MATHS

BOOKS - S CHAND MATHS (ENGLISH)

COMPLEX NUMBER

Example

1. If x =1 +i, then the value of $x^6+x^4+x^2+1$ is

A. 6i-3

B. $-6i+3$

C. $-6i-3$

D. $6i+3$

Answer: c

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2. If $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}, xy, \in R$ then y-x equals

A. 91

B. 85

C. -85

D. -91

Answer: a

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3. The argument of $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is 60^0 b. 120^0 c. 210^0 d. 240^0

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $-\frac{2\pi}{3}$

D. $\frac{4\pi}{3}$

Answer: c

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4. If z = $\frac{(1+i)^2}{\alpha-i}$,$\alpha \in R$ has magnitude $\sqrt{\frac{2}{5}}$ then the value of α is (i) 3 only (ii) -3 only (iii) 3 or -3 (iv) none of these

A. 3 only

B. -3 only

C. 3 or -3

D. none of these

Answer: c

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5. If z is a non -zero complex number then $\frac{|\bar{z}|^2}{z\bar{z}}$ is equal to

A. $\frac{|\bar{z}|}{|z|}$

B. $|\bar{z}|$

C. z

D. none of these

Answer: a

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6. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right| = 1$ lies on the

A. y -axis

B. x-axis

C. line x+y=1

D. circle $x^2 + y^2 = 1$

Answer: B

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7. If $1, \omega, \omega^2$ are cube roots of unity then the value of $\left(5+2\omega+5\omega^2\right)^3$ is

A. 27

B. -9

C. -27

D. -81

Answer: c

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8. Simplify : i^{38}

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9. Simplify : i^{15}

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10. Simplify : i^{-6}

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11. Simplify : $\frac{1}{i}$

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12. Show that i is neither 0, nor greater than 0, nor less than 0

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13. Simplify the following

$(5i) \times 7$

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14. Simplify the following

(3i) (4i)

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15. Simplify the following

$$\frac{21}{14i}$$

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16. Simplify the following

$$\frac{5}{i^3}$$

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17. Simplify the following

$$\sqrt{-9} + \sqrt{-16}$$

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18. Simplify the following

$$\frac{21}{4}\sqrt{-48} - 5\sqrt{-27}$$

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19. Simplify the following

$\sqrt{-18}.\sqrt{-2}$

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20. Simplify the following

$\frac{20}{\sqrt{-5}}$

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21. Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^{3}$

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22. Find the values of x and y if $2x + 4iy = -i^3x - y + 3$

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23. Write the values of x and y if $(3 - 4i)(x + yi) = 1 + i(0)$

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24. Represent the following complex numbers in the complex plane

$2 + 3i$

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25. Represent the following complex numbers in the complex plane

$3 - 5i$

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26. Represent the following complex numbers in the complex plane

$0 + 0i$

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27. Represent the following complex numbers in the complex plane

i

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28. Can two different points in the complex plane represent the same complex number? Give reasons for you answer

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29. Express the following in the form a+bi

$(8 + 7i)(8 - 7i)$

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30. Express the following in the form a+bi

$(5 - 6i)^2$

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31. Express the following in the form a+bi

$(2+3i)(3+7i)$

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32. Express the following in the form a+bi

$\left(-2-\frac{1}{3}i\right)^{3}$

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33. Express the following in the form a+bi

$(1-i)^{4}$

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34. Express the following in the form a+bi

$\left(\sqrt{3}+5i\right)\left(\sqrt{3}-5i\right)^2+(-4+5i)^2$

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35. If $(x+yi)^3=u+vi$, prove that $\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$

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36. If $x=-5+\sqrt{-16}$, find the value of $x^4+9x^3+35x^2-x+4$

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37. Express the following in the form a+bi, where a and b are real numbers

$$\frac{1+i}{1-i}$$

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38. Express the following in the form a+bi, where a and b are rea numbers

$$\left(\frac{1-i}{1+i}\right)^2$$

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39. Express the following in the form a+bi, where a and b are real numbers

$$\left(\frac{1+i}{1-i}\right)^3$$

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40. Express the following in the form a+bi, where a and b are real numbers

$$\frac{\left(3+\sqrt{5}i\right)\left(3-\sqrt{5}i\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-\sqrt{2}i\right)}$$

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41. Find the multiplicative inverse of $\frac{3+4i}{4-5i}$

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42. Find the smallest positive integer n, for which $\left(\frac{1+i}{1-i}\right)^n=1$

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43. If $a+bi=\frac{c+i}{c-i}, a, b, c \in R,$ show that $a^2+b^2=1 \text{ and } \frac{b}{a}=\frac{2c}{c^2-1}$

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44. Solve the equation $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i, xy \in R, i=\sqrt{-1}$

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45. Write the conjugate of

$\sqrt{-16}-3$

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46. Write the conjugate of

i^7

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47. Write the conjugate of

$(3+4i)^2$

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48. Write the conjugate of

$\sqrt{-25}\left(7+\sqrt{-576}\right)$

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49. Write the conjugate of

$$\frac{1-i}{1+i}$$

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50. Find the real numbers x and y if $(x-yi)(3+5i)$ is the conjugate of $-6-24i$

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51. Given $z_1 = 1-i, z_2 = -2+4i$, calculate the values of a and b if $a+bi = \frac{z_1 z_2}{z_1}$

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52. If z be a non-zero complex number, show that

$$\left(\overline{z^{-1}}\right) = (\bar{z})^{-1}$$

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53. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{107} + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{107}$, then show that

Im(z)=0

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54. If $\frac{(a+i)^2}{2a-i} = p + qi$, show that $p^2 + q^2 = \frac{(a^2+1)^2}{4a^2+1}$

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55. Find the modulus of the following complex numbers

$8 - 6i^7$

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56. Find the modulus of the following complex numbers

$$\frac{2 + 3i}{3 + 2i}$$

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57. Find the modulus of the following complex numbers

$$\frac{(3 + 2i)^2}{(4 - 3i)}$$

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58. Find the modulus of the following complex numbers

$(3+2i)(5-4i)$

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59. Find the modulus of the following complex numbers

$\frac{1+i}{1-i} - \frac{1-i}{1+i}$

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60. If z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$, then is it necessary that $z_1 = z_2$

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61. Show that the points representing the complex numbers $(3+3i), (-3-3i)$ and $\left(-3\sqrt{3}+3\sqrt{3}i\right)$ on the Argand plane are the vertices of an equilateral triangle

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62. If $|z|=1$, then prove that $\frac{z-1}{z+1}(z \neq -1)$ is a purely imaginary number. What is the conclusion if z=1?

