MATHS

BOOKS - S CHAND MATHS (ENGLISH)

COMPLEX NUMBERS

Example

1. Simplify : i^{38}

Watch Video Solution

2. Simplify : i^{15}

Watch Video Solution

3. Simplify : i^{-6}

Watch Video Solution

4. Simplify : $\frac{1}{i}$

Watch Video Solution

5. Show that i is neither 0, nor greater than 0, nor less than 0

Watch Video Solution

6. Simplify the following

$(5i) \times 7$

Watch Video Solution

7. Simplify the following

(3i) (4i)

Watch Video Solution

8. Simplify the following

$$\frac{21}{14i}$$

Watch Video Solution

9. Simplify the following

$$\frac{5}{i^3}$$

Watch Video Solution

10. Simplify the following

$$\sqrt{-9}+\sqrt{-16}$$

Watch Video Solution

11. Simplify the following

$$\frac{21}{4}\sqrt{-48}-5\sqrt{-27}$$

Watch Video Solution

12. Simplify the following

$$\sqrt{-18}.\sqrt{-2}$$

Watch Video Solution

13. Simplify the following

$$\frac{20}{\sqrt{-5}}$$

Watch Video Solution

14. Evaluate: $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^{3}$

Watch Video Solution

15. Find the values of x and y if $2x + 4iy = -i^3x - y + 3$

Watch Video Solution

16. Write the values of x and y if $(3 - 4i)(x + yi) = 1 + i(0)$

Watch Video Solution

17. Represent the following complex numbers in the complex plane

$2 + 3i$

Watch Video Solution

18. Represent the following complex numbers in the complex plane

$3 - 5i$

Watch Video Solution

19. Represent the following complex numbers in the complex plane

$0 + 0i$

Watch Video Solution

20. Represent the following complex numbers in the complex plane

i

Watch Video Solution

21. Can two different points in the complex plane represent the same complex number? Give reasons for you answer

Watch Video Solution

22. Express the following in the form a+bi

$(8 + 7i)(8 - 7i)$

Watch Video Solution

23. Express the following in the form a+bi

$(5 - 6i)^2$

Watch Video Solution

24. Express the following in the form a+bi

$(2 + 3i)(3 + 7i)$

Watch Video Solution

25. Express the following in the form a+bi

$$\left(-2-\frac{1}{3}i\right)^3$$

Watch Video Solution

26. Express the following in the form a+bi

$$(1-i)^4$$

Watch Video Solution

27. Express the following in the form a+bi

$$\left(\sqrt{3}+5i\right)\left(\sqrt{3}-5i\right)^2+(-4+5i)^2$$

Watch Video Solution

28. If $(x+yi)^3 = u + vi$, prove that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

Watch Video Solution

29. If $x = -5 + \sqrt{-16}$, find the value of $x^4 + 9x^3 + 35x^2 - x + 4$

Watch Video Solution

30. Express the following in the form a+bi, where a and b are real numbers

$$\frac{1+i}{1-i}$$

Watch Video Solution

31. Express the following in the form a+bi, where a and b are rea numbers

$$\left(\frac{1-i}{1+i}\right)^2$$

Watch Video Solution

32. Express the following in the form a+bi, where a and b are real numbers

$$\left(\frac{1+i}{1-i}\right)^3$$

Watch Video Solution

33. Express the following in the form a+bi, where a and b are real numbers

$$\frac{\left(3+\sqrt{5}i\right)\left(3-\sqrt{5}i\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-\sqrt{2}i\right)}$$

Watch Video Solution

34. Find the multiplicative inverse of $\dfrac{3+4i}{4-5i}$

Watch Video Solution

35. Find the smallest positive integer n, for which $\left(\dfrac{1+i}{1-i}\right)^n=1$

Watch Video Solution

36. If $a+bi=\dfrac{c+i}{c-i}, a, b, c \in R,$ show that $a^2+b^2=1$ and $\dfrac{b}{a}=\dfrac{2c}{c^2-1}$

Watch Video Solution

37. Solve the equation $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i, xy \in R, i=\sqrt{-1}$

Watch Video Solution

38. Write the conjugate of

$\sqrt{-16}-3$

Watch Video Solution

39. Write the conjugate of

i^7

Watch Video Solution

40. Write the conjugate of

$(3+4i)^2$

Watch Video Solution

41. Write the conjugate of

$\sqrt{-25}\left(7+\sqrt{-576}\right)$

Watch Video Solution

42. Write the conjugate of

$$\frac{1-i}{1+i}$$

Watch Video Solution

43. Find the real numbers x and y if $(x - yi)(3 + 5i)$ is the conjugate of $-6 - 24i$

Watch Video Solution

44. Given $z_1 = 1 - i, z_2 = -2 + 4i$, calculate the values of a and b if $a + bi = \frac{z_1 z_2}{z_1}$

Watch Video Solution

45. If z be a non-zero complex number, show that $\left(\overline{z^{-1}}\right) = (\bar{z})^{-1}$

Watch Video Solution

46. If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{107}$, then show that

Im(z)=0

Watch Video Solution

47. If $\frac{(a+i)^2}{2a-i}=p+qi$, show that $p^2+q^2=\frac{(a^2+1)^2}{4a^2+1}$

Watch Video Solution

48. Find the modulus of the following complex numbers

$8-6i^7$

Watch Video Solution

49. Find the modulus of the following complex numbers

$$\frac{2+3i}{3+2i}$$

Watch Video Solution

50. Find the modulus of the following complex numbers

$$\frac{(3+2i)^2}{(4-3i)}$$

Watch Video Solution

51. Find the modulus of the following complex numbers

$$(3+2i)(5-4i)$$

Watch Video Solution

52. Find the modulus of the following complex numbers

$$\frac{1+i}{1-i} - \frac{1-i}{1+i}$$

Watch Video Solution

53. If z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$, then is it necessary that $z_1 = z_2$

Watch Video Solution

54. Show that the points representing the complex numbers $(3+3i), (-3-3i)$ and $\left(-3\sqrt{3}+3\sqrt{3}i\right)$ on the Argand plane are the vertices of an equilateral triangle

