MATHS

BOOKS - KALYANI PUBLICATION

INEQUALITIES AND INEQUATIONS

Example

1. For $a > 0$ prove that $\left(a + \frac{1}{a}\right) \geq 2$.

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2. For all values of x prove that $(2x^2 - 8x + 9) > 0.$

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3. For $x \geq -3$ prove that $x^3 + 27 + 9x^2 + 27x \geq 0.$

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4. For $x > 1$ and $y > 1$ prove that $(x+1)(y+1) < 2(xy+1).$

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5. if $a > b > c > 0$,then prove that

$$\frac{a+b}{2} > \frac{a+b+c}{3} > \frac{b+c}{2}$$

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6. If a,b,x,y are four unequal real numbers and $x^2 + y^2 = 5, a^2 + b^2 = 3$,then prove that $ax + by < 4$.

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7. If a,b,c are three positive numbers,then`. a/b + b/c + c/a

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8. Prove that the perimeter of any quadrilateral is greater than the sum of the diagonals.

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9. If a and b are two unequal positive real numbers,then prove that $(a+b)\left(\frac{5}{a}+\frac{1}{5b}\right)>4.$

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10. If x,y,z are all positive real number and $x+y+z=s$,then prove that $(s-x)(s-y)(s-z) \geq 8xyz.$

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11. Prove that $\sqrt{7}+\sqrt{6}>\sqrt{8}+\sqrt{5}.$

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12. If $a, b > 0, a \neq b$, and $n > 1$, where n is an integer, then prove that $a^n + b^n > a^{n-1}b + ab^{n-1}$.

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13. If a,b,c are three positive real numbers then show that $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$.

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14. If $4x + y = 5$, then find the greatest value of $\frac{1}{3}(xy)$.

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15. If $\frac{3}{5}xy = 15$,then find the least value of $2(x + y)$.

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16. Find the solution sets of the following inequations.

$5(1 - x) < 3(x - 2) - 29.$

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17. Find the solution sets of the following inequations.

$$\frac{9x+7}{2} > \left(x - \frac{x-2}{7}\right) + 36.$$

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18. Find the solution set of the following system of in—equation.

$$2x - 3 < 0, 4 - 5x < 0.$$

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19. Find the solution sets of the following inequation

$x^2 - x - 2 > 0$

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20. Find the solution sets of the following inequation

$3 - x^2 - 2x > 0.$

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21. Find the solution set of the inequations $4 \leq x^2 < 16$.

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22. Solve the equation $|x - 3| = 2$

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23. Express the following removing the absolute sign OR solve the following inequations.

$|x| \leq 5$.

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24. Express the following removing the absolute sign OR solve the following inequations.

$|x+2|<5.$

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25. Express in absolute value

$-6<x<6.$

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26. Express in absolute value

$x > 5$ or $x < -1$

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27. Express in absolute value

$-\sqrt{5}+2 \leq x \leq \sqrt{5}+2.$

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28. Find the solution set of the inequation

$\frac{x}{2}+\frac{y}{3} > 2.$

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29. Find the simultaneous solution sets of the following sets of inequations.

$(3x + 4y) < 24$

$x - 2y < 4.$

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30. Find the simultaneous solution sets of the following sets of inequations.

$\frac{x}{3} + \frac{y}{4} < 1$

$x \geq 0, y \geq 0.$

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Exercise

1. State whether the following statements are true or false. If false give the correct answer.

$a > a + m.$

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2. State whether the following statements are true or false. If false give the correct answer.

$a > b \Rightarrow ma > mb.$

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3. State whether the following statements are true or false. If false give the correct answer.

$a^3 > a.$

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4. State whether the following statements are true or false. If false give the correct answer.

$ma > mb \Rightarrow a^2 > b^2.$

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5. State whether the following statements are true or false. If false give the correct answer.

$a > b \Rightarrow -\frac{1}{a} > -\frac{1}{b}.$

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6. State whether the following statements are true or false. If false give the correct answer.

$x > 2, x > 3 \Rightarrow x > 2.$

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7. State whether the following statements are true or false. If false give the correct answer.

$x < 3, x < 1 \Rightarrow x < 1.$

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8. State whether the following statements are true or false. If false give the correct answer.

