MATHS

BOOKS - NAGEEN PRAKASHAN ENGLISH

RELATIONS AND FUNCTIONS

Solved Examples

1. If (2x + 3,y-1) =(3,5) , then find x and y.

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2. If $A = \{1, 4\}$ and $B = \{2, 3, 4\}$, then evaluate $A \times B$.

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3. If A={1,2} and B={2,3} , then show that:

$$A \times B \neq B \times A$$

4. If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$, then find A and B

5. A and B are two sets and $n(A \times B)$ =6 . If there elements of $A \times B$ are (1,2) ,(2,3),(3,3), then find $A \times B$ and $B \times A$.

6. If $A = \{a, b\}, B = \{2, 3\}$ and $C = \{3, 4\}$, then evaluate:

(i) $A \times (B \cap C)$ $(ii) A \times (B \cup C)$

7. If $A \subseteq B$ and C be any set , then prove that:

$$A \times C \subseteq B \times C$$

8. If $A \subseteq B$ and $C \subseteq D$, then prove that:

$$A \times C \subseteq B \times D$$

9. For any sets A, B, C, D prove that:

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

10. If n elements are common in sets A and B, then prove that n^2 elements will be common in the set $A \times B$ and $B \times A$.

11. Following relations from the set of natural number N to N are given:

$(a) R_1 = \{(1, 1), (4, 2), (9, 3), (16, 4)\}$

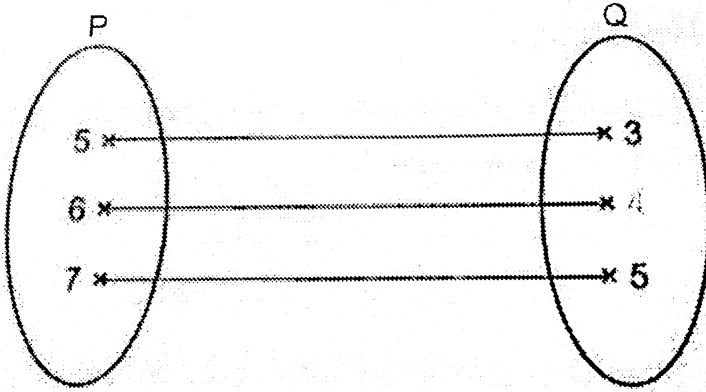(b) $R_2 = \{(3, 1), (4, 2), (5, 3), (6, 4)\}$

Represent them in set builder form.

12. A relation R is represent from set P to Q in the following diagram:

Write it in Roaster and Set builder form. Write the domain and range of the relation.

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13. If $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$, then which of the following is a relation from set A to B? Give reason:

(i) $R_1 = \{(1, 5), (2, 4), (3, 5)\}$

(ii) $R_2 = \{(4, 1), (2, 6), (5, 1), (2, 4)\}$

(iii) $R_3 = \{(1, 4), (2, 5), (3, 4), (2, 6), (3, 6)\}$

(iv) $A \times B$

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14. Let R be a relation from set Q to Q defined as:

$$R = \{(a, b) : a, b \in Q \text{ and } a - b \in Z\}$$

Prove that

(i) For each $a \in Q, (a, a) \in R$

$(ii) (a, b) \in R \Rightarrow (b, c) \in R$ where $a, b \in Q$

(iii) $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$, where $a, b, c \in Q$

15. Check whether the following relations are functions or not:

$$R_1 = \{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$$

$$R_2 = \{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$$

$$R_3 = \{(1, 3), (1, 5), (2, 5)\}$$

16. If A={1,2,3} and B={2,3,4} , find which of the following are the functions from A to B?

(i) $f = \{(1, 2), (2, 3), (3, 4)\}$

(ii) $g = \{(1, 2), (1, 3), (2, 3), (3, 4)\}$

(iii) $h = \{(1, 3), (2, 4)\}$

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17. Let R be a relation on the set of natural numbers N, defined as:

$R = \{(x, y) : y = 2x, x, y \in N\}.$

Is R a function from $N \times N$? If yes find the domain, co-domain and range of R.

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18. If $f = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$, is a linear function from Z to Z , then find f(x).

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19. Let A be the set of two positive integers. Let $f: A \xrightarrow{} Z^+$ (set of positive integers) be defined by $f(n) = p,$ where p is the highest prime factor of n. If range of $f = \{3\}$. Find set A. Is A uniquely determined?