Watch Video Solution

55. If $|z|=1$, then prove that $\frac{z-1}{z+1}(z \ne -1)$ is a purely imaginary number. What is the conclusion if z=1?

Watch Video Solution

56. Solve the equation $2z=|z|+2i$ is complex numbers

Watch Video Solution

57. If $z=x+yi$ and $\frac{|z-1-i|+4}{3|z-1-i|-2}=1$, show that $x^2+y^2-2x-2y-7=0$

Watch Video Solution

58. Prove that $|z_1+z_2|^2+|z_1-z_2|^2=2|z_1|^2+2|z_2|^2$

Watch Video Solution

59. For any two complex numbers z_1 and z_2 and any real numbers a and b, prove that $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)\left[|z_1|^2 + |z_2|^2\right]$

Watch Video Solution

60. Prove that the representative points of the complex numbers $1 + 4i, 2 + 7i, 3 + 10i$ are collinear

Watch Video Solution

61. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, zi and $z + zi$ is $= \frac{1}{2}|z|^2$

Watch Video Solution

62. The complex numbers z_1, z_2 and the origin are the vertices of an equilateral triangle in the Argand plane Prove that $z_1^2 + z_2^2 = z_1 . z_2$

Watch Video Solution

63. If z_1 and z_2 are complex numbers of two points, then prove that the complex number of the point, which divides the distance between them internally in the ratio $l : m$ is given by $\frac{lz_2 + mz_1}{l + m}$

Watch Video Solution

64. If z_1, z_2, z_3 are three complex numbers representing three vertices of a triangle, then centroid of the triangle be $\dfrac{z_1 + z_2 + z_3}{3}$

Watch Video Solution

65. If the complex numbers z_1, z_2, z_3 represent the vertices of an equilateral triangle, and $|z_1| = |z_2| = |z_3|$, prove that $z_1 + z_2 + z_3 = 0$

Watch Video Solution

66. If z_1, z_2, z_3, z_4 are complex numbers, show that they are vertices of a parallelogram In the Argand diagram if and only if $z_1 + z_3 = z_2 + z_4$

Watch Video Solution

67. If $z = 4 + 3i$, then verify that

$|z| = |\bar{z}|$

Watch Video Solution

68. If $z = 4 + 3i$, then verify that

$-|z| \leq Re(z) < |z|$

Watch Video Solution

69. If $z = 4 + 3i$, then verify that

$-|z| < Im(z) \leq |z|$

Watch Video Solution

70. If $z = 4 + 3i$, then verify that

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

Watch Video Solution

71. If $z_1 = 2 + 7i \text{ and } z_2 = 1 - 5i$, then verify that

$$|z_1 z_2| = |z_1||z_2|$$

Watch Video Solution

72. If $z_1 = 2 + 7i \text{ and } z_2 = 1 - 5i$, then verify that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

Watch Video Solution

73. If $z_1 = 2 + 7i$ and $z_2 = 1 - 5i$, then verify that

$|z_1 + z_2| \leq |z_1| + |z_2|$

Watch Video Solution

74. If $z_1 = 2 + 7i$ and $z_2 = 1 - 5i$, then verify that

$|z_1 - z_2| > |z_1| - |z_2|$

Watch Video Solution

75. If $z_1 = 2 + 3i$ and $z_2 = 3 + i$ plot the number $z_1 + z_2$.

Also show that $|z_1| + |z_2| > |z_1 + z_2|$

Watch Video Solution

76. Find the modulus of $\frac{(3+2i)(1+i)(2+3i)}{(3+4i)(4+5i)}$

Watch Video Solution

77. If $x-yi=\frac{a-bi}{c-di}$, prove that $x^2+y^2=\frac{a^2+b^2}{c^2+d^2}$

Watch Video Solution

78. If $x-yi=\frac{a-bi}{c-di}$, prove that $x^2+y^2=\frac{a^2+b^2}{c^2+d^2}$

Watch Video Solution

79. If $x-yi=\frac{a-bi}{c-di}$, prove that $x^2+y^2=\frac{a^2+b^2}{c^2+d^2}$

Watch Video Solution

80. Given that $(1+i)(1+2i)(1+3i)\ldots(1+ni)=x+iy$, show that $2.5.10\ldots(1+n^2)=x^2+y^2$

Watch Video Solution

81. Given that $\frac{1+i}{1+2^2i}\times\frac{1+3^2i}{1+4^2i}\times\ldots\times\frac{1+(2n-1)^2i}{1+(2n)^2i}=\frac{a+bi}{c+di}$, show that $\frac{2}{17}\times\frac{82}{257}\times\ldots\times\frac{(2n-1)^4+1}{(2n)^4+1}=\frac{a^2+b^2}{c^2+d^2}$

Watch Video Solution

82. If $|z_1|=|z_2|=\ldots=|z_n|=1$, prove that $|z_1+z_2+\ldots+z_n|=\left|\frac{1}{z_1}+\frac{1}{z_2}+\ldots\frac{1}{z_n}\right|$

Watch Video Solution

Watch Video Solution

83. Find the greatest value of the moduli of complex numbers z satisfying the equation $\left|z-\frac{4}{z}\right|=2$. What is the minimum value ?