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63. Solve the equation $2z=|z|+2i$ is complex numbers

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64. If $z = x + yi$ and $\frac{|z - 1 - i| + 4}{3|z - 1 - i| - 2} = 1$, show that $x^2 + y^2 - 2x - 2y - 7 = 0$

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65. Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$

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66. For any two complex numbers z_1 and z_2 and any real numbers a and b, prove that $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)\left[|z_1|^2 + |z_2|^2\right]$

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67. Prove that the representative points of the complex numbers $1+4i, 2+7i, 3+10i$ are collinear

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68. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, zi and $z+zi$ is $=\frac{1}{2}|z|^2$

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69. The complex numbers z_1, z_2 and the origin are the vertices of an equilateral triangle in the Argand plane Prove that $z_1^2+z_2^2=z_1. z_2$

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70. If z_1 and z_2 are complex numbers of two points, then prove that the complex number of the point, which divides the distance between them internally in the ratio $l : m$ is given by $\frac{lz_2 + mz_1}{l + m}$

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71. If z_1, z_2, z_3 are three complex numbers representing three vertices of a triangle, then centroid of the triangle be $\frac{z_1 + z_2 + z_3}{3}$

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72. If the complex numbers z_1, z_2, z_3 represent the vertices of an equilateral triangle, and $|z_1| = |z_2| = |z_3|$, prove that

$z_1 + z_2 + z_3 = 0$

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73. If z_1, z_2, z_3, z_4 are complex numbers, show that they are vertices of a parallelogram In the Argand diagram if and only if

$z_1 + z_3 = z_2 + z_4$

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74. If $z = 4 + 3i$, then verify that

$|z| = |\bar{z}|$

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75. If $z = 4 + 3i$, then verify that

$-|z| \leq Re(z) < |z|$

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76. If $z = 4 + 3i$, then verify that

$-|z| < Im(z) \leq |z|$

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77. If $z = 4 + 3i$, then verify that

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

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78. If $z_1 = 2 + 7i \text{ and } z_2 = 1 - 5i$, then verify that

$|z_1 z_2| = |z_1||z_2|$

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79. If $z_1 = 2 + 7i \text{ and } z_2 = 1 - 5i$, then verify that

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

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80. If $z_1 = 2 + 7i \text{ and } z_2 = 1 - 5i$, then verify that

$|z_1 + z_2| \leq |z_1| + |z_2|$

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81. If $z_1 = 2 + 7i$ and $z_2 = 1 - 5i$, then verify that

$|z_1 - z_2| > |z_1| - |z_2|$

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82. If $z_1 = 2 + 3i$ and $z_2 = 3 + i$ plot the number $z_1 + z_2$.

Also show that $|z_1| + |z_2| > |z_1 + z_2|$

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83. Find the modulus of $\dfrac{(3 + 2i)(1 + i)(2 + 3i)}{(3 + 4i)(4 + 5i)}$

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84. If $x - yi = \frac{a - bi}{c - di}$, prove that $x^2 + y^2 = \frac{a^2 + b^2}{c^2 + d^2}$

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85. If $x - yi = \frac{a - bi}{c - di}$, prove that $x^2 + y^2 = \frac{a^2 + b^2}{c^2 + d^2}$

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86. If $x - yi = \frac{a - bi}{c - di}$, prove that $x^2 + y^2 = \frac{a^2 + b^2}{c^2 + d^2}$

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87. Given that $(1 + i)(1 + 2i)(1 + 3i)\dots(1 + ni) = x + iy$, show that $2.5.10\dots(1 + n^2) = x^2 + y^2$

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88. Given that $\frac{1+i}{1+2^2 i} \times \frac{1+3^2 i}{1+4^2 i} \times \dots . \times \frac{1+(2n-1)^2 i}{1+(2n)^2 i} = \frac{a+bi}{c+di}$, show that $\frac{2}{17} \times \frac{82}{257} \times \dots . \times \frac{(2n-1)^4+1}{(2n)^4+1} = \frac{a^2+b^2}{c^2+d^2}$

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89. If $|z_1| = |z_2| = \dots . = |z_n| = 1$, prove that $|z_1 + z_2 + \dots + z_n| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \dots \frac{1}{z_n}\right|$

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90. Find the greatest value of the moduli of complex numbers z satisfying the equation $\left|z - \frac{4}{z}\right| = 2$. What is the minimum

value ?

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91. If $z = x + yi$ and $\omega = \frac{1 - zi}{z - i}$ show that $|\omega| = 1 \Rightarrow z$ is purely real

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92. Find the modulus and amplitude of

$$\frac{-1}{2} + \frac{\sqrt{3}}{2}i$$

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93. Find the modulus and amplitude of

$-4i$

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94. Find the modulus and amplitude of $\dfrac{2+3i}{3+2i}$

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95. Find the modulus and amplitude of $\dfrac{2+i}{4i+(1+i)^2}$.

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96. Represent the complex numbers

$z = 1 + \sqrt{3}i$ into polar form

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97. Represent the complex numbers

$2 - 2i$

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98. Represent the complex numbers

$\frac{1+7i}{(2-i)^2}$ in polar form

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99. Change the complex number $4(\cos 300^\circ + i \sin 300^\circ)$ to cartesian form

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100. Represent on complex plane the complex numbers $w = 3 + 4i$ and $z = 6 - 3i$ together with $w + z$ and $w - z$. Obtain the modulus and argument of w and z.

