MATHS

BOOKS - CENGAGE

REMAINDER THEOREM

Worked Examples

1. Determine the quotient and the remainder if $f(x) = x^4 + 3x^2 - 2x^2 - 4x - 3$ is divided by $x - 3$.

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2. Determine the quotient and the remainder if $f(x) = 3x^2 + 6x - 12 + x^3$ is divided by $x + 3$.

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3. Determine the quotient and the remainder if $x^5 + 7$ is divided by $x + 1$.

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Test Yourself Level 1

1. Use synthetic division to find the quotient and remainder and verify by actual division.

$x^3 + 6x^2 + 3x + 1 + x - 2$

2. Use synthetic division to find the quotient and remainder and verify by actual division.

$2x^3 - 5x + 1 + x + 1$

3. Use synthetic division to find the quotient and remainder and verify by actual division.

$$x^3 - 11x + 10 \div x + 3$$

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4. Use synthetic division to find the quotient and remainder and verify by actual division.

$$x^3 - 4x^2 + 3x - 11 \div x - 4$$

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5. Use synthetic division to find the quotient and remainder and verify by actual division.

$$x^6 - y^6 \div x - y$$

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6. Use synthetic division to find the quotient and remainder and verify by actual division.

$$x^5 + y^5 \div x + y$$

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7. Use synthetic division to find the quotient and remainder and verify by actual division.

$$x^5 - 32 \div x - 2$$

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8. Use synthetic division to find the quotient and remainder and verify by actual division.

$2x^3 - 13x + 11 + x - 1$

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9. Use synthetic division to find the quotient and remainder and verify by actual division.

$2x^3 - 4x^2 + 3x - 6 \div x - 2$

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10. Use synthetic division to find the quotient and remainder and verify by actual division.

$$1 + x^2 + x^3 + x^4 + x^5 \div x + 1$$

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Test Yourself Level 2

1. Factorise $2x^2 + 11x + 14$ and deduce the prime factors of 21, 114.

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2. Determine the value of k for which $x^3 - 6x + k$ may be divisible by $x - 2$, by the synthetic division method.

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3. $f(x) \div x^2 - 3x + 2$ leaves the remainder $ax + b$. If $f(1) = 4$ and $f(2) = 2$, determine a and b.

4. If $x - 2$ is a factor of $x^3 - 6x^2 + 11x + k$, determine k.

5. If $x + 1$ divides $5x^3 + 11x^2 + 4x - p$, determine p.

6. If $4x - 1$ divides $4x^3 - 5x^2 - kx + 3$, determine k.

7. Find the remainder when $f(x) = 3x^3 + 6x^2 - 4x - 5$

is divided by $x + 3$.

8. Find the value of l and m if $8x^2 + lx^2 - 27x + m$ is

an integral multiple of $2x^2 - x - 6$.

9. Find the remainder and the quotient in the following:

$3x^2 - 7x^2 + 6x - 2$ is divided by $x - 3$.

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10. Find the remainder and the quotient in the following:

$3x^4 - 20x^2 - 5x - 18$ is divided by $x + 3$.

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Test Yourself Level 3

1. Find p and q if $2x^3 + 6px^2 - 2qx - q - 2p + 1$ may contain the factors $2x - 1$ and $x + 3$.

2. Find l and m if $2x^3 - (2l + 1)x^2 + (l + m)x + m$

may be exactly divisible by $2x^2 - x + 3$.

3. Without substituting, find the remainder if

$2x^9 - 6x^8 + 5x^7 - 15x^6 + 26x^5 - 78x^4 + 2lx^3 - 63x^2 + 7$

is divided by $x - 3$.

4. Do the following for the given expressions:

(i) Find the remainder when $f(x)$ is divided by the respective factors of the divisor using the formula.

(ii) Divide $f(x)$ by the synthetic division method in succession by the factors of the divisor and find the complete remainder.

