MATHS

BOOKS - NAGEEN PRAKASHAN ENGLISH

BINOMIAL THEOREM

Example

1. Using binomial theorem, write down the expansions of the following: $(2x + 3y)^5$

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2. Expand $(3x - 2y)^4$

3. Simplify with the help of binomial theorm. (x+1)^5 + (x-1)^5

4. Find an approximation of $(0.\,99)^5$ using the first three terms of its expansion.

5. Using binomial theorem, prove that $(101)^{50} > 100^{50} + 99^{50}$.

6. If number of terms in the expansion of $(x - 2y + 3z)^n$ are 45, then n is equal to

7. Prove that $\sum_{r=0}^{n} {}^nC_r.4^r = 5^n$

8. If $\left(1 - x + x^2\right)^4 = 1 + P_1x + P_2x^2 + P_3x^3 + \ldots + P_8x^8$, then prove that : $P_2 + P_4 + P_6 + P_8 = 40$ and $P_1 + P_3 + P_5 + P_7 = -40$.

9. If o be the sum of odd terms and E that of even terms in the expansion of $(x + a)^n$ prove that: (i) $O^2 - E^2 = (x^2 - a^2)^n$ (ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$ (iii) $2(O^2 + E^2) = (x + a)^{2n} + (x - a)^{2n}$

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10. Find the 8th term in the expansion of $\left(\dfrac{2x}{3} - \dfrac{3}{5x} \right)^{12}$

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11. Find the 13th term in the expansion of $\left(9x - \dfrac{1}{3\sqrt{x}} \right)^{18}, x \neq 0$

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12. Find the 15th term in the expansion of $(\sqrt{x} - \sqrt{y})^{17}$

13. Find the middle term in the expansion of $\left(3x - \dfrac{1}{2x}\right)^{16}$

14. Find the middle term in the expansion of $\left(1 + 2x + x^2\right)^{10}$

15. Find the 4th term from the end in the expansion of $(1 - 3x)^{10}$

16. Show that the middle term in the expansion of $(1 + x)^{2n} is \dfrac{(1.\,3.\,5(2n - 1))}{n!} 2^n x^n$, where n is a positive integer.

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19. Find the coefficient of x^6 in the expansion of $\left(2x^3 - \dfrac{1}{3x^3}\right)^{10}$

20. Find the coefficient of x^7 in the expansion of $\left(2x^2 - \dfrac{1}{x}\right)^{20}$

21. Find the coefficient of x^{-25} in the expansion of $\left(\dfrac{x^2}{2} - \dfrac{3}{x^3}\right)^{15}$

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22. Find the coefficient of $x^6 . y^3$ in the expansion of $(2x + y)^9$

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23. Find the constant term in the expansion of $\left(2x^4 - \dfrac{1}{3x^7}\right)^{11}$

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24. Find the terms independent of x in the expansion of $\left(\sqrt{x} + \dfrac{1}{3x^2} \right)^{10}$

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25. Find the term in the expansion of $\left(2x^2 - \dfrac{3}{x} \right)^{11}$ Which contains x^6

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26. If the coefficient of x^2 and x^3 are equal in the expansion of $(3 + ax)^9$, then find the value of 'a'

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27. If m and n are positive integers, then prove that the coefficients of x^m and x^n are equal in the expansion of $(1 + x)^{m+n}$

28. Find the coefficient of x^5 in the product $(1 + 2x)^6 (1 - x)^7$ using binomial theorem.

29. If the coefficients of a^{r-1}, a^r and a^{r+1} in the binomial expansion of $(1 + a)^n$ are in A.P., prove that $n^2 - on(4r + 1) + 4r^2 - 2 = 0$.

30. Evaluate :

$$1 + {}^{15}C_1 + {}^{15}C_2 + {}^{15}C_3 + \ldots\ldots + {}^{15}C_{15}$$

31. If

$$(1 + x)^n = C_0 + C_1 . x + C_2 . x^2 + C_3 . x^3 + \ldots\ldots + C_n . x^n,$$

then   prove   that

$$C_0 + 2C_1 + 4C_2 + 6C_3 + \ldots + 2n . C_n = 1 + n \cdot 2^n$$

32. If $C_0, C_1, C_2 \ldots, C_n$, denote the binomial coefficients in the expansion of $(1 + x)^n$, then $\dfrac{C_1}{2} + \dfrac{C_3}{4} + \dfrac{C_5}{6} + \ldots\ldots$ is

equal to

33. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$

then $C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2$ is equal to

34. If $(1+x)^n = C_0 + C_1 x m + C_2 x^2 + \ldots \ldots + C_n x^n$,

then

$C_0 C_1 + C_1 C_2 + C_2 C_3 + \ldots \ldots + C_{n-1} C_n$ is equal to :

35. Expand $(2X + 5Y)^5$

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36. Expand $(3x - 2y)^4$

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37. Simplify with the help of binomial theorm. $(x + 1)^5 + (x - 1)^5$

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38. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

39. Using binomial theorem, prove that $(101)^{50} > 100^{50} + 99^{50}$.

40. If number of terms in the expansion of $(x - 2y + 3z)^n$ are 45, then n is equal to

41. Prove that $\sum_{r=0}^{n} {}^n C_r . 4^r = 5^n$

42.