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101. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| + |z - z_2| = |z_1 - z_2|$

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102. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| = |z - z_2|$

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103. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| = k|z - z_2|, k \in R^+, k \neq 1$

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104. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| + |z - z_2|$= constant $\neq (|z_1 - z_2|)$

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105. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| - |z - z_2| =$ constant $(\neq |z_1 - z_2|)$

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106. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| - |z - z_2| = |z_1 - z_2|$

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107. Illustrate in the complex plane the following set of points and explain your answer

$|Z| = 3$

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108. Illustrate in the complex plane the following set of points and explain your answer

$|z| < 5$

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109. Illustrate in the complex plane the following set of points and explain your answer

$|z - 4| < 1$

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110. Illustrate in the complex plane the following set of points and explain your answer arg $(Z)=\frac{\pi}{6}$

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111. Illustrate in the complex plane the set of points z satisfying

$|z+i-2|\leq 2$

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112. A variable complex number z is such that the amplitude of

$\frac{z-1}{z+1}$ is always equal to $\frac{\pi}{4}$

Illustrate the locus of z in the Argand plane

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113. Find the locus of a complex number $z = x + yi$ satisfying the relation arg $(z - a) = \frac{\pi}{4}, a \in R$

Illustrate the locus of z in the Argand plane

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114. Given $z_1 = 1 + 2i$. Determine the region in the complex plane represented by $1 < |z - z_1| \leq 3$. Represent it with the help of an Argand diagram

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115. Find the radius and centre of the circle $z\bar{z} - (2 + 3i)z - (2 - 3i)\bar{z} + 9 = 0$ where z is a complex variable

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116. If $z \neq 1$ and $\dfrac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

A. either on the real axis or on a circle passing through the origin

B. on a circle with centre at the origin

C. either on the real axis or on a circle not passing through the origin

D. on the imaginary axis

Answer: A

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117. Find the square root of the complex number

$5 + 12i$

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118. Find the square root of the complex number

$-4 - 3i$

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119. Find the square root of the complex number

18i

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120. If $1, \omega, \omega^2$ are the cube roots of unity, prove that

$$(1+\omega)^3 - \left(1+\omega^2\right)^3 = 0$$

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121. If $1, \omega, \omega^2$ are the cube roots of unity, prove that

$$(x-y)(x\omega - y)\left(x\omega^2 - y\right) = x^3 - y^3$$

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122. If $1, \omega, \omega^2$ are three cube roots of unity, show that

$$\left(a+\omega b+\omega^2 c\right)\left(a+\omega^2 b+\omega c\right) = a^2+b^2+c^2-ab-bc-ca$$

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123. If α and β are the complex cube roots of unity, then prove that $(1+\alpha)(1+\beta)(1+\alpha)^2(1+\beta)^2 = 1$

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124. If, $1, \omega, \omega^2$ are cube roots of unity, show that $\dfrac{p+q\omega+r\omega^2}{r+p\omega+q\omega^2} = \omega^2$

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125. Given that $1, \omega, \omega^2$ are cube roots of unity. Show that $(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5 = 32$

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126. If $1, \omega, \omega^2$ are cube roots of unity, prove that $(x+y)^2 + (x\omega + y\omega^2)^2 + (x\omega^2 + y\omega)^2 = 6xy$

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127. If $x = a + b, y = a\alpha + b\beta, z = a\beta + b\alpha$, where α and β are complex cube roots of unity, then show that $xyz = a^3 + b^3$

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128. If $x = a + b, y = a\omega^2 + b\omega, z = a\omega + b\omega^2$, then show that $x^3 + y^3 + z^3 = 3(a^3 + b^3)$

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129. Show that

$$\sqrt{\left[-1\sqrt{\left\{-1-\sqrt{-1+\ldots\text{to}\infty}\right\}}\right]}=\omega,\quad \text{or}\ \omega^2$$

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Multiple Choice Questions

1. If $8x+i(2x-y)=3-8i$ and $x, y \in R$ then the values of x and y are

A. $x=\frac{3}{8}, y=\frac{35}{4}$

B. $x=-\frac{3}{8}, y=\frac{35}{4}$

C. $x=\frac{3}{8}, y=-\frac{35}{4}$

D. $x=-\frac{3}{8}, y=-\frac{35}{4}$

Answer: A

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2. The value of $1 + i + i^2 + \ldots + i^n$ is (i) positive (ii) negative (iii) 0 (iv) cannot be determined

A. positive

B. negative

C. 0

D. cannot be determined

Answer: D

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3. If z = x +iy satisfies |z+1|=1 then

A. (a) x=0

B. (b) $(x-1)^2 + y^2 = 1$

C. (c) $y = 0$

D. (d) $(x+1)^2 + y^2 = 1$

Answer: D

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4. If z =x +iy satisfies |z+1-i|=|z-1+i| then

A. (a) y=x

B. (b) y=-x

C. (c) x-y+1=0

D. (d) x+y-1=0

Answer: A

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5. Number of solutions of the equation $z^2 + |z|^2 = 0$ is (i) 1 (ii) 2 (iii) 3 (iv) infinitely many

A. 1

B. 2

C. 3

D. infinitely many

Answer: D

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6. The amplitude of $\sin\frac{\pi}{5}+i\left(1-\cos\frac{\pi}{5}\right)$ is

A. (a) $\frac{2\pi}{5}$

B. (b) $\frac{\pi}{5}$

C. (c) $\frac{\pi}{15}$

D. (d) $\frac{\pi}{10}$

Answer: D

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7. The multiplicative inverse of 3+4i is

A. (a) 3-4i

B. (b) $\frac{3+4i}{25}$

C. (c) $\frac{3-4i}{25}$

D. (d) $-3+4i$

Answer: C

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8. If z = $\bar{z}$ then z lies on

A. (a) x - axis

B. (b) y - axis

C. (c) origin

D. (d) none of these

Answer: A

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9. The principal argument $\left(1 + i\sqrt{3}\right)^2$ is

A. $\frac{\pi}{3}$

B. $-\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $-\frac{2\pi}{3}$

Answer: C

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10. The polar form of 1+i$\sqrt{3}$ is

