



# MATHS

# **BOOKS - NAGEEN PRAKASHAN ENGLISH**

# QUADRILATERALS

**Solved Examples** 

**1.** The angles of a quadrilateral are in the ratio 3:4:5:6. Find all

its angles.



2. Three angles of a quadrilateral are in the ratio 4:6:3. If the

fourth angle is  $100^{\,\circ}$  find the three angles of the quadrilateral.



5. In a rectangle ABCD, diagonals AC and BD intersect at O. If

 $\angle OAB = 35^{\circ}, \; \mathsf{find}:$ 

 $(a) \angle ABC(b) \angle ABO(c) \angle COD(d) \angle BOC)$ 

6. In the given figure, ABCD is a square. Find x.



7. In quadrilateral ABCD, AO and BO are the bisectors of  $\angle A$  and  $\angle B$  respectively. Prove that  $\angle AOB = \frac{1}{2}(\angle C + \angle D)$ .

**8.** In a parallelogram ABCD, the bisectors of  $\angle A$  and  $\angle B$  intersect

each other at point P. Prove that  $\angle APB = 90^{\circ}$ .

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**9.** Find the remaining angle of a parallelogram if one of its angles is

 $110^{\circ}$  .

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**10.** PQRS is a parallclogram such that PQ is parallel to SR and SP is parallel to RQ. The length of side PQ is 20 cm. M is point between P and Q such that the length of PM is 3 cm. N is a points between points S and R. Find the length of SN such that segmen MN divides

the paralelogram in two regions with equal areas.



**11.** In the given figure, ABCD is a parallelogram in which AN and CP are perpendiculars on diagonal BD. Prove that :

(i)  $\Delta ADN = \Delta CBP$ 

(ii)AN = CP



12. ABCD is a quadrilateral in which AB||DC and AD = BC. Prove that

 $\angle A = \angle B$  and  $\angle C = \angle D$ .



**13.** In the adjoining figure,  $\Box ABCD$  and  $\Box APQR$  are two parallelograms. Prove that :



14. In the given figure,  $\Box$  ABCD is a parallelogram. If DM  $\perp$  AC and

BN  $\perp$  AC, then show that  $\Box BNDM$  is a parallelogram.





15. In a ABC median AD is produced to X such that AD = DX .

Prove that ABXC is a parallelogram.

**16.** ABCD is a parallelogram. Two points P and Q are taken on sides AD and BC respectively such that AP  $\frac{1}{3}AD$  and  $CQ = \frac{1}{3}BC$ . Prove that  $\Box AQCP$  is a parallelogram. 17. A cyclic polygon has n sides such that each of its interior angle measures  $114^{\circ}$ . What is the measure of he angle subtended by each of its sides at the geometrical centre of the polygon?



**18.** The number of diagonals of a regular polygon is 27. Then, find the measure of each of the interior angles of the polygon.

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**19.** P is the mid-point of side AB of a parallelogram ABCD. A Line through B parallel to PD meets DC at Q and AD produced at R. Prove that (i) AR = 2BC (ii) BR = 2BQ. **20.** L and M are the mid-points of sides AB and DC respectively of parallelogram ABCD. Prove that segments DL and BM trisect diagonal AC.

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**21.** In the adjoining figure D, E and F are the mid-points of the sides BC, CA and AB of the equilateral  $\Delta ABC$ . Prove that  $\Delta DEF$  is also

an equilateral triangle.



the mid-point of AD, if EF||DC, then show that



**23.** If the mid -point of the consecvitve sides of any quadrilateral are connected by staight line segments, prove that the resulting quadrilteral is a parallelogram .



**24.** The diagonals of a quadrilateral ABCD are mutually perpendicular . Prove that the quadrilateral formed by joining the mid-points of its consecutive sides is a reactangle.





**25.** The figure formed by joining the mid-points of the adjacent sides of a rectangle is square (b) rhombus (c) trapezium (d) none of these



26. Show that the line segments joining the mid-points of opposite

sides of a quadrilateral bisects each other.



**27.** Prove by vector method that the line segment joining the midpoints of the diagonals of a trapezium is parallel to the parallel sides and equal to half of their difference.

