

MATHS

BOOKS - S CHAND MATHS (ENGLISH)

COMPLEX NUMBER

Example

1. If x =1 +i, then the value of $x^6+x^4+x^2+1$ is

A. 6i-3

B. -6i + 3

 $\mathsf{C.}-6i-3$

D. 6i + 3

Answer: c



Watch Video Solution

2. If $\left(-2-rac{1}{3}i
ight)^3=rac{x+iy}{27}, xy, \ \in R$ then y-x equals

A. 91

B. 85

C. - 85

D. - 91

Answer: a



3. The argument of $\dfrac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is 60^0 b. 120^0 c. 210^0 d. 240^0

A.
$$\frac{\pi}{3}$$

$$\mathrm{B.}\ \frac{2\pi}{3}$$

$$\mathsf{C.} - \frac{2\pi}{3}$$

D. $\frac{4\pi}{2}$

Answer: c



Watch Video Solution

4. If $z=\frac{(1+i)^2}{\alpha-i}$, $\alpha\in R$ has magnitude $\sqrt{\frac{2}{5}}$ then the value of α is (i) 3 only (ii) -3 only (iii) 3 or -3 (iv) none of these

A. 3 only

- B.-3 only
- C. 3 or -3
- D. none of these

Answer: c



- **5.** If z is a non -zero complex number then $\dfrac{\left|\bar{z}\right|^2}{z\bar{z}}$ is equal to
 - A. $\frac{|z|}{|z|}$
 - B. $|ar{z}|$
 - C. z
 - D. none of these

Answer: a



Watch Video Solution

6. The complex number z which satisfies the condition

$$\left|rac{i+z}{i-z}
ight|=1$$
 lies on the

A. y -axis

B. x-axis

C. line x+y=1

D. circle $x^2 + y^2 = 1$

Answer: B



7. If $1,\omega,\omega^2$ are cube roots of unity then the value of $\left(5+2\omega+5\omega^2\right)^3$ is

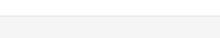
$$B. - 9$$

$$\mathsf{C.}-27$$

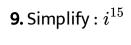
$$\mathsf{D.}-81$$

Answer: c

8. Simplify : i^{38}









Watch Video Solution

10. Simplify: i^{-6}



11. Simplify: $\frac{1}{i}$



Watch Video Solution

12. Show that i is neither 0, nor greater than 0, nor less than 0



13. Simplify the following

$$(5i) \times 7$$



Watch Video Solution

14. Simplify the following

(3i)(4i)



Watch Video Solution

15. Simplify the following

 $\frac{21}{14i}$



16. Simplify the following

$$\frac{5}{i^3}$$



Watch Video Solution

17. Simplify the following

$$\sqrt{-9} + \sqrt{-16}$$



Watch Video Solution

18. Simplify the following

$$\frac{21}{4}\sqrt{-48} - 5\sqrt{-27}$$



19. Simplify the following

$$\sqrt{-18}$$
. $\sqrt{-2}$



Watch Video Solution

20. Simplify the following

$$\frac{20}{\sqrt{-5}}$$



21. Evaluate: $\left[i^{18} + \left(rac{1}{i} ight)^{25} ight]^3$



22. Find the values of x and y if $2x+4iy={}-i^3x-y+3$



23. Write the values of x and y if (3-4i)(x+yi)=1+i(0)



24. Represent the following complex numbers in the complex plane

2+3i



25. Represent the following complex numbers in the complex plane





26. Represent the following complex numbers in the complex plane

0 + 0i



27. Represent the following complex numbers in the complex plane

i

28. Can two different points in the complex plane represent the same complex number? Give reasons for you answer



29. Express the following in the form a+bi

$$(8+7i)(8-7i)$$



30. Express the following in the form a+bi

$$(5-6i)^2$$



31. Express the following in the form a+bi

$$(2+3i)(3+7i)$$



32. Express the following in the form a+bi

$$\bigg(-2-rac{1}{3}i\bigg)^3$$



Watch Video Solution

33. Express the following in the form a+bi

$$(1-i)^4$$



34. Express the following in the form a+bi

$$\left(\sqrt{3}+5i
ight)\left(\sqrt{3}-5i
ight)^2+\left(\,-4+5i
ight)^2$$



35. If $\left(x+yi ight)^3=u+vi$, prove that $\dfrac{u}{x}+\dfrac{v}{y}=4ig(x^2-y^2ig)$



36. If $x=-5+\sqrt{-16}$, find the value of $x^4+9x^3+35x^2-x+4$

37. Express the following in the form a+bi, where a and b are real numbers

$$\frac{1+i}{1-i}$$



Watch Video Solution

38. Express the following in the form a+bi, where a and b are rea numbers

$$\left(\frac{1-i}{1+i}\right)^2$$



Watch Video Solution

39. Express the following in the form a+bi, where a and b are real numbers

$$\left(rac{1+i}{1-i}
ight)^3$$



40. Express the following in the form a+bi, where a and b are real numbers

$$rac{\left(3+\sqrt{5}i
ight)\!\left(3-\sqrt{5}i
ight)}{\left(\sqrt{3}+\sqrt{2}i
ight)-\left(\sqrt{3}-\sqrt{2}i
ight)}$$



41. Find the multiplicative inverse of $\frac{3+4i}{4-5i}$



42. Find the smallest positive integer n, for which
$$\left(\frac{1+i}{1-i}\right)^n=1$$

43. If
$$a+bi=rac{c+i}{c-i}, a,b,c\in R$$
, show that $a^2+b^2=1$ and $rac{b}{a}=rac{2c}{c^2-1}$

the

equation



$$rac{(1+i)x-2i}{3+i} + rac{(2-3i)y+i}{3-i} = i, xy \in R, i = \sqrt{-1}$$

Solve

44.