A. $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

B. $2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

C. $2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

D. $2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$

Answer: B

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11. The complex numbers $\sin x+i\cos 2x$ and $\cos x-i\sin 2x$ are conjugate to each other for

A. x=nx

B. $x=\left(n+\frac{1}{2}\right)\frac{\pi}{2}$

C. x=0

D. no value of x

Answer: D

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12. The real value of α for which the expression $\dfrac{1 - i \sin \alpha}{1 + 2i \sin \alpha}$ is purely real is

A. $(n+1)\dfrac{\pi}{2}n, \ \in N$

B. $(2n+1)\dfrac{\pi}{2}n, \ \in N$

C. $n\pi, n \in N$

D. none of these

Answer: C

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13. If z =x +iy lies in the third quadrant then $\frac{\bar{z}}{z}$ also lies in third quadrant if (i) $x > y > 0$ (ii) $x < y < 0$ (iii) $y < x < 0$ (iv) $y > x > 0$

A. $x > y > 0$

B. $x < y < 0$

C. $y < x < 0$

D. $y > x > 0$

Answer: C

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14. The value of $(z+3)(\bar{z}+3)$ is equal to (i) $|z+3|^2$ (ii) $|z-3|$ (iii) z^2+3 (iv) none of these

A. $|z+3|^2$

B. $|z-3|$

C. z^2+3

D. none of these

Answer: A

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15. If $\left(\frac{1+i}{1-i}\right)^x = 1 \forall n \in N$ is (i) x = 2n+1 (ii) x =4n (iii) x=2n (iv) x=4n+1

A. x = 2n+1

B. x =4n

C. x=2n

D. x=4n+1

Answer: B

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16. The argument of $\frac{1+i}{1-i}$ is (i) 0 (ii) $-\frac{\pi}{2}$ (iii) $\frac{\pi}{2}$ (iv) π

A. 0

B. $-\frac{\pi}{2}$

C. $\frac{\pi}{2}$

D. π

Answer: C

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17. If $(1+2i)(2+3i)(3+4i)=x+iy, x, y \in R$ then x^2+y^2 is

A. 1450

B. 1625

C. 1575

D. 1725

Answer: B

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18. The polar form of sin $75^\circ + i\cos 75^\circ$ is

A. (a) sin $75^\circ + i\cos 75^\circ$

B. (b) $\sin 15^\circ + i\cos 15^\circ$

C. (c) $\cos 15^\circ + i \sin 15^\circ$

D. (d) $\cos 75^\circ + i \sin 75^\circ$

Answer: C

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19. The modulus of $\frac{(1+2i)(3-4i)}{(4+3i)(2-3i)}$ is

A. $\sqrt{\frac{5}{13}}$

B. $\sqrt{\frac{13}{5}}$

C. 1

D. -1

Answer: A

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20. If a +ib = $\frac{(x+i)^2}{2x-1}$ then a^2+b^2 is equal to

A. $\frac{(x+1)^4}{4x^2-1}$

B. $\frac{(x^2+1)^2}{(2x-1)^2}$

C. $\frac{(x+1)^4}{4x^2+1}$

D. $\frac{(x+1)^2}{4x^2-1}$

Answer: C

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21. If z=x +iy is purely real number such that $x<0$ then arg (z) is

A. 0

B. $\frac{\pi}{2}$

C. π

D. $-\pi$

Answer: C

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22. If z is a purely imaginary number then arg (z) may be

A. 0 or π

B. $-\pi$ or 0

C. $\pi \text{ or } \pi$

D. $-\frac{\pi}{2}$ or $\frac{\pi}{2}$

Answer: D

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23. If z is a complex number then

A. $|z^2| > |z|^2$

B. $|z|^2 > |z^2|$

C. $|z^2| = |z|^2$

D. $|z^2| \geq |z|^2$

Answer: C

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24. If f(z) $= \frac{7-z}{1-z^2}$ where z=1 +2i then |f(z)| is

A. $\frac{|z|}{2}$

B. $|z|$

C. 2|z|

D. none of these

Answer: A

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25. A real value of of x satisfies the equation $\frac{3-4ix}{3+4ix} = \alpha - i\beta(\alpha, \beta \in R)$ if $\alpha^2 + \beta^2)$ =

A. 1

B. -1

C. 2

D. -2

Answer: A

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26. The real vaule of θ for which the expression $\dfrac{1+i\cos\theta}{1-2i\cos\theta}$ is a purely imaginary number is

A. $n\pi \pm \dfrac{\pi}{6}$

B. $n\pi \pm \dfrac{\pi}{3}$

C. $(2n+1)\dfrac{\pi}{2}$

D. $n\pi \pm \dfrac{\pi}{4}$

Answer: B

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27. The amplitude of $\frac{1}{i}$ is

A. (a) 0

B. (b) π

C. (c) $\frac{\pi}{2}$

D. (d) $-\frac{\pi}{2}$

Answer: D

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28. The amplitude of $\frac{-2}{1+i\sqrt{3}}$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $-\frac{2\pi}{3}$

Answer: C

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29. If z = $\frac{1}{(2-3i)^2}$ then |z| is equal to

A. $\frac{1}{13}$

B. $\frac{1}{12}$

C. $\frac{1}{5}$

D. 13

Answer: A

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30. If z = 1 -cos $\theta + i \sin \theta$ then |z| is equal to

A. $2 \sin \frac{\theta}{2}$

B. $2 \cos \frac{\theta}{2}$

C. $2\left|\sin \frac{\theta}{2}\right|$

D. $2\left|\cos \frac{\theta}{2}\right|$

Answer: C

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31. If z $= \frac{1-i}{1+i}$ then z^4 equals

A. 1

B. -1

C. 0

D. none of these

Answer: A

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32. If x +iy $= \frac{3+5i}{7-6i}$ then y is equal to

A. $\frac{9}{85}$

B. $-\frac{9}{85}$

C. $\frac{53}{85}$

D. $-\frac{53}{85}$

Answer: C

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33. If $1, \omega, \omega^2$ are cube roots of unity then the value of $(3 + 5\omega + 3\omega^2)^3$ is

A. 6

B. 8

C. 12

D. 16

Answer: B

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34. If $1, \omega, \omega^2$ are cube roots of unity then the value of $(2 + 2\omega - 3\omega^2)^3$ is

A. 125

B. -125

C. 27

D. -27

Answer: B

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Exercise A

1. Express each of the following in the form b or bi, where b is a real number

3i.2

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2. Express each of the following in the form b or bi, where b is a real number

$i(-i)$

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3. Express each of the following in the form b or bi, where b is a real number

$-i(-i)$

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4. Express each of the following in the form b or bi, where b is a real number

$5i(-8i)$.