20. If $f(x) = x^2 + 5x - 3$, then evaluate $f(4)$

21. If $f(x) = 2(1 + \sin x),$ then evaluate $f\left(\frac{\pi}{2}\right).$

22. If $f: R \to R$ is defined as:

$$f(x) = \begin{cases} 2x + 5, & x > 9 \\ x^2 - 1, & -9 < x < 9 \\ x - 4, & x < -9 \end{cases}$$

then evaluate (i) $f(2)$, $(ii)\, f(10)$, $(iii)\, f(-12)$ and $(iv)\, f[f(3)]$.

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23. If $f(x) = x^3 - \dfrac{1}{x^3}$, then find the value of $f(x) + f(-x)$.

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24. Let (x) is a real function, defines as $f(x) = \dfrac{x-1}{x+1}$, then prove that $f(2x) = \dfrac{3f(x) + 1}{f(x) + 3}$.

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25. If y = f(x) = $\dfrac{x+2}{x-1}$, $x \neq 1$, then show that x = f(y) .

26. If for $x \neq 0,\, af(x) + bf\left(\dfrac{1}{x}\right) = \dfrac{1}{x} - 5,\, a \neq b,$ then f (x).

27. If f (x) = cot x, then prove that :

$$f(-x) = -f(x)$$

28. If $f(x) = \log_e\left(\dfrac{1-x}{1+x}\right);$ prove that $f(a) + f(b) = f\left(\dfrac{a+b}{1+ab}\right)$

29. If f (x) $= \dfrac{x-1}{x+1},$ then prove that $f\{f(x)\} = -\dfrac{1}{x}.$

30. If f(x) $= x^2$ and $g(x) = 2x + 1$ are two real valued function, then evaluate :

$$(f + g)(x), (f - g)(x), (fg)(x), \frac{f}{g}(x)$$

31. Find the domain and range of $f(x) = \dfrac{x - 3}{4 - x}$.

32. Find the domain and range of $f(x) = \dfrac{1}{\sqrt{x - 2}}$

33. Find the domain and range of

$$f(x) = \dfrac{x}{1 + x^2}$$

34. Find the domain and range of the function $f(x) = \dfrac{x^2 - 9}{x - 3}$

35. Find the domain and range of $f(x) = \dfrac{1}{2 - \sin 3x}$.

Exercise 2 A

1. If $\left(\dfrac{x}{3} + 1, y - \dfrac{2}{3} \right) = \left(\dfrac{5}{3}, \dfrac{1}{3} \right)$, then find the values of x and y.

2. Find the values of x and y from each of the following

(i) $(x + y, x - 2y) = (7, 1)$

(ii) $(2x, x + 3y) = (4, 5)$

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3. If $A = \{2, -3\}$ and $B = \{1, 3, 5\}$, then find $A \times B$ and $B \times A$.

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4. If A ={1,2}, then find $A \times A \times A$.

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5. If $A = \{x, y, z\}$ and B={2,3}, then find $A \times B$.

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6. Let $A = \{1, 2, 3\}$ and $B = \{x : x \in N, x$ is prime less than $5\}$. Find $A \times B$

7. If $A \times B = \{(a, 3), (a, 5), (b, 3), (b, 5), (c, 3), (c, 5)\}$, then find $B \times A$.

8. State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly. (i) If $P = \{m, n\}$ and $Q = \{n, m\}$, then $P \times Q =$. (ii) If A

9. The Cartesian product $A \times A$ has 9 elements among which are found $(1, \quad 0) \quad and \quad (0, \quad 1)$. Find the set A and the remaining elements of $A \times A$.

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10. If $A \times B = \{(a, 2), (b, 4), (a, 6), (b, 2), (b, 6), (a, 4)\}$, then find A and B.

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11. If $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$, then find each of the following :

$(i) A \times (B \cap C) \qquad (ii)(A \times B) \cap (A \times C)$

$(iii) A \times (B \cup C) \qquad (iv)(A \times B) \cup (A \times C)$

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12. If A={1,2,3,4,5}, B={1,2,3,6,7} and C{8}, then find (A-B) $\times C$.