$If\left(1 - x + x^2\right)^4 = 1 + P_1 x + P_2 x^2 + P_3 x^3 + â€¦â€¦ + P_8 x^8,$

then prove that :

$P_2 + P_4 +_6 + P_8 = 40 \quad \text{and} \quad P_1 + P_3 + P_5 + P_7 = -40$

43. If o be the sum of odd terms and E that of even terms in the expansion of $(x + a)^n$ prove that: (i) $O^2 - E^2 = \left(x^2 - a^2\right)^n$ (ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$ (iii) $2\left(O^2 + E^2\right) = (x + a)^{2n} + (x - a)^{2n}$

44. Find the 8th term in the expansion of $\left(\dfrac{2x}{3} - \dfrac{3}{5x} \right)^{12}$

45. Find the 13th term in the expansion of $\left(9x - \dfrac{1}{3\sqrt{x}} \right)^{18}, \ x \neq 0$

46. Find the 15th term in the expansion of $\left(\sqrt{x} - \sqrt{y} \right)^{17}$

47. Find the middle term in the expansion of $\left(3x - \dfrac{1}{2x}\right)^{16}$

48. Find the middle term in the expansion of $\left(1 + 2x + x^2\right)^{10}$

49. Find the 4th term from the end in the expansion of $(1 - 3x)^{10}$

50. Show that the middle term in the expansion of $(1 + x)^{2n}$ is $\dfrac{1.3.5\ldots(2n-1)}{n!} \cdot 2^n \cdot x^n$, where $n \in N$.

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53. Find the coefficient of x^6 in the expansion of $\left(2x^3 - \dfrac{1}{3x^3} \right)^{10}$

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54. Find the coefficient of x^7 in the expansion of $\left(2x^2 - \dfrac{1}{x} \right)^{20}$

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55. Find the coefficient of x^{-25} in the expansion of $\left(\dfrac{x^2}{2} - \dfrac{3}{x^3} \right)^{15}$

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56. Find the coefficient of $x^6 y^3$ in the expansion of $(x + 2y)^9$.

57. Find the constant term in the expansion of

$$\left(2x^4 - \frac{1}{3x^7} \right)^{11}$$

58. Find the constant term in the expansion of

$$\left(\sqrt{x} + \frac{1}{3x^2} \right)^{10}.$$

59. Prove that there is no term involving x^6 in the expansion of $\left(2x^2 - \dfrac{3}{x} \right)^{11}$, where $\neq 0$.

60. If the coefficients of $x^2 \, and \, x^3$ in the expansion o $(3 + ax)^9$ are the same, then the value of a is $-\dfrac{7}{9}$ b. $-\dfrac{9}{7}$ c. $\dfrac{7}{9}$ d. $\dfrac{9}{7}$

61. In the binomial expansion of $(1 + a)^{m+n}$, prove that the coefficient of $a^m \, and \, a^n$ are equal.

62. Find the coefficient of x^5 in the expansion of $(1 + 2x)^6 (1 - x)^7$.

63. If the coefficients of a^{r-1}, $a^r and\ a^{r+1}$ in the binomial expansion of $(1 + a)^n$ are in A.P., prove that $n^2 - on(4r + 1) + 4r^2 - 2 = 0.$

64. Evaluate :

$$1 +^{15} C_1 +^{15} C_2 +^{15} C_3 + \ldots\ldots +^{15} C_{15}$$

65.

if $(1 + x)^n = C_0 + C_1.\, x + C_2.\, x^2 + C_3.\, x^3 + \ldots\ldots + C_n.\, x^n$,

then prove that

$C_0 + 2C_1 + 4C_2 + 6C_3 + \ldots\ldots + 2n.\, C_n = 1 + n.2^n$

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66.   Prove   that

$.^n\, C_0 + \dfrac{.^n\, C_1}{2} + \dfrac{.^n\, C_2}{3} + \ldots\ldots + \dfrac{.^n\, C_n}{n+1} = \dfrac{2^{n+1} - 1}{n+1}.$

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67. If $C_0, C_1, C_2…………C_n$ are the binomial coefficient in the expansion of $(1 + x)^n$ then prove that:

$$C_0^2 + C_1^2 + C_2^2 + â€¦â€¦ .. + C_n^2 = \frac{\lfloor 2n}{\lfloor n \lfloor n}$$

68. if $C_0 C_1 C_2, â€¦â€¦ C_n$ are the binomial coefficients in the expansion of $(1 + x)^n$ then prove that:

$$C_0 C_2 + C_1 C_3 + C_2 C_4 + â€¦â€¦ + C_{n-2} C_n = \frac{\lfloor 2n}{\lfloor n - 2 \lfloor n + 2}$$

Exercise 8 A

1. Expand using binomial theorem:

$(i) (1 - 2x)^4$ $\qquad (ii) \left(1 + \frac{1}{x^2}\right)^4$

2. Evaluate using binomial theorem:

$(i)\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6$

$(ii)\left(\sqrt{5}+\sqrt{2}\right)^4 - \left(\sqrt{5}-\sqrt{2}\right)^4$

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3. Find the value of $\left(\sqrt{2}+1\right)^6 - \left(\sqrt{2}-1\right)^6$.

