

MATHS

BOOKS - S CHAND MATHS (ENGLISH)

COMPOUND AND MULTIPLE ANGLES

Example

1. Computer $\sin 75^\circ$, $\cos 75^\circ$ and $\tan 15^\circ$, from the functions of 30° and 45°



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2. If α and β are acute angles and $\sin \alpha = \frac{3}{5}$, $\sin \beta = \frac{8}{17}$, find the values of $\sin(\alpha + \beta)$, $\cos(\alpha \pm \beta)$, and $\tan(\alpha \pm \beta)$.



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3. Prove that $\sin(\alpha + 30^\circ) = \cos \alpha + \sin(\alpha - 30^\circ)$

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4. Prove that

$$\sin(x + y)\sin(x - y) + \sin(y + z)\sin(y - z) + \sin(z + x)\sin(z - x) = 0.$$

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5. Find the value of $\cos^2 45^\circ - \sin^2 15^\circ$.

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6. Prove that $\tan 11A \tan 7A \tan 4A = \tan 11A - \tan 7A - \tan 4A$.

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7. Prove that $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$.



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8. If $\sin(\alpha + \beta) = \frac{4}{5}$, $\cos(\alpha - \beta) = \frac{12}{13}$ $0 < \alpha < \frac{\pi}{4}$, $0 < \beta < \frac{\pi}{4}$, find $\tan 2\alpha$.



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9. Find the value of $\sin 75^\circ \sin 15^\circ$.



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10. Prove that $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$.



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11. Convert the following into products:

(i) $\sin 53^\circ + \sin 21^\circ$, (ii) $\cos 83^\circ - \cos 17^\circ$, (iii) $\sin 50^\circ - \cos 80^\circ$



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12. Prove that $\frac{\sin 7A - \sin 5A}{\cos 5A + \cos 7A} = \tan A$.



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13. Prove that

$$\frac{\cos 3A + 2 \cos 5A + \cos 7A}{\cos A + 2 \cos 3A + \cos 5A} = \cos 2A - \sin 2A \tan 3A.$$



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14. Prove that $\frac{\cos \theta + \cos 2\theta + \cos 3\theta + \cos 4\theta}{\sin \theta + \sin 2\theta + \sin 3\theta + \sin 4\theta} = \cot \frac{5\theta}{2}$.



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$$15. \sin 10x = 2 \sin 5x \cos 5x$$



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$$16. \cos 6x = \cos^2 3x - \sin^2 3x$$



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$$17. \tan 8x = \frac{2 \tan 4x}{1 - \tan^2 4x}$$



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18. Given that $\sin A = \frac{3}{5}$ and that A is an acute angle, find without using tables, the values of $\sin 2A$, $\cos 2A$ and $\tan 2A$. Hence find the value of $\sin 4A$.



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19. Prove that

$$\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = \tan x, x \in I \text{ or } III \text{ quad.}$$



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20. Prove that

$$\tan A - \cot A = -2 \cot 2A$$



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21. Prove that

$$\frac{\cos A}{1 - \sin A} = \tan\left(45^\circ + \frac{A}{2}\right),$$



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22. Prove that

$$\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = 2 \sec \theta.$$



23. Prove that

$$\sin^2\left(\frac{\pi}{8} + \frac{\theta}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{\theta}{2}\right) = \sin \frac{\pi}{4} \sin \theta.$$



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24. Prove that $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8\theta}}} = 2 \cos \theta$, where $\theta \in \left[\frac{-\pi}{8}, \frac{\pi}{8}\right]$



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25. If $7A = \pi$, show that $\cos A \cos 2A \cos 3A = \frac{1}{8}$.



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26. Prove that $\cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{3\pi}{9} \cos \frac{4\pi}{9} = \frac{1}{2^4}$.



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27. Prove that

$$\cos^2 A + \cos^2(A + 120^\circ) + \cos^2(A - 120^\circ) = \frac{3}{2}$$



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28. Prove that

$$\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8} = \frac{3}{2}$$



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29. Prove that $\cos \theta \cos 2\theta \cos 4\theta \dots \cos 2^{n-1}\theta = \frac{\sin(2^n\theta)}{2^n(\sin \theta)}$.