$x > -1, x < 3 \Rightarrow -1 < x < 3.$

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9. State whether the following statements are true or false. If false give the correct answer.

$\times \leq 1, x \geq -1 \Rightarrow x \geq 4.$

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10. State whether the following statements are true or false. If false give the correct answer.

$x > 2, x < -2 \Rightarrow -2 < x < 2.$

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11. Compare the pairs of real number given below

$a^3 + 1, a^2 + 1.$

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12. Compare the pairs of real number given below

$a^3 + b^3, a^2b + ab^2.$

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13. Compare the pairs of real number given below

$ab + 1, a + b.$

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14. Prove the following where $a, b, xm, n > 0$

$m^5 + n^5 \geq m^4 n - mn^4.$

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15. Prove the following where $a, b, xm, n > 0$

$a^6 + b^6 \geq a^5 b + ab^5.$

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16. Prove the following where $a, b, xm, n > 0$

$$a^{m+n} + b^{m+n} \geq a^m b^n + a^n b^m.$$

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17. Prove the following where $m, n > 0$

$$m^3 - n^3 > (m-n)^3, (m > n).$$

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18. Prove the following where $a, b, m, n > 0$

$$\frac{m+2n}{m+3n} \geq \frac{m+n}{m+2n}.$$

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19. Prove the following where $a, b, xm, n > 0$

$$\frac{m^3 + n^3}{m^2 + n^2} \geq \frac{m^2 + n^2}{m + n}.$$

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20. Prove the following where $a, b, xm, n > 0$

$$\frac{a - c}{b} \geq \frac{b - c}{a} + \frac{a - b}{b} (0 < a < b < c).$$

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21. Prove the following where $a, b, xm, n > 0$

$$\frac{a^3+b^3}{a^2+b^2} > \frac{a^2+b^2}{a+b} > \frac{a+b}{2}(a \neq b).$$

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22. Prove that

$$\frac{1}{a+b} + \frac{1}{a-b} > \frac{1}{\sqrt{a^2-b^2}}(a > b).$$

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23. Prove that

$$\frac{a^2+b^2}{2} > \left(\frac{a+b}{2}\right)(a \neq b).$$

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24. Prove that

$(a+b)^2 > 14(a+b) - 49 (a > 0, b > 0).$

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25. Find the value of x for which the following expressions aquire greatest value ? Also determine their greatest values.

$24x - 8 - 9x^2.$

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26. Find the value of x for which the following expressions acquire greatest value ? Also determine their greatest values.

$8x - x^2 - 13$

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27. Find the value of x for which the following expressions are minimum. Also determine their minimum values.

$4x^2 - 12x + 13.$

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28. Find the value of x for which the following expressions are minimum. Also determine their minimum values.

$x^2 - 6x + 40.$

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29. Find the greatest value of x in order that.

$7x^2 + 11 \geq x^3 + 17x.$

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30. Find the least value of x in order that.

$3x^3 + 7x \geq 8x^2 + 2.$

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31.

$$\frac{b^3 + c^3}{b + c} + \frac{c^3 + a^3}{c + a} + \frac{a^3 + b^3}{a + b} > ab + bc + ca$$

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32. If a,b,c are positive unequal real numbers then prove that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{1}{\sqrt{b}c}+\frac{1}{\sqrt{a}c}+\frac{1}{\sqrt{a}b}.$$

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33. If a,b,c are positive unequal real numbers then prove that

$$\frac{bc}{a}+\frac{ca}{b}+\frac{ab}{c} \geq a+b+c.$$

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34. If a,b,c are positive unequal real numbers then prove that

$$b^2c^2+c^2a^2+a^2b^2 > abc(a+b+c).$$

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35. If a,b,c are positive unequal real numbers then prove that

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)>9$$

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36. If a,b,c,e,f,g are positive unequal real numbers then prove that

$$\left(\frac{a}{e}+\frac{b}{f}+\frac{c}{g}\right)\left(\frac{e}{a}+\frac{f}{b}+\frac{g}{c}\right)>9$$

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37. If a,b,c are positive unequal real numbers then prove that

$a^2 + b^2 + c^2 > ab + bc + ca.$

Hence prove that $a^3 + b^3 + c^3 > 3abc$

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38. prove that

$\sqrt{7} + \sqrt{12} > \sqrt{6} + \sqrt{13}.$

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39. If a,b,c are positive unequal real numbers then prove that

$\sqrt{14}+\sqrt{7}>\sqrt{16}+\sqrt{5}.$

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40. Prove that in any triangle, the semi perimeter is greater than each of its sides.