45. Write the conjugate of

 $\sqrt{-16} - 3$

46. Write the conjugate of

$$i^7$$



Watch Video Solution

47. Write the conjugate of

$$(3+4i)^2$$



Watch Video Solution

48. Write the conjugate of

$$\sqrt{-25}\big(7+\sqrt{-576}\big)$$



49. Write the conjugate of

$$\frac{1-i}{1+i}$$



Watch Video Solution

50. Find the real numbers x and y if (x-yi)(3+5i) is the conjugate of -6-24i



51. Given $z_1=1-i, z_2=-2+4i$, calculate the values of a and b if $a+bi=rac{z_1z_2}{z_1}$



52. If z be a non-zero complex number, show that $\left(\overline{z^{-1}}\right) = (\bar{z})^{-1}$



53. If
$$z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{107}$$
 , then show that Im(z)=0



54. If
$$\dfrac{\left(a+i
ight)^2}{2a-i}=p+qi$$
, show that $p^2+q^2=\dfrac{\left(a^2+1
ight)^2}{4a^2+1}$



55. Find the modulus of the following complex numbers

 $8 - 6i^{7}$



Watch Video Solution

56. Find the modulus of the following complex numbers

$$\frac{2+3i}{3+2i}$$



Watch Video Solution

57. Find the modulus of the following complex numbers

$$\frac{\left(3+2i\right)^2}{\left(4-3i\right)}$$



58. Find the modulus of the following complex numbers

$$(3+2i)(5-4i)$$



Watch Video Solution

59. Find the modulus of the following complex numbers

$$\frac{1+i}{1-i} - \frac{1-i}{1+i}$$



60. If z_1 and z_2 are two complex numbers such that

 $|z_1|=|z_2|$, then is it necessary that $z_1=z_2$



61. Show that the points representing the complex numbers (3+3i), (-3-3i) and $(-3\sqrt{3}+3\sqrt{3}i)$ on the Argand plane are the vertices of an equilateral triangle



62. If |z|=1, then prove that $\frac{z-1}{z+1}(z\neq -1)$ is a purely imaginary number. What is the conclusion if z=1?



63. Solve the equation 2z=|z|+2i is complex numbers



64. If
$$z=x+yi$$
 and $\frac{|z-1-i|+4}{3|z-1-i|-2}=1$, show that $x^2+y^2-2x-2y-7=0$



65. Prove that
$$\left|z_{1}+z_{2}\right|^{2}+\left|z_{1}-z_{2}\right|^{2}=2{\left|z_{1}\right|}^{2}+2{\left|z_{2}\right|}^{2}$$



66. For any two complex numbers z_1 and z_2 and any real numbers a and b, prove that $\left|az_1-bz_2\right|^2+\left|bz_1+az_2\right|^2=\left(a^2+b^2\right)\left\lceil\left|z_1\right|^2+\left|z_2\right|^2\right\rceil$



67. Prove that the representative points of the complex numbers $1+4i,\, 2+7i,\, 3+10i$ are collinear



Watch Video Solution

68. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, zi and z+zi is $=rac{1}{2}|z|^2$



69. The complex numbers z_1, z_2 and the origin are the vertices of an equilateral triangle in the Argand plane Prove that $z_1^2+z_2^2=z_1,z_2$



70. If z_1 and z_2 are complex numbers of two points, then prove that the complex number of the point, which divides the distance between them internally in the ratio $l\!:\!m$ is given by lz_2+mz_1



l+m

Watch Video Solution

71. If z_1, z_2, z_3 are three complex numbers representing three vertices of a triangle, then centroid of the triangle be $\frac{z_1+z_2+z_3}{3}$



Watch Video Solution

72. If the complex numbers $z_1,\,z_2,\,z_3$ represent the vertices of an equilateral triangle, and $|z_1|=|z_2|=|z_3|$, prove that

$$z_1 + z_2 + z_3 = 0$$



73. If z_1, z_2, z_3, z_4 are complex numbers, show that they are vertices of a parallelogram In the Argand diagram if and only if $z_1+z_3=z_2+z_4$



74. If z=4+3i, then verify that

$$|z| = |\bar{z}|$$



75. If z=4+3i, then verify that

$$-|z| \leq Re(z) < |z|$$



76. If z=4+3i, then verify that

$$-|z| < Im(z) \le |z|$$



77. If z=4+3i, then verify that

$$z^{-1} = \frac{\bar{z}}{\left|z\right|^2}$$



78. If $z_1=2+7i \; ext{and} \; z_2=1-5i$, then verify that

$$|z_1z_2|=|z_1||z_2|$$



Watch Video Solution

79. If $z_1=2+7i \ ext{ and } \ z_2=1-5i$, then verify that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$



80. If $z_1 = 2 + 7i$ and $z_2 = 1 - 5i$, then verify that

$$|z_1+z_2| \leq |z_1|+|z_2|$$



81. If $z_1=2+7i \; ext{and} \; z_2=1-5i$, then verify that

$$|z_1 - z_2| > |z_1| - |z_2|$$



82. If $z_1=2+3i$ and $z_2=3+i$ plot the number z_1+z_2 .

Also show that $|z_1|+|z_2|>|z_1+z_2|$



- **83.** Find the modulus of $\frac{(3+2i)(1+i)(2+3i)}{(3+4i)(4+5i)}$
 - **Natch Video Solution**

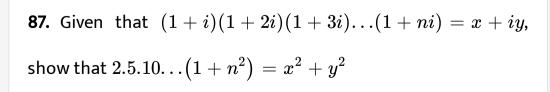


85. If
$$x-yi=rac{a-bi}{c-di}$$
, prove that $x^2+y^2=rac{a^2+b^2}{c^2+d^2}$



86. If
$$x-yi=rac{a-bi}{c-di}$$
, prove that $x^2+y^2=rac{a^2+b^2}{c^2+d^2}$







$$rac{1+i}{1+2^2i} imes rac{1+3^2i}{1+4^2i} imes \ldots imes rac{1+(2n-1)^2i}{1+(2n)^2i} = rac{a+bi}{c+di},$$
 show that $rac{2}{17} imes rac{82}{257} imes \ldots imes rac{(2n-1)^4+1}{(2n)^4+1} = rac{a^2+b^2}{c^2+d^2}$



89. If
$$|z_1|=|z_2|=\ldots=|z_n|=1$$
, prove that $|z_1+z_2+\ldots+z_n|=\left|rac{1}{z_1}+rac{1}{z_2}+\ldotsrac{1}{z_n}
ight|$



90. Find the greatest value of the moduli of complex numbers z satisfying the equation $\left|z-\frac{4}{z}\right|=2$. What is the minimum



Watch Video Solution

91. If z=x+yi and $\omega=rac{1-zi}{z-i}$ show that $|\omega|=1\Rightarrow z$ is purely real



Watch Video Solution

92. Find the modulus and amplitude of

$$\frac{-1}{2} + \frac{\sqrt{3}}{2}i$$



- **93.** Find the modulus and amplitude of
- -4i
 - Watch Video Solution

- **94.** Find the modulus and amplitude of $\dfrac{2+3i}{3+2i}$
 - Watch Video Solution

- **95.** Find the modulus and amplitude of $\frac{2+i}{4i+(1+i)^2}$.
 - Watch Video Solution

96. Represent the complex numbers

 $z=1+\sqrt{3}i$ into polar form



97. Represent the complex numbers

$$2-2i$$



 $rac{1+7i}{\left(2-i
ight)^2}$ in polar form



98. Represent the complex numbers

99. Change the complex number $4(\cos 300^{\circ} + i \sin 300^{\circ})$ to cartesian form



100. Represent on complex plane the complex numbers w=3+4i and z=6-3i together with w+z and w-z. Obtain the modulus and argument of w and z.