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5. Express each of the following in the form b or bi, where b is a real number

$\frac{20i}{4}$

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6. Express each of the following in the form b or bi, where b is a real number

$\sqrt{-25}$

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7. Express the following in the form a+bi,

$\sqrt{-8}$

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8. Express each of the following in the form b or bi, where b is a real number

$$\sqrt{\frac{-1}{3}}$$

Watch Video Solution

9. Express each of the following in the form b or bi, where b is a real number

$$\frac{1}{2}\sqrt{\frac{-3}{4}}$$

Watch Video Solution

10. Express each of the following in the form b or bi, where b is a real number

$$\frac{6}{-i}$$

Watch Video Solution

11. Express each of the following in the form b or bi, where b is a real number

$\sqrt{-144}$

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12. Express each of the following in the form b or bi, where b is a real number

$\frac{x}{i}$

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13. Simplify:

i^{13}

Watch Video Solution

14. Simplify:

i^{28}

Watch Video Solution

15. Simplify: i^{18}

Watch Video Solution

16. Simplify:

i^{23}

Watch Video Solution

17. Simplify:

$$\sqrt{-4}+\sqrt{-16}-\sqrt{-25}$$

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18. Simplify:

$$\sqrt{-20}+\sqrt{-12}$$

Watch Video Solution

19. Simplify:

$$-\sqrt{\frac{-7}{4}}-\sqrt{\frac{-1}{7}}$$

Watch Video Solution

20. Simplify:

$$\frac{\sqrt{-2}}{\sqrt{-8}}$$

Watch Video Solution

21. Simplify:

$$\frac{1}{i}+\frac{1}{i^{2}}+\frac{1}{i^{3}}+\frac{1}{i^{4}}$$

Watch Video Solution

22. Simplify:

$$\frac{1}{i}-\frac{1}{i^{2}}+\frac{1}{i^{3}}-\frac{1}{i^{4}}$$

Watch Video Solution

23. Simplify:

$$i + 2i^2 + 3i^3 + i^4$$

Watch Video Solution

24. Simplify:

$$\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$$

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25. $\sqrt{\frac{-x}{4}} + \sqrt{\frac{-x}{16}} - \sqrt{\frac{-x}{64}}$, where x is a positive real number

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26. $\sqrt{-5x^8} - \sqrt{-20x^8} + \sqrt{-45x^8}$, where x is a positive real number

Watch Video Solution

27. If $i = \sqrt{-1}$, prove that following

$(x+1+i)(x+1-i)(x-1+i)(x-1-i) = x^4 + 4$

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Exercise B

1. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$3 + 7i, i$

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2. In each of the following find $r + s, r - s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$-i, 5 + 2i$

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3. In each of the following find $r + s, r - s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$3i, 1 - i$

Watch Video Solution

4. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$-7, \ -1-3i$

Watch Video Solution

5. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$7+3i, 3i-7$

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6. Solve each of the following equation for real x and y :

$(x + yi) + (3 - 2i) = 1 + 4i$

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7. Solve each of the following equations for real x and y :

$(x + yi) - (7 + 4i) = 3 - 5i$

Watch Video Solution

8. Solve each of the following equations for real x and y :

$2x + yi = 1 + (2 + 3i)$

Watch Video Solution

9. Solve each of the following equations for real x and y :

$x + 2yi = i - (- 3 + 5)$

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10. Determine the conjugate and the reciprocal of each complex number given below:

i

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11. Determine the conjugate and the reciprocal of each complex number given below:

i^3

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12. Determine the conjugate and the reciprocal of each complex number given below:

$3 - i$

Watch Video Solution

13. Determine the conjugate and the reciprocal of each complex number given below:

$\sqrt{-1} - 3$

Watch Video Solution

14. Determine the conjugate and the reciprocal of each complex number given below:

$\sqrt{-9} - 1$

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15. Simplify: $(3 - 7i)^2$

Watch Video Solution

16. Simplify: $\left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right)^2$

Watch Video Solution

17. Simplify: $(9 + 4i)\left(\frac{3}{2} - i\right)(9 - 4i)$

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18. Determine real values of x and y for which each statement is true

$$\frac{x+y}{i}+x-y+4=0$$

Watch Video Solution

19. Determine real values of x and y for which each statement is true

$$-(x+3y)i+(2-y+1)=\frac{8}{i}$$

Watch Video Solution

20. Determine real values of x and y

$$(x-yi)=\frac{2+i}{1+i}$$

Watch Video Solution

21. Determine real values of x and y for which each statement is true

$(3-4i)(x+yi)=1+0i$

Watch Video Solution

22. Determine real values of x and y for which each statement is true

$(x-yi)(2+3i)=\frac{x-2i}{1-i}$

Watch Video Solution

23. Determine real values of x and y for which each statement is true

$(x^4 + 2xi) - (3x^2 + yi) = (3 - 5i) + (1 + 2yi)$

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24. Write the conjugate of $(6 + 5i)^2$

Watch Video Solution

25. Write the additive inverse of the following

$-2 + 3i$

Watch Video Solution

26. Write the additive inverse of the following

$3 - 4i$

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Watch Video Solution

27. Find the multiplicative inverse of each of the following complex numbers when it exists.

$2 + 2i$

Watch Video Solution

28. Find the multiplicative inverse of each of the following complex numbers when it exists.

$-7 + 0i$

Watch Video Solution

29. Find the multiplicative inverse of each of the following complex numbers when it exists.

$0 + 0i$

Watch Video Solution

30. Find the multiplicative inverse of each of the following complex numbers when it exists.

-16

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31. Find the multiplicative inverse of each of the following complex numbers when it exists.

$$\frac{i}{1+i}$$

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32. Find the multiplicative inverse of each of the following complex numbers when it exists.