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30. Prove that $16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta = \cos 5\theta$.



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31. Prove that

$$\sin 3A = 4 \sin A \sin (60^\circ - A) \sin (60^\circ + A).$$



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32. Prove that

$$\sin \frac{\pi}{9} \sin \frac{2\pi}{9} \sin \frac{3\pi}{9} \sin \frac{4\pi}{9} = \frac{3}{16}.$$



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33. Prove that $\cos \theta + \cos(120^\circ + \theta) + \cos(120^\circ - \theta) = 0$.

Hence

deduce

that

$$\cos^3 \theta + \cos^3(120^\circ + \theta) + \cos^3(120^\circ - \theta) = \frac{3}{4} \cos 3\theta.$$



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34. Find $\cos \frac{7\pi}{8}$.

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35. Using the value of $\cos 315^\circ$, find the value of
 $\sin 157\frac{1}{2}^\circ$ and $\cos 157\frac{1}{2}^\circ$.

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36. Find the value of
 $\sin 22\frac{1}{2}^\circ$

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37. Find the value of
 $\cos 22\frac{1}{2}^\circ$

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38. Find the value of

$$\tan 22\frac{1}{2}^\circ$$



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39. Find the value of

$$\sin 7\frac{1}{2}^\circ$$



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40. Find the value of

$$\cos 7\frac{1}{2}^\circ$$



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41. Find the value of

$$\tan 11\frac{1}{4}^\circ$$





42. Prove that

$$\tan 7\frac{1}{2}^\circ = \sqrt{2} - \sqrt{3} - \sqrt{4} + \sqrt{6} = (\sqrt{3} - \sqrt{2})(\sqrt{2} - 1).$$



43. Prove that

$$\cot 7\frac{1}{2}^\circ = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6} = (\sqrt{3} + \sqrt{2})(\sqrt{2} + 1).$$



Exercise 5 A

1. Compute

$\sin 15^\circ$ from the functions 60° and 45°



2. Compute

$\cos 345^\circ$ from the functions of 300° and 45°



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3. Compute

$\tan 105^\circ$ from the functions of 45° and 60°



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4. Compute

$\sin 135^\circ$ from the functions of 180° and 45°



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5. Compute

$\cos 195^\circ$ from the functions of 150° and 45° ,





6. Compute

$$\cos ec(13\pi/12)$$



7. Simplify by reducing to a single term :

$$\sin 3\alpha \cos 2\alpha + \cos 3\alpha \sin 2\alpha.$$



8. Simplify by reducing to a single term :

$$\cos 5\theta \cos 2\theta - \sin 5\theta \sin 2\theta.$$



9. Simplify by reducing to a single term :

$$\sin 22^\circ \cos 38^\circ + \cos 22^\circ \sin 38^\circ.$$



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10. Simplify by reducing to a single term :

$$\sin 80^\circ \cos 20^\circ - \cos 80^\circ \sin 20^\circ.$$



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11. Simplify by reducing to a single term :

$$\sin(x - y)\cos x - \cos(x - y)\sin x.$$



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12. Simplify by reducing to a single term :

$$\cos(\theta + \alpha)\cos(\theta - \alpha) - \sin(\theta + \alpha)\sin(\theta - \alpha).$$





13. Simplify by reducing to a single term :

$$\frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$$



14. Simplify by reducing to a single term :

$$\frac{\tan \alpha - \tan(\alpha - \beta)}{1 + \tan \alpha \tan(\alpha - \beta)}$$



15.

Prove

that

$$(\sin \alpha \cos \beta + \cos \alpha \sin \beta)^2 + (\cos \alpha \cos \beta - \sin \alpha \sin \beta)^2 = 1.$$



$$16. \sin(60^\circ + \theta) - \sin(60^\circ - \theta) = \sin \theta.$$



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$$17. \text{Prove that } \sin(\theta+30^\circ) + \cos(\theta+60^\circ) = \cos \theta.$$



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$$18. \sin(240^\circ + \theta) + \cos(330^\circ + \theta) = 0$$



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$$19. \sin(A - 45^\circ) = \frac{1}{\sqrt{2}}(\sin A - \cos A)$$