Watch Video Solution

101. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$$|z-z_1|+|z-z_2|=|z_1-z_2|$$



102. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$$|z-z_1| = |z-z_2|$$



103. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$$|z-z_1|=k|z-z_2|, k\in R^+, k
eq 1$$



104. If $z_1 \ {
m and} \ z_2$ are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$$|z-z_1|+|z-z_2|$$
= constant $\neq (|z_1-z_2|)$



find the locus of a point z in each of the following

105. If z_1 and z_2 are two fixed points in the Argand plane, then

106. If z_1 and z_2 are two fixed points in the Argand plane, then

$$|z-z_1|-|z-z_2|= {\sf constant}\ (\
eq |z_1-z_2|)$$

find the locus of a point z in each of the following



 $|z-z_1|-|z-z_2|=|z_1-z_2|$

107. Illustrate in the complex plane the following set of points and explain your answer

|Z| = 3



108. Illustrate in the complex plane the following set of points and explain your answer



Watch Video Solution

109. Illustrate in the complex plane the following set of points and explain your answer

$$|z - 4| < 1$$



Watch Video Solution

110. Illustrate in the complex plane the following set of points and explain your answer

 $\operatorname{arg}\left(Z
ight)=rac{\pi}{\kappa}$



Watch Video Solution

111. Illustrate in the complex plane the set of points z satisfying

$$|z+i-2|\leq 2$$



Watch Video Solution

112. A variable complex number z is such that the amplitude of

$$\frac{z-1}{z+1}$$
 is always equal to $\frac{\pi}{4}$

Illustrate the locus of z in the Argand plane



113. Find the locus of a complex number z=x+yi satisfying the relation arg $(z-a)=rac{\pi}{4}, a\in R$



Watch Video Solution

Illustrate the locus of z in the Argand plane

114. Given $z_1 = 1 + 2i$. Determine the region in the complex plane represented by $1 < |z-z_1| \le 3$. Represent it with the help of an Argand diagram



variable

Watch Video Solution

115. Find the radius and centre of the circle $zar{z}-(2+3i)z-(2-3i)ar{z}+9=0$ where z is a complex **116.** If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

A. either on the real axis or on a circle passing through the origin

B. on a circle with centre at the origin

C. either on the real axis or on a circle not passing through the origin

D. on the imaginary axis

Answer: A



117. Find the square root of the complex number

5 + 12i



Watch Video Solution

118. Find the square root of the complex number

-4-3i



Watch Video Solution

119. Find the square root of the complex number

18i



120. If $1, \omega, \omega^2$ are the cube roots of unity, prove that

$$(1+\omega)^3 - \left(1+\omega^2\right)^3 = 0$$



Watch Video Solution

121. If $1, \omega, \omega^2$ are the cube roots of unity, prove that

$$(x-y)(x\omega-y)(x\omega^2-y)=x^3-y^3$$



122. If $1, \omega, \omega^2$ are three cube roots of unity, show that $(a+\omega b+\omega^2 c)(a+\omega^2 b+\omega c)=a^2+b^2+c^2-ab-bc-ca$



123. If lpha and eta are the complex cube roots of unity, then prove that $(1+lpha)(1+eta)(1+lpha)^2(1+eta)^2=1$



Watch Video Solution

124. If, $1,\omega,\omega^2$ are cube roots of unity, show that $\frac{p+q\omega+r\omega^2}{r+p\omega+q\omega^2}=\omega^2$



125. Given that $1,\omega,\omega^2$ are cube roots of unity. Show that $\left(1-\omega+\omega^2\right)^5+\left(1+\omega-\omega^2\right)^5=32$



126. If $1,\omega,\omega^2$ are cube roots of unity, prove that $(x+y)^2+\left(x\omega+y\omega^2\right)^2+\left(x\omega^2+y\omega\right)^2=6xy$



127. If $x=a+b, y=a\alpha+b\beta, z=a\beta+b\alpha$, where α and β are complex cube roots of unity, then show that $xyz=a^3+b^3$



128. If $x=a+b, y=a\omega^2+b\omega, z=a\omega+b\omega^2$, then show that $x^3+y^3+z^3=3(a^3+b^3)$



$$\sqrt{\left[-1\sqrt{\left\{-1-\sqrt{-1+... ext{to}\infty}
ight\}}
ight]}=\omega, \ ext{or} \ \omega^2$$



Watch Video Solution

Multiple Choice Questions

1. If 8x+i(2x-y)=3-8i and $x,y\in R$ then the values of x and y are

A.
$$x = \frac{3}{8}, y = \frac{35}{4}$$

B.
$$x = -\frac{3}{8}, y = \frac{35}{4}$$

C.
$$x = \frac{3}{8}, y = -\frac{35}{4}$$

D.
$$x = -\frac{3}{8}, y = -\frac{35}{4}$$

Answer: A



Watch Video Solution

- **2.** The value of $1+i+i^2+\ldots+i^n$ is (i) positive (ii) negative (iii) 0 (iv) cannot be determined
 - A. positive
 - B. negative
 - C. 0
 - D. cannot be determined

Answer: D



3. If z = x + iy satisfies |z+1|=1 then

A. (a) x=0

B. (b) $(x-1)^2 + y^2 = 1$

C. (c) y=0

D. (d) $(x+1)^2 + y^2 = 1$

Answer: D



- **4.** If z = x + iy satisfies |z+1-i|=|z-1+i| then
 - A. (a) y=x
 - B. (b) y=-x
 - C. (c) x-y+1=0

Answer: A



Watch Video Solution

- **5.** Number of solutions of the equation $z^2 + \left|z\right|^2 = 0$ is (i) 1 (ii)
- 2 (iii) 3 (iv) infinitely many
 - A. 1
 - B. 2
 - C. 3
 - D. infinitely many