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4. If $x = \sqrt{5}+\sqrt{3}$ and $y = \sqrt{5}-\sqrt{3}$, then $x^4 - y^4$

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5. Find the values of the following using binomial theorem:

$(i)\,49^4$ $(ii)\,(1.1)^4$

$(iii)\,101^3$ $(iv)\,(0.9)^5$

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6. By using binomial theorem find which number is greater

$(1.2)^{3000}$ or   600?

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7. Prove that $\Sigma_{r=0}^{n}\,{}^{n}C_{r}.3^{r} = 4^{n}$

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8. If n is a positive integer then find the number of terms in the expansion of $(x + y - 2z)^n$

9. Find the number of terms in the expansion of $\left(1 + 3x + 3x^2 + x^3\right)^{15}$

10. If $\left(1 + x + x^2\right)^n = 1 + a_1 x + a_2 x^2 + a_3 x^3$

$+ â€¦â€¦. . + a_{2n}. x^{2n}$ then prove that:

$(i) a_1 + a_3 + a_5 + â€¦. . + a_{2n-1} = \dfrac{3^n - 1}{2}$

$(ii) a_2 + a_4 + a_6 + â€¦â€¦ + a_{2n} = \dfrac{3^n - 1}{2}$

11. Using binomial theorem, prove that $2^{3n} - 7n - 1$ is divisible by 49, where $n \in N.$

12. Expand using binomial theorem:

$(i)(1 - 2x)^4$

13. Evaluate using binomial theorem:

$(i)\left(\sqrt{2} + 1\right)^6 + \left(\sqrt{2} - 1\right)^6$

$(ii)\left(\sqrt{5} + \sqrt{2}\right)^4 - \left(\sqrt{5} - \sqrt{2}\right)^4$

14. Using binomial theorem, expand $\left\{(x+y)^5 + (x-y)^5\right\}$. and hence find the value of $\left\{(\sqrt{2}+1)^5 + (\sqrt{2}-1)^5\right\}$.

15. Expand $(x+y)^4 - (x-y)^4$. Hence find the value of $(3+\sqrt{5})^4 - (3-\sqrt{5})^4$.

16. Find the values of the following using binomial theorem:

$(i)\, 49^4$ $(ii)\,(1.1)^4$

$(iii)\, 101^3$ $(iv)\,(0.9)^5$

17. By using binomial theorem find which number is greater $(1.2)^{3000}$ or 600?

18. Prove that $\Sigma_{r=1}{}^n C_r . 3^r = 4^n$

19. If n is a positive integer then find the number of terms in the expansion of $(x + y - 2z)^n$

20. Find the number of terms in the expansion of $\left(1 + 3x + 3x^2 + x^3\right)^{15}$

21. If

$$\left(1 - x + x^2\right)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{2n} x^{2n}, \text{ find}$$

the value of $a_0 + a_2 + a_4 + \ldots\ldots + a_{2n}.$

22. By using binomial theorem prove that $\left(2^{3n} - 7n - 1\right)$ is divisible by 49 where n is a positive integer.

Exercise 8 B

1. Find the 4th term in the expansion of $(x - 2y)^{12}$.

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2. Find the 7th term in the expansion of $\left(\dfrac{4x}{5} - \dfrac{5}{2x} \right)^9$.

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3. Find the 15th term in the expansion of $\left(2y - \dfrac{x}{2} \right)^{18}$

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4. Find the 10th term in the binomial expansion of $\left(2x^2 + \dfrac{1}{x}\right)^{12}$.

5. Find the $(r+1)$th term in the expansion of $\left(\dfrac{x}{a} - \dfrac{a}{x}\right)^{2n}$

6. Find the 7th term from the end in the expansion of $\left(x + \dfrac{1}{x}\right)^{11}$

7. Find the 3rd term the end in the expansion of $(2 - 3x)^8$

8. Find the 4th term from the end in the expansion of $\left(\dfrac{x}{2} - \dfrac{4}{x} \right)^{15}$

9. Find the middle term in the following expansion:

$(i)\left(x^2 - \dfrac{1}{x^2} \right)^{10}$

10. In the expansion of $(1 + x)^{2n} (n \in N)$, the coefficients of $(p + 1)^{th}$ and $(p + 3)^{th}$ terms are equal, then

11. If the coefficients of the $(2r + 4)th, (r + 2)th$ term in the expansion of $(1 + x)^{18}$ are equal, then the value of r is.

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13. Find a if 17th and 18th terms in the expansion of $(2 + a)^{50}$ are equal.

14. If the coefficient of 2nd, 3rd and 4th terms in the expansion of $(1 + x)^{2n}$ are in A.P. , show that $2n^2 - 9n + 7 = 0$.

15. If n is an odd positive integer, prove that the coefficients of the middle terms in the expansion of $(x + y)^n$ are equal.

16. If 3rd, 4th, 5th terms in the expansion of $(x + a)^n$ be 84, 280 and 560, Find x, a and n.

17. Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.