$(1+i)^2$

Watch Video Solution

33. Find the multiplicative inverse of each of the following complex numbers when it exists.

$$\frac{3+4i}{4-5i}$$

Watch Video Solution

34. Find the multiplicative inverse of each of the following complex numbers when it exists.

$(6+5i)^2$

Watch Video Solution

35. Find the multiplicative inverse of each of the following complex numbers when it exists.

$$\frac{(2+3i)(3+2i)i}{5+i}$$

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36. Simiplify : $(1+i)^{-1}$

Watch Video Solution

37. Simiplify : $\sqrt{-\frac{49}{25}}\sqrt{-\frac{1}{9}}$

Watch Video Solution

38. Simiplify : $\sqrt{-64}.\ (3+\sqrt{-361})$

Watch Video Solution

39. Simiplify : $(3-7i)^2$

Watch Video Solution

40. Simiplify : $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^2$

Watch Video Solution

41. Simiplify : $\frac{(1-i)^3}{(1-i^3)}$

Watch Video Solution

42. Simiplify : $\left(\frac{1+i}{1-i}\right)^{4n+1}$ (n is a positive integer)

Watch Video Solution

43. Simiplify : $\frac{\sqrt{(5+12i)}+\sqrt{(5-12i)}}{\sqrt{(5+12i)}-\sqrt{(5-12i)}}$

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44. Prove that $\left[\left(\frac{3+2i}{2-5i}\right)+\left(\frac{3-2i}{2+5i}\right)\right]$ is rational

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45. Show that $\frac{1+2i}{3+4i}\times\frac{1-2i}{3-4i}$ is real

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46. Perform the indicated operation and give your answer in the form $x + yi$, where x and y are real numbers and $i = \sqrt{-1}$

$(3 + 4i)^{-1}$

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47. Perform the indicated operation and give your answer in the form $x + yi$, where x and y are real numbers and $i = \sqrt{-1}$

$$\frac{2 - \sqrt{-25}}{1 - \sqrt{-16}}$$

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48. Perform the indicated operation and give your answer in the form $x + yi$, where x and y are real numbers and $i = \sqrt{-1}$

$$\frac{5+2i}{-1+\sqrt{3}i}$$

Watch Video Solution

49. Perform the indicated operation and give your answer in the form $x+yi$, where x and y are real numbers and $i=\sqrt{-1}$

$$\frac{5-3i}{6+i}$$

Watch Video Solution

50. Perform the indicated operation and give your answer in the form $x+yi$, where x and y are real numbers and $i=\sqrt{-1}$

$$\left(\sqrt{5}-7i\right)\left(\sqrt{5}-7i\right)^2+(-2+7i)^2$$

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51. If $x + yi = \frac{u + vi}{u - yi}$, prove that $x^2 + y^2 = 1$

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52. Prove that : $\left[4 + 3\sqrt{-20}\right]^{\frac{1}{2}} + \left[4 - 3\sqrt{-20}\right]^{\frac{1}{2}} = 6$

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53. Express the following in the form a+ bi

$$\sqrt{\frac{5(2 + i)}{2 - i}}$$

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54. Express the following in the form a+ bi

$$\frac{(3 - i)^2}{2 + i}$$

Watch Video Solution

55. Express the following in the form a+ bi

$$(1+i)^{-3}$$

Watch Video Solution

56. Express the following in the form a+ bi

$$\frac{\left(4i^3-i\right)^2}{2i+1}$$

Watch Video Solution

57. Express the following in the form a+ bi

$$\frac{i-1}{i+1}$$

Watch Video Solution

Watch Video Solution

58. Express the following in the form a+ bi

$$\frac{2+i}{(3-i)(1+2i)}$$

Watch Video Solution

59. Express the following in the form a+ bi

$$\frac{5}{2i-7i^2}$$

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60. Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^3$ is a positive integer

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61. If one of the values of x of the equation $2x^2 - 6x + k = 0$ be $\frac{1}{2}(a + 5i)$, find the values of a and k.

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62. Define conjugate complex numbers and show that their sum and product are real numbers.

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63. If $\bar{z} = -z \neq 0$, show that z is necessarily a purely imaginary number

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64. z and z' are complex numbers such that their product zz' $= 3 - 4i$. Given that z' is $5 + 3i$, express z in the form $a + bi$ where a and b are rational numbers.

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65. If $a + bi = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Watch Video Solution

66. Let $z_1 = 2 - I, z_2 = -2 + i$, find (i) Re $\left(\frac{z_1 z_2}{\bar{z}_1}\right)$, (ii) Im $\left(\frac{1}{z_1 \bar{z}_2}\right)$

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67. If $z_1 = 3 + 5i$ and $z_2 = 2 - 3i$, then verify that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

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68. If $x = -2 - \sqrt{3}i$, where $i = \sqrt{-1}$, find the value of $2x^4 + 5x^3 + 7x^2 - x + 41$

Watch Video Solution

69. If $z = -3 + \sqrt{2}i$, then prove that $z^4 + 5z^3 + 8z^2 + 7z + 4$ is equal to -29

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Exercise C

1. If $(-2+\sqrt{-3})(-3+2\sqrt{-3})=a+bi,$ find the real numbers a and b with values of a and b, also find the modulus of $a+bi$

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2. Find the modulus of $(1-i)^{-2}+(1+i)^{-2}$

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3. If $z=6+8i$, verify that

$|z|=|\bar{z}|$

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4. If $z = 6 + 8i$, verify that $-|z| \leq \text{ Re } (z) \leq |z|$

Watch Video Solution

5. If $z = 6 + 8i$, verify that

$-|z| < \text{Im}(z) < |z|$

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6. If $z = 6 + 8i$, verify that $z^{-1} = \frac{\bar{z}}{|z|^2}$