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$$20. \text{Prove that :} \cos\left(\frac{\pi}{3} + x\right) = \frac{\cos x - \sqrt{3} \sin x}{2}$$



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21. Prove: $\tan(45^\circ + \theta) = \frac{1 + \tan \theta}{1 - \tan \theta}$



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22. Prove: $\tan(45^\circ - \theta) = \frac{1 - \tan \theta}{1 + \tan \theta}$



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23. $\frac{\sin(\theta + \phi)}{\sin \theta \cos \phi} = \cot \theta \tan \phi + 1.$



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24. $\frac{\sin(\theta - \phi)}{\sin \theta \sin \phi} = \cot \phi - \cot \theta.$



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25. Prove that $\frac{\sin(A - B)}{\sin A \sin B} + \frac{\sin(B - C)}{\sin B \sin C} + \frac{\sin(C - A)}{\sin C \sin A} = 0$



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26. Prove that: $\sin 105^\circ + \cos 105^\circ = \cos 45^\circ$



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27. Find the value of $\sin(\alpha + \beta)$, $\cos(\alpha + \beta)$, and $\tan(\alpha + \beta)$, given

$$\sin \alpha = \frac{3}{5}, \cos \beta = \frac{5}{13}, \alpha \text{ and } \beta \text{ in Quadrant I.}$$



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28. Find the value of $\sin(\alpha + \beta)$, $\cos(\alpha + \beta)$, and $\tan(\alpha + \beta)$, given

$$\cos \alpha = \frac{-12}{13}, \cot \beta = \frac{24}{7}, \alpha \text{ in Quadrant II, } \beta \text{ in Quadrant III.}$$



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29. Find the value of $\sin(\alpha - \beta)$, $\cos(\alpha - \beta)$ and $\tan(\alpha - \beta)$, given

$$\sin \alpha = \frac{8}{17}, \tan \beta = \frac{5}{12}, \alpha \text{ and } \beta \text{ in Quadrant I.}$$



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30. Find the value of $\sin(\alpha - \beta)$, $\cos(\alpha - \beta)$ and $\tan(\alpha - \beta)$, given

$$\cos \alpha = \frac{-12}{13}, \cot \beta = \frac{24}{7}, \alpha \text{ in Quadrant II, } \beta \text{ in Quadrant I.}$$



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31. If A and B are acute angles, find $(A + B)$ given

$$\sin A = \frac{1}{\sqrt{5}}, \sin B = \frac{1}{\sqrt{10}}$$



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32. If A and B are acute angles, find $(A + B)$ given

$$\tan A = \frac{5}{6}, \tan B = \frac{1}{11}.$$





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33. Given that $\tan \alpha = \frac{m}{m+1}$, $\tan \beta = \frac{1}{2m+1}$ then what is the value of $\alpha + \beta$?



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34. In the ΔABC the foot of the perpendicular from A to BC is D. Given that $\tan B = \frac{4}{3}$, $\cos C = \frac{15}{17}$ and that $AB = 20\text{cm}$, calculate without using tables

- (i) the lengths of the sides AC and BC,
- (ii) the value of $\sin A$.



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35. Given that $\tan(A + B) = 1$ and $\tan(A - B) = \frac{1}{7}$, find without using tables the values of $\tan A$ and $\tan B$.



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36. If $\tan(A + B) = x$ and $\tan B = \frac{1}{2}$ prove that $\tan A = \frac{2x - 1}{x + 2}$ and obtain an expression for $\tan(A - B)$ in terms of x . If $\tan(A - B) = \frac{1}{3}$ and A is acute, find A without using tables,



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37. If $\sin(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$, $\alpha + \beta, \alpha - \beta$ being acute angles prove that $\tan 2\alpha = \frac{63}{16}$.



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38. Prove that $1 + \tan 2\theta \tan \theta = \sec 2\theta$.



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39. Show that $4 \sin\left(\frac{\pi}{3} - \theta\right) \sin\left(\frac{\pi}{3} + \theta\right) = 3 - 4 \sin^2 \theta$



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$$40. \frac{\cos 17^\circ + \sin 17^\circ}{\cos 17^\circ - \sin 17^\circ} = \tan 62^\circ$$



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$$41. \tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A.$$



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$$42. \tan 75^\circ - \tan 30^\circ - \tan 75^\circ \tan 30^\circ = 1.$$



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43.