Watch Video Solution

84. If $z=x+yi$ and $\omega=\frac{1-zi}{z-i}$ show that $|\omega|=1 \Rightarrow z$ is purely real

Watch Video Solution

85. Find the modulus and amplitude of

$$\frac{-1}{2}+\frac{\sqrt{3}}{2}i$$

Watch Video Solution

Watch Video Solution

86. Find the modulus and amplitude of

$-4i$

Watch Video Solution

87. Find the modulus and amplitude of $\frac{2+3i}{3+2i}$

Watch Video Solution

88. Find the modulus and amplitude of $\frac{2+i}{4i+(1+i)^2}$.

Watch Video Solution

89. Represent the complex numbers

$z = 1 + \sqrt{3}i$ into polar form

Watch Video Solution

90. Represent the complex numbers

$2 - 2i$

Watch Video Solution

91. Represent the complex numbers

$\dfrac{1 + 7i}{(2 - i)^2}$ in polar form

Watch Video Solution

92. Change the complex number $4(\cos 300^\circ + i\sin 300^\circ)$ to cartesian form

Watch Video Solution

93. Represent on complex plane the complex numbers $w = 3 + 4i$ and $z = 6 - 3i$ together with $w + z$ and $w - z$. Obtain the modulus and argument of w and z.

Watch Video Solution

94. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| + |z - z_2| = |z_1 - z_2|$

Watch Video Solution

95. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| = |z - z_2|$

Watch Video Solution

96. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| = k|z - z_2|, k \in R^+, k \neq 1$

Watch Video Solution

97. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| + |z - z_2|$= constant $\neq (|z_1 - z_2|)$

Watch Video Solution

98. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| - |z - z_2| =$ constant $(\neq |z_1 - z_2|)$

Watch Video Solution

99. If z_1 and z_2 are two fixed points in the Argand plane, then find the locus of a point z in each of the following

$|z - z_1| - |z - z_2| = |z_1 - z_2|$

Watch Video Solution

100. Illustrate in the complex plane the following set of points and explain your answer

$|Z| = 3$

Watch Video Solution

101. Illustrate in the complex plane the following set of points and explain your answer

$|z| < 5$

Watch Video Solution

102. Illustrate in the complex plane the following set of points and explain your answer

$|z - 4| < 1$

Watch Video Solution

103. Illustrate in the complex plane the following set of points and explain your answer

arg $(Z) = \frac{\pi}{6}$

Watch Video Solution

104. Illustrate in the complex plane the set of points z satisfying $|z + i - 2| \leq 2$

Watch Video Solution

105. A variable complex number z is such that the amplitude of $\frac{z-1}{z+1}$ is always equal to $\frac{\pi}{4}$

Illustrate the locus of z in the Argand plane

Watch Video Solution

106. Find the locus of a complex number $z = x + yi$ satisfying the relation arg $(z - a) = \frac{\pi}{4}, a \in R$

Illustrate the locus of z in the Argand plane

Watch Video Solution

107. Given $z_1 = 1 + 2i$. Determine the region in the complex plane represented by $1 < |z - z_1| \leq 3$. Represent it with the help of an Argand diagram

Watch Video Solution

108. Find the radius and centre of the circle

$z\bar{z} - (2+3i)z - (2-3i)\bar{z} + 9 = 0$ where z is a complex variable

Watch Video Solution

109. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

A. either on the real axis or on a circle passing through the origin

B. on a circle with centre at the origin

C. either on the real axis or on a circle not passing through the origin

D. on the imaginary axis

Answer: A

Watch Video Solution

110. Find the square root of the complex number

$5 + 12i$

Watch Video Solution

111. Find the square root of the complex number

$-4 - 3i$

Watch Video Solution

112. Find the square root of the complex number

18i

Watch Video Solution

113. If $1, \omega, \omega^2$ are the cube roots of unity, prove that

$(1+\omega)^3 - (1+\omega^2)^3 = 0$

Watch Video Solution

114. If $1, \omega, \omega^2$ are the cube roots of unity, prove that

$(x-y)(x\omega - y)(x\omega^2 - y) = x^3 - y^3$

Watch Video Solution

115. If $1, \omega, \omega^2$ are three cube roots of unity, show that $(a + \omega b + \omega^2 c)(a + \omega^2 b + \omega c) = a^2 + b^2 + c^2 - ab - bc - ca$

Watch Video Solution

116. If α and β are the complex cube roots of unity, then prove that $(1 + \alpha)(1 + \beta)(1 + \alpha)^2(1 + \beta)^2 = 1$

Watch Video Solution

117. If, $1, \omega, \omega^2$ are cube roots of unity, show that $\frac{p + q\omega + r\omega^2}{r + p\omega + q\omega^2} = \omega^2$

Watch Video Solution

118. Given that $1, \omega, \omega^2$ are cube roots of unity. Show that $\left(1 - \omega + \omega^2\right)^5 + \left(1 + \omega - \omega^2\right)^5 = 32$

Watch Video Solution

119. If $1, \omega, \omega^2$ are cube roots of unity, prove that $(x + y)^2 + \left(x\omega + y\omega^2\right)^2 + \left(x\omega^2 + y\omega\right)^2 = 6xy$