Answer: D



6. The amplitude of
$$\sin \frac{\pi}{5} + i \Big(1 - \cos \frac{\pi}{5} \Big)$$
 is

A. (a)
$$\frac{2\pi}{5}$$

B. (b)
$$\frac{\pi}{5}$$

C. (c)
$$\frac{\pi}{15}$$

D. (d)
$$\frac{\pi}{10}$$

Answer: D



Watch Video Solution

7. The multiplicative inverse of 3+4i is

A. (a) 3-4i

B. (b) $\frac{3+4i}{25}$

D. (d) none of these

Watch Video Solution

8. If $z = \overline{z}$ then z lies on

A. (a) x - axis

C. (c) origin

Answer: A

Watch Video Solution

C. (c) $\frac{3-4i}{25}$

D. (d) -3 + 4i

Answer: C

9. The principal argument $\left(1+i\sqrt{3}\right)^2$ is

A.
$$\frac{\pi}{3}$$

$$\mathsf{B.}-\frac{\pi}{3}$$

$$\operatorname{C.}\frac{2\pi}{3}$$

$$\mathrm{D.}-\frac{2\pi}{3}$$

Answer: C



Watch Video Solution

10. The polar form of $1+i\sqrt{3}$ is

A.
$$2\Bigl(\cos{\pi\over 6}+i\sin{\pi\over 6}\Bigr)$$

B.
$$2\Bigl(\cosrac{\pi}{3}+i\sinrac{\pi}{3}\Bigr)$$

$$\mathsf{C.}\,2\bigg(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\bigg)$$

D.
$$2\Bigl(\cos\Bigl(-rac{\pi}{6}\Bigr)+i\sin\Bigl(-rac{\pi}{6}\Bigr)\Bigr)$$

Answer: B



Watch Video Solution

are conjugate to each other for

11. The complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$

B.
$$x = \left(n + \frac{1}{2}\right)\frac{\pi}{2}$$

D. no value of x

Answer: D



Watch Video Solution

12. The real value of lpha for which the expression $\dfrac{1-i\sinlpha}{1+2i\sinlpha}$ is purely real is

A.
$$(n+1)rac{\pi}{2}n, \ \in N$$

B.
$$(2n+1)rac{\pi}{2}n, \ \in N$$

C.
$$n\pi, n \in N$$

D. none of these

Answer: C



13. If z =x +iy lies in the third quadrant then $\frac{\overline{z}}{z}$ also lies in third quadrant if (i) x>y>0 (ii) x< y<0 (iii) y< x<0 (iv) y > x > 0

A.
$$x > y > 0$$

B.
$$x < y < 0$$

C.
$$y < x < 0$$

D.
$$y > x > 0$$

Answer: C



Watch Video Solution

14. The value of $(z+3)(ar{z}+3)$ is equal to (i) $|z+3|^2$ (ii)

$$|z-3|$$
 (iii) z^2+3 (iv) none of these

A.
$$|z + 3|^2$$

B.
$$|z-3|$$

$$c. z^2 + 3$$

D. none of these

Answer: A



15. If
$$\left(\frac{1+i}{1-i}\right)^x=1\,orall\,n\in N$$
 is (i) x = 2n+1 (ii) x =4n (iii) x=2n (iv) x=4n+1

A.
$$x = 2n+1$$

B.
$$x = 4n$$

Answer: B



Watch Video Solution

16. The argument of $\frac{1+i}{1-i}$ is (i) 0 (ii) $-\frac{\pi}{2}$ (iii) $\frac{\pi}{2}$ (iv) π

A. 0

$$\mathsf{B.}-\frac{\pi}{2}$$

C.
$$\frac{\pi}{2}$$

D.
$$\pi$$

Answer: C



17. If $(1+2i)(2+3i)(3+4i) = x+iy, x,y \in R$ then x^2+y^2 is

A. 1450

B. 1625

C. 1575

D. 1725

Answer: B



Watch Video Solution

18. The polar form of $\sin 75^{\circ} + i \cos 75^{\circ}$ is

A. (a) $\sin 75^{\circ} + i \cos 75^{\circ}$

B. (b) $\sin 15^{\circ} + i \cos 15^{\circ}$

C. (c) $\cos 15^{\circ} + i \sin 15^{\circ}$

D. (d) $\cos 75^{\circ} + i \sin 75^{\circ}$

Answer: C



Watch Video Solution

19. The modulus of $\frac{(1+2i)(3-4i)}{(4+3i)(2-3i)}$ is

$$\frac{13}{13}$$

D. - 1

C. 1

Answer: A



20. If a +ib =
$$\frac{(x+i)^2}{2x-1}$$
 then a^2+b^2 is equal to

A.
$$\dfrac{\left(x+1\right)^4}{4x^2-1}$$

B.
$$\frac{\left(x^2+1\right)^2}{\left(2x-1\right)^2}$$

C.
$$\frac{(x+1)^4}{4x^2+1}$$

D.
$$\frac{(x+1)^2}{4x^2-1}$$

Answer: C



Watch Video Solution

21. If z=x +iy is purely real number such that x < 0 then arg (z)

is

- A. 0
- $\mathsf{B.}\;\frac{\pi}{2}$
- C. π
- $\mathsf{D.}-\pi$

Answer: C



- 22. If z is a purely imaginary number then arg (z) may be
 - A. 0 or π
 - B. $-\pi$ or 0
 - $\mathsf{C}.\,\pi\,\,\mathrm{or}\,\,\pi$
 - D. $-\frac{\pi}{2}$ or $\frac{\pi}{2}$

