



MATHS

BOOKS - PSEB

PROOFS IN MATHEMATICS

Exercise

1. State whether the following statements are always true, always false or ambiguous. Justify

your answer : All mathematics textbooks are interesting.



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2. State whether the following statements are always true, always false or ambiguous. Justify your answer : The distance from the Earth to the Sun is approximately 1.5×10^8 km.



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3. State whether the following statements are always true, always false or ambiguous. Justify your answer : All human beings grow old.



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4. State whether the following statements are always true, always false or ambiguous. Justify your answer : The journey from Uttarkashi to Harsil is tiring.



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5. State whether the following statements are always true, always false or ambiguous. Justify your answer : The woman saw an elephant through a pair of binoculars.



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6. State whether the following statements are true or false. Justify your answer. : All hexagons are polygons.



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7. State whether the following statements are true or false. Justify your answer. : Some polygons are pentagons.



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8. State whether the following statements are true or false. Justify your answer. : Not all even numbers are divisible by 2.



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9. State whether the following statements are true or false. Justify your answer. : Some real numbers are irrational.



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10. State whether the following statements are true or false. Justify your answer. : Not all real numbers are rational.



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11. Let a and b be real numbers such that $ab \neq 0$. Then which of the following statements are true ? Justify your answer :

Both a and b must be non-zero.



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12. Let a and b be real numbers such that $ab \neq 0$. Then which of the following statements are true ? Justify your answer :

Both a and b must be non-zero.





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13. Let a and b be real numbers such that $ab \neq 0$. Then which of the following statements are true ? Justify your answer :
Either a or b must be non-zero.



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14. Restate the following statements with appropriate conditions, so that they become true. : If $a^2 > b^2$, then $a > b$.



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15. Restate the following statements with appropriate conditions, so that they become true.: If $x^2 = y^2$, then $x = y$.



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16. Restate the following statements with appropriate conditions, so that they become true.: If $(x + y)^2 = x^2 + y^2$, then $x = 0$.



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17. Restate the following statements with appropriate conditions, so that they become true. : The diagonals of a quadrilateral bisect each other.



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18. Given that all women are mortal, and suppose that A is a woman, what can we conclude about A ?



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19. Given that the product of two rational numbers is rational, and suppose a and b are rationals, what can you conclude about ab ?



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20. Given that the decimal expansion of irrational numbers is non-terminating, non-recurring, and $\sqrt{17}$ is irrational, what can we

conclude about the decimal expansion of $\sqrt{17}$

?



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21. Given that $y = x^2 + 6$ and $x = -1$, what can we conclude about the value of y ?



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22. Given that ABCD is a parallelogram and $\angle B = 80^\circ$. What can you conclude about the

other angles of the parallelogram ?



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23. Given that PQRS is a cyclic quadrilateral and also its diagonals bisect each other. What can you conclude about the quadrilateral ?



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24. Given that \sqrt{p} is irrational for all prime p and also suppose that 3721 is a prime. Can you

conclude that $\sqrt{3721}$ is an irrational number ?

Is your conclusion correct ? Why or why not ?



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25. Prove that the sum of the two consecutive odd numbers is divisible by 4.



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26. Take two consecutive odd numbers. Find the sum of their squares, and then add 6 to

the result. Prove that the new number is always divisible by 8.



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27. In the following question, we ask you to prove a statement. List the step in proof, and give the reason for step. If $p \geq 5$ is a prime number, show that $p^2 + 2$ is divisible by 3.



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28. Let x and y be rational numbers. Show that xy is a rational number.



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29. If a and b are positive integers, then you know that $a = bq + r$, such that r is less than or equal to b and r is less than b , where q is a whole number. Prove that $\text{HCF}(a, b) = \text{HCF}(b, r)$.



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30. A line parallel to BC of a triangle ABC, intersects AB and AC at D and E respectively.

Prove that $(AD / DB) = (AE / EC)$.



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31. State the negations for the following statements : Man is mortal.



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32. State the negations for the following statements : Line l is parallel of line m .



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33. State the negations for the following statements : This chapter has many exerises.