Prove

that

$$\cos 2\theta \cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi) = \cos(2\theta + 2\phi).$$



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44. If $\sin(\theta + \alpha) = \cos(\theta + \alpha)$, prove that $\tan \theta = \frac{1 - \tan \alpha}{1 + \tan \alpha}$



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45. Given that $A = B + C$. prove that

$$\tan A - \tan B - \tan C = \tan A \tan B \tan C.$$



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46. If $A + B = 45^\circ$, show that $\tan A + \tan B + \tan A \tan B = 1$.

Hence or otherwise, express $\tan 22^\circ 30'$ in surd form.



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47. Use the expansion of $\tan(A - B)$ to find $\tan 15^\circ$ without the use of tables, leaving your answer in surd form with an integral denominator.



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48. Prove that $\frac{2 \tan A}{1 + \tan^2 A} = \sin 2A.$



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49. If $A + B = 225^\circ$, prove that $\tan A + \tan B = 1 - \tan A \tan B$



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Exercise 5 B

1. Convert the products into sum or difference.

$$2\sin 48^\circ \cos 12^\circ$$



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2. Convert the products into sum or difference. If angles are given in degrees, evaluate from tables.

$$2\sin 54^\circ \sin 66^\circ$$



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3. Convert the products into sum or difference. If angles are given in degrees, evaluate from tables.

$$2\cos 5\theta \cos 3\theta$$



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4. Convert the products into sum or difference. If angles are given in degrees, evaluate from tables.

$$2\cos 72^\circ \sin 56^\circ$$



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5. Convert the products into sum or difference. If angles are given in degrees, evaluate from tables.

$$\cos(\alpha + \beta)\cos(\alpha - \beta)$$



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6. Convert the products into sum or difference. If angles are given in degrees, evaluate from tables.

$$\sin \frac{A + B}{2} \cos \frac{A - B}{2}$$



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7. Convert the sums or differences into products:

$$\sin 12A + \sin 4A$$



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8. Convert the sums or differences into products:

$$\sin 37^\circ + \sin 21^\circ$$



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9. Convert the sums or differences into products:

$$\sin 12A - \sin 4A$$



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10. Convert the sums or differences into products:

$$\cos 79^\circ + \cos 11^\circ$$



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11. Convert the sums or differences into products:

$$\cos 12\alpha + \cos 8\alpha$$





12. Convert the sums or differences into products:

$$\cos 25^\circ - \cos 37^\circ$$



13. Convert the sums or differences into products:

$$\sin 61^\circ - \cos 39^\circ$$



14. Convert the sums or differences into products:

$$\sin 4x + \cos 2x$$



$$15. \text{ Prove that: } \frac{\sin A + \sin B}{\cos A + \cos B} = \tan\left(\frac{A + B}{2}\right)$$



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$$16. \frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = \frac{1}{\sqrt{3}}$$



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$$17. \frac{\sin 7x + \sin 3x}{\cos 7x + \cos 3x} = \tan 5x.$$



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$$18. \frac{\cos 2B - \cos 2A}{\sin 2A + \sin 2B} = \tan(A - B).$$



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$$19. \frac{\sin(4A - 2B) + \sin(4B - 2A)}{\cos(4A - 2B) + \cos(4B - 2A)} = \tan(A + B)$$



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$$20. \frac{\cos \alpha + 2 \cos 3\alpha + \cos 5\alpha}{\cos 3\alpha + 2 \cos 5\alpha + \cos 7\alpha} = \cos 3\alpha \sec 5\alpha.$$



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$$21. \frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A} = \tan 4A.$$



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$$22. \cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$$



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23.

Prove

that

:

$$\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = \sin 70^\circ + \sin 80^\circ.$$



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24. Prove that $\cos 15^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$



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25. Prove that $\sin 36^\circ + \cos 36^\circ = \sqrt{2}\cos 9^\circ$.



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26. Prove that $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$.