13. If A={1,2} and B={x,y}, then find AXB. Also find the number of its subsets.

14. If $A = \{ -1, 1\}$, find $A \times A \times A$.

15. If A={1,2,3} and B={1,2}, then represent $A \times B$ graphically.

16. If A ={x,y,z} and B={5,6}, then represent each of the following by arrow diagram:

$(i) A \times B$ $(ii) B \times A$

$(iii) A \times A$ $(iv) B \times B$

17. The set A and B has 3 common elements . If n(A) =5 and n(B)=4, then find $n(A \times B)$ and $n\{(A \times B) \cap (B \times A)\}$

18. If $A \subseteq B,$ show that $A \times A \subseteq (A \times B) \cap (B \times A).$

19. Let A be a non-empty set such that $A \times B = A \times C$. Show that $B = C$.

20. If A and B are any two non-empty sets, then prove that: $A \times B = B \times A' \Rightarrow A = B$.

Exercise 2 B

1. If A={a,b,c,d} and B={w,xy,z} , then which of the following is a relation from set A to B? Given reason:

$(i) R_1 = \{(a, w), (a, x), (a, y), (a, z)\}$

$(ii) R_2 = \{(w, b), (z, c), (y, d)\}$

$(iii) R_3 = \{(a, y), (b, y), (w, y), (d, y)\}$

$(iv) R_4 = \{(b, z), (b, y), (c, z), (c, w)\}$

2. Set A has 5 eleements and set B has 3 elements. Find the number of relations from set A to B.

3. If A={a,y} and B={2,5} , then find the number of relations from set A to B.

4. If A={1,2,3,4} and B={5,7,8,11,15}, are two sets and a relation R from A to B is defined as follows:

$_xR_y \Leftrightarrow y = 2x + 3$, where $x \in A, y \in B$

(i) Express R in Roaster form.

(ii) Find the domain and range of R.

(iii) Find R^{-1}.

(iv) Represent R by arrow diagram.

5. If A={2,4,6,8} and B={1,3,5}, then find the domain and range of the relation:

$$R\{(x, y) : x \in A, y \in B \text{ and } x > y\}$$

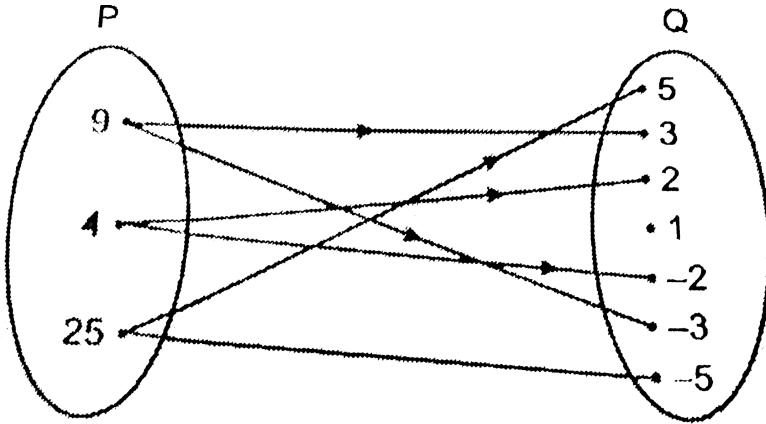**6.** If `A={1,2,3,4,5,6}, then define a relation R={(x,y):y=x-1}, from A to A.

7. A relation R is shown from sets P to Q in the following diagram:

write this relation in Roaster form and in Set builder form. Also find the domain, co-domain and range of the relation.

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8. Write the relation $R = \{(x, x^3) : x$ is a prime number less than 10$\}$ in roster form.

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9. A is a set of first 10 natural numbers and R is a relation from A to A defined as:

$(x, y) \in R \Leftrightarrow x + 2y = 10$ when $x, y \in A$

(i)Express R in the form of a set of ordered pairs.

(ii) Find the domain and range of R.

(iii) Find R^{-1}.

10. For the relation R_1 defined on R by the rule $(a, b) \in R_{11} + ab > 0$.

Prove that: $(a, b) \in R_1 and (b, c) \in R_1 (a, c) \in R_1$ is not true for all

$a, b, c \in R$.

11. Let R be a relation from N to N defined by $R = \{(a, b) : a\dot b \in N and$

$a = b^2)$.   Are   the   following   true?(i) $(a, a) \in R, \forall a \in N$(ii)

$(a, b) \in R, implies(b, a) \in R$(iii) $(a, b) \in R$, (b , c) $\in$ R implies (a , c) $\in$ R .

12. If set A={1, 2, 3, 4} and a relation R is defined from A to A as follows:

$$R = \{(x, y) : x > 1, y = 3\}$$

Find the domain and range of R.

13. A relation R is defined from the set of integer Z to Z as follows:

$$(x, y) \in Z \Leftrightarrow x^2 + y^2 = 25$$

(i) Express R and R^{-1} as the set of ordered pairs.