18. If a. b, c and d are the coefficients of 2nd, 3rd, 4th and 5th terms respectively in the binomial expansion of $(1 + x)^n$, then prove that $\dfrac{a}{a + b} + \dfrac{c}{c + d} = 2\dfrac{b}{b + c}$

19. The coefficient of three consecutive terms in the expansion of $(1 + x)^k$. Are in the ratio $1 : 7 : 42$ find the value of k.

20. Find the 4th term in the expansion of $(x - 2y)^{12}$.

21. Find the 6th term in the expansion of $\left(\dfrac{4x}{5} - \dfrac{5}{2x} \right)^9$.

22. Find the 15th term in the expansion of $\left(2y - \dfrac{x}{2}\right)^{18}$

23. (i) Find the 9th term in the expansion of $\left(\dfrac{x}{a} - \dfrac{2a}{x^2}\right)^{12}$

(ii) Find the 8th term in the expansion of $\left(2x^2 + \dfrac{1}{x}\right)^{12}$

24. Find the $(r+1)$th term in the expansion of $\left(\dfrac{x}{a} - \dfrac{a}{x}\right)^{2n}$

25. Find the 7th term from the end in the expansion of

$$\left(x + \frac{1}{x}\right)^{11}$$

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26. Find the 3rd term from the end in the expansion of

$$(2 - 3x)^{8}$$

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27. Find the 4th term from the end in the expansion of

$$\left(\frac{x}{2} - \frac{4}{x}\right)^{15}$$

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28. Find the middle term in the following expansion:

(i) $\left(\dfrac{x}{a} + \dfrac{a}{x}\right)^{12}$

29. In the expansion of $(1 + x)^{2n} (n \in N)$, the coefficients of

$(p + 1)^{th}$ and $(p + 3)^{th}$ terms are equal, then

30. If the coefficients of the $(2r + 4)th, (r + 2)th$ term in

the expansion of $(1 + x)^{18}$ are equal, then the value of r is.

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32. Find a if 17th and 18th terms in the expansion of $(2 + a)^{50}$ are equal.

33. If the coefficient of 2nd, 3rd and 4th terms in the expansion of $(1 + x)^{2n}$ are in A.P. , show that $2n^2 - 9n + 7 = 0$.

34. If n is an odd positive integer, prove that the coefficients of the middle terms in the expansion of $(x + y)^n$ are equal.

35. If 3rd, 4th, 5th terms in the expansion of $(x + a)^n$ be 84, 280 and 560, Find x, a and n.

36. Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.

37. If the coefficients of flour consecutive terms in the expansion of $(1 + x)^n$ are a, b, c, d respectively then prove that:

$$\frac{a}{a+b} + \frac{C}{c+d} = \frac{2b}{b+c}$$

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38. The coefficient of three consecutive terms in the expansion of $(1 + x)^k$. Are in the ratio $1 : 7 : 42$ find the value of k.

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Exercise 8 C

1. Find the coefficient of x^9 in the expansion of $\left(x^2 - \dfrac{1}{3x} \right)^9$.

2. Find the coefficient of x^{10} in the expansion of $\left(1 - x^2\right)^{10}$

3. The coefficient of x^{-17} in the expansion of $\left(x^4 - \dfrac{1}{x^3} \right)^{15}$ is

4. Find the coefficient of x^{40} in the expansion of $\left(1 + 2x + x^2\right)^{27}$.

5. If 'n' is a positive integer then prove that the coefficient fo x^m in the expansion of $\left(x^2 + \dfrac{1}{x} \right)^{2n}$ is :

$$\dfrac{\lfloor 2n}{\left\lfloor \dfrac{4n-m}{3} \right. \cdot \left\lfloor \dfrac{2n+m}{3} \right.}$$

6. Find the term independent of x (constant term) in the following expansion:

$(i)\left(x^2 - \dfrac{1}{3x} \right)^9$ $\qquad$ $(ii)\left(x - \dfrac{1}{x} \right)^{10}$

7. Prove that the term independent of x in the expansin of $\left(x + \dfrac{1}{x}\right)^{2n}$ $is \dfrac{1.\ 3.\ 5(2n-1)}{n!} \cdot 2^{n}.$

8. Find the coefficient of $a^5 b^7 \in (a - 2b)^{12}$

9. Find the coefficient of $x^2 . y^7$ in the expansion of $(x + 2y)^9$

10. Prove that the ratio of the coefficient of x^{10} in $(1 - x^2)^{10}$ & the term independent of x in $\left(x - \dfrac{2}{x}\right)^{10}$ is $1:32$

11. Prove that he coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$

12. Find a positive value of m for which the coefficient of x^2 in the expansion of $(1 + x)^m$ is 6.

13. The sum of the coefficients of x^{32} and x^{-17} in $\left(x^4 - \dfrac{1}{x^3}\right)^{15}$ is

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14. If the coefficient of x^7 in $\left[ax^2 + \left(\dfrac{1}{b}x\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\dfrac{1}{bx^2}\right)\right]^{11}$ then a and b satisfy the relation

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16. Find the coefficient of x^{10} in the expansion of $\left(1 - x^2\right)^{10}$

