE (MAIN AND ADVANCED) MATHEMATICS

RANDOM VARIABLES

Solved Examples

1. A cubical die is thrown. Find the mean and variance of X , giving the number on the face shows up.

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2. A random variable X has the following probability distribution.

$X = x_i$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) Mean (iii) $P(0 < X < 5)$



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3. If X is a random variable with probability distribution

$$P(X = k) = \frac{(k + 1)C}{2}, \quad K = 0, 1, 2, \dots \text{ then find } C.$$



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4. The range of a random variable X is $\{1, 2, 3, \dots\}$ and

$P(X = k) = \frac{C^k}{k!}$ where $k = 1, 2, 3, \dots$. Find the value of C and

$P(0 < X < 3)$.



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5. Let S be the sample space of the random experiment of rolling a fair die. Define $X: S \rightarrow R$ by

$X(n) = \begin{cases} 0 & \text{If } n \text{ is even} \\ 1 & \text{If } n \text{ is odd} \end{cases}$ Find P .



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6. 8 coins are tossed simultaneously. Find the probability of getting at least 6 heads.



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7. The mean and variance of a Binomial variate are 2.4 and 1.44 respectively. Find the parameters, $P(X = 2)$ and $P(1 < X \leq 4)$.

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8. If the difference between the mean and variance of a Binomial variate is $5/9$ then find the probability for the event of 2 successes when the experiment is conducted 5 times.

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9. When a coin is tossed n times if the probability for getting 6 heads is equal to the probability for getting 8 heads then find the value of n .



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10. If 3 coins are tossed simultaneously and the number of heads turned up is denoted by the variable X , then find mean and variance of X .



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11. One in 9 ships is likely to be wrecked when they are set on a sail. When 6 ships are on sail, find the probability for

(i) atleast one will arrive safely.

(ii) exactly three will arrive safely



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12. The least number of times a fair coin must be tossed so that the probability of getting at least one head is at least 0.8 is



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13. If for a Binomial distribution, $\mu = 10$ and $\sigma^2 = 5$, then find $P(X > 6)$.



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14. In a box containing 15 identical bulbs, 5 are defective. If 5 bulbs are drawn at random from the box with replacement, find the probability that

- (i) none is defective
- (ii) only one of them is defective
- (iii) atleast one of them is defective



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15. In the experiment of tossing a coin n times, if the variable X denotes the number of heads and $P(X = 4), P(X = 5), P(X = 6)$ are in arithmetic progression then find n .



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16. For a poisson variate X , $P(X = 2) = P(X = 3)$ find variance of X .



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17. If X is a poisson variate such that $P(X = 0) = P(X = 1) = K$, then show that $K = 1/e$



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18. In a large consignment of apples are rotten. What is the probability that a carton of 48 apples contain less than two rotten ones ? Answer the questions using

(i) Binomial distribution

(ii) Poisson distribution

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19. The number of persons joining a cinema ticket counter in a minute has poisson distribution with parameter 6. Find the probability that (i) no one joins the queue in a particular minute (ii) two or more persons join the queue in a minute

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20. A factory produces razor blades and 1 in 500 blades is estimated to be defective. The blades are supplied in packets of 10. IN a consignment of 10,000 packets, using poisson distribution, find approximately the number of packets which contain no defective blades. (Given $e^{-0.02} = 0.9802$)

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21. Five coins are tossed 3200 times using poisson distribution, find the probability of getting five heads 2 times.

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Exercise 4 1

$X = x$	-2	-1	0	1	2	3
$P(X = x)$	0.1	k	0.2	$2k$	0.3	k

1. is the probability distribution of random varibale X. Find k and variance of X.

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2. Find the mean and variance of the random variable X which follows the following distribution

$X = x$	1	2	3	4
$P(X = x)$	0.1	0.2	0.3	0.4



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3. The probability distribution of a random variable X is given below.

$X = x$	1	2	3	4	5
$P(X = x)$	k	$2k$	$3k$	$4k$	$5k$

Find mean and variance of X .



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4. The range of a random variable X is $\{0, 1, 2\}$. Given that

$$P(X = 0) = 3c^3, P(X = 1) = 4c - 10c^2, P(X = 2) = 5c - 1$$

where c is constant.

Find (i) the value of c (ii) $P(X < 1)$

(iii) $P(1 < X \leq 2)$ (iv) $P(0 < X \leq 3)$



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5. Let X be a random variable such that

$$P(X = -2) = P(X = -1) = P(X = 2) = P(X = 1) = \frac{1}{6}$$

$$\text{and } P(X = 0) = \frac{1}{3}.$$



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6. If X is a random variable with the following distribution

$X = x_i$	a	b
$P(X = x_i)$	p	q

where $p + q = 1$ then show that the variance of X is $pq(a - b)^2$.



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7. The range of a random variable $X = \{1, 2, 3, \dots\}$ and probabilities are given by $P(X = k) = \frac{3^{Ck}}{k}$ for $k = 1, 2, 3, \dots$ and C is a constant. Find the value of C .



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8. The range of a random variable $X = \{1, 2, 3, \dots\}$ and the probability distribution of X is given by

$$P(X = n) = \frac{k(n+1)}{2^n} \quad (n = 1, 2, 3, \dots) \text{ find } k.$$



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9. If a random variable X takes the value

$$x_k = \frac{(-1)^k 2^k}{k} \quad (k = 1, 2, 3, \dots) \quad \text{with probability}$$

$$P(X = x_k) = 2^{-k}. \text{ Show that mean is } -\log_e 2.$$



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10. Two dice are rolled and the probability distribution of the sum of the numbers on the dice is formed. Find the mean of

the sum.