(ii) Write the domain of R and R^{-1}.

14. Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\left\{ \left(\overset{.}{a}b \right) : a, b \in A, b \text{ is exactly divisible by a} \right\}$.(i) Write R in roster form(ii) Find the domain of R(iii) Find the range of R.

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1. Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z to Z? Justify your answer.

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2. Examine each of the following relations given below and state in each case, giving reasons whether it is a function or not?(i) $R = \{(2, 1), (3, 1), (4, 2)\}$, (ii) $R = \{(2, 2), (2, 4), (3, 3), (4, 4)\}$(iii) `R= {(1, 2

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3. Let X = {1, 2, 3, 4, 5} and Y = {1, 3, 5, 7, 9}. Which of the following is/are not relations from X to Y?

4. Let R be a relation defined on the set of natural numbers as:

$$R = \{(x, y) : y = 3x, y \in N\}$$

Is R a function from N to N? If yes find its domain, co-domain and range.

5. Express each relation in the form of ordered pair and check whether is it a function or not?

(i) $f = \{(x, y) : y = 3x, x \in \{1, 2, 3\}, y \in \{3, 6, 9, 12\}\}$

$(ii) g = \{(x, y) : y > x + 1, x \in \{1, 2\}, y \in \{2, 4, 6\}\}$

(iii) $h = \{(x, y) : x + y = 3, x, y \in \{0, 1, 2, 3\}\}$

6. A function $f: R \rightarrow R$ is defined as:

$$f(x) = \begin{cases} 1 & x \in Q \\ -1 & x \notin Q \end{cases}$$

Evaluate each of the following:

(i) $f(2), f(\pi)$ (ii) Range of f

$(iii) f^{-1}(-1)$ $(iv) f^{-1}(1)$

7. Find which of the following are the function on set $A = \{a, b, c\}$? Give reason.

(i) $R_1 = \{(a, b), (b, a), (a, c)\}$

(ii) $R_2 = \{(b, b), (c, c), (a, b)\}$

8. A function $f: R \to R$ is defined as $f(x) = x^2 + 1$. Evaluate each of the following:

$(i) f^{-1}(-2)$ $\quad$ $(ii) f^{-1}(17)$

$(iii) f^{-1}(26, 50)$ $\quad$ $(iv) f^{-1}(0)$

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9. $\quad$ If $\quad$ A={1,2,3,4},B={1,5,9,11,15,16} $\quad$ and

$f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$ are the following statements true ?

(i) f is a relation from A to B .

(ii) f is a function from A to B

Justify your answer.

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10. Let $A = \{9, 10, 11, 12, 13\}$ and let $f: A \to N$ be defined by f(n) = the highest prime factor of n. Find the range of f.

11. If A={1,2} and B={3,6} and two functions $f: A \to B$ and $g: A \to B$ are defined respectively as :

$f(x) = x^2 + 2$ and $g(x) = 3x$

Find whether $f = g$

12. Express the following functions in the form of order pairs and find their range:

(i) $f: A \to R$, $f(x) = x^2 + 5$, where $A = \{-2, -1, 0, 1\}$

$g: A \to N$, $g(x) = 3x$, where $A: \{x: x \in N, x < 4\}$

13. If a function $f: R \to R$ is defined as $f(x) = x^2 + 1$, then find each of the following :

$(i)\{x, f(x) = 26\}$

(ii) Pre-image of 17 under f

(iii) Pre-image of 2 under f

14. Let a function $f : R^+ \to R$ is defined as $f(x) = \log_e x,$ then find each of the following:

(i) Range of f

(ii) f(x)=-2

(iii) Is $f(xy) = f(x) + f(y)$ true ?

15. A function $F(c)$ is defined as :

$$F(c) = \frac{9}{5} \cdot c + 32.$$

Evaluate each of the following:

(i) F(0)

(ii) F(28)

(iii) f(-10)