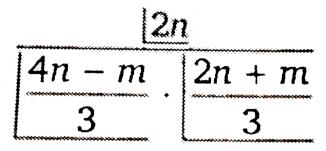**17.** about to only mathematics

18. Find the coefficient of x^{40} in the expansion of $\left(1 + 2x + x^2\right)^{27}$.

19. If 'n' is a positive integer then prove that the coefficient fo

x^m in the expansion of $\left(x^2 + \dfrac{1}{x}\right)^{2n}$ is :

$$\frac{\lfloor 2n}{\left\lfloor \dfrac{4n-m}{3} \right.} \cdot \frac{}{\left\lfloor \dfrac{2n+m}{3} \right.}$$

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20. Find the term independent of x in $\left(\dfrac{3x^2}{2} - \dfrac{1}{3x}\right)^9$

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21. Prove that the term independent of x in the expansin of $\left(x + \dfrac{1}{x}\right)^{2n}$ is $\dfrac{1.\ 3.\ 5(2n-1)}{n!} \cdot 2^n.$

22. Find the coefficient of $a^5 b^7 \in (a - 2b)^{12}$

23. Find the coefficient of $x^2 . y^7$ in the expansion of $(x + 2y)^9$

24. Prove that the ratio of the coefficient of x^{10} in $\left(1 - x^2\right)^{10}$

& the term independent of x in $\left(x - \dfrac{2}{x}\right)^{10}$ is $1 : 32$

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25. The coefficient of x^n in the expansion of $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ are in the ratio

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26. Find a positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6

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27. Find the coefficients of $x^{32} and x^{-7}$ in the expansion of $\left(x^4 - \dfrac{1}{x^3}\right)^{15}$.

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28. Find the coefficients of x^7 in $\left(ax^2 + \dfrac{1}{bx}\right)^{11} and x^{-7} \in \left(a\dfrac{x^{-1}}{bx^2}\right)^{11}$ and find the relation between $a and b$ so that coefficients are equal.

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Exercise 8 D

1. Evaluate the following :

$(i) 1 + .^{20}C_1 + {}^{20}C_2 + {}^{20}C_3 + ... + {}^{20}C_{19} + {}^{20}C_{20}$

$(ii) {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + ... + {}^{10}C_9$

$(iii) {}^{25}C_1 + {}^{25}C_3 + {}^{25}C_5 + ... + {}^{25}C_{25}$

$(iv) {}^{18}C_2 + {}^{18}C_4 + {}^{18}C_4 + {}^{18}C_6 + ... + {}^{18}C_{18}$

2. If $(1 + x)^n = C_0 + C_1. x + C_2. x^2 + â€¦ + C_n. x^n.$ then

prove that

$(i) C_0 + 2C_1 + 3C_2 + â€¦ + (n-1)C_n = (n+2).2^{n-1}$

$(ii) C_0 + 3C_1 + 5C_2 + ... + (2n+1)C_n = (n+1).2^n$

3. Prove that:

$$^2C_2 +{}^3C_2 +{}^4C_2 + â€¦.. +{}^{n+1}C_2 = \frac{1}{6}n(n+1)(n+2)$$

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4. Evaluate the following :

$(i)\, 1 + {}.^{20}C_1 +{}^{20}C_2 +{}^{20}C_3 + ... +{}^{20}C_{19} +{}^{20}C_{20}$

$(ii)\,{}^{10}C_1 +{}^{10}C_2 +{}^{10}C_3 + ... +{}^{10}C_9$

$(iii)\,{}^{25}C_1 +{}^{25}C_3 +{}^{25}C_5 + ... +{}^{25}C_{25}$

$(iv)\,{}^{18}C_2 +{}^{18}C_4 +{}^{18}C_4 +{}^{18}C_6 + ... +{}^{18}C_{18}$

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5. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, prove that

$$C_0 + 2C_1 + 3C_2 + \ldots + (n+1)C_n = (n+2)2^{n-1}.$$

6. Prove that:

$$.^2 C_2 +^3 C_2 +^4 C_2 + \ldots\ldots +^{n+1} C_2 = \frac{1}{6}n(n+1)(n+2)$$

Exercise 8 E

1. No. of terms in the expansion of $\left(1 + 3x + 3x^2 + x^3\right)^{10}$ is:

A. 31

B. 32

C. 10

D. 11

Answer: A

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2. Find $(x + 1)^6 + (x - 1)^6$. Hence or otherwise evaluate $\left(\sqrt{2} + 1\right)^6 + \left(\sqrt{2} - 1\right)^6$.

A. 184

B. 192

C. 198

D. 202

Answer: C

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3. 15th term in the expansion of $\left(\sqrt{x} - \sqrt{y}\right)^{17}$ is :

A. $860x^{3/2}y^7$

B. $680x^7y^{3/2}$

C. $680x^{3/2}y^7$

D. $860x^3y^{7/2}$

Answer: C

4. If the coefficients of the $(n+1)^{th}$ term and the $(n+3)^{th}$ term in the expansion of $(1+x)^{20}$ are equal , then the value of n is 10 b. 8 c. 9 d. none of these

A. P

B. $P+1$

C. $P+2$

D. $P+3$

Answer: B

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5. Find a if 17th and 18th terms in the expansion of $(2+a)^{50}$ are equal.