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41. Prove that in any quadrilateral the sum of the diagonals is greater than its semi—perimeter.

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42. If $\left(a^2+b^2\right)\left(x^2+y^2\right)=(ax+by)^2$ then prove that $\frac{x}{a}=\frac{y}{b}$.

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43. If a,b,c are positive unequal real numbers then prove that

$a^2+b^2+c^2=1=x^2+y^2+z^2$,then prove that $az+bx+cy\leq 1$.

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44. If a,b, c and x, y, z are all positive and unequal, then prove that,

$(ax + by)(ab + xy) > 4abxy.$

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45. If a,b, c and x, y, z are all positive and unequal, then prove that,

$(b + c)(c + a)(a + b) > 8abc.$

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46. If x, y, z are all positive real numbers, then prove that,

If $x+y+z=1$, then prove that $(1-x)(1-y)(1-z) \geq 8xyz$.

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47. If a,b, c and x, y, z are all positive and unequal, then prove that,

$(xy+1)(yz+1)(zx+1) > 8xyz\left[\sqrt{x}y + \frac{1}{\sqrt{x}y} > 2\right.$

etc]

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48. If $x + y = 5$, then find the greatest value of 10xy.

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49. If $2x + 3y = 5$, then find the greatest value of 8xy.

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50. If $\frac{7}{12}xy = 21$, then find the minimum value of $5(x + y)$.

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51. Find the solution sets of the following inequations.

$7(3-2x)+5(x-1)<34.$

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52. Find the solution sets of the following inequations.

$5(x-11)+4>2(x-15).$

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53. Find the solution sets of the following inequations.

$8(2x - 7) - 9(3x - 14) > 15.$

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54. Find the solution sets of the following inequations.

$13x - 4(5x - 8) + 17 > 0.$

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55. Find the solution sets of the following inequations.

$14(x-4)+3(x+5)<6(7-2x)+4.$

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56. Find the solution sets of the following inequations.

$a^2(x-a)+b^2(x-b)\geq abx.$

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57. Find the solution sets of the following inequations.

$$\frac{x-6}{5}+\frac{x-4}{3}\leq 8-\frac{x-2}{7}.$$

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58. Find the solution sets of the following inequations.

$$\frac{x}{10}+\frac{2x-13}{9}>8-\frac{4x-35}{15}.$$

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59. Find the solution sets of the following system of inequations.

$4x + 3 > 2x + 5$

$5x - 6 < 2x + 3.$

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60. Find the solution sets of the following system of inequations.

$5(1 - x) > 3(x - 2) - 29$

$4(2 - x) + 2(3 - 2x) < 30$

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61. Find the solution sets of the following system of inequations.

$3x + 2 \leq x + 6$

$15x - 9 \geq 11x - 25.$

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62. Find the solution sets of the following system of inequations.

$4(x - 3) \leq 2(x - 6).$

$19 - 3x < 5x + 35.$

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63. Find the solution sets of the following system of inequations.

$3(x-2)+7(2x-3) \geq 5(1-2x)-59.$

$13x-4(5x-8)+17>0.$

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64. Find the solution set of the following system of inequations.

$49+13(5x+27)<8(5+x)-3x.$

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65. Find the solution sets of the following system of inequations.

$14(x-4)+3(x+5)<6(7-2x)+4.$

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66. Find the solution sets of the following inequations.

$x^2-7x+12>0.$

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67. Find the solution sets of the following inequations.

$x^2 - x - 20 < 0.$

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68. Find the solution sets of the following inequations.

$x^2 + x - 20 > 0.$

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69. Find the solution sets of the following inequations.

$x^2 + 2x - 15 < 0.$

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70. Find the solution sets of the following inequations.

$x - x^2 + 6 \geq 0.$

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71. Find the solution sets of the following inequations.

$7x - x^2 - 6 \leq 0.$

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72. Find the solution sets of the following inequations.

$x^2 < 9.$

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73. Find the solution sets of the following inequations.