**28.** In the adjoining figure. AD and BE are two medians of  $\Delta ABC$ .

IfDF||BE, then prove that  $CF=rac{1}{4}AC.$ 



**29.** In the adjoining figure, PQRS is a parallelogram. A and B are the mid-points of PQ of SR respectively. If PS= BR, then prove that

quadrilateral ADBC is a reactangle.





## Problems From Ncert Exemplar

1. Show that if the diagonals of a quadrilateral bisect each other at

right angles, then it is a rhombus.

**2.** Diagonal AC of a paraleligram ABCD bisects  $\angle A$  (sec figure). Show

that:

(i) it bisects  $\angle C$  also (ii) ABCD is a rhombus.



3. ABCD is a rectangle in which diagonal AC bisects  $\angle A$  as well as

 $\angle C$ . Show that

(i) ABCD is a square

(ii) diagonal BD bisects  $\angle B$ 



4. In parallelogram ABCD two points P and Q are taken on diagonal

BD such that DP = BQ (set figure). Show that:



**5.** P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that AC  $\perp$  BD. Prove that PQRS is a rectangle.



**6.** Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.



7. Through A, B and C lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a  $\Delta$ ABC as shown in

figure. Show that  $BC = rac{1}{2}QR$ .



8. In figure, P is the mid-point of side BC of a parallelogram ABCD such that  $\angle BAP = \angle DAP$ . Prove that AD = 2CD.



**9.** P and Q are the mid-point of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at

R. Show that PQRS is a parallelogram.



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**10.** In figure, AB||DE, AB=DE, AC||DF and AC=DF. Prove that BC||EF and

BC=EF.



**11.** Prove that the quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.



**1.** The angles of a quadrilateral are  $89^{\circ}$  and  $113^{\circ}$  If the other two angles are equal, find the equal angles.



**3.** In quadrilateral ABCD, side AB is parallel to side DC. If  $\angle A : \angle D = 1 : 2$  and  $\angle C : \angle B = 4 : 5$ .

(i) Calculate each angle of the quadrilateral.

(ii) Assign a special name to quadrialateral ABCD.



**4.** Find the values of x and y from adjoining parallelogram.



5. Find x,y and z in each of the following figure:



**6.** In the given figure, find  $(i) \angle XOD(ii) \angle XOC$ 



- 7. In the given figure, ABCD is a square and  $\angle PQR = 90^{\circ}$ . IfPB = QC = DR, prove that.
- (i) QB = RC
- (ii) PQ = QR
- (iii)  $\angle QPR = 45^{\circ}$



8. In a square ABCD, diagonals meet at O. P is point on BC such that

OB = BP.Show that (i) 
$$\angle POC = \left(22rac{1}{2}
ight)^\circ$$

(ii)  $\angle BDC = 2 \angle POC$ 

(iii)  $\angle BOP = 3 \angle COP$ 

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**9.** The give figure shows a square ABCD and an equilayeral teiangle APB. Calculate :



10. In the given figure, ABCD is a rohombus with  $A=67^{\circ}$  . If DEC is an equilateral triangle, calculate

(i)  $\angle CBE$ 



Find all the angles of parallelogram.

**12.** Prove that the sum of two consecutive angles of a parallelogram

is  $180^{\circ}$  .

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<b>13.</b> One angle of a parallelogram is $60^\circ$ . Find its remaining angles.
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14. One diagonal of a parallelogram biscets its one of the angles.
Show tht it will also bisec the opposite angle.
S Watch Video Solution

**15.** The opposite angles of a parallelogram are  $(3x-2)^{\circ}$  and  $(150-x)^{\circ}$ . Find each angle of the parallelogram.

- 16. In the adjoining figure, ABCD is a parallelogram. If  $\angle ABC = 125^{\circ}$  ,
- $\angle ACD = 28^{\circ}$ , then fine $\angle DAC$ .



**17.** In a parallelogram, one angleis twice of its consecutiv angle.

Find all the angles of the parallelogram.

Watch Video Solution 18. In a parallelogram ABCD, AX and CY are the bisectors of  $\angle A$  and  $\angle C$  respectively. Prove that AX || CY. Watch Video Solution

**19.** In a parallelogram PQRS, PX and QY are the perpendiculars drawn from P and Q respectively so SR and SR produced. Prove that PX= QY.