(iii) Compare the results in (a) and (b) and verify the answer using division algorithm.

$x^3 - 8x^2 + 7x + 5 \div (x - 2)(x - 1)$

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5. Do the following for the given expressions:

(i) Find the remainder when $f(x)$ is divided by the

respective factors of the divisor using the formula.

(ii) Divide $f(x)$ by the synthetic division method in succession by the factors of the divisor and find the complete remainder.

(iii) Compare the results in (a) and (b) and verify the answer using division algorithm.

$$x^3 - 8x^2 + 8x + 5 \div (x + 2)(x + 1)$$

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6. Do the following for the given expressions:

(i) Find the remainder when $f(x)$ is divided by the respective factors of the divisor using the formula.

(ii) Divide $f(x)$ by the synthetic division method in succession by the factors of the divisor and find the

complete remainder.

(iii) Compare the results in (a) and (b) and verify the answer using division algorithm.

$$x^3 + 7x^2 - 8x - 21 \div (x - 2)(x - 3)$$

7. Do the following for the given expressions:

(i) Find the remainder when $f(x)$ is divided by the respective factors of the divisor using the formula.

(ii) Divide $f(x)$ by the synthetic division method in succession by the factors of the divisor and find the complete remainder.

(iii) Compare the results in (a) and (b) and verify the

answer using division algorithm.

$$2x^3 - 8x^2 + 7x + 20 \div (x + 2)(x + 3)$$

8. Do the following for the given expressions:

(i) Find the remainder when $f(x)$ is divided by the respective factors of the divisor using the formula.

(ii) Divide $f(x)$ by the synthetic division method in succession by the factors of the divisor and find the complete remainder.

(iii) Compare the results in (a) and (b) and verify the answer using division algorithm.

$$3x^3 - 4x^2 - 35 \div (x - 2)(x + 3)$$

9. If $(x + 1)$ and $(x - 1)$ are factors of $x^4 + px^4 + 2x^2 - 3x - q$, then

    A. $p = -1, q = l$

    B. $p = l, q = -2$

    C. $p = 3, q = -3$

    D. $p = -3, q = 3$

Answer: C

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10. When $x^{100} + 1$ is divisible by $x^2 - 1$, the remainder is

A. $x + 2$

B. $2x + 1$

C. 2

D. 1

Answer: C

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Test Yourself Multiple Choice Questions

1. The remainder when $x^3 - 4x^2 + 3x - 11$ is divided by $x - 4$ is

A. 0

B. 1

C. 3

D. 3

Answer: B

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2. Which of the following has $(x - 1)$ as a factor?

A. $2x^3 - 13x + 11$

B. $2x^3 - 5x + 1$

C. $x^2 + x + 1$

D. $x^5 + x^4 + x^3 + x^2 + x + 1$

Answer: A

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3. What is the value of k for which the given polynomial $x^3 - 6x + k$ is divisible by $x - 2$?

A. 1

B. 2

C. 3

D. 4

Answer: D

4. If the polynomial

$ax^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + a$ have the same

remainder when divided by $x - 3$, then the value of a is

A. 0

B. 1

C. -1

D. 2

Answer: C

5. If the polynomial $P(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $(x + 1)$ leaves the remainder 19, then what is the remainder when $P(x)$ is divided by $x + 2$?

A. 60

B. 61

C. 62

D. 63

Answer: C

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6. If both $(x - 2)$ and $\left(x - \dfrac{1}{2}\right)$ are the factors of $px^2 + 5x + r$, then which of the following is correct?