(iv) Value of c when F(c)=212

16. If f(x)=2x-5 , then evaluate the following:

(i) $f(0)$

(ii) $f(7)$

(iii) $f(-3)$

17. If $f(x) = x^2$, then evaluate: $\dfrac{f(1 \cdot 2) - f(1)}{1 \cdot 2 - 1}$

18. If $f(x) = x^2$, then evaluate : $\dfrac{f(x+1) - f(x-1)}{4x}$

19. If $f(x) = \dfrac{x}{x-1}$, then evaluate : $\dfrac{f(a/b)}{f(b/a)}$

20. If $f(x) = \dfrac{x-1}{x+1}$, then prove that:

$$\dfrac{f(b) - f(a)}{1 + f(b) \cdot f(a)} = \dfrac{b-a}{1+ab}$$

21. If $f(x) = \dfrac{1}{1-x}$, then prove that : $f[f\{f(x)\}] = x$

22. If $(x) = \tan x$, the prove that : $f(x) + f(-x) = 0$

23. If $f(x) = x + \dfrac{1}{x}$, then prove that :

$$\{f(x)\}^3 = f(x^3) + 3 \cdot f\left(\dfrac{1}{x}\right)$$

24. If $y = f(x) = \dfrac{ax - b}{bx - a}$, the prove that : $x = f(y)$

25. If $f(x) = \sin x + \cos^2 x$, then prove that:

$$f(x) = f(\pi - x)$$

26. If $f(x) = \dfrac{1 - x^2}{1 + x^2}$, then prove that:

$$f(\tan \theta) = \cos 2\theta$$

27. If $f(x) = x^2 + x + 1$, then find the value of 'x' for which f(x-1) =f(x)

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28. If $f(x) = \log_e x$, then prove that : $f(xyz) = f(x) + f(y) + f(z)$

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29. If $f(x) = \log_e x$ and $g(x) = e^x$, then prove that :

$f(g(x)\} = g\{f(x)\}$

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30. If $f(x) = \sqrt{\dfrac{1-x}{1+x}}$, then evalaute : $f(\cos 2\theta)$

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31. If $f(x) = \log\dfrac{1+x}{1-x}$, then prove that:

$f\left(\dfrac{2x}{1+x^2}\right) = 2f(x)$

32. If $f: R \to R$ is defined as:

$$f(x) = \begin{cases} 2x + 1 & \text{if } x > 2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 2x & \text{if } x < -2 \end{cases}$$

then evaluate the following:

(i) $f(1)$

(ii) $f(5)$

(iii) $f(-3)$

33. If $f(x) = \cos(\log x)$ then $f(x)f(y) - \dfrac{1}{2}\left[f\left(\dfrac{x}{y}\right) + f(xy)\right]$ has the value

34. If $f(x) = 3\cos x$ and $g(x) = \sin^2 x$, the evaluate:

$(f + g)\left(\dfrac{\pi}{2}\right)$

35. If $f(x) = x^2$ and $g(x) = 2x$, then evaluate,

(i) $(f + g)(3)$ $(ii)(f - g)(2)$

(iii) $(f \cdot g)(1)$ $(iv)\left(\dfrac{f}{g}\right)(5)$

Exercise 2 D

1. Find the domain of the following functions:

$\dfrac{1}{x - 3}$

2. Find the domain of the following functions:

$$\frac{1}{x + 1}$$

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3. Find the domain of the following functions:

$$\frac{x - 2}{x + 4}$$

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4. Find the domain of the following functions:

$$\frac{1}{x^2 - 4}$$

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5. Find the domain of the following functions:

$$\sqrt{x - 4}$$

6. Find the domain of the following functions:

$$\frac{1}{\sqrt{x - 6}}$$

7. Find the domain of the following functions:

$$\frac{1}{\sqrt{4 - x}}$$

8. Find the domain of the following functions:

$$\sqrt{(x - 2)(4 - x)}$$

9. Find the domain of the following functions:

$$\frac{1}{\sqrt{25 - x^2}}$$

A. $x > 5$ or $x < -5$

B. $x > 4$ or $x < -5$

C. $x > -5$ or $x < 5$

D. $x > 3$ or $x < -5$

Answer: [-5 , 5]

10. Find the domain of the following functions:

$$\frac{x^2 + 5x + 30}{x^2 - 8x + 7}$$

11. Find the domain and range of the following functions:

$$\frac{x - 3}{2 - x}$$

12. Find the domain and range of the following functions:

$$\frac{1}{\sqrt{x - 3}}$$

13. Find the domain and range of the following functions:

$$\frac{x^2}{1 + x^2}$$

    A. Domain = R ,   Range = [0,1)

    B.

    C.

    D.