**20.** In a parallelogram ABCD, the bisector of  $\angle A$  bisects the line B at

point X. Prove that AD = 2AB.

**21.** In a parallelogram ABCD,  $\angle BCD = 60^{\circ}$  If the bisectors AP and BP of  $\angle A$  and  $\angle B$  respectively, meet the side CD at point P, then prove that CP = PD.



**22.** In the adjoining figure,  $\Delta PQR$  is formed by the sides PQ, QR and RP which are drawn parallel to sides AB,BC and CA respectively of  $\Delta ABC$ . Prove that

PQ + QR + RP = 2(AB + BC + CA).



23. X and Y are the mid-points of the opposite sides AB and DC of a

parallelogram ABCD. Prove that  $\Box AXCY$  is a parallelogram.

24. Two points X and Y lie on the diagonal BD of a parallelogram

ABCD such that DX = BY. Prove that  $\Box AXCY$  is a parallelogram.



**25.** In the adjoining figure,  $\Delta ABC$  is an isosceles triangle in which AB = AC. Side CP is parallel to AB and AP is the bisector of exterior angle CAD of  $\Delta$  ABC. Prove that  $\angle PAC = \angle BCA$  and  $\Box ABCP$ 

### is a parallelogram.



**26.** AB and CD are two parallel lines and a transversal 'l' intersects these lines at X and Y respectively Prove that the bisectors of interior angles from a parallelogram whose each angle is  $90^{\circ}$ .

**27.** In the adjoining figure  $\Box ABCD$  is a parallelogeam. Points X and Y lie on the sides AD and BC respectively and  $AX = \frac{1}{4}AD$  and  $CY = \frac{1}{4}BC$ . Show that  $\Box XBYD$  is a parallelogram.



28. In the adjoining figure, ABCDEF is a regular hexagon. Prove that

 $\Box ABDE$ ,  $\Box ACDF$  and  $\Box AGDH$  are parallelograms.



**29.** Two triangles  $\Delta ABC$  and  $\Delta DEF$  are given such that AB||DE, BC||EF and AB = DE, BC = EF. "Show

that"AC"||"DFandAC=DE`





(i) Line segment AD is perpedicular to line segment EF.

(ii) Line segment AD bisects the line segment EF.

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2. Show that the quadrialteral formed by joining the mid-points of

the consecutive sides of a rhombus, is a rectangle.



**3.** E is the mid-point of the median AD of  $\Delta ABC$ . Line segment BE meets AC at point F when produce, prove that  $AF = \frac{1}{3}AC$ .



4. Show that the quadrilateral, formed by joining the mid-points of

the sides of a square is also a square.

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5. Show that, in a parallelogram ABCD, the internal and external

bisectors of  $\angle A$  and  $\angle B$  from a rectangle.

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**6.** If the mid -point of the consecvitve sides of any quadrilateral are connected by staight line segments, prove that the resulting quadrilteral is a parallelogram .

7. In a right-angled triangle  $ABC, \angle ABC = 90^\circ$  and D is midpoint of AC. Prove that  $BD = \frac{1}{2}AC$ .

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8.  $\Box PQRS$  is a reactangle. If A, B and C are the mid-points of PQ,

PS and QR respectively, then prove that

$$AB+AC=rac{1}{2}(PR+SQ).$$

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9. P, Q and R are, respectively, the mid-points of sides BC, CAand AB of a triangle ABC, PR and BQ meet at XCR and PQmeet at Y. Prove that  $XY = \frac{1}{4}BC$ . 10. In  $\Delta PQR, Pq = PR$  and S is the mid-point of PQ. A line drawn

from S parallel to QR, intersects the line PR at T. Prove that PS = PT.



**11.** M and N divide the side AB of a triangle ABC into three equal parts . Through M and N, lines are drawn parallel to BC and intersecting AC at points P and Q respectively. Prove that P and Q divide AC into three equal parts.

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12. In the adjoining figure, two points A and B lie on the same side of a line 'l'. C is the mid-point of AB. If AD  $\perp l$  and  $BE \perp l$ , then prove that CD = CE.





**13.** AB and CD are the parallel sides of a trapeziuml. E is the midpoint of AD. A line through E and parallel to side AB meets the line BC at point F. Prove that F is the mid-point of BC.