Watch Video Solution

120. If $x = a + b$, $y = a\alpha + b\beta$, $z = a\beta + b\alpha$, where α and β are complex cube roots of unity, then show that $xyz = a^3 + b^3$

Watch Video Solution

121. If $x = a + b, y = a\omega^2 + b\omega, z = a\omega + b\omega^2,$ then show that $x^3 + y^3 + z^3 = 3(a^3 + b^3)$

Watch Video Solution

122. Show that

$$\sqrt{\left[-1\sqrt{\left\{-1-\sqrt{-1+\ldots\text{to}\infty}\right\}}\right]} = \omega, \text{ or } \omega^2$$

Watch Video Solution

Exercise A

1. Express each of the following in the form b or bi, where b is a real number

3i.2

Watch Video Solution

2. Express each of the following in the form b or bi, where b is a real number

$i(-i)$

Watch Video Solution

3. Express each of the following in the form b or bi, where b is a real number

$-i(-i)$

Watch Video Solution

4. Express each of the following in the form b or bi, where b is a real number

$5i(-8i)$.

Watch Video Solution

5. Express each of the following in the form b or bi, where b is a real number

$\frac{20i}{4}$

Watch Video Solution

6. Express each of the following in the form b or bi, where b is a real number

$\sqrt{-25}$

Watch Video Solution

7. Express the following in the form a+bi,

$$\sqrt{-8}$$

Watch Video Solution

8. Express each of the following in the form b or bi, where b is a real number

$$\sqrt{\frac{-1}{3}}$$

Watch Video Solution

9. Express each of the following in the form b or bi, where b is a real number

$$\frac{1}{2}\sqrt{\frac{-3}{4}}$$

Watch Video Solution

10. Express each of the following in the form b or bi, where b is a real number

$$\frac{6}{-i}$$

Watch Video Solution

11. Express each of the following in the form b or bi, where b is a real number

$$\sqrt{-144}$$

Watch Video Solution

12. Express each of the following in the form b or bi, where b is a real number

$$\frac{x}{i}$$

Watch Video Solution

13. Simplify:

i^{13}

Watch Video Solution

14. Simplify:

i^{28}

Watch Video Solution

15. Simplify: i^{18}

Watch Video Solution

16. Simplify:

i^{23}

Watch Video Solution

17. Simplify:

$\sqrt{-4}+\sqrt{-16}-\sqrt{-25}$

Watch Video Solution

18. Simplify:

$$\sqrt{-20} + \sqrt{-12}$$

Watch Video Solution

19. Simplify:

$$-\sqrt{\frac{-7}{4}} - \sqrt{\frac{-1}{7}}$$

Watch Video Solution

20. Simplify:

$$\frac{\sqrt{-2}}{\sqrt{-8}}$$

Watch Video Solution

21. Simplify:

$$\frac{1}{i}+\frac{1}{i^2}+\frac{1}{i^3}+\frac{1}{i^4}$$

Watch Video Solution

22. Simplify:

$$\frac{1}{i}-\frac{1}{i^2}+\frac{1}{i^3}-\frac{1}{i^4}$$

Watch Video Solution

23. Simplify:

$$i+2i^2+3i^3+i^4$$

Watch Video Solution

24. Simplify:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^{3}$$

Watch Video Solution

25. $\sqrt{\frac{-x}{4}}+\sqrt{\frac{-x}{16}}-\sqrt{\frac{-x}{64}}$, where x is a positive real number

Watch Video Solution

26. $\sqrt{-5x^{8}}-\sqrt{-20x^{8}}+\sqrt{-45x^{8}}$, where x is a positive real number

Watch Video Solution

27. If $i = \sqrt{-1}$, prove that following

$(x+1+i)(x+1-i)(x-1+i)(x-1-i) = x^4 + 4$

Watch Video Solution

Exercise B

1. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$3+7i, i$

Watch Video Solution

2. In each of the following find $r + s, r - s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$-i, 5 + 2i$

Watch Video Solution

3. In each of the following find $r + s, r - s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$3i, 1 - i$

Watch Video Solution

4. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$-7, \ -1-3i$

Watch Video Solution

5. In each of the following find $r+s, r-s, rs, \frac{r}{s}$ if r denotes the first complex number and s denotes the second complex number