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11. Four bad apples are mixed with 20 good apples. If 2 apples are drawn at random at a time then find mean of number of bad apples



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12. A box contain 6 tickets. Two of the tickets carry a price of Rs 5/- each and the other 4 are the price of Rs 1 each. If one ticket is drawn at random, what is the mean price.



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13. A business man gets a profit of Rs. 2800 with probability 0.5, loss of Rs. 5000 with probability 0.3. and neither profit nor loss with probability 0.2. Find mean of his income.



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14. Two cards are drawn from pack of 52 cards one after another with replacement. Find the mean of the number of kings.



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15. Two cards are drawn from pack of 52 cards one after another without replacement. Find the mean of the random variable X where X is number of Aces.

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16. A pair of fair dice is rolled 2 times. Find the mean of the number of doublets on the dice. Find also variance of the number of doublets on the dice.

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17. A coin is loaded such that $P(H) = 3P(T)$. It is tossed 3 times. Let X be the random variable which indicates the number of heads which occur. Find the mean of X , variance of X .

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18. Two coins whose faces are marked 1 and 2 are tossed. Let X be the total value of the numbers. Find mean and variance of X .



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Exercise 4 2

1. If $X \sim B(10, 0.6)$ then find mean and variance of X .



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2. In a Binomial distribution mean is 4 and variance is 3 find $P(X \geq 1)$.



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3. $X \sim B(400, 1/5)$ then find its standard deviation



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4. $X \sim B(n, p)$ and mean and variance of X are $\frac{15}{2}$ and $\frac{15}{4}$.

Find n and p .



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5. In a binomial distribution, the parameter $n = 6$. If $9P(X = 4) = P(X = 2)$, then $p =$



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6. It is given that 10% of the electric bulbs manufactured by a company are defective. In a sample of 20 bulbs, find the probability that more than 2 are defective.



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7. On an average, rain falls 12 days in every 30 days. Find the probability that rain will fall on just 3 days of a given week.



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8. Find the probability of guessing atleast one out of 10 answers in multiple choice question with 4 possible answers.



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9. The probability that a person chosen at random is left handed (in hand writing) is 0.1 what is the probability that in a group of ten people there is one and only one who is left handed.



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10. A coin is tossed a given number of times. IF the probability of getting 7 heads is equal to that of 8 heads. Find the probability of getting 2 heads.



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11. A man makes a forward step with probability 0.6. Find the probability that at the end of eleven steps, he is one step

away from the starting point.



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12. The probability of a bomb hitting a bridge is $\frac{1}{2}$ and three direct hits (not necessarily consecutives) are needed to destroy it. Find the minimum number of bombs required so that the probability of the bridge being destroyed is greater than 0.9.



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13. Out of 10000 families with 4 children each, find the number of families all of whose children are daughters.



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14. Out of 10000 families with 4 children each, find the frequencies of distribution of number of male children.



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Exercise 4 3

1. If the mean of poisson distribution is 2.56 then find the standard deviation.



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2. If a random variable X has a poisson distribution with parameter $1/2$ then find $P(X = 2)$

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3. A random variable X has a poisson distribution with parameter 2 then find $P(X > 1.5)$.

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4. If X is a poisson variate such that $P(X = 0) = P(X = 1)$ then find the parameter λ .

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5. In a poisson distribution, $P(X = 0) = 2P(X = 1)$ then find standard deviation.

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6. If X is a poisson variate such that $P(X = 2) = 9P(X = 4) + 90P(X = 6)$ then find mean of X .



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7. If 2% of a given lot of manufactured parts are defective then find the probability that a sample of 100 items has no defective item.



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8. In a book of 450 pages, it is found that there are 400 typing errors. Assume that poisson law holds for the number of errors per page, find the probability that a random sample of 5 pages will contain no errors.



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9. Suppose on an average 1 house in 1000 in a certain district has a fire during a year. If there are 2000 houses in that district, then find the probability that exactly 5 houses will have a fire during the year.



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10. The incidence of an occupational disease to the workers of a factory is found to be $\frac{1}{5000}$. If there are 1000 workers in a factory then find the probability that none of them will get the disease.



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11. Deficiency of red cells in the blood cells is determined by examining a specimen of blood under microscope. Suppose a small fixed volume on an average 20 red cells for normal persons. Using the poisson distribution find the probability that a specimen of blood taken from a normal person will contain less than 15 red cells.



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12. In a city 10 accidents take place in a span of 50 days. Assuming that the number of accidents follows the poisson distribution. Find the probability that there will be 3 or more accidents in a day.



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Exercise 4 4

1. A random variable X takes four values with probabilities $\frac{1 + 3x}{4}$, $(1 - x)$, $\frac{1 + 2x}{4}$ and $\frac{1 - 4x}{4}$. Find the set of values of x for the variable X to have probability distribution.



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2. For a random variable X given that $P(X = k) = 2^{-k}$ find the mean and variance of X .



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3. If X represents difference between the number of heads and number of tails obtained when a fair coin is tossed 3 times. Then find mean and variance of X .



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4. The probability of a missile hitting a target is $1/2$ and two direct hits are required to destroy it. Find the least number of missiles, so that the probability of the target destroyed is greater than 0.9.



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5. A gunman has four bullets, he fires till he makes first hit on the target. The probability of a hit for each shot is 0.7, find the probability distribution of the number of bullets used.



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6. The probability of a gun man hitting a target is $\frac{1}{3}$, he fires ten shots, the random variable X is the number of hits. Find the mean and variance of X .



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7. A telephone exchange receives on an average 180 calls per hour. Find the probability that it will receive only 2 calls in a given minute.



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8. A book of 500 pages has 50 misprints. Find the probability that there are not less than 3 misprints on a given page.



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9. A car hire firm, hires 2 cars everyday. The number of demands for car per day on an average is 1.5. Find the expected number of days it can reject a demand in 100 working days.



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