Answer: D



Watch Video Solution

23. If z is a complex number then

A.
$$\left|z^{2}\right|>\left|z\right|^{2}$$

B.
$$|z|^2 > |z^2|$$

$$\mathsf{C.}\left|z^{2}\right|=\left|z\right|^{2}$$

$$\left| z^2 \right| \geq \left| z \right|^2$$

Answer: C



24. If f(z)
$$=$$
 $\frac{7-z}{1-z^2}$ where z=1 +2i then |f(z)| is

A.
$$\frac{|z|}{2}$$

B.
$$|z|$$

D. none of these

Answer: A



$$rac{3-4ix}{3+4ix}=lpha-ieta(lpha,eta\in R)$$
 if $lpha^2+eta^2ig)$ =

$$B.-1$$

$$\mathsf{D.}-2$$

Answer: A



Watch Video Solution

26. The real vaule of θ for which the expression $\dfrac{1+i\cos\theta}{1-2i\cos\theta}$ is a purely imaginary number is

A.
$$n\pi\pmrac{\pi}{6}$$

B.
$$n\pi\pmrac{\pi}{3}$$

C.
$$(2n+1)rac{\pi}{2}$$

D.
$$n\pi\pmrac{\pi}{4}$$

Answer: B



Watch Video Solution

27. The amplitude of $\frac{1}{i}$ is

- A. (a) 0
- B. (b) π
- C. (c) $\frac{\pi}{2}$
- D. (d) $-\frac{\pi}{2}$

Answer: D



28. The amplitude of
$$\dfrac{-2}{1+i\sqrt{3}}$$
 is

A.
$$\frac{\pi}{4}$$

$$\operatorname{B.}\frac{\pi}{3}$$

$$\mathsf{C.}\,\frac{2\pi}{3}$$

 $\mathrm{D.}-\frac{2\pi}{3}$

Answer: C



29. If
$$z = \frac{1}{(2-3i)^2}$$
 then |z| is equal to

A.
$$\frac{1}{13}$$

B.
$$\frac{1}{12}$$

Answer: A

c. $\frac{1}{5}$



Watch Video Solution

30. If z = 1 -cos
$$heta + i \sin heta$$
 then |z| is equal to

A.
$$2\sin\frac{\theta}{2}$$

$$\mathsf{B.}\,2\cos\frac{\theta}{2}$$

$$|C.2|\sin\frac{\theta}{2}|$$

D.
$$2\left|\cos\frac{\theta}{2}\right|$$

Answer: C



31. If
$$z = \frac{1-i}{1+i}$$
 then z^4 equals

A. 1

B.-1

C. 0

D. none of these

Answer: A



32. If x +iy
$$=$$
 $\frac{3+5i}{7-6i}$ then y is equal to

A.
$$\frac{9}{85}$$

B.
$$-\frac{9}{85}$$

c. $\frac{53}{85}$ $\mathrm{D.}-\frac{53}{85}$

Answer: C



Watch Video Solution

- **33.** If $1, \omega, \omega^2$ are cube roots of unity then the value of $\left(3+5\omega+3\omega^2
 ight)^3$ is
 - A. 6

B. 8

- C. 12
 - D. 16

Answer: B



Watch Video Solution

34. If $1,\omega,\omega^2$ are cube roots of unity then the value of $\left(2+2\omega-3\omega^2\right)^3$ is

A. 125

B. - 125

C. 27

D. - 27

Answer: B



1. Express each of the following in the form b or bi, where b is a real number

3i.2



Watch Video Solution

2. Express each of the following in the form b or bi, where b is a real number



i(-i)

3. Express each of the following in the form b or bi, where b is a real number





4. Express each of the following in the form b or bi, where b is a real number

$$5i(-8i)$$
.



5. Express each of the following in the form b or bi, where b is a real number

20i
4



6. Express each of the following in the form b or bi, where b is a real number

$$\sqrt{-25}$$



7. Express the following in the form a+bi,

$$\sqrt{-8}$$



8. Express each of the following in the form b or bi, where b is a real number





Watch Video Solution

9. Express each of the following in the form b or bi, where b is a real number

$$\frac{1}{2}\sqrt{\frac{-3}{4}}$$



Watch Video Solution

10. Express each of the following in the form b or bi, where b is

a real number

$$\frac{6}{-i}$$



11. Express each of the following in the form b or bi, where b is

a real number

$$\sqrt{-144}$$



Watch Video Solution

12. Express each of the following in the form b or bi, where b is

a real number

$$\frac{x}{\cdot}$$

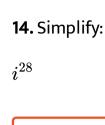


Watch Video Solution

13. Simplify:

 i^{13}







15. Simplify: i^{18}



16. Simplify:

 i^{23}



17. Simplify:

$$\sqrt{-4}+\sqrt{-16}-\sqrt{-25}$$



Watch Video Solution

18. Simplify:

$$\sqrt{-20} + \sqrt{-12}$$



Watch Video Solution

19. Simplify:

$$-\sqrt{\frac{-7}{4}}-\sqrt{\frac{-1}{7}}$$



20. Simplify:

$$\frac{\sqrt{-2}}{\sqrt{2}}$$



Watch Video Solution

21. Simplify:

$$\frac{1}{i} + \frac{1}{i^2} + \frac{1}{i^3} + \frac{1}{i^4}$$



Watch Video Solution

22. Simplify:

$$rac{1}{i} - rac{1}{i^2} + rac{1}{i^3} - rac{1}{i^4}$$



23. Simplify:

$$i+2i^2+3i^3+i^4$$



Watch Video Solution

24. Simplify:

$$\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$$



Watch Video Solution

25. $\sqrt{\frac{-x}{4}} + \sqrt{\frac{-x}{16}} - \sqrt{\frac{-x}{64}}$, where x is a positive real number



26. $\sqrt{-5x^8}-\sqrt{-20x^8}+\sqrt{-45x^8}$, where x is a positive real number



Watch Video Solution

27. If $i=\sqrt{-1}$, prove that following $(x+1+i)(x+1-i)(x-1+i)(x-1-i)=x^4+4$



Exercise B

1. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex

number

3 + 7i, i



Watch Video Solution

2. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

-i, 5+2i



Watch Video Solution

3. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

3i, 1-i

4. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$$-7, -1-3i$$



5. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$$7 + 3i, 3i - 7$$



6. Solve each of the following equation for real x and y:

$$(x+yi) + (3-2i) = 1+4i$$



Watch Video Solution

7. Solve each of the following equations for real x and y:

$$(x+yi) - (7+4i) = 3-5i$$



8. Solve each of the following equations for real x and y:

$$2x + yi = 1 + (2 + 3i)$$



9. Solve each of the following equations for real x and y:

$$x + 2yi = i - (-3 + 5)$$



10. Determine the conjugate and the reciprocal of each complex number given below:



11. Determine the conjugate and the reciprocal of each complex number given below:

 i^3



12. Determine the conjugate and the reciprocal of each complex number given below:

3-i



13. Determine the conjugate and the reciprocal of each complex number given below:

$$\sqrt{-1} - 3$$



14. Determine the conjugate and the reciprocal of each complex number given below:

$$\sqrt{-9} - 1$$



Watch Video Solution

15. Simplify: $(3-7i)^2$



Watch Video Solution

16. Simplify: $\left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right)^2$



Watch Video Solution

17. Simplify: $(9+4i)\Big(rac{3}{2}-i\Big)(9-4i)$



18. Determine real values of x and y for which each statement is true

$$\frac{x+y}{i} + x - y + 4 = 0$$



19. Determine real values of x and y for which each statement is

$$-(x+3y)i + (2-y+1) = \frac{8}{i}$$



20. Determine real values of x and y

$$(x-yi)=rac{2+i}{1+i}$$

true



21. Determine real values of x and y for which each statement is true

$$(3-4i)(x+yi) = 1+0i$$



22. Determine real values of x and y for which each statement is true

$$(x-yi)(2+3i)=\frac{x-2i}{1-i}$$



23. Determine real values of x and y for which each statement is true

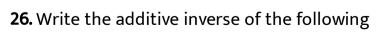
 $\left(x^4+2xi
ight)-\left(3x^2+yi
ight)=(3-5i)+(1+2yi)$