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$$27. \sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}.$$



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$$28. 4\cos 12^\circ \cos 48^\circ \cos 72^\circ = \cos 36^\circ$$



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$$29. \tan 20^\circ \tan 40^\circ \tan 80^\circ = \tan 60^\circ$$



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$$30. 2\cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0.$$



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$$31. \tan(A + 30^\circ) + \cot(A - 30^\circ) = \frac{1}{\sin 2A - \sin 60^\circ}.$$



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32. $\cos A + \cos(120^\circ - A) + \cos(120^\circ + A) = 0$



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33. $\cos(A + B) + \sin(A - B) = 2 \sin(45^\circ + A) \cos(45^\circ + B)$.



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34. If $\cos A + \cos B = \frac{1}{3}$ and $\sin A + \sin B = \frac{1}{4}$ prove that
 $\tan \frac{1}{2}(A + B) = \frac{3}{4}$.



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35. What is the value of $\frac{\cos 10^\circ + \sin 20^\circ}{\cos 20^\circ - \sin 10^\circ}$?



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Exercise 5 C

1. Evaluate :

$$2\sin 15^\circ \cos 15^\circ$$



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2. Evaluate :

$$1 - 2\sin^2 22.5^\circ$$



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3. Evaluate :

$$2\cos^2 157.5^\circ - 1$$



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4. Evaluate :

$$\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$$



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5. Evaluate :

$$\frac{1}{2} - \sin^2 \frac{7\pi}{12}$$



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6. Evaluate :

$$\cos \frac{\pi}{8} \sin \frac{\pi}{8}$$



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7. Evaluate :

$$\frac{2 \tan 22\frac{1}{2}^\circ}{1 - \tan^2 22\frac{1}{2}^\circ}$$



8. Evaluate :

$$8 \cos^3 \frac{\pi}{9} - 6 \cos \frac{\pi}{9}$$



9. Find the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given :

- (i) $\sin \theta = \frac{3}{5}$, θ in Quadrant I.
- (ii) $\sin \theta = \frac{3}{5}$, θ in Quadrant II.
- (iii) $\sin \theta = -\frac{1}{2}$, θ in Quadrant IV.
- (iv) $\tan \theta = -\frac{1}{5}$, θ in Quadrant II.



10. ABC is an acute-angled triangle inscribed in a circle of radius 5 cm and centre O . The sine of angle A is equal to $\frac{3}{5}$. Calculate without using tables :

(i) the length of BC

(ii) $\sin OBC$

(iii) $\sin BOC$

(iv) $\cos BOC$



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11. Derive function of 120° from functions of 60° and check by using relations between functions of supplementary angles.



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12. If $\sin \theta = a$ and $\sin 2\theta = b$, find an expression for $\cos \theta$ in terms of a and b. Hence find a relation between a and b not involving θ .



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13. Given that $\tan A = \frac{1}{5}$. find the values of $\tan 2A$, $\tan 4A$ and $\tan(45^\circ - 4A)$.



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14. If A is an obtuse angle whose sine is $\frac{5}{13}$ and B is an acute angle whose tangent is $\frac{3}{4}$, without using tables find the values of

- (a) $\sin 2B$,
- (b) $\tan(A - B)$.



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15. Express $\cos 6\alpha$ in terms of $\cos 3\alpha$.



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16. $\sin 100^\circ$ in terms of functions of 5θ ,





17. write $\tan 8\alpha$ in terms of $\tan 4\alpha$.



18. $\cos 2\theta$ in terms of $\cos 4\theta$,



19. $\tan 4\phi$ in terms of $\cos 8\phi$,



20. Express $\sin \frac{5\pi}{2}$ in terms of $\cos 5\pi$,



21. $\cos 20\theta$ in terms of $\sin 5\theta$.



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22. Using the half angle formulas, find the exact value of
(i) $\sin 15^\circ$ (ii) $\sin 22\frac{1}{2}^\circ$.



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23. In the triangle ABC , in which C is the right angle, prove that :

$$\sin 2A = \frac{2ab}{c^2}, \cos 2A = \frac{b^2 - a^2}{c^2}, \sin \frac{A}{2} = \sqrt{\frac{c - b}{2c}}, \cos \frac{A}{2} = \sqrt{\frac{c + b}{2c}}.$$



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24. If $\cos \alpha = \frac{3}{5}$, $\cos \beta = \frac{4}{5}$, find the value of $\cos \left(\frac{\alpha - \beta}{2} \right)$, assuming α and β to be acute angles.