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34. State the negations for the following statements : All integers are rational numbers.



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35. State the negations for the following statements : Some prime numbers are odd.



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36. State the negations for the following statements : No student is lazy.



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37. State the negations for the following statements : Some cats are not black.



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38. State the negations for the following statement : There is no real number x , such that $\sqrt{x} = -1$



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39. State the negations for the following statements : 2 divides the positive integer a .



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40. State the negations for the following statements : Integers a and b are coprime.



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41. In each of the following questions, there are two statements. State if the second is the negation of the first or not. : Mumtaz is hungry. Mumtaz is not hungry.



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42. In each of the following questions, there are two statements. State if the second is the negation of the first or not. : Some cats are black. Some cats are brown.



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43. In each of the following questions, there are two statements. State if the second is the negation of the first or not. : All elephants are huge. One elephant is not huge.



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44. In each of the following questions, there are two statements. State if the second is the negation of the first or not. : All fire engine are red. All fire engines are not red.



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45. In each of the following questions, there are two statements. State if the second is the

negation of the first or not. : No man is a cow.

Some men are cows.



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46. Write the converses of the following statements. : If it is hot in Tokyo, then Sharan sweats a lot.



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47. Write the converses of the following statements. : If Shalini is hungry, then her stomach grumbles.



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48. Write the converses of the following statements. : If Jaswant has scholarship, then she can get a degree.



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49. Write the converses of the following statements. : If a plant has flowers, then it is alive.



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50. Write the converses of the following statements. : If an animal is a cat, then it has a tail.



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51. Write the converses of the following statements. Also, decide in each case whether the converse is true or false. : If triangle ABC is isosceles, then its base angles are equal.



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52. Write the converses of the following statements. Also, decide in each case whether the converse is true or false. : If an integer is odd, then its square is an odd integer.



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53. Write the converses of the following statements. Also, decide in each case whether the converse is true or false. : If $x^2 = 1$, then $x = 1$.



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54. Write the converses of the following statements. Also, decide in each case whether the converse is true or false. : If ABCD is a

parallelogram, then its diagonals bisect each other.



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55. Write the converses of the following statements. Also, decide in each case whether the converse is true or false. : If a and b are whole numbers, that

$$a + (b + c) = (a + b) + c.$$



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56. Write the converses of the following statements. Also, decide in each case whether the converse is true or false. : If x and y are two odd numbers, then their sum is an even number.



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57. Write the converses of the following statements. Also, decide in each case whether the converse is true or false. : If vertices of a

parallorgam PQRS lie on a circle, then it is rectangle.



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58. Suppose $a + b = c + d$, and $a < c$. Use proof by contradiction to show $b \geq d$.



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59. Let r be a rational number and x be an irrational number. Use proof by contradiction

to show that $r + x$ is an irrational number.



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60. Use proof by contradiction to prove that if for an integer a , a^2 is even, then so is a .



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61. Use proof by contradiction to prove that if for an integer a , a^2 is divisible by 3, then a is divisible by 3.



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62. Use proof by contradiction to show that there is no value of n for which 6^n ends with the digit zero.



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63. Prove by contradiction that two lines in a plane cannot intersect in more than one point.



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Example

1. State whether the following statement is always true, always false or ambiguous. Justify your answer. The Sun orbits the Earth.



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2. State whether the following statement is always true, always false or ambiguous. Justify your answer. Vehicles have four wheels.



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3. State whether the following statement is always true, always false or ambiguous. Justify your answer. The speed of light is approximately 3×10^5 km/s.



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4. State whether the following statement is always true, always false or ambiguous. Justify

your answer. The Sun orbits the Earth.



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5. State whether the following statement is always true, always false or ambiguous. Justify your answer. All humans are mortal.



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6. State whether the following statement is true or false, and justify your answers. All

equilateral triangles are isosceles.



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7. State whether the following statement is true or false, and justify your answers. Some isosceles triangles are equilateral.



[Watch Video Solution](#)

8. State whether the following statement is true or false, and justify your answers. All

isosceles triangles are equilateral.