24. Write the conjugate of $(6+5i)^2$







25. Write the additive inverse of the following



3-4i

-2 + 3i

Watch Video Solution

27. Find the multiplicative inverse of each of the following complex numbers when it exists.

2+2i



28. Find the multiplicative inverse of each of the following complex numbers when it exists.

-7 + 0i



29. Find the multiplicative inverse of each of the following complex numbers when it exists.



30. Find the multiplicative inverse of each of the following complex numbers when it exists.

-16



31. Find the multiplicative inverse of each of the following complex numbers when it exists.

$$\frac{\iota}{1+i}$$



32. Find the multiplicative inverse of each of the following complex numbers when it exists. $(1+i)^2$



33. Find the multiplicative inverse of each of the following complex numbers when it exists.



34. Find the multiplicative inverse of each of the following complex numbers when it exists.

 $(6+5i)^2$



35. Find the multiplicative inverse of each of the following complex numbers when it exists.

$$\frac{(2+3i)(3+2i)i}{5+i}$$



- **36.** Simiplify: $(1+i)^{-1}$
 - Watch Video Solution

- **37.** Simiplify: $\sqrt{-\frac{49}{25}}\sqrt{-\frac{1}{9}}$
 - Watch Video Solution

38. Simiplify: $\sqrt{-64}$. $(3 + \sqrt{-361})$

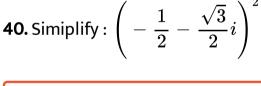


Watch Video Solution

39. Simiplify: $(3-7i)^2$



Watch Video Solution





Watch Video Solution

41. Simiplify : $\frac{(1-i)^3}{(1-i^3)}$



42. Simiplify : $\left(\frac{1+i}{1-i}\right)^{4n+1}$ (n is a positive integer)



43. Simiplify :
$$\frac{\sqrt{(5+12i)}+\sqrt{(5-12i)}}{\sqrt{(5+12i)}-\sqrt{(5-12i)}}$$



44. Prove that
$$\left\lfloor \left(\frac{3+2i}{2-5i} \right) + \left(\frac{3-2i}{2+5i} \right) \right
floor$$
 is rational



- **45.** Show that $rac{1+2i}{3+4i} imesrac{1-2i}{3-4i}$ is real
 - Watch Video Solution

46. Perform the indicated operation and give your answer in the form x+yi, where x and y are real numbers and $i=\sqrt{-1}$ $\left(3+4i\right)^{-1}$



Watch Video Solution

47. Perform the indicated operation and give your answer in the form x+yi, where x and y are real numbers and $i=\sqrt{-1}$ $2-\sqrt{-25}$

$$\frac{2-\sqrt{-25}}{1-\sqrt{-16}}$$



Watch Video Solution

48. Perform the indicated operation and give your answer in the form x+yi, where x and y are real numbers and $i=\sqrt{-1}$

$$\frac{5+2i}{-1+\sqrt{3}i}$$



49. Perform the indicated operation and give your answer in the form x+yi, where x and y are real numbers and $i=\sqrt{-1}$ $\frac{5-3i}{6+i}$



50. Perform the indicated operation and give your answer in the form x+yi, where x and y are real numbers and $i=\sqrt{-1}$ $\left(\sqrt{5}-7i\right)\left(\sqrt{5}-7i\right)^2+\left(-2+7i\right)^2$





52. Prove that :
$$\left[4+3\sqrt{-20}\right]^{\frac{1}{2}}+\left[4-3\sqrt{-20}\right]^{\frac{1}{2}}=6$$

53. Express the following in the form a+ bi

51. If $x+yi=\dfrac{u+vi}{u-vi}$, prove that $x^2+y^2=1$



$$\sqrt{rac{5(2+i)}{2-i}}$$



54. Express the following in the form a+ bi
$$\frac{(3-i)^2}{2+i}$$



56. Express the following in the form a+ bi

 $(1+i)^{-3}$

$\frac{\left(4i^3-i\right)^2}{2i+1}$



 $rac{i-1}{i+1}$

Water video Solution

58. Express the following in the form a+ bi

$$\frac{2+i}{(3-i)(1+2i)}$$



Watch Video Solution

59. Express the following in the form a+ bi

$$\frac{5}{2i-7i^2}$$



Watch Video Solution

60. Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^3$ is a positive integer



61. If one of the values of x of the equation $2x^2-6x+k=0$ be $\frac{1}{2}(a+5i)$, find the values of a and k.



62. Define conjugate complex numbers and show that their sum and product are real numbers.



63. If $ar{z}=-z
eq 0$, show that z is necessarily a purely imaginary number



64. z and z' are complex numbers such that their product zz'

$$=3-4i$$
. Given that z' is $5+3i$, express z in the form $a+bi$

where a and b are rational numbers.



65. If
$$a+bi=\displaystyle\frac{\left(x+i\right)^2}{2x^2+1}$$
, prove that $a^2+b^2=\displaystyle\frac{\left(x^2+1\right)^2}{\left(2x^2+1\right)^2}$

66. Let
$$z_1=2-I, z_2=-2+i$$
, find (i) Re $\left(\frac{z_1z_2}{\bar{z}_1}\right)$, (ii) Im $\left(\frac{1}{z_1\bar{z}_2}\right)$



67. If $z_1=3+5i$ and $z_2=2-3i$, then verify that

$$\overline{\left(rac{z_1}{z_2}
ight)}=rac{ar{z}_1}{ar{z}_2}$$



Watch Video Solution

68. If $x = -2 - \sqrt{3}i$, where $i = \sqrt{-1}$, find the value of

$$2x^4 + 5x^3 + 7x^2 - x + 41$$



Watch Video Solution

69. If $z=-3+\sqrt{2}i$, then prove that

$$z^4+5z^3+8z^2+7z+4$$
 is equal to -29



1. If $(-2+\sqrt{-3})(-3+2\sqrt{-3})=a+bi$, find the real numbers a and b with values of a and b, also find the modulus of a + bi



- **2.** Find the modulus of $(1-i)^{-2}+(1+i)^{-2}$
 - **Watch Video Solution**

3. If z=6+8i, verify that

$$|z| = |\bar{z}|$$



4. If z=6+8i, verify that $-|z|\leq \operatorname{Re}_{-}(z)\leq |z|$



Watch Video Solution

5. If z=6+8i, verify that

$$-|z|<\mathrm{Im}(z)<|z|$$



6. If z=6+8i, verify that $z^{-1}=rac{ar{z}}{\leftert z
ightert ^{2}}$



7. If $z_1=3+4i, z_2=8-15i$, verify that

$$|-z_1|=|z_1|$$



8. If $z_1=3+4i, z_2=8-15i$, verify that

$$\left|z_{1}^{2}
ight|=\left|z_{2}
ight|^{2}$$



9. If $z_1=3+4i, z_2=8-15i$, verify that

$$|z_1 z_2| = |z_1| |z_2|$$



10. If
$$z_1 = 3 + 4i, z_2 = 8 - 15i$$
, verify that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$



Watch Video Solution

11. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$|z_1+z_2|<|z_1|+|z_2|$$



Watch Video Solution

12. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$|z_2-z_1|>||z_2|-|z_1||$$