Answer: Domain = R ,   Range = [0,1[

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14. Find the domain and range of the following functions:

$$\frac{x^2 - 4}{x - 2}$$

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15. Find the domain and range of the real function $f(x) = \sqrt{9 - x^2}$

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16. Find the domain and range of the following functions:

$$\sqrt{2 - x}$$

17. Find the domain and range of the following functions:

$$\frac{1}{\sqrt{x}}$$

18. Find the domain and range of the following functions:

$$\sqrt{1 - x^2}$$

Exercise 2 E

1. If $f(x) = x$ and $g(x) = |x|$, then define the following functions:

$(i) f + g$ $(ii) f - g$

$(iii) f \cdot g$ $(iv) \dfrac{f}{g}$

2. Find the domain of $f(x) = \dfrac{1}{x - |x|}$, when $x \in R$

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3. Find the domain of the function $\log\left|4 - x^2\right|$.

A. $(-\infty, \infty)$

B. $(-\infty, 2)$

C. $(-2, \infty)$

D. $(-\infty, \infty) - \{-2, 2\}$

Answer: D

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4. Find the domain and range of $f(x) = -|x|$.

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5. Find the domain and range of $f(x) = |x - 1|$

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1. If f = {(2,3) ,(3,4) , (4,5}, then its inverse is :

A. {(3,4) , (4,5) , (3,2)}

B. {(3,2) , (4,3) , (5,4)}

C. {(2,3) , (4, 3) , (5,4)}

D. None of these

Answer: b

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2. If $f(x) = \dfrac{x+1}{x-1}$, then the value of f{f(3)} is :

A. 3

B. 2

C. 6

D. None of these

Answer: a

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3. If $f(x) = \dfrac{x^2}{1+x^2}$, then the value of f{f(2)} is :

A. $\dfrac{9}{41}$

B. $\dfrac{25}{41}$

C. $\dfrac{16}{25}$

D. $\dfrac{16}{41}$

4. If f(x) = x - $\dfrac{1}{x}$, then the value of f(x) + $f\left(\dfrac{1}{x}\right)$ is :

A. 0

B. 2x

C. $\dfrac{2}{x}$

D. None of these

5. If f(x) = $\log_e\left(\dfrac{1-x}{1+x}\right)$, then $f\left(\dfrac{2x}{1+x^2}\right)$ is equal to :

A. $[f(x)]^2$

B. 2f(x)

C. 4f(x)

D. None of these

Answer: b

6. The domain of f(x) = $\dfrac{x^2}{x^2 - 3x + 2}$ is :

A. R

B. R - {1}

C. R - {1,2}

D. R - {-1,-2}

Answer: c

7. The range of f(x) = $\dfrac{|x|}{x}$, $x \neq 0$ is :

A. [0,1]

B. [-1,1]

C. (-1,1)

D. {-1,1}

Answer: D

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8. The domain of f(x) = $\dfrac{1}{\sqrt{x-3}}$ is :

A. $(-\infty, 3)$

B. $(3, \infty)$

C. $[3, \infty)$

D. None of these

Answer: b

9. The range of function f(x) = $|x + 2|$ is :

A. R

B. $[0, \infty[$

C. $] - \infty, 0]$

D. $]0, \infty[$

Answer: b

10. If f(x) = $\dfrac{1}{1 - x}$, $x \neq 1$ and g(x) = $\dfrac{x - 1}{x}$, $x \neq 0$, then the value of g[f(x)]

is :

A. $-x$

B. x

C. 2x

D. None of these

Answer: b

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Exercise 2 G

1. If $f(x) = \cos(\log x)$ then $f(x)f(y) - \dfrac{1}{2}\left[f\left(\dfrac{x}{y}\right) + f(xy)\right]$ has the value

A. 0

B. $f(x + y)$

C. $\dfrac{1}{2} \cdot f(x)f(y)$

D. None of these

Answer: a

2. If $f(x) = (a - x^n)^{1/n} m$ where $a > 0$ and $n \in N$, then of $fof(x)$ is equal to:

A. x

B. a

C. x^n

D. None of these

Answer: a

3. If $f(x) = |x + 1|$ then the true statement from the following is :

A. $f(x^2) = \{f(x)\}^2$

B. $f(x + y) = f(x) + f(y)$

C. $f(|x|) = |f(x)|$

D. None of these

Answer: d

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4. If $f(x) = 3x + |x|$, then the value of $f(3x) + f(- x) - f(x)$ is:

A. $3(x + |x|)^2$

B. $3(x + |x|)$

C. $(x - |x|)^3$

D. None of these

Answer: b

5. If $A = \left\{ x : \dfrac{\pi}{6} < x < \dfrac{\pi}{3} \right\}$ and $f(x) = \cos x - x(1+x)$, then $f(A)$ is equal to :