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7. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$|-z_1| = |z_1|$

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8. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$|z_1^2| = |z_2|^2$

Watch Video Solution

9. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$|z_1 z_2| = |z_1||z_2|$

Watch Video Solution

10. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

Watch Video Solution

11. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$|z_1 + z_2| < |z_1| + |z_2|$$

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12. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$|z_2 - z_1| > ||z_2| - |z_1||$$

Watch Video Solution

13. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right)$$

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14. Find the modulus of the following using the property of modulus

$(3 + 4i)(8 - 6i)$

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15. Find the modulus of the following using the property of modulus

$$\frac{8 + 15i}{8 - 6i}$$

Watch Video Solution

16. Find the modulus of the following using the property of modulus

$$\frac{3+2i}{2-5i}+\frac{3-2i}{2+5i}$$

Watch Video Solution

17. Find the modulus of the following using the property of modulus

$$\frac{(2-3i)(4+5i)}{(1-4i)(2-i)}$$

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18. Let z be a complex number such that $\left|\frac{z-5i}{z+5i}\right|=1$, then show that z is purely real

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19. Find the complex number z satisfying the equation $\left|\frac{z-12}{z-8i}\right|=\frac{5}{3}, \left|\frac{z-4}{z-8}\right|=1$

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20. If z is a complex number such that $|z-1|=|z+1|$, show that Re(z)= 0

Watch Video Solution

21. Solve $|z|+z=2+i$, where z is a complex number

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Exercise D

1. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$\sqrt{3} + i$

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2. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-\sqrt{3} + i$

Watch Video Solution

3. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-2 + 2\sqrt{3}i$

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4. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-1 - i$

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5. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-2i$

Watch Video Solution

6. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-1 - \sqrt{3}i$

Watch Video Solution

7. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

-2

Watch Video Solution

8. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{(1+i)^{13}}{(1-i)^{7}}$$

Watch Video Solution

9. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$(3+i)(4+i)$

Watch Video Solution

10. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{(1+i)(2+i)}{(3+i)}$$

Watch Video Solution

11. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{5-i}{2-3i}$$

Watch Video Solution

12. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{(3+4i)(4+5i)}{(4+3i)(6+7i)}$$

Watch Video Solution

13. Change the following complex numbers into polar form

$-4+4\sqrt{3}i$

Watch Video Solution

14. Change the following complex numbers into polar form

$$\frac{1+3i}{1-2i}$$

Watch Video Solution

15. Change the following complex numbers into polar form

$$\frac{1+2i}{1-(1-i)^2}$$

Watch Video Solution

16. Change the following complex numbers into polar form

$$\frac{1+7i}{(2-i)^2}$$

Watch Video Solution

17. Given the complex number $z=\frac{-1+\sqrt{3}i}{2}$ and $w=\frac{-1-\sqrt{3}i}{2}$ (where $i=\sqrt{-1}$)

Prove that each of these complex numbers is the square of the other

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18. Given the complex number $z=\frac{-1+\sqrt{3}i}{2}$ and $w=\frac{-1-\sqrt{3}i}{2}$ (where $i=\sqrt{-1}$)

Calculate the modulus and argument of w and z

Watch Video Solution

19. Given the complex number $z = \frac{-1+\sqrt{3}i}{2}$ and $w = \frac{-1-\sqrt{3}i}{2}$ (where $i = \sqrt{-1}$)

Calculate the modulus and argument of $\frac{w}{z}$

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20. Given the complex number $z = \frac{-1+\sqrt{3}i}{2}$ and $w = \frac{-1-\sqrt{3}i}{2}$ (where $i = \sqrt{-1}$)

Represent z and w accurately on the complex plane.

Watch Video Solution

Exercise E

1. Illustrate in the complex plane, the set of points satisfying the following condition. Explain your answer

$|z| \leq 3$

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2. Illustrate in the complex plane, the set of points satisfying the following condition. Explain your answer

$\arg(z-2) = \frac{\pi}{3}$

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3. Illustrate in the complex plane, the set of points satisfying the following condition. Explain your answer

$|i - 1 - 2z| > 9$

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4. Illustrate and explain the region of the Argand's plane represented by the inequality $|z+i| \geq |z+2|$

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5. Illustrate and explain the set of points z in the Argand diagram, which represents $|z-z_1| \leq 3$ where $z_1 = 3-2i$

Watch Video Solution

6. If $z = x + yi$ and $\omega = \frac{(1-zi)}{z-i}$, then $|\omega| = 1$ implies that in the complex plane

A. z lies on the imaginary axis

B. z lies on the real axis

C. z lies on the unit circle

D. None of these

Answer: B

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7. Find the locus of a complex number z such that arg $\left(\frac{z-2}{z+2}\right)=\frac{\pi}{3}$

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8. If the amplitude of $z-2-3i$ is $\frac{\pi}{4}$, then find the locus of $z=x+yi$

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9. Find the locus of z if $\omega=\frac{z}{z-\frac{1}{3}i}, |\omega|=1$

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10. A variable complex number z is such that the amplitude of $\frac{z-1}{z+1}$ is always equal to $\frac{\pi}{4}$. Illustrate the locus of z in the Argand plane

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11. Find the radius and centre of the circle

$z\bar{z} + (1 - i)z + (1 + i)\bar{z} - 7 = 0$

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12. What is the region represented by the inequality

$3 < |z - 2 - 3i| < 4$ in the Argand plane

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Exercise F

1. Find the square root of the following complex numbers

$3 + 4i$

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2. Find the square root of the following complex numbers

$-8+6i$

Watch Video Solution

3. Find the square root of the following complex numbers

$-40-42i$

Watch Video Solution

4. Find the square root of the following complex numbers

i

Watch Video Solution

5. Find the square root of the following complex number

$$\left(\frac{2+3i}{5-4i}+\frac{2-3i}{5+4i}\right)$$

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6. If ω is a cube root of unity, then

$\omega+\omega^2=$ .....

Watch Video Solution

7. If ω is a cube root of unity, then

$1+\omega$= .....