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25. Given that $\cos \frac{A}{2} = \frac{12}{13}$, calculate without the use of tables, the values of $\sin A$, $\cos A$ and $\tan A$.

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26. Given that $\tan x = \frac{12}{5}$, $\cos y = \frac{-3}{5}$, and the angles x and y are in the same quadrants, calculate without the use of tables the values of
(i) $\sin(x + y)$, (ii) $\cos \frac{y}{2}$.

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27. Given that $\sin^2 \beta = \sin \alpha \cos \alpha$, show that
 $\cos 2\beta = 2 \cos^2 \left(\frac{\pi}{4} + \alpha \right)$.

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28. Derive formulas for the following in terms of functions of 2θ and θ and then of θ .

(i) $\sin 4\theta$

(ii) $\cos 4\theta$,

(iii) $\tan 4\theta$



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29. If $\sin \alpha = \frac{3}{5}$, find value of (i) $\sin 3\alpha$, (ii) $\cos 3\alpha$ (iii) $\tan 3\alpha$.



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30. If $2 \cos \theta = x + \frac{1}{x}$, prove that $2 \cos 3\theta = x^3 + \frac{1}{x^3}$.



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Exercise 5 D

1. Prove: $(\sin \phi - \cos \phi)^2 = 1 - \sin 2\phi$.



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2. Prove that $\left(\cos \frac{\theta}{2} + \sin \frac{\theta}{2}\right)^2 = 1 + \sin \theta$.



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3. Prove: $\left(\sin \theta \times \tan \frac{\theta}{2}\right) = 2 \sin^2 \frac{\theta}{2}$



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4. $\frac{1}{\sec 2\theta} = \cos^4 \theta - \sin^4 \theta$.



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5. Prove: $\frac{2}{\cot \alpha \tan 2\alpha} = 1 - \tan^2 \alpha$.



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6. Prove that $\frac{\sin 2A}{1 - \cos 2A} = \cot A$



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7. Prove: $\frac{\sin 2A}{1 + \cos 2A} = \tan A$



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8. Prove that $\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} = 2$



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9. Prove that $\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = \frac{2 + \sin 2A}{2}$.



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10. Prove that $\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \theta.$



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11. Prove that $\cot \alpha - \tan \alpha = 2 \cot 2\alpha.$



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12. Prove that $\cos ec A - 2 \cot 2A \cos A = 2 \sin A.$



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13. $\frac{1 + \sin 2x + \cos 2x}{\cos x + \sin x} = 2 \cos x$



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14. Show that '(1-cos 2x + sin x)/(sin 2x + cos x) = tan x'



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$$15. \frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} = \tan \theta.$$



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$$16. \cot A = \frac{1}{2} \left(\cot \frac{A}{2} - \tan \frac{A}{2} \right).$$



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$$17. \frac{\cos A + \sin A}{\cos A - \sin A} - \frac{\cos A - \sin A}{\cos A + \sin A} = 2 \tan 2A.$$



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$$18. \frac{\cos A - \sin A}{\cos A + \sin A} = \sec 2A - \tan 2A.$$



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$$19. \text{ Prove that } \frac{\sin 2A}{1 - \cos 2A} \cdot \frac{1 - \cos A}{\cos A} = \tan \frac{A}{2}$$



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$$20. \text{ Prove that : } 2 \cos A = \sqrt{2 + \sqrt{2(1 + \cos 4A)}}, A \in I \text{ or } IV \text{ Quadrant.}$$



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$$21. \tan 2A = (\sec 2A + 1) \sqrt{(\sec^2 A - 1)}, A \text{ belongs to I or IV Quad.}$$



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$$22. \text{ Prove that } \frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}.$$



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23. Prove that $\tan 2A - \sec A \sin A = \tan A \sec 2A$.



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24. Prove that $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4 \cos^2\left(\frac{A - B}{2}\right)$.



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25. Prove that $(\cos A - \cos B)^2 + (\sin A - \sin B)^2 = 4 \sin^2\left(\frac{A - B}{2}\right)$.