$7+3i, 3i-7$

Watch Video Solution

6. Solve each of the following equation for real x and y :

$$(x + yi) + (3 - 2i) = 1 + 4i$$

Watch Video Solution

7. Solve each of the following equations for real x and y :

$$(x + yi) - (7 + 4i) = 3 - 5i$$

Watch Video Solution

8. Solve each of the following equations for real x and y :

$$2x + yi = 1 + (2 + 3i)$$

Watch Video Solution

9. Solve each of the following equations for real x and y :

$x + 2yi = i - (- 3 + 5)$

Watch Video Solution

10. Determine the conjugate and the reciprocal of each complex number given below:

i

Watch Video Solution

11. Determine the conjugate and the reciprocal of each complex number given below:

i^3

Watch Video Solution

12. Determine the conjugate and the reciprocal of each complex number given below:

$3 - i$

Watch Video Solution

13. Determine the conjugate and the reciprocal of each complex number given below:

$\sqrt{-1} - 3$

Watch Video Solution

14. Determine the conjugate and the reciprocal of each complex number given below:

$\sqrt{-9} - 1$

Watch Video Solution

15. Simplify: $(3 - 7i)^2$

Watch Video Solution

16. Simplify: $\left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right)^2$

Watch Video Solution

17. Simplify: $(9 + 4i)\left(\frac{3}{2} - i\right)(9 - 4i)$

Watch Video Solution

18. Determine real values of x and y for which each statement is

true

$$\frac{x+y}{i}+x-y+4=0$$

Watch Video Solution

19. Determine real values of x and y for which each statement is

true

$$-(x+3y)i+(2-y+1)=\frac{8}{i}$$

Watch Video Solution

20. Determine real values of x and y

$$(x-yi)=\frac{2+i}{1+i}$$

Watch Video Solution

21. Determine real values of x and y for which each statement is true

$(3 - 4i)(x + yi) = 1 + 0i$

Watch Video Solution

22. Determine real values of x and y for which each statement is true

$(x - yi)(2 + 3i) = \frac{x - 2i}{1 - i}$

Watch Video Solution

23. Determine real values of x and y for which each statement is true

$$\left(x^4 + 2xi\right) - \left(3x^2 + yi\right) = (3 - 5i) + (1 + 2yi)$$

Watch Video Solution

24. Write the conjugate of $(6 + 5i)^2$

Watch Video Solution

25. Write the additive inverse of the following

$-2 + 3i$

Watch Video Solution

26. Write the additive inverse of the following

$3 - 4i$

Watch Video Solution

Watch Video Solution

27. Find the multiplicative inverse of each of the following complex numbers when it exists.

$2 + 2i$

Watch Video Solution

28. Find the multiplicative inverse of each of the following complex numbers when it exists.

$-7 + 0i$

Watch Video Solution

29. Find the multiplicative inverse of each of the following complex numbers when it exists.

$0 + 0i$

Watch Video Solution

30. Find the multiplicative inverse of each of the following complex numbers when it exists.

-16

Watch Video Solution

31. Find the multiplicative inverse of each of the following complex numbers when it exists.

$$\frac{i}{1+i}$$

Watch Video Solution

32. Find the multiplicative inverse of each of the following complex numbers when it exists.

$(1 + i)^2$

Watch Video Solution

33. Find the multiplicative inverse of each of the following complex numbers when it exists.

$\frac{3 + 4i}{4 - 5i}$

Watch Video Solution

34. Find the multiplicative inverse of each of the following complex numbers when it exists.

$(6 + 5i)^2$

Watch Video Solution

35. Find the multiplicative inverse of each of the following complex numbers when it exists.

$$\frac{(2+3i)(3+2i)i}{5+i}$$

Watch Video Solution

36. Simiplify : $(1+i)^{-1}$

Watch Video Solution

37. Simiplify : $\sqrt{-\frac{49}{25}}\sqrt{-\frac{1}{9}}$

Watch Video Solution

38. Simiplify : $\sqrt{-64}.\ (3+\sqrt{-361})$

Watch Video Solution

39. Simiplify : $(3-7i)^2$

Watch Video Solution

40. Simiplify : $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^2$

Watch Video Solution

41. Simiplify : $\frac{(1-i)^3}{(1-i^3)}$

Watch Video Solution

42. Simiplify : $\left(\frac{1+i}{1-i}\right)^{4n+1}$ (n is a positive integer)

Watch Video Solution

43. Simiplify : $\frac{\sqrt{(5+12i)}+\sqrt{(5-12i)}}{\sqrt{(5+12i)}-\sqrt{(5-12i)}}$

Watch Video Solution

44. Prove that $\left[\left(\frac{3+2i}{2-5i}\right)+\left(\frac{3-2i}{2+5i}\right)\right]$ is rational

Watch Video Solution

45. Show that $\frac{1+2i}{3+4i}\times\frac{1-2i}{3-4i}$ is real

Watch Video Solution

46. Perform the indicated operation and give your answer in the form $x + yi$, where x and y are real numbers and $i = \sqrt{-1}$

$(3 + 4i)^{-1}$

Watch Video Solution

47. Perform the indicated operation and give your answer in the form $x + yi$, where x and y are real numbers and $i = \sqrt{-1}$

$$\frac{2 - \sqrt{-25}}{1 - \sqrt{-16}}$$

Watch Video Solution

48. Perform the indicated operation and give your answer in the form $x + yi$, where x and y are real numbers and $i = \sqrt{-1}$

$$\frac{5+2i}{-1+\sqrt{3}i}$$

Watch Video Solution

49. Perform the indicated operation and give your answer in the form $x+yi$, where x and y are real numbers and $i=\sqrt{-1}$

$$\frac{5-3i}{6+i}$$

Watch Video Solution

50. Perform the indicated operation and give your answer in the form $x+yi$, where x and y are real numbers and $i=\sqrt{-1}$

$$\left(\sqrt{5}-7i\right)\left(\sqrt{5}-7i\right)^{2}+(-2+7i)^{2}$$

Watch Video Solution

51. If $x + yi = \frac{u + vi}{u - yi}$, prove that $x^2 + y^2 = 1$

Watch Video Solution

52. Prove that : $[4 + 3\sqrt{-20}]^{\frac{1}{2}} + [4 - 3\sqrt{-20}]^{\frac{1}{2}} = 6$

Watch Video Solution

53. Express the following in the form a+ bi

$$\sqrt{\frac{5(2+i)}{2-i}}$$

Watch Video Solution

54. Express the following in the form a+ bi

$$\frac{(3-i)^2}{2+i}$$

Watch Video Solution

55. Express the following in the form a+ bi

$$(1+i)^{-3}$$

Watch Video Solution

56. Express the following in the form a+ bi

$$\frac{(4i^3 - i)^2}{2i+1}$$

Watch Video Solution

57. Express the following in the form a+ bi

$$\frac{i-1}{i+1}$$

Watch Video Solution

Watch Video Solution

58. Express the following in the form a+ bi

$$\frac{2+i}{(3-i)(1+2i)}$$

Watch Video Solution

59. Express the following in the form a+ bi

$$\frac{5}{2i-7i^2}$$

Watch Video Solution

60. Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^3$ is a positive integer