**14.** Prove that a line drawn from the vertex of a triangle to its base is bisected by the line joining the mid points of the remaining two



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**15.** In a parallelogram ABCD, E and F are the mid-points of sides BC and AD respectively. Show that the line segment BF and ED trisect the diagonal AC.

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**Revision Exercise Very Short Answer Questions** 

1. Three angles of aquadrilateral are respectively  $90^\circ, 91^\circ, 69^\circ.$ 

Find the fourth angle.



**2.** Find the other angles of a parallelogram if its one angle is  $60^\circ$ 



3. Find the angles of the parallelogram ABCD, if

$$\angle C = rac{2}{3} \angle D$$

**4.** Find  $\angle A$  of the given figure.



5. In the given figure, if ABCD is a rectangle and x : y = 2:7 find x and y.



**6.** In a  $\Delta ABC$ , D, E and F are respectively the mid-points of BC, CA and AB. If the lengths of side AB, BC and CA are 7 cm, 8 cm and 9 cm respectively, find the permeter of  $\Delta DEF$ .



7. If the bisectors of two adjacent angles A and B of a quadrilateral ABCD intersect at a point O such that

 $\angle C + \angle D = k \angle AOB$ , then find the value of k.



**9.** In a rhombus ABCD if  $\angle ACB = 40^{\,\circ}\,,\,\,$  then fine  $\angle ADC.$ 

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10. The diagonals of a parallelogram ABCD intersect at O . If  $\angle BOC = 90^0 \ and \ \angle BDC = 50^0$ , then  $\angle OAB =$  (a)  $40^0$  (b)  $50^0$  (c)  $10^0$  (d)  $90^0$ 

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**Revision Exercise Short Answer Questions** 

**1.** In an isosceles-trapezium, show that the opposite angles are supplementary.



2. In the given figure ABCD is a parallelogram. AB = (2x+25) cm, CD =

(3x+14)

 $egin{array}{lll} egin{array}{lll} egin{array}{lll} egin{array}{lll} egin{array}{lll} B = z^o, egin{array}{lll} BAC = 24^\circ, egin{array}{lll} DAC = 3y + 5^\circ \end{array} & ext{and} \ egin{array}{lll} egin{array}{lll} DCA = y + 9^\circ, \end{array} \end{array}$ 

find the values of x, y and z.



**3.** ABCD is a parallelogram and AE and CF bisect  $\angle A$  and  $\angle C$  respectively. Prove that AE||FC.



4. In the given figure, AM bisects angle A and DM bisects angle D of

parallelogram ABCD. Prove that  $\angle AMD = 90^{\circ}$  .





5. In the given figure ABCD is a parallelogram. Prove that AB = 2BC.





6. E and F are points on diagonal AC of a parallelogram ABCD such

that AE=CF. Show that BFDE is a parallelogram.



7. In a quadrilateral ABCD, AB= AD and CB= CD. Prove that:

AC is perpendicular bisector of BD



**8.** In the adjoining figure, ABCD is a rhombus and ABE is an equilateral triangle. If  $\angle BCD = 70^{\circ}$ , find

 $(a) \angle ADE \quad (b) \angle BDE \quad (c) \angle BED$ 



9. In a trapezium ABCD, if E and F be the mid-points of diagonal AC

and BD respectively. Prove that  $EF = rac{1}{2}(AB - CD).$ 



**10.** In a quadrilateral ABCD the linesegment bisecting  $\angle C$  and  $\angle D$ 

meet at E. Prove that  $\angle A + \angle B = 2 \angle CED$ .

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**Revision Exercise Long Answer Questions** 

1. In the adjoining figure, ABCD and PBCQ are paralelograms. Prove

that

 $\Delta ABP \cong \Delta DCQ$ 



**2.** A transverals cuts two parallel lines at A and B. The two interior angles at A are bisected and so are the two interior angles at B, the

four bisectors from a quadrilateral ACBD, prove that ABCD is parallelogram.



**3.** Prove that the quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.





**4.** In a square ABCD, A is joined to a point X on BC and D is joined to

a point Y on AB. If AX = DY, prove that AX is perpendicular to DY.



5. ABCD is a rhombus. RABS is a straight line such that RA = AB = BS.

Prove that RD and SC when produced meet at right angles.