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26. Prove that: $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$.



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27. $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$.



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$$28. \cos \alpha \cos(60^\circ - \alpha) \cos(60^\circ + \alpha) = \frac{1}{4} \cos 3\alpha.$$



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$$29. \frac{\cos^3 \alpha - \cos 3\alpha}{\cos \alpha} + \frac{\sin^3 \alpha + \sin 3\alpha}{\sin \alpha} = 3.$$



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$$30. \cos^2 2\theta - \sin^2 \theta = \cos \theta \cdot \cos 3\theta.$$



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$$31. 1 + \frac{\cos 2\theta + \cos 6\theta}{\cos 4\theta} = \frac{\sin 3\theta}{\sin \theta}.$$



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$$32. \text{ Prove that } \cos^3\left(x - \frac{2\pi}{3}\right) + \cos^3 x + \cos^3\left(x + \frac{2\pi}{3}\right) = \frac{3}{4} \cos 3x.$$



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$$33. \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4.$$



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$$34. \tan 70^\circ - \tan 20^\circ - 2\tan 40^\circ = 4\tan 10^\circ.$$



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$$35. \tan \theta + 2 \tan 2\theta + 4 \tan 4\theta + 8 \cot 8\theta = \cot \theta.$$



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$$36. 4(\cos^3 20^\circ + \cos^3 40^\circ) = 3(\cos 20^\circ + \cos 40^\circ)$$



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37. Prove that $4(\cos^3 10^\circ + \sin^3 20^\circ) = 3(\cos 10^\circ + \sin 20^\circ)$.



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38. Prove that $\cos^3 x \sin^2 x = \frac{1}{16}(2 \cos x - \cos 3x - \cos 5x)$.



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39. $\cot A + \cot(60^\circ + A) + \cot(120^\circ + A) = 3 \cot 3A$.



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40. Prove that : $\cos 36^\circ - \sin 18^\circ = \frac{1}{2}$.



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$$41. \cos^2 36^\circ + \sin^2 18^\circ = \frac{3}{4}.$$



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$$42. 3\cos 72^\circ - 4\sin^3 18^\circ = \cos 36^\circ.$$



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$$43. \cos^2 48^\circ - \sin^2 12^\circ = \frac{\sqrt{5} + 1}{8}.$$



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$$44. \text{Prove that } \sin \frac{1}{10}\pi \cos \frac{1}{5}\pi = \frac{1}{4}.$$



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$$45. \sec 72^\circ - \sec 36^\circ = 2.$$



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$$46. \sin^2 24^\circ - \sin^2 6^\circ = \frac{\sqrt{5} - 1}{8}.$$



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$$47. \sin 12^\circ \sin 48^\circ \sin 54^\circ = \frac{1}{8}.$$



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$$48. \text{Prove that } 4\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ = \frac{1}{4}.$$



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$$49. \sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \frac{4\pi}{5} \sin \frac{3\pi}{5} = \frac{5}{16}.$$



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50. Prove that $\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{14\pi}{15} = \frac{1}{16}$.



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51. $\cos^2 \frac{\pi}{10} + \cos^2 \frac{2\pi}{5} + \cos^2 \frac{3\pi}{5} + \cos^2 \frac{9\pi}{10} = 2$



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52. $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$



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53. $\cos \alpha \cos 2\alpha \cos 4\alpha \cos 8\alpha = \frac{1}{16}$, if $\alpha = 24^\circ$



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54. $\cos 12^\circ \cos 24^\circ \cos 36^\circ \cos 48^\circ \cos 72^\circ \cos 84^\circ = \frac{1}{2^6}$



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55. If $\theta = \frac{\pi}{2^n + 1}$ prove that $2^n \cos \theta \cos 2\theta \cos 2^2\theta \dots \cos 2^{n-1}\theta = 1$



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56. If $\tan \frac{x-y}{2}, \tan z, \tan \frac{x+y}{2}$ are in G.P. then show that
 $\cos x = \cos y \cdot \cos 2z$



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57. If $\sin \theta$ is G.M. of $\sin \phi$ and $\cos \phi$, then prove that
 $\cos 2\theta = 2 \cos^2 \left(\frac{\pi}{4} + \phi \right)$.