$x^2 > 16.$

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74. Find the solution sets of the following inequations.

$x^2 < 36.$

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75. Find the solution sets of the following inequations.

$9 < x^2 < 25.$

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76. Find the solution sets of the following inequations.

$1 < x^2 < 4.$

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77. Evaluate the following

$|-2|+|1|$.

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78. Evaluate the following

$|-4-3|$.

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79. Evaluate the following

$|2-7|$.

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80. Evaluate the following

$|(-2)(4)|$

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81. Evaluate the following

$\left|-\frac{6}{2}\right|$.

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82. State whether true or false.

$-|-x| = -(-x).$

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83. State whether true or false.

$(-x)|x| = x^2.$

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84. State whether true or false.

$|(-5) + (-7)| = |-5||-7|.$

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85. State whether true or false.

$|2x| = |-x||x|.$

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86. State whether true or false.

$\left|5\left(-x^2\right)\right| = 5|x|^2.$

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87. State whether true or false.

$|5x| - |3x| = |2x|.$

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88. Solve the equation

$|x - 4| = 2.$

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89. Solve the equation

$|2x - 1| = 3.$

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90. Solve the equation

$|x + 4| = 3.$

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91. Solve the equation

$|4 - 3x| = 2.$

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92. Solve the equation

$|6 - x| = -2.$

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93. Solve the equation

$|x - 1| + |x - 3| = 6.$

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94. Solve the equation

$|x + 2| + |x + 5| = 7$

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95. Solve the equation

$$|x-3| = |x+1|$$

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96. Solve the equation

$$|x-3| = |x-1|$$

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97. Solve the equation

$|x+2| = |x-2|$

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98. Express the following by removing the absolute value sign.

$|x| < \sqrt{5}.$

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99. Express the following by removing the absolute value sign.

$|x - 2| < 6.$

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100. Express the following by removing the absolute value sign.

$|x + 3| \leq 5.$

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101. Express the following by removing the absolute value sign.

$|2x + 1| > 3.$

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102. Express the following by removing the absolute value sign.

$|5 - x| \geq 0.$

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103. Express the following by removing the absolute value sign.

$|3x - 2| > 2.$

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104. Express the following by removing the absolute value sign.

$|3x - 1|.$

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105. Express the following by removing the absolute value sign.

$|4x - 3| > x.$

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106. Express the following by removing the absolute value sign.

$|x - 1||x + 1| < \frac{1}{4}.$

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107. Express the following in terms of absolute value.

$-\sqrt{6} < x < \sqrt{6}.$

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108. Express the following in terms of absolute value.

$1 < x < 7.$

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109. Express the following in terms of absolute value.

$-5 < x < 1.$

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110. Express the following in terms of absolute value.

$x > 8 \text{ or } x < 2.$

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111. Express the following in terms of absolute value.

$-7 \leq x \leq -3.$

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112. Express the following in terms of absolute value.

$x \geq -2$ or $x \leq -6.$

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113. Express the following in terms of absolute value.

$-8 < (x + 2) < 12.$

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114. Express the following in terms of absolute value.

$(a - b) < x < a.$

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115. Express the following in terms of absolute value.

$x > a - b$ or $x < a$.

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116. Express the following in terms of absolute value.

$-8 < (x + 2) < 10$.

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117. Find the solution sets of the following inequations.

$2x + 1 > 3x.$

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118. Find the solution sets of the following inequations.

$5x \geq 3x + 1.$

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119. Find the solution sets of the following inequations.

$x + 2x < 2.$

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120. Find the solution sets of the following inequations.

$2x < 8.$

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121. Find the solution sets of the following inequations.

$x - 1 > 5.$

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122. Find the simultaneous solution sets of the following sets of inequations.

$3y > (2x - 6).$

$x > 0.$

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123. Find the simultaneous solution sets of the following sets of inequations.

$y < 2x + 5$

$y \geq 0.$

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124. Find the simultaneous solution sets of the following sets of inequations.

$(2x + y + 4) \geq 0$

$x \leq 0, y \leq 0.$

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125. Find the simultaneous solution sets of the following sets of inequations.

$y > x - 12$

$x \geq 0, y \leq 4.$

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126. Find the simultaneous solution sets of the following sets of inequations.

$y \leq 4x, x < 6, y > 0.$

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