A. 1/3

B. 1/2

C. 1

D. None of these

Answer: C

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6. Find the coefficient of x^{-25} in the expansion of $\left(\dfrac{x^2}{2} - \dfrac{3}{x^3} \right)^{15}$

A. $\dfrac{-1365}{16} \times 3^{11}$

B. $\dfrac{1365}{16} \times 3^{11}$

C. $\dfrac{-16}{1365} \times 3^{11}$

D. None of these

Answer: A

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7. The reamainder left out when $8^{2n} - (62)^{2n+1}$ is divided by

9 is

A. 0

B. 2

C. 4

D. none of these

Answer: B

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8. No. of terms in the expansion of $(1 + 2x)^9 + (1 - 2x)^9$ is :

A. 10

B. 9

C. 7

D. 5

Answer: D

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9. Find the middle term in the expansion of : $\left(x - \dfrac{1}{x}\right)^{10}$

A. 126

B. -126

C. -252

D. 252

Answer: C

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10. if the coefficient of $(2r + 1)$th term and $(r + 2)$th term in the expansion of $(1 + x)^{43}$ are equal then r=?

A. 14

B. 30

C. 41

D. 42

Answer: A

11. Find the middle term in the expansion of :

$$\left(1 + 3x + 3x^2 + x^3\right)^{2n}$$

A. 31

B. 32

C. 10

D. 11

Answer: A

12. Find $(x + 1)^6 + (x - 1)^6$. hence, or otherwise evaluate $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$

A. 184

B. 192

C. 198

D. 202

Answer: C

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13. 15th term in the expansion of $\left(\sqrt{2} - \sqrt{y}\right)^{17}$ is :

A. $87040y^7$

B. $-87040y^{3/2}$

C. $680y^7$

D. $-860y^{7/2}$

Answer: B

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14. If the coefficients of the $(n+1)^{th}$ term and the $(n+3)^{th}$ term in the expansion of $(1+x)^{20}$ are equal , then the value of n is 10 b. 8 c. 9 d. none of these

A. P

B. $P+1$

C. $P+2$

D. $P + 3$

Answer: B

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15. Find a if 17th and 18th terms in the expansion of $(2 + a)^{50}$

are equal.

A. $1/3$

B. $1/2$

C. 1

D. None of these

Answer: C

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16. Find the coefficient of x^{-25} in the expansion of $\left(\dfrac{x^2}{2} - \dfrac{3}{x^3}\right)^{15}$

A. $\dfrac{-1365}{16} \times 3^{11}$

B. $\dfrac{1365}{16} \times 3^{11}$

C. $\dfrac{-16}{1365} \times 3^{11}$

D. None of these

Answer: A

17. The reamainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is

    A. 0

    B. 2

    C. 4

    D. none of these

Answer: B

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18. No. of terms in the expansion of $(1 + 2x)^9 + (1 - 2x)^9$ is :

A. 10

B. 9

C. 7

D. 5

Answer: D

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19. Find the middle term in the expansion of : $\left(x - \dfrac{1}{x} \right)^{10}$

A. 126

B. -126

C. -252

D. 252

Answer: C

20. If the coefficient of $(2r + 1)$ th and $(r + 2)$ th terms in the expansion of $(1 + x)^{43}$ are equal, then the value of $r(r \neq 1)$ is

A. 14

B. 30

C. 41

D. 42

Answer: A

1. Find the coefficient of x^4 in the expansion of $\left(2 - x + 3x^2\right)^6$.

    A. -5051

    B. 4632

    C. -4631

    D. none of these

Answer: A

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2. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is a. 924 b. 792 c. 1594 d. none of these

A. 792

B. 924

C. 1048

D. 2096

Answer: B

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3. If the second, third and fourth terms in the expansion of $(x + y)^n$ be 135, 30 and 10/3 respectively, then

A. 5

B. 6

C. 7

D. 9

Answer: A

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4. Find the coefficient of x^4 in the expansion of $\left(1 + x + x^2 + x^3\right)^{11}$.

A. 900

B. 909

C. 990

D. 999

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5. If a. b, c and d are the coefficients of 2nd, 3rd, 4th and 5th terms respectively in the binomial expansion of $(1 + x)^n$, then prove that $\dfrac{a}{a + b} + \dfrac{c}{c + d} = 2\dfrac{b}{b + c}$

A. $\dfrac{b}{b + c}$

B. $\dfrac{b}{2(b + c)}$

C. $\dfrac{2b}{b + c}$

D. $\dfrac{2c}{b + c}$

Answer: C

6. If the coefficients of x^7 and x^8 in the expansion of $\left[2 + \dfrac{x}{3}\right]^n$ are equal, then the value of n is : (A) 15 (B) 45 (C) 55 (D) 56

A. 15

B. 45

C. 55

D. 60

Answer: C

7. If A and B denote the coefficients of x^n in the binomial expansions of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then

A. $A = B$

B. $2A = B$

C. $A = 2B$

D. None of these

Answer: C

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8. Find the greatest term in the expansion of $\sqrt{3}\left(1 + \dfrac{1}{\sqrt{3}}\right)^{20}$.