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58. If $m \tan(\theta - 30^\circ) = n \tan(\theta + 120^\circ)$, show that

$$\cos 2\theta = \frac{m+n}{2(m-n)}.$$



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59. If $\sin \alpha = \lambda \sin(\theta - \alpha)$ then prove that

$$\tan\left(\alpha - \frac{\theta}{2}\right) = \frac{\lambda - 1}{\lambda + 1} \tan \frac{\theta}{2}.$$



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60. If $\tan \frac{\alpha}{2} = \sqrt{\left\{ \frac{1-e}{1+e} \right\}} \tan \frac{\beta}{2}$, Prove that $\cos \beta = \frac{\cos \alpha - e}{1 - e \cos \alpha}$.



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Chapter Test

1. Show that $\tan 75^\circ = (\sqrt{3}) + \frac{1}{\sqrt{3}-1} = 2 + \sqrt{3}$. Hence deduce that $\tan 75^\circ - \cot 75^\circ = 4\sin 60^\circ$.

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2. Prove that

$$\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$$

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3. if $A + B + C = \pi$, and $\cos A = \cos B \cos C$, show that $2 \cot B \cot C = 1$.

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4. Show that $\frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = 2$, given that $\tan \alpha = 2 \tan \beta$.

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5. Show that $\frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ} = \tan 55^\circ$.



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6. If $\sin 2A = \frac{4}{5}$, find the value of $\tan A$, $(0^\circ \leq A \leq \frac{\pi}{3})$



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7. Express $\cot A$ in terms of $\cos 2A$



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8. Write $\cos 4\theta$ in terms of $\cos \theta$.



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9. A positive acute angle is divided into two parts whose tangents are $\frac{1}{2}$ and $\frac{1}{3}$. Show that the angle is $\frac{\pi}{4}$.

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10. Show that $\cos 10^\circ + \cos 110^\circ + \cos 130^\circ = 0$

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11. Show that $\frac{\sin 5A + 2\sin 8A + \sin 11A}{\sin 8A + 2\sin 11A + \sin 14A} = \frac{\sin 8A}{\sin 11A}$.

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12. Show that $\frac{1}{2\sin 10^\circ} - 2\sin 70^\circ = 1$.

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13.

Show

that

$$\sin 19^\circ + \sin 41^\circ + \sin 83^\circ = \sin 23^\circ + \sin 37^\circ + \sin 79^\circ.$$



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14. If $\sin A = \frac{1}{\sqrt{3}}$ and $\sin B = \frac{1}{\sqrt{5}}$ find the value of $\tan \frac{1}{2}(A+B) \cdot \cot \frac{1}{2}(A-B)$.



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15. If $\sin \theta = n \sin(\theta + 2\alpha)$, show that $(n-1)\tan(\theta + \alpha) + (n+1)\tan \alpha = 0$.



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16. If $\tan \frac{\alpha}{2}$ and $\tan \frac{\beta}{2}$ are the roots of the equation $8x^2 - 26x + 15 = 0$, then find the value of $\cos(\alpha + \beta)$.



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17.

Prove

that

$$\left(\frac{\cos A + \cos B}{\sin A - \sin B} \right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B} \right)^n = \begin{cases} 2 \cot^n \left(\frac{A-B}{2} \right) & \text{,if } n \text{ is even.} \\ 0 & \text{,if } n \text{ is odd.} \end{cases}$$



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18. Find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ in each of the case:

$$\sin x = \frac{1}{4}, x \text{ in II quadrant.}$$



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19. Find $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ in each of the case:

$$\tan x = \frac{-4}{3}, x \text{ in II quadrant.}$$



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20. Prove that $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$.



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21. Prove that $\sin^2 72^\circ - \sin^2 60^\circ = \frac{\sqrt{5} - 1}{8}$.



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22. If $\tan x, \tan y, \tan z$ are in G.P., show that $\cos 2y = \frac{\cos(x+z)}{\cos(x-z)}$.



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23. If $\tan \frac{\alpha}{2} : \tan \frac{\beta}{2} = 1 : \sqrt{3}$ show that $\cos \beta = \frac{2 \cos \alpha - 1}{2 - \cos \alpha}$.



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