Watch Video Solution

61. If one of the values of x of the equation $2x^2 - 6x + k = 0$ be $\frac{1}{2}(a + 5i)$, find the values of a and k.

Watch Video Solution

62. Define conjugate complex numbers and show that their sum and product are real numbers.

Watch Video Solution

63. If $\bar{z} = -z \neq 0$, show that z is necessarily a purely imaginary number

Watch Video Solution

64. z and z' are complex numbers such that their product zz' $= 3 - 4i$. Given that z' is $5 + 3i$, express z in the form $a + bi$ where a and b are rational numbers.

Watch Video Solution

65. If $a + bi = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Watch Video Solution

66. Let $z_1 = 2 - I, z_2 = -2 + i$, find (i) Re $\left(\frac{z_1 z_2}{\bar{z}_1}\right)$, (ii) Im $\left(\frac{1}{z_1 \bar{z}_2}\right)$

Watch Video Solution

67. If $z_1 = 3 + 5i$ and $z_2 = 2 - 3i$, then verify that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

Watch Video Solution

68. If $x = -2 - \sqrt{3}i$, where $i = \sqrt{-1}$, find the value of $2x^4 + 5x^3 + 7x^2 - x + 41$

Watch Video Solution

69. If $z = -3 + \sqrt{2}i$, then prove that $z^4 + 5z^3 + 8z^2 + 7z + 4$ is equal to -29

Watch Video Solution

Exercise C

1. If $(-2+\sqrt{-3})(-3+2\sqrt{-3}) = a+bi,$ find the real numbers a and b with values of a and b, also find the modulus of $a+bi$

Watch Video Solution

2. Find the modulus of $(1-i)^{-2}+(1+i)^{-2}$

Watch Video Solution

3. If $z=6+8i$, verify that

$|z| = |\bar{z}|$

Watch Video Solution

4. If $z = 6 + 8i$, verify that $-|z| \leq \text{ Re } (z) \leq |z|$

Watch Video Solution

5. If $z = 6 + 8i$, verify that

$-|z| < \text{Im}(z) < |z|$

Watch Video Solution

6. If $z = 6 + 8i$, verify that $z^{-1} = \frac{\bar{z}}{|z|^2}$

Watch Video Solution

7. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$|-z_1| = |z_1|$

Watch Video Solution

8. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$|z_1^2| = |z_2|^2$

Watch Video Solution

9. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$|z_1 z_2| = |z_1||z_2|$

Watch Video Solution

10. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

Watch Video Solution

11. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$|z_1 + z_2| < |z_1| + |z_2|$$

Watch Video Solution

12. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$|z_2 - z_1| > ||z_2| - |z_1||$$

Watch Video Solution

13. If $z_1 = 3 + 4i$, $z_2 = 8 - 15i$, verify that

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right)$$

Watch Video Solution

14. Find the modulus of the following using the property of modulus

$(3 + 4i)(8 - 6i)$

Watch Video Solution

15. Find the modulus of the following using the property of modulus

$$\frac{8 + 15i}{8 - 6i}$$

Watch Video Solution

16. Find the modulus of the following using the property of modulus

$$\frac{3+2i}{2-5i}+\frac{3-2i}{2+5i}$$

Watch Video Solution

17. Find the modulus of the following using the property of modulus

$$\frac{(2-3i)(4+5i)}{(1-4i)(2-i)}$$

Watch Video Solution

18. Let z be a complex number such that $\left|\frac{z-5i}{z+5i}\right|=1$, then show that z is purely real

Watch Video Solution

19. Find the complex number z satisfying the equation $\left|\frac{z-12}{z-8i}\right|=\frac{5}{3}, \left|\frac{z-4}{z-8}\right|=1$