A. $\left[(\pi), (6), \dfrac{\pi}{3} \right]$

B. $\left[\dfrac{-\pi}{3}, \dfrac{-\pi}{6} \right]$

C. $\left[\dfrac{1}{2} - \dfrac{\pi}{3}\left(1 + \dfrac{\pi}{3}\right), \dfrac{\sqrt{3}}{2} - \dfrac{\pi}{6}\left(1 + \dfrac{\pi}{6}\right) \right]$

D. None of these

Answer: c

6. If $f(x) = \dfrac{3x + 2}{5x - 3}$, then $f[f(x)]$ is equal to:

A. $-x$

B. x

C. 0

D. None of these

Answer: b

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7. If $f(x) = a^x$, then the false statement from the following is:

A. $f(-x) \cdot f(x) = 1$

B. $f(x + 3) - 2f(x + 2) + f(x + 1) = (a - 2)^2 f(x + 1)$

C. $f(x + y) = f(x) \cdot f(y)$

D. $f(x) \div f(y) = f\left(\dfrac{x}{y}\right)$

Answer: b

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8. If $f(x) = 10x - 7$ and $(fog)(x) = x$, then $g(x)$ is equal to:

A. $\dfrac{x + 7}{10}$

B. $\dfrac{x - 7}{10}$

C. $\dfrac{1}{10x - 7}$

D. None of these

Answer: a

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9. If $f(x) = \cos(\log x)$ then $f(x)f(y) - \dfrac{1}{2}\left[f\left(\dfrac{x}{y}\right) + f(xy)\right]$ has the value

A. $\dfrac{1}{2}$

B. -1

C. -2

D. None of these

Answer: d

10. If $f(x) = \log\left(\dfrac{1 + x}{1 - x}\right) and g(x) = \left(\dfrac{3x + x^3}{1 + 3x^2}\right)$, then $f(g(x))$ is equal to (a)$f(3x)$ (b) $\{f(x)\}^3$ (c) $3f(x)$ (d) $- f(x)$

A. $- f(x)$

B. $3f(x)$

C. $[f(x)]^2$

D. None of these

Answer: b

Exercise 2 1

1. If $\left(\dfrac{x}{3} + 1, y - \dfrac{2}{3} \right) = \left(\dfrac{5}{3}, \dfrac{1}{3} \right)$, find the values of x and y.

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2. If the set A has 3 elements and the set $B = \{3, 4, 5\}$, then find the number of elements in $(A \times B)$.

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3. If $G = \{7, 8\}$ and $H = \{5, 4, 2\}$, find $G \times H$ and $H \times G$.

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4. Find whether the following statements are true or false. If the statement is false, then write its correct statement:

(i) If P={m,n} and Q={n,m},then $P \times Q = \{(m, n), (n, m)\}$.

(ii) If A and B are non-empty sets, then $A \times B$ is a non-empty set of the ordered pairs (x,y) such that $x \in A$ and $y \in B$.

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5. If $A = \{ -1, 1 \}$ find $A \times A \times A$.

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6. If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B.

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7. Let A={1,2},B={1,2,3,4},C={5,6}and D={5,6,7,8}. Verify that:

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

(ii) $A \times C$ is a subset of $B \times D$.

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8. Let $A = \{1, 2\} and B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have? List them.

9. Let A and B be two sets such that $n(A) = 3 \, and \, n(B) = 2$. if $(x, 1), (y, 2), (z, 1) are \in A \times B$, find $A \, and \, B, \, where \, x, y, z$ are distinct elements.

10. The Cartesian product $A \times A$ has 9 elements among which are found $(-1, \quad 0) \quad and \quad (0, \quad 1)$. Find the set A and the remaining elements of $A \times A$.

Exercise 2 2

1. Let $A = \{1, 2, 3, 14\}$. Define a relation on a set A by $R = \{(x, y): 3x - y = 0. \, where \, x, y \in A\}$. Depict this relationship using an arrow diagram. Write down its domain, co-domain and range.

2. Define a relation R on the set N of natural numbers by $R = \{(x, \quad y) \quad : \quad y \quad = \quad x \quad + \quad 5$, x is a natural number less than 4; x, $y \in N$ }. Depict this relationship using roster form. Write down the domain and the range.