Watch Video Solution

8. If ω is a cube root of unity, then

$1+\omega^2 =$ .....

Watch Video Solution

9. If ω is a cube root of unity, then

ω^3= ......

Watch Video Solution

10. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$(1+\omega^2)^4 = \omega$

Watch Video Solution

11. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(1+\omega-\omega^2\right)^3 = \left(1-\omega+\omega^2\right)^3 = -8$$

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12. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$(1-\omega)\left(1-\omega^2\right) = 3$$

Watch Video Solution

13. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\frac{1}{1+\omega} + \frac{1}{1+\omega^2} = 1$$

Watch Video Solution

14. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(1-\omega-\omega^2\right)^6=64$$

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15. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(1+\omega-\omega^2\right)\left(1-\omega+\omega^2\right)=4$$

Watch Video Solution

16. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(3+5\omega+3\omega^2\right)^6=\left(3+5\omega^2+3\omega\right)^6=64$$

Watch Video Solution

17. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$\omega^{28} + \omega^{29} + 1 = 0$

Watch Video Solution

18. Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$ is equal to 2 if n be a multiple of 3 and is equal to -1 if n be any other integer

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19. If $1, \omega, \omega^2$ are the cube roots of unity, prove that $\omega^n + \omega^{2n} = 2 \text{ or } -1$ according as n is a multiple of 3 or any other integer.

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20. Prove the following

$(1-\omega+\omega^2)(1+\omega-\omega^2)(1-\omega-\omega^2)=8$

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21. Prove the following

$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$....to 2n factors = 1

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22. Prove the following

$(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)$ .... to 2n factors

$=2^{2n}$ where , ω is the cube root of unity.

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23. Prove the following

$$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}=\omega$$

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24. Prove the following

$$\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}=-1$$

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25. If ω is a cube root of unity and n is a positive integer which is not a multiple of 3, then show that $\left(1+\omega^n+\omega^{2n}\right)=0$

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26. Show that $\left(x+\omega y+\omega^2 z\right)\left(x+\omega^2 y+\omega z\right)=x^2+y^2+z^2-yz-zx-xy$

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27. Show that $x^3+y^3=(x+y)\left(\omega x+\omega^2 y\right)\left(\omega^2 x+\omega y\right)$

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28. If $1, \omega, \omega^2$ are cube roots of unity, prove that $1, \omega, \omega^2$ are vertices of an equilateral triangle

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Chapter Test

1. Find the square root of $5 - 12i$

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2. Find the locus of a complex number $z = x + yi$, satisfying the relation $|z + i| = |z + 2|$. Illustrate the locus of z in the Argand plane

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3. Express $\frac{13i}{2 - 3i}$ in the form $A + Bi$

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4. If $z = x + yi$ and $\frac{|z - 1 - i| + 4}{3|z - 1 - i| - 2} = 1$, show that $x^2 + y^2 - 2x - 2y - 7 = 0$

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5. If ω and ω^2 are cube roots of unity, prove that $(2 - \omega + 2\omega^2)(2 + 2\omega - \omega^2) = 9$

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6. If $z_1, z_2 \in C$ (set of complex numbers), prove that $|z_1 + z_2| \leq |z_1| + |z_2|$

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7. If $z = x + yi$, $\omega = \dfrac{2 - iz}{2z - i}$ and $|\omega| = 1$, find the locus of z in the complex plane

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8. Simplify: $(1 - 3\omega + \omega^2)(1 + \omega - 3\omega^2)$

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9. Find the locus of z satisfying $\left|\dfrac{z-3}{z+1}\right| = 3$ in the complex plane.

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10. Given that $\frac{2\sqrt{3}\cos 30^\circ - 2i\sin 30^\circ}{\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)} = A + Bi$, find the values of A and B.

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11. Simplify : $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8)$

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12. Find the locus of a complex number $z = x + yi$, satisfying the relation $|2z + 3i| \geq |2z + 5|$. Illustrate the locus in the Argand plane.

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13. Find the real values of x and y satisfying the equality

$$\frac{x-2+(y-3)i}{1+i}=1-3t$$

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14. If $i=(\sqrt{-1})$, prove that following

$$(x+1+i)(x+1-i)(x-1-i)(x-1+i)=x^4+4$$

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15. If $z=x+yi$ and $|2z+1|=|z-2i|$, show that

$$3(x^2+y^2)+4(x-y)=3$$

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16. Find the amplitude of the complex number $\sin\frac{6\pi}{5} + i\left(1 - \cos\frac{6\pi}{5}\right)$

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17. Express $\frac{1-2i}{2+i} + \frac{3+i}{2-i}$ in the form $a + bi$

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18. Find the value of x and y given that $(x + yi)(2 - 3i) = 4 + i$

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19. If the ratio $\frac{z-i}{z-1}$ is purely imaginary, prove that the point z lies on the circle whose centre is the point $\frac{1}{2}(1+i)$ and radius is $\frac{1}{\sqrt{2}}$

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20. If $(-2+\sqrt{-3})(-3+2\sqrt{-3})=a+bi$, find the real numbers a and b. With these values of a and b, also find the modulus of $a+bi$

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21. If $1, \omega, \omega^2$ are the three cube roots of unity, then simplify: $(3+5\omega+3\omega^2)^2(1+2\omega+\omega^2)$

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22. Find the locus of a complex number $z = x + yi$, satisfying the relation $|3z - 4i| \leq |3z + 2|$. Illustrate the locus in the Argand plane

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23. Find the modulus and argument of the complex number $\frac{2+i}{4i + (1+i)^2}$

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24. If $|z - 3 + i| = 4$, then the locus of z is

A. $x^2 + y^2 - 6 = 0$

B. $x^2 + y^2 - 3x + y - 6 = 0$

C. $x^2 + y^2 - 6x - 2 = 0$

D. $x^2 + y^2 - 6x + 2y - 6 = 0$

Answer: D

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25. The locus of the point z is the Argand plane for which $|z+1|^2 + |z-1|^2 = 4$ is a

A. Straight line

B. Pair of straight lines

C. Parabola

D. Circle

Answer: D

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