13. If $z_1=3+4i, z_2=8-15i$, verify that

$$\left|z_{1}+z_{2}
ight|^{2}+\left|z_{1}-z_{2}
ight|^{2}=2\Big(\left|z_{1}
ight|^{2}+\left|z_{2}
ight|^{2}\Big)$$



14. Find the modulus of the following using the property of modulus

$$(3+4i)(8-6i)$$

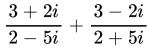


15. Find the modulus of the following using the property of modulus

$$\frac{8+15i}{8-6i}$$



16. Find the modulus of the following using the property of





17. Find the modulus of the following using the property of modulus

$$rac{(2-3i)(4+5i)}{(1-4i)(2-i)}$$



18. Let z be a complex number such that $\left| \frac{z-5i}{z+5i} \right| = 1$, then show that z is purely real

19. Find the complex number z satisfying the equation

$$\left|rac{z-12}{z-8i}
ight|=rac{5}{3}, \left|rac{z-4}{z-8}
ight|=1$$



20. If z is a complex number such that |z-1|=|z+1|, show that Re(z)= 0



21. Solve |z|+z=2+i, where z is a complex number



1. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\sqrt{3}+i$$



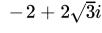
Watch Video Solution

2. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$-\sqrt{3}+i$$



3. Find the modulus and amplitude of the following complex numbers and hence express them into polar form





4. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$-1 - i$$

-2i



5. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

6. Find th	ie modulus	and amp	litude o	f the	following	complex
numbers	and hence e	express th	em into	polar	form	



 $-1-\sqrt{3}i$

-2

7. Find the modulus and amplitude of the following complex numbers and hence express them into polar form



8. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$rac{{{{\left({1 + i}
ight)}^{13}}}}{{{{{\left({1 - i}
ight)}^7}}}$$



9. Find the modulus and amplitude of the following complex numbers and hence express them into polar form (3+i)(4+i)



10. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{(1+i)(2+i)}{(3+i)}$$

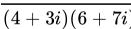
11. Find the modulus and amplitude of the following complex numbers and hence express them into polar form





12. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{(3+4i)(4+5i)}{(4+3i)(6+7i)}$$





13. Change the following complex numbers into polar form

$$-4+4\sqrt{3}i$$



Watch Video Solution

14. Change the following complex numbers into polar form

$$\frac{1+3i}{1-2i}$$



Watch Video Solution

15. Change the following complex numbers into polar form

$$1+2i$$

$$\frac{1+2i}{1-\left(1-i\right)^2}$$



16. Change the following complex numbers into polar form

$$rac{1+7i}{\left(2-i
ight)^2}$$



Watch Video Solution

Given the complex number **17**.

$$z=rac{-1+\sqrt{3}i}{2} \, ext{ and }\, w=rac{-1-\sqrt{3}i}{2}$$
 (where $i=\sqrt{-1}$)

Prove that each of these complex numbers is the square of the other



Watch Video Solution

the complex 18. Given number

$$z=rac{-1+\sqrt{3}i}{2} \ ext{ and } \ w=rac{-1-\sqrt{3}i}{2}$$
 (where $i=\sqrt{-1}$)

Calculate the modulus and argument of w and z

19. Given the complex number
$$z=rac{-1+\sqrt{3}i}{2}$$
 and $w=rac{-1-\sqrt{3}i}{2}$ (where $i=\sqrt{-1}$)

Calculate the modulus and argument of $\frac{w}{z}$



20. Given the complex number
$$z=rac{-1+\sqrt{3}i}{2}$$
 and $w=rac{-1-\sqrt{3}i}{2}$ (where $i=\sqrt{-1}$)

Represent z and w accurately on the complex plane.

1. Illustrate in the complex plane, the set of points satisfying the following condition. Explain your answer

$$|z| \leq 3$$



2. Illustrate in the complex plane, the set of points satisfying the following condition. Explain your answer

$${\sf arg}\ (z-2) = rac{\pi}{3}$$



3. Illustrate in the complex plane, the set of points satisfying the following condition. Explain your answer

$$|i-1-2z|>9$$

4. Illustrate and explain the region of the Argand's plane represented by the inequality $|z+i| \geq |z+2|$



Watch Video Solution

5. Illustrate and explain the set of points z in the Argand diagram, which represents $|z-z_1| \leq 3$ where $z_1=3-2i$



6. If $z=x+yi \ ext{ and } \ \omega=\dfrac{(1-zi)}{z-i}$, then $|\omega|=1$ implies that in the complex plane

A. z lies on the imaginary axis

B. z lies on the real axis

C. z lies on the unit circle

D. None of these

Answer: B



Watch Video Solution

7. Find the locus of a complex number z such that arg

$$\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$$



8. If the amplitude of z-2-3i is $\frac{\pi}{4}$, then find the locus of

$$z = x + yi$$



9. Find the locus of z if $\omega = \frac{z}{z - \frac{1}{2}i}, |\omega| = 1$



10. A variable complex number z is such that the amplitude of

$$\frac{z-1}{z+1}$$
 is always equal to $\frac{\pi}{4}$. Illustrate the locus of z in the

Argand plane



 $z ar{z} + (1-i)z + (1+i)ar{z} - 7 = 0$

12. What is the region represented by the inequality 3<|z-2-3i|<4 in the Argand plane

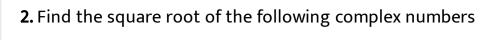
Find the radius and centre of the circle

Watch Video Solution

Exercise F

11.