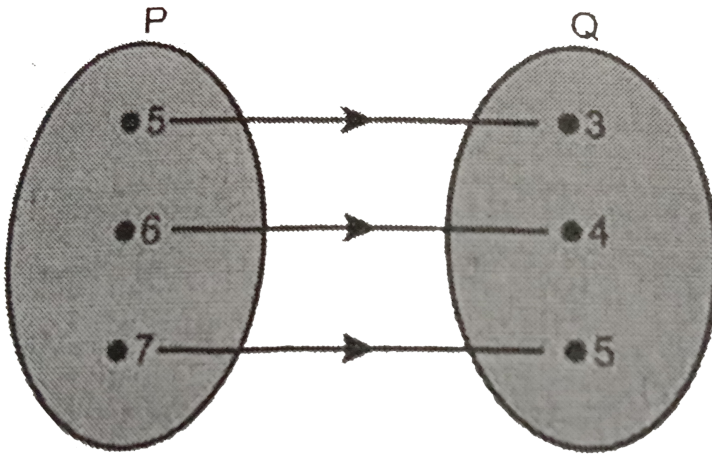**3.** A = { 1,2,3,5} and B = {4,6,9} A relation R is defined from A to B by R = { (x,y) : the difference between x & y is odd}. Writer R in roster form.

4. The figure shows a relationship between the sets P and Q. Write this relation :

(i) in set-builder from

(ii) Roaster from. What is its domain and range?

5. Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\{(ab) : a, b \in A, b$ is exactly divisible by a$\}$.(i) Write R in roster form(ii) Find the domain of R(iii) Find the range of R.

6. Determine the domain and range of the relation R defined by

$R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}.$

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7. Write the relation $R = \{(x, x^3) : x$ is a prime number less than 10} in

roster form.

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8. Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A

to B.

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9. Let R be the relation on Z defined by $R = \{(a, b) : a, b \in Z, a$ is an

integer}.Find the domain and range of R.

Exercise 2 3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.(i) $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$(ii) `$\{(2, 1), (4, 2), (6, 3), (8, 4),$

2. Find the domain and range of the following real functions:(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9 - x^2}$

3. A function f is defined by $f(x) = 2x$ 5 . Write down the values of (i) $f(0)$, (ii) $f(7)$, (iii) $f(3)$.

4. The function t which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \dfrac{9C}{5} + 32$. Find (i) t (0) (ii) t(28) (iii) $t(\quad 10)$ (iv) The value of C, when $t(C) \quad = \quad 212$.

5. Find the rage of each of the following functions. (i) $f(x) \quad = \quad 2 \quad 3x, \quad x \in R, x \quad > \quad 0$ (ii) $f(x) \quad = x^2 + \quad 2$, x is a real number. (iii) $f(x) \quad = \quad x$, x is a real number.

Miscellaneous Exercise

1. The relation f is defined by $f(x) = \{x^2, 0 \le x \le 33x, 3 \le x \le 10$The relating g is defined by $g(x) = \{x^2, 0 \le x \le 33x, 2 \le x \le 10$Show that

f is a function and g is not a function.

2. If $f(x) = x^2$, $f \in d \dfrac{f(1.1)f(1)}{(1.1) - 1}$.

3. Find the domain of each of the following real valued functions of real variable: $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$

4. Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x - 1)}$.

5. Find the domain and the range of the real function/defined by

$$f(x) = |x - 1|$$

6. Let $f = \left\{ \left(x, \dfrac{x^2}{1 + x^2} \right) : x \in R \right\}$ be a function from R into R. Determine the rage of f.

7. Let f, g : R $\to$ R be defined, respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\dfrac{f}{g}$.

8. Let $f = \{(1, 1), (2, 3), (0, 1), (1, 3)\}$ be a function from $Z \to Z$ defined by $f(x) = ax + b$, for some integers a, b.

Determine a, b.

9. Let R be a relation from N to N defined by $R = \{(a, b) : a\dot b \in N$ and $a = b^2)$. Are the following true?(i) $(a, a) \in R, f$ or $all a \in N$(ii) $(a, b) \in R, implies(b, a) \in R$(iii) `(a ,

10. If A={1,2,3,4},B={1,5,9,11,15,16} and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$ are the following statements true ?

(i) f is a relation from A to B .

(ii) f is a function from A to B

Justify your answer.

11. Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z to Z? Justify your answer.

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12. Let $A = \{9, 10, 11, 12, 13\}$ and let $f: A \to N$ be defined by f(n) = the highest prime factor of n. Find the range of f.

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