Watch Video Solution

20. If z is a complex number such that $|z-1|=|z+1|$, show that Re(z)= 0

Watch Video Solution

21. Solve $|z|+z=2+i$, where z is a complex number

Watch Video Solution

Exercise D

1. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$\sqrt{3}+i$

Watch Video Solution

2. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-\sqrt{3}+i$

Watch Video Solution

3. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-2 + 2\sqrt{3}i$

Watch Video Solution

4. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-1 - i$

Watch Video Solution

5. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-2i$

Watch Video Solution

6. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$-1-\sqrt{3}i$

Watch Video Solution

7. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

-2

Watch Video Solution

8. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{(1+i)^{13}}{(1-i)^{7}}$$

Watch Video Solution

9. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$(3+i)(4+i)$$

Watch Video Solution

10. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{(1+i)(2+i)}{(3+i)}$$

Watch Video Solution

11. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{5-i}{2-3i}$$

Watch Video Solution

12. Find the modulus and amplitude of the following complex numbers and hence express them into polar form

$$\frac{(3+4i)(4+5i)}{(4+3i)(6+7i)}$$

Watch Video Solution

13. Change the following complex numbers into polar form

$-4 + 4\sqrt{3}i$

Watch Video Solution

14. Change the following complex numbers into polar form

$$\frac{1+3i}{1-2i}$$

Watch Video Solution

15. Change the following complex numbers into polar form

$$\frac{1+2i}{1-(1-i)^2}$$

Watch Video Solution

16. Change the following complex numbers into polar form

$$\frac{1+7i}{(2-i)^2}$$

Watch Video Solution

17. Given the complex number $z=\frac{-1+\sqrt{3}i}{2}$ and $w=\frac{-1-\sqrt{3}i}{2}$ (where $i=\sqrt{-1}$)

Prove that each of these complex numbers is the square of the other

Watch Video Solution

18. Given the complex number $z=\frac{-1+\sqrt{3}i}{2}$ and $w=\frac{-1-\sqrt{3}i}{2}$ (where $i=\sqrt{-1}$)

Calculate the modulus and argument of w and z

Watch Video Solution

19. Given the complex number $z = \frac{-1 + \sqrt{3}i}{2}$ and $w = \frac{-1 - \sqrt{3}i}{2}$ (where $i = \sqrt{-1}$)

Calculate the modulus and argument of $\frac{w}{z}$

Watch Video Solution

20. Given the complex number $z = \frac{-1 + \sqrt{3}i}{2}$ and $w = \frac{-1 - \sqrt{3}i}{2}$ (where $i = \sqrt{-1}$)

Represent z and w accurately on the complex plane.

Watch Video Solution

Exercise E

1. Illustrate in the complex plane, the set of points satisfying the following condition. Explain your answer

$|z| \leq 3$

Watch Video Solution

2. Illustrate in the complex plane, the set of points satisfying the following condition. Explain your answer

$\arg(z-2) = \frac{\pi}{3}$

Watch Video Solution

3. Illustrate in the complex plane, the set of points satisfying the following condition. Explain your answer

$|i-1-2z| > 9$

Watch Video Solution

4. Illustrate and explain the region of the Argand's plane represented by the inequality $|z+i| \geq |z+2|$

Watch Video Solution

5. Illustrate and explain the set of points z in the Argand diagram, which represents $|z-z_1| \leq 3$ where $z_1 = 3-2i$

Watch Video Solution

6. If $z = x + yi$ and $\omega = \dfrac{(1-zi)}{z-i}$, then $|\omega| = 1$ implies that in the complex plane

A. z lies on the imaginary axis

B. z lies on the real axis

C. z lies on the unit circle

D. None of these

Answer: B

Watch Video Solution

7. Find the locus of a complex number z such that arg $\left(\frac{z-2}{z+2}\right)=\frac{\pi}{3}$

Watch Video Solution

8. If the amplitude of $z - 2 - 3i$ is $\frac{\pi}{4}$, then find the locus of $z = x + yi$

Watch Video Solution

9. Find the locus of z if $\omega = \frac{z}{z - \frac{1}{3}i}, |\omega| = 1$

Watch Video Solution

10. A variable complex number z is such that the amplitude of $\frac{z-1}{z+1}$ is always equal to $\frac{\pi}{4}$. Illustrate the locus of z in the Argand plane

Watch Video Solution

11. Find the radius and centre of the circle

$z\bar{z}+(1-i)z+(1+i)\bar{z}-7=0$

Watch Video Solution

12. What is the region represented by the inequality

$3<|z-2-3i|<4$ in the Argand plane

Watch Video Solution

Exercise F

1. Find the square root of the following complex numbers

$3+4i$

Watch Video Solution

2. Find the square root of the following complex numbers

$-8+6i$

Watch Video Solution

3. Find the square root of the following complex numbers

$-40-42i$

Watch Video Solution

4. Find the square root of the following complex numbers

i

Watch Video Solution

5. Find the square root of the following complex number

$$\left(\frac{2+3i}{5-4i}+\frac{2-3i}{5+4i}\right)$$

Watch Video Solution

6. If ω is a cube root of unity, then

$\omega+\omega^2=$ .....

Watch Video Solution

7. If ω is a cube root of unity, then

$1+\omega$= .....

Watch Video Solution

8. If ω is a cube root of unity, then

$1+\omega^{2} = \ldots\ldots$

Watch Video Solution

9. If ω is a cube root of unity, then

ω^{3}= ......

Watch Video Solution

10. If $1, \omega, \omega^{2}$ are three cube roots of unity, prove that

$\left(1+\omega^{2}\right)^{4}=\omega$

Watch Video Solution

11. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(1+\omega-\omega^2\right)^3 = \left(1-\omega+\omega^2\right)^3 = -8$$

Watch Video Solution

12. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$(1-\omega)\left(1-\omega^2\right) = 3$$

Watch Video Solution

13. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\frac{1}{1+\omega} + \frac{1}{1+\omega^2} = 1$$

Watch Video Solution

14. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(1 - \omega - \omega^2\right)^6 = 64$$

Watch Video Solution

15. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(1 + \omega - \omega^2\right)\left(1 - \omega + \omega^2\right) = 4$$

Watch Video Solution

16. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$$\left(3 + 5\omega + 3\omega^2\right)^6 = \left(3 + 5\omega^2 + 3\omega\right)^6 = 64$$

Watch Video Solution

17. If $1, \omega, \omega^2$ are three cube roots of unity, prove that

$\omega^{28} + \omega^{29} + 1 = 0$

Watch Video Solution

18. Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$ is equal to 2 if n be a multiple of 3 and is equal to -1 if n be any other integer

Watch Video Solution

19. If $1, \omega, \omega^2$ are the cube roots of unity, prove that $\omega^n + \omega^{2n} = 2 \text{ or } -1$ according as n is a multiple of 3 or any other integer.