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9. State whether the following statement is true or false, and justify your answers. Some rational numbers are integers.



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10. State whether the following statement is true or false, and justify your answers. Some

rational numbers are not integers.



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11. State whether the following statement is true or false, and justify your answers. Not all integers are rational.



[Watch Video Solution](#)

12. State whether the following statements are true or false. Justify your answer. : Not all real

numbers are rational.



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13. If $x < 4$, which of the following statements are true? Justify your answers.:- $2x > 8$



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14. If $x < 4$, which of the following statements are true? Justify your answers.:- $2x < 6$



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15. If $x < 4$, which of the following statements are true? Justify your answers.:- $2x < 8$



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16. Restate the following statement with appropriate conditions, so that they become true statement: If the diagonals of a quadrilateral are equal, then it is a rectangle.



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17. Restate the following statement with appropriate conditions, so that they become true statement: A line joining two mid-points on two sides of a triangle is parallel to the third side.



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18. Restate the following statement with appropriate conditions, so that they become

true statement: All quadratic equations have two real roots.



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19. Given that Bijapur is in the state of Karnataka, and suppose Shabana lives in Bijapur. In which state does Shabana live?



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20. Given that all mathematics textbooks are interesting, and suppose you are reading a mathematics textbook. What can we conclude about the textbook you are reading?



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21. Given that $y = -6x + 5$ and suppose $x = 3$. What is y ?



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22. Given that ABCD is a parallelogram, and suppose $AD = 5$ cm, $AB = 7$ cm (see Fig. A1.1).

What can you conclude about the lengths of DC and BC?



Fig. A1.1



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23. Given that \sqrt{p} is irrational for all primes p , and suppose that 19423 is a prime. What can you conclude about $\sqrt{19423}$?



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24. The sum of two rational numbers is a rational number.



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25. Every prime number greater than 3 is of the form $6k + 1$ or $6k + 5$, where k is some integer.



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26. State the negations for the following statement:- Mike's dog does not have a black tail.



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27. State the negations for the following statement:-All irrational numbers are real numbers.



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28. State the negations for the following statement:- $\sqrt{2}$ is irrational.



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29. State the negations for the following statement:- Some rational numbers are integers.



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30. State the negations for the following statement:- Not all teachers are males.



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31. State the negations for the following statement:- Some horses are not brown.



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32. State the negations for the following statements : There is no real number x , such that $x^2 = -1$.



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33. Write the converses of the following statement : If Jamila is riding a bicycle, then 17 August falls on a Sunday.



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34. Write the converses of the following statement : If 17 August is a Sunday, then Jamila is riding a bicycle.



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35. Write the converses of the following statement : If Pauline is angry, then her face turns red.



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36. Write the converses of the following statement : If a person has a degree in education, then she is allowed to teach.



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37. Write the converses of the following statement : If a person has a viral infection, then he runs a high temperature.



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38. Write the converses of the following statement : If Ahmad is in Mumbai, then he is in India.



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39. Write the converses of the following statement : If triangle ABC is equilateral, then all its interior angles are equal.



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40. Write the converses of the following statement : If x is an irrational number, then the decimal expansion of x is non-terminating non-recurring.



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41. Write the converses of the following statement : If $x - a$ is a factor of the polynomial $p(x)$, then $p(a) = 0$.



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42. State the converses of the following statement. In each case, also decide whether the converse is true or false:- If n is an even integer, then $2n + 1$ is an odd integer.



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43. State the converses of the following statement. In each case, also decide whether the converse is true or false.:- If the decimal expansion of a real number is terminating, then the number is rational.



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44. State the converses of the following statement. In each case, also decide whether

the converse is true or false.:- If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.



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45. State the converses of the following statement. In each case, also decide whether the converse is true or false.:- If each pair of opposite sides of a quadrilateral is equal, then the quadrilateral is a parallelogram.



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46. State the converses of the following statement. In each case, also decide whether the converse is true or false.:- If two triangles are congruent, then their corresponding angles are equal.



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47. The product of a non-zero rational number and an irrational number is irrational.



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