   A. $p = r$

   B. $p = 2r$

   C. $r = 2p$

   D. None of these

Answer: A

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7. When $x^{50} + 1$ is divisible by $x^2 - 4$ then remainder will be '

A. $2x + 2^{50} + 1$

B. $2^{50} + 1$

C. $\left(2^{50} + 1\right)x + 5$

D. None of these

Answer: B

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8. If $x^{100} - 1$ is divided by $x^2 - 1$, then remainder will be

A. 0

B. -1

C. 1

D. 2

Answer: A

9.   If $(x^2 - 3x + 2)$ is   factor   of $(x^4 + ax^3 + 2x^2 + 5x + b)$, then the values of a and b are

A. $a = -\dfrac{26}{7}, b = -\dfrac{30}{7}$

B. $a = \dfrac{26}{7}, b = \dfrac{30}{7}$

C. $a = -\dfrac{26}{7}, b = \dfrac{30}{7}$

D. None of these

Answer: A

10. Polynomial $f(x)$ when divided by $x^2 - 3x + 2$ leaves the remainder $3ax + 2b$. If $f(2) = 8$ and $f(1) = 9$, then what are the values of a and b, respectively?

A. $\dfrac{1}{3}, 5$

B. $-\dfrac{1}{3}, 5$

C. $5, \dfrac{1}{3}$

D. $-5, \dfrac{1}{3}$

Answer: B

11. If $(x - 2)$ is a factor of $x^4 + ax^3 - bx^2 + 18$, then which of the following is correct?

    A. $2a + b + 17 = 0$

    B. $4a - 2b + 17 = 0.$

    C. $2a + b + \dfrac{17}{2} = 0$

    D. $2a - b - \dfrac{17}{2} = 0$

Answer: B

12. $(x + 1)$ is a factor of the polynomial

A. $x^3 + x^2 - x + 1$

B. $x^3 + x^2 + x + 1$

C. $x^4 + x^3 + x^2 + 1$

D. $4x^4 + 3x^3 + 3x^2 + x + 1$

Answer: B

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13. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is

A. $5 + x$

B. $5 - x$

C. $5x - 1$

D. $10x$

Answer: D

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14. Find p and q if $2x^3 + 6px^2 - 2qx - q - 2p + 1$ has $(x - 2)$ and $(x - 1)$ as its factors.

A. $p = -\dfrac{18}{23}, q = -\dfrac{1}{23}$

B. $q = -(2)(23), p = -\dfrac{75}{92}$

C. $p = -\dfrac{1}{23}, q = -\dfrac{18}{23}$

D. $p = -\dfrac{75}{32}, q = -\dfrac{1}{23}$

Answer: A

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15. When $(x + 1)$ divides $5x^4 - 4x^3 + ax^2 + b$, it leaves

remainder 10. Which of the following may be correct ?

A. $a = 10, b = -9$

B. $a = -10, b = -9$

C. $a = 10, b = 9$

D. None of these

Answer: A

16. What is k, if $(4x - 1)$ is an integral multiple of $4x^3 - 5x^2 - kx + 32$?

A. 27

B. 127

C. -127

D. -27

Answer: B

17. When

$P(x) = ax^3 + 8x^2 + 7$ and $Q(x) = 9x^2 + 7x + b$ are

divided by $(x - 1)$, both leave the same remainder.

which of the following may be correct?

    A. $a = 4, b = 3$

    B. $a = 3, b = 3$

    C. $a = 3, b = 4$

    D. $a = 4, b = 4$

Answer: A

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18. If $ax^3 + bx^2 + x - 6$ has $(x + 2)$ as a factor and leaves a remainder 4 when divided by $(x - 2)$, then what are the values of a and b?

A. $a = 0, b = 2$

B. $a = 2, b = 0$

C. $a = 1, b = 1$

D. $a = 2, b = 2$

Answer: A

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19. R_1 and R_2 are the remainders when the polynomials $x^3 - 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $(x + 1)$ and $(x - 2)$, respectively. If $2R_1 + R_2 = 6$, then what is the value of a?

A. 1

B. 0

C. 2

D. 4

Answer: C

20. When

$P(x) = ax^2 - 3x + 5$ and $Q(x) = 3x^2 + 2a + 5$ are

divided by $(x - 1)$ and $(x - 3)$, respectively, both leave

the same remainder. What is the value of a?