A. $\dfrac{25840}{9}$

B. $\dfrac{24840}{9}$

C. $\dfrac{26840}{9}$

D. None of these

Answer: A

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9. If the coefficient of the rth, $(r+1)th$ and $(r+2)th$ terms in the expansion of $(1+x)^n$ are in A.P., prove that

$$n^2 - n(4r + 1) + 4r^2 - 2 = 0.$$

A. $n^2 - n(4r + 1) + 4r^2 - 2 = 0$

B. $n^2 + n(4r + 1) + 4r^2 - 2 = 0$

C. $n^2 + n(4r + 1) + 4r^2 + 2 = 0$

D. $n^2 + n(4r + 1) + 4r^2 + 2 = 0$

Answer: A

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10. if the coefficients of x^5 and x^{15} in the expansion of $\left(x^2 + \dfrac{a}{x^3}\right)^{10}$ are equal then then the positive value of 'a' is:

A. $2\sqrt{3}$

B. 1

C. $\dfrac{1}{\sqrt{3}}$

D. $\dfrac{1}{2\sqrt{3}}$

Answer: D

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11. Find the coefficient of x^4 in the expansion of $\left(2 - x + 3x^2\right)^6$.

A. -5051

B. 4632

C. -4631

D. none of these

Answer: A

---

12. If the sum of the coefficients in the expansion of $(a + b)^n$

is 4096, then the greatest coefficient in the expansion is a.

924 b. 792 c. 1594 d. none of these

   A. 792

   B. 924

   C. 1048

   D. 2096

Answer: B

13. If the second, third and fourth terms in the expansion of $(x + y)^n$ be 135, 30 and 10/3 respectively, then

A. 5

B. 6

C. 7

D. 9

Answer: A

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14. Find the coefficient of x^4 in the expansion of $\left(1 + x + x^2 + x^3\right)^{11}$.

A. 900

B. 909

C. 990

D. 999

Answer: C

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15. if a,b,c and d are the coefficient of four consecutive terms in the expansion of $(1+x)^n$ then $\dfrac{a}{a+b} + \dfrac{c}{c+d} = ?$

A. $\dfrac{b}{b+c}$

B. $\dfrac{b}{2(b+c)}$

C. $\dfrac{2b}{b+c}$

D. $\dfrac{2c}{b+c}$

Answer: C

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16. If the coefficients of x^7 and x^8 in the expansion of $\left(2 + \dfrac{x}{3}\right)^n$ are equal then n is

A. 15

B. 45

C. 55

D. 60

Answer: C

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17. If A and B are the coefficients of x^n in the expansion $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then

A. $A = B$

B. $2A = B$

C. $A = 2B$

D. None of these

Answer: C

18. Find the greatest term in the expansion of $\sqrt{3}\left(1 + \dfrac{1}{\sqrt{3}}\right)^{20}$.

A. $\dfrac{25840}{9}$

B. $\dfrac{24840}{9}$

C. $\dfrac{26840}{9}$

D. None of these

Answer: A

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19. If the coefficients of the rth, $(r + 1)th$, $(r + 2)th$ terms is the expansion of $(1 + x)^{14}$ are in A.P, then the largest value

of r is.

A. $n^2 - n(4r + 1) + 4r^2 - 2 = 0$

B. $n^2 + n(4r + 1) + 4r^2 - 2 = 0$

C. $n^2 + n(4r + 1) + 4r^2 + 2 = 0$

D. $n^2 + n(4r + 1) + 4r^2 + 2 = 0$

Answer: A

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20. if the coefficients of x^5 and x^{15} in the expansion of $\left(x^2 + \dfrac{a}{x^3}\right)^{10}$ are equal then then the positive value of 'a' is:
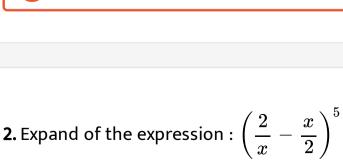
A. $2\sqrt{3}$

B. 1

C. $\dfrac{1}{\sqrt{3}}$

D. $\dfrac{1}{2\sqrt{3}}$

Answer: D

Exericse 8 1

1. Expand of the expression : $(1 - 2x)^5$

2. Expand of the expression : $\left(\dfrac{2}{x} - \dfrac{x}{2}\right)^5$

3. Expand of the expression : $(2x - 3)^6$

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4. Expand of the expression : $\left(\dfrac{x}{3} + \dfrac{1}{x} \right)^5$

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5. Expand $\left(x + \dfrac{1}{x} \right)^6 . (x \neq 0)$

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6. Using binomial theorem, evaluate : $(96)^3$

7. Using binomial theorem, evaluate : $(102)^5$

8. Using binomial theorem, evaluate : $(101)^4$

9. Using binomial theorem, evaluate : $(99)^5$

10. Using binomial theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

11. Find $(a+b)^4 - (a-b)^4$. Hence evaluate $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$

12. Find $(x+1)^6 + (x-1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$.

13. Show that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

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14. Prove that $\displaystyle\sum_{r=0}^{n} 3^r \ {}^n C_r = 4^n$.