- 1. Find the square root of the following complex numbers
- 3+4i
 - Watch Video Solution



$$-8 + 6i$$



3. Find the square root of the following complex numbers

$$-40-42i$$



4. Find the square root of the following complex numbers



5. Find the square root of the following complex number

$$\left(\frac{2+3i}{5-4i}+\frac{2-3i}{5+4i}\right)$$



Watch Video Solution

6. If ω is a cube root of unity, then

$$\omega + \omega^2 = \dots$$



Watch Video Solution

7. If ω is a cube root of unity, then

$$1 + \omega$$
=



8. If ω is a cube root of unity, then

$$1 + \omega^2 =$$



Watch Video Solution

9. If ω is a cube root of unity, then

$$\omega^3$$
=



Watch Video Solution

10. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(1+\omega^2\right)^4=\omega$$



11. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(1+\omega-\omega^2\right)^3=\left(1-\omega+\omega^2\right)^3=\ -8$$



Watch Video Solution

12. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$(1-\omega)(1-\omega^2)=3$$



Watch Video Solution

13. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\frac{1}{1+\omega}+\frac{1}{1+\omega^2}=1$$



14. If $1,\,\omega,\,\omega^2$ are three cube roots of unity, prove that

$$\left(1-\omega-\omega^2
ight)^6=64$$



Watch Video Solution

15. If $1,\,\omega,\,\omega^2$ are three cube roots of unity, prove that

$$(1+\omega-\omega^2)(1-\omega+\omega^2)=4$$



Watch Video Solution

16. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(3 + 5\omega + 3\omega^2\right)^6 = \left(3 + 5\omega^2 + 3\omega\right)^6 = 64$$



17. If $1,\,\omega,\,\omega^2$ are three cube roots of unity, prove that

$$\omega^{28} + \omega^{29} + 1 = 0$$



- **18.** Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^n+\left(\frac{-1-i\sqrt{3}}{2}\right)^n$ is equal to
- 2 if n be a multiple of 3 and is equal to -1 if n be any other integer
 - Watch Video Solution

19. If $1,\omega,\omega^2$ are the cube roots of unity, prove that $\omega^n+\omega^{2n}=2$ or -1 according as n is a multiple of 3 or any other integer.

20. Prove the following

$$\left(1-\omega+\omega^2
ight)\left(1+\omega-\omega^2
ight)\left(1-\omega-\omega^2
ight)=8$$



21. Prove the following

$$(1+\omega)\left(1+\omega^2
ight)\left(1+\omega^4
ight)\left(1+\omega^8
ight)$$
....to 2n factors = 1



22. Prove the following

$$\left(1-\omega+\omega^2
ight)\left(1-\omega^2+\omega^4
ight)\left(1-\omega^4+\omega^8
ight)$$
 to 2n factors

$$=2^{2n}$$
 where , ω is the cube root of unity.



23. Prove the following

$$\frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2} = \omega$$



Watch Video Solution

24. Prove the following

$$rac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+rac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}= \ -1$$



Watch Video Solution

25. If ω is a cube root of unity and n is a positive integer which is not a multiple of 3, then show that $\left(1+\omega^n+\omega^{2n}
ight)=0$



Show

$$\left(x+\omega y+\omega^2 z
ight)\left(x+\omega^2 y+\omega z
ight)=x^2+y^2+z^2-yz-zx-xy$$

that

Watch Video Solution

27. Show that $x^3+y^3=(x+y)ig(\omega x+\omega^2 yig)ig(\omega^2 x+\omega yig)$



Watch Video Solution

28. If $1,\,\omega,\,\omega^2$ are cube roots of unity, prove that $1,\,\omega,\,\omega^2$ are vertices of an equilateral triangle



1. Find the square root of 5-12i



Watch Video Solution

2. Find the locus of a complex number z=x+yi, satisfying the relation |z+i|=|z+2|. Illustrate the locus of z in the Argand plane



Watch Video Solution

3. Express $\dfrac{13i}{2-3i}$ in the form A+Bi



4. If z=x+yi and $\dfrac{|z-1-i|+4}{3|z-1-i|-2}=1$, show that $x^2+y^2-2x-2y-7=0$



5. If ω and ω^2 are cube roots of unity, prove that $(2-\omega+2\omega^2)\,(2+2\omega-\omega^2)=9$



6. If $z_1,z_2\in C$ (set of complex numbers), prove that $|z_1+z_2|\leq |z_1|+|z_2|$



7. If $z=x+yi, \omega=rac{2-iz}{2z-i}$ and $|\omega|=1$, find the locus of z



Watch Video Solution

in the complex plane

- **8.** Simplify: $(1-3\omega+\omega^2)(1+\omega-3\omega^2)$
 - **Watch Video Solution**

- **9.** Find the locus of z satisfying $\left| \dfrac{z-3}{z+1} \right| = 3$ in the complex plane.
 - Watch Video Solution

10. Given that $\dfrac{2\sqrt{3}{\cos30^\circ-2i{\sin30^\circ}}}{\sqrt{2}(\cos45^\circ+i{\sin45^\circ})}=A+Bi$, find the values of A and B.



11. Simplify :
$$(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^8\right)$$



12. Find the locus of a complex number z=x+yi, satisfying the relation $|2z+3i|\geq |2z+5|.$ Illustrate the locus in the Argand plane.



13. Find the real values of x and y satisfying the equality 2x + (x + 2):

$$\frac{x-2+(y-3)i}{1+i} = 1-3t$$



14. If
$$i=(\sqrt{-1})$$
, prove that following $(x+1+i)(x+1-i)(x-1-i)(x-1+i)=x^4+4$

15. If z = x + yi and |2z + 1| = |z - 2i|, show that



 $3(x^2+y^2)+4(x-y)=3$

16. Find the amplitude of the complex number
$$\sin\!\frac{6\pi}{5} + i\!\left(1-\cos\!\frac{6\pi}{5}\right)$$



- 17. Express $\frac{1-2i}{2+i}+\frac{3+i}{2-i}$ in the form a+bi
 - Watch Video Solution

- **18.** Find the value of x and y given that (x+yi)(2-3i) = 4+i
 - Watch Video Solution

19. If the ratio $\frac{z-i}{z-1}$ is purely imaginary, prove that the point z lies on the circle whose centre is the point $\frac{1}{2}(1+i)$ and radius is $\frac{1}{\sqrt{2}}$



20. If $(-2+\sqrt{-3})(-3+2\sqrt{-3})=a+bi$, find the real numbers a and b. With these values of a and b, also find the modulus of a+bi



21. If $1,\omega,\omega^2$ are the three cube roots of unity, then simplify: $\left(3+5\omega+3\omega^2\right)^2\left(1+2\omega+\omega^2\right)$



22. Find the locus of a complex number z=x+yi, satisfying the relation $|3z-4i|\leq |3z+2|.$ Illustrate the locus in the Argand plane



Watch Video Solution

23. Find the modulus and argument of the complex number

$$rac{2+i}{4i+\left(1+i
ight)^2}$$



Watch Video Solution

24. If |z-3+i|=4, then the locus of z is

A. $x^2 + y^2 - 6 = 0$

B.
$$x^2 + y^2 - 3x + y - 6 = 0$$

C.
$$x^2 + y^2 - 6x - 2 = 0$$

D.
$$x^2 + y^2 - 6x + 2y - 6 = 0$$

Answer: D



Watch Video Solution

25. The locus of the point z is the Argand plane for which

$$|z+1|^2 + |z-1|^2 = 4$$
 is a

A. Straight line

B. Pair of straight lines

C. Parabola

D. Circle

Answer: D