A. 30

B. -28

C. 28

D. -30

Answer: D

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1. What is the remainder when x^{37} is divided by $x^2 - 4x + 3$?

A. $\left(\dfrac{3^{37} - 1}{3}\right) x + \left(\dfrac{3 - 3^{37}}{2}\right)$

B. $\left(\dfrac{3^{37} - 1}{2}\right) x + \left(\dfrac{3 - 3^{37}}{2}\right)$

C. $\left(\dfrac{3^{37} - 1}{2}\right) x + \left(\dfrac{3^{37} - 3}{2}\right)$

D. $\left(\dfrac{3^{37} - 1}{2}\right) x - \left(\dfrac{3 - 3^{37}}{2}\right)$

Answer: B

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2. $5x\&(2) - 2x + 1$ is a factor of the expression $ax^3 - 19x^2 + bx - 3$. If a and b are integers, then the value of b is

A. 10

B. -8

C. 8

D. 0

Answer: C

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3. $P(x)$ is a polynomial of degree 2. When $P(x)$ is divided by $(x - 1)(x + 2)$ and $(x - 3)$, it leaves $2, 5$ and 1 as remainders, respectively. $P(x)$ is

A. $\dfrac{1}{10} \left(x^2 - 9x - 28 \right)$

B. $\dfrac{1}{5} \left(x^2 - 9x + 14 \right)$

C. $\dfrac{1}{20} \left(x^2 - 9x + 28 \right)$

D. $\dfrac{1}{10} \left(x^2 - 9x + 28 \right)$

Answer: D

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4. The polynomials $x^2 - x + a$ and $2x^2 \div 5x + b$ have a common factor. Which of the following is correct?

A. $4a^2 + b^2 - 4ab + 35a + 7b = 0$

B. $4a^2 + b^2 \div 4ab + 35a + 7b = 0$

C. $4a^2 + b^2 - 4ab - 35a + 7b = 0$

D. $4a^2 + b^2 - 2ab + 35a + 7b = 0$

Answer: A

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5. If a third degree polynomial $f(y)$ is divided by $(y + 2)$, the quotient and the remainder obtained are

$P(y)$ and 0. Also, if $f(y)$ is divided by $[P(y) + y - 1]$, the quotient and the remainder obtained are $(y - 3)$ and $Q(y)$. Then $Q(y)$ is

A. $5P(y) - y^2 + 4y$

B. $5P(y) - y^2 - 4y - 3$

C. $5P(y) - y^2 + 4y - 3$

D. $P(y) - y^2 \div 4y - 3$.

Answer: C

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6. When the expression $x^2 - 3x - 3$ is divided by $(x - a)$, the remainder is 1. The value of a is

A. 1

B. -1

C. -2

D. -3

Answer: B

7. The expression $ax^3 + 3x^2 - 5x + b$ is divisible by the expression $x^2 + 3x + 2$. The values of a and b are

A. $a = 2, b = -6$

B. $a = -2, b = -6$

C. $a = 2/3, b = -22/3$

D. $a = -6, b = 2$

Answer: A

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8. What should be added to the expression $2x^3 - 5x^2 - x + 5$ so that the sum is completely divisible by $2x - 3$?

A. -1

B. 1

C. -2

D. 2

Answer: B

9. If the expression $12x^2 - 25x + 12$ is divisible by $4x - 3$ then the other factor of the expression is

A. $3x + 4$

B. $4x + 3$

C. $x - 2$

D. $3x - 4$

Answer: D

10. The expression $px^2 + qx + r$ leaves the remainders $1, 0$ and -1 when divided by $(x + 1), (x - 2)$ and $(x - 3)$, respectively. The value of $p + q + r$ is

A. $\dfrac{2}{3}$

B. $\dfrac{1}{2}$

C. $\dfrac{5}{6}$

D. 0

Answer: A