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15. Expand of the expression : $(1 - 2x)^5$

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16. Expand of the expression : $\left(\dfrac{2}{x} - \dfrac{x}{2}\right)^5$

17. Expand of the expression : $(2x - 3)^6$

18. Expand of the expression : $\left(\dfrac{x}{3} + \dfrac{1}{x}\right)^5$

19. Expand $\left(x + \dfrac{1}{x}\right)^6 . (x \neq 0)$

20. Using binomial theorem, evaluate : $(96)^3$

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21. Using binomial theorem, evaluate : $(102)^5$

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22. Using binomial theorem, evaluate : $(101)^4$

▶ **Watch Video Solution**

23. Using binomial theorem, evaluate : $(99)^5$

▶ **Watch Video Solution**

24. Using binomial theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

25. Find $(a+b)^4 - (a-b)^4$. Hence evaluate $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$

26. Find $(x+1)^6 + (x-1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$.

27. Show that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

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28. Prove that $\Sigma_{r=0}^{n} 3^{rn} C_r = 4^n$

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Exericse 8 2

1. Find the coefficient of $x^5 \in (x+3)^8$

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2. Find the coefficient of $a^5 b^7 \in (a - 2b)^{12}$

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3. Write the general term in the expansion of $\left(x^2 - y\right)^6$.

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4. Write the general term in the expansion of $\left(x^2 - yx\right)^{12}, x \neq 0$

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5. Find the 4th term in the expansion of $(x - 2y)^{12}$.

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6. Find 13th term in the expansion of $\left(9x - \dfrac{1}{3x} \right)^{18}$, $x \neq 0$.

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7. Find the middle terms in the expansion of $\left(3 - \dfrac{x^3}{6} \right)^{7}$.

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8. Find the middle term in the expansion of : $\left(\dfrac{x}{3} + 9y \right)^{10}$

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9. In the binomial expansion of $(1 + a)^{m+n}$, prove that the coefficient of a^m and a^n are equal.

10. The coefficients of the $(r - 1)^{th}$, r^{th} and $(r + 1)^{th}$ terms in the expansion of $(x + 1)^n$ are in the ratio $1:3:5$ .Find n and r.

11. The coefficient of x^n in the expansion of $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ are in the ratio

12. Find a positive value of m for which the coefficient of x^2 in the expansion of $(1 + x)^m$, is 6.

13. Find the coefficient of $x^5 \in (x + 3)^8$

14. Find the coefficient of $a^5 b^7 \in (a - 2b)^{12}$

15. Write the general term in the expansion of $(x^2 - y)^6$.

16. Write the general term in the expansion of $\left(x^2 - yx\right)^{12}$, $x \neq 0$

17. Find the 4th term in the expansion of $(x - 2y)^{12}$.

18. Find the 13th term in the expansion of $\left(9x - \dfrac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$

19. Find the middle term in the expansion of $\left(3 - \dfrac{x^3}{6}\right)^7$

20. Find the middle term in the expansion of : $\left(\dfrac{x}{3} + 9y\right)^{10}$

21. In the expansion of $(1 + a)^{m+n}$,prove that coefficients of a^m and a^n are equal.

22. The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio $1:3:5$ .Find n and r.

23. prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$

24. Find a positive value of m for which the coefficient of x^2 in the expansion of $(1+x)^m$ is 6.

Miscellaneous Exericse

1. Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.

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2. If the coefficients of x^2 and x^3 in the expansion o $(3 + ax)^9$ are the same, then the value of a is $-\frac{7}{9}$ b. $-\frac{9}{7}$ c. $\frac{7}{9}$ d. $\frac{9}{7}$

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3. Find the coefficient of a^4 in the product $(1 + a)^4(2 - a)^5$ using binomial theorem.

4. If a and b are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, whenever n is a positive integer.

5. Evaluate $\left(\sqrt{3} + \sqrt{2}\right)^6 - \left(\sqrt{3} - \sqrt{2}\right)^6$.

6. Find the value of $\left(a^2 + \sqrt{a^2 - 1}\right)^4 + \left(a^2 - \sqrt{a^2 - 1}\right)^4$.

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7. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

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8. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(24 + \dfrac{1}{34}\right)^n$ is $\sqrt{6}:1$.

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9. Using binomial theorem expand $\left(1 + \dfrac{x}{2} - \dfrac{2}{x}\right)^4$, $x \neq 0$.

10. Find the expansion of $\left(3x^2 - 2ax + 3a^2\right)^3$ using binomial theorem.

11. Find a, b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375, respectively.

12. If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are the same, then the value of a is $-\dfrac{7}{9}$ b. $-\dfrac{9}{7}$ c. $\dfrac{7}{9}$ d. $\dfrac{9}{7}$

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13. Find the coefficient of x^5 in the expansion of $(1 + 2x)^6(1 - x)^7$.

▶ Watch Video Solution

14. If a and b are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, whenever n is a positive integer.

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15. Evaluate $\left(\sqrt{3} + \sqrt{2}\right)^6 - \left(\sqrt{3} - \sqrt{2}\right)^6$.

16. Find the value of $\left(a^2 + \sqrt{a^2 - 1}\right)^4 + \left(a^2 - \sqrt{a^2 - 1}\right)^4$.

17. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

18. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(24 + \dfrac{1}{34} \right)^n$ is $\sqrt{6} : 1$.

19. Expand using Binomial Theorem $\left(1 + \dfrac{x}{2} - \dfrac{2}{x} \right)^4 , x \neq 0.$

20. Find the expansion of $\left(3x^2 - 2ax + 3a^2 \right)^3$ using binomial theorem.