Watch Video Solution

Watch Video Solution

20. Prove the following

$$\left(1-\omega+\omega^2\right)\left(1+\omega-\omega^2\right)\left(1-\omega-\omega^2\right)=8$$

Watch Video Solution

21. Prove the following

$(1+\omega)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)$....to 2n factors = 1

Watch Video Solution

22. Prove the following

$\left(1-\omega+\omega^2\right)\left(1-\omega^2+\omega^4\right)\left(1-\omega^4+\omega^8\right)$ .... to 2n factors

$=2^{2n}$ where , ω is the cube root of unity.

Watch Video Solution

Watch Video Solution

23. Prove the following

$$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}=\omega$$

Watch Video Solution

24. Prove the following

$$\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}=-1$$

Watch Video Solution

25. If ω is a cube root of unity and n is a positive integer which is not a multiple of 3, then show that $\left(1+\omega^n+\omega^{2n}\right)=0$

Watch Video Solution

26. Show that

$\left(x+\omega y+\omega^2 z\right)\left(x+\omega^2 y+\omega z\right)=x^2+y^2+z^2-yz-zx-xy$

Watch Video Solution

27. Show that $x^3+y^3=(x+y)\left(\omega x+\omega^2 y\right)\left(\omega^2 x+\omega y\right)$

Watch Video Solution

28. If $1, \omega, \omega^2$ are cube roots of unity, prove that $1, \omega, \omega^2$ are vertices of an equilateral triangle

Watch Video Solution

Chapter Test

1. Find the square root of $5-12i$

Watch Video Solution

2. Find the locus of a complex number $z=x+yi$, satisfying the relation $|z+i|=|z+2|$. Illustrate the locus of z in the Argand plane

Watch Video Solution

3. Express $\frac{13i}{2-3i}$ in the form $A+Bi$

Watch Video Solution

4. If $z = x + yi$ and $\frac{|z - 1 - i| + 4}{3|z - 1 - i| - 2} = 1,$ show that $x^2 + y^2 - 2x - 2y - 7 = 0$

Watch Video Solution

5. If ω and ω^2 are cube roots of unity, prove that $(2 - \omega + 2\omega^2)(2 + 2\omega - \omega^2) = 9$

Watch Video Solution

6. If $z_1, z_2 \in C$ (set of complex numbers), prove that $|z_1 + z_2| \leq |z_1| + |z_2|$

Watch Video Solution

7. If $z = x + yi, \omega = \frac{2 - iz}{2z - i}$ and $|\omega| = 1$, find the locus of z in the complex plane

Watch Video Solution

8. Simplify: $(1 - 3\omega + \omega^2)(1 + \omega - 3\omega^2)$

Watch Video Solution

9. Find the locus of z satisfying $\left|\frac{z - 3}{z + 1}\right| = 3$ in the complex plane.

Watch Video Solution

10. Given that $\frac{2\sqrt{3}\cos 30^\circ - 2i\sin 30^\circ}{\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)} = A + Bi$, find the values of A and B.

Watch Video Solution

11. Simplify : $(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^8\right)$

Watch Video Solution

12. Find the locus of a complex number $z = x + yi$, satisfying the relation $|2z + 3i| \geq |2z + 5|$. Illustrate the locus in the Argand plane.

Watch Video Solution

13. Find the real values of x and y satisfying the equality

$$\frac{x-2+(y-3)i}{1+i}=1-3t$$

Watch Video Solution

14. If $i=(\sqrt{-1})$, prove that following

$$(x+1+i)(x+1-i)(x-1-i)(x-1+i)=x^4+4$$

Watch Video Solution

15. If $z=x+yi$ and $|2z+1|=|z-2i|$, show that

$$3\left(x^2+y^2\right)+4(x-y)=3$$

Watch Video Solution

16. Find the amplitude of the complex number $\sin\frac{6\pi}{5} + i\left(1 - \cos\frac{6\pi}{5}\right)$

Watch Video Solution

17. Express $\frac{1-2i}{2+i} + \frac{3+i}{2-i}$ in the form $a + bi$

Watch Video Solution

18. Find the value of x and y given that $(x + yi)(2 - 3i) = 4 + i$

Watch Video Solution

19. If the ratio $\frac{z-i}{z-1}$ is purely imaginary, prove that the point z lies on the circle whose centre is the point $\frac{1}{2}(1+i)$ and radius is $\frac{1}{\sqrt{2}}$

Watch Video Solution

20. If $(-2+\sqrt{-3})(-3+2\sqrt{-3})=a+bi$, find the real numbers a and b. With these values of a and b, also find the modulus of $a+bi$

Watch Video Solution

21. If $1, \omega, \omega^2$ are the three cube roots of unity, then simplify: $(3+5\omega+3\omega^2)^2(1+2\omega+\omega^2)$

Watch Video Solution

22. Find the locus of a complex number $z = x + yi$, satisfying the relation $|3z - 4i| \leq |3z + 2|$. Illustrate the locus in the Argand plane

Watch Video Solution

23. Find the modulus and argument of the complex number $\dfrac{2+i}{4i + (1+i)^2}$

Watch Video Solution

24. If $|z - 3 + i| = 4$, then the locus of z is

A. $x^2 + y^2 - 6 = 0$

B. $x^2+y^2-3x+y-6=0$

C. $x^2+y^2-6x-2=0$

D. $x^2+y^2-6x+2y-6=0$

Answer: D

Watch Video Solution

25. The locus of the point z is the Argand plane for which $|z+1|^2+|z-1|^2=4$ is a

A. Straight line

B. Pair of straight lines

C. Parabola

D. Circle

Answer: D

Watch Video Solution