MATHS

BOOKS - S CHAND MATHS (ENGLISH)

PROPERTIES OF TRIANGLE

Examples

1. In a $\triangle ABC$, a = 2, b = 3 and sinA = $\frac{2}{3}$, find $\angle B$.

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2. The angles of a triangle ABC are in A.P. and $b : c = \sqrt{3} : \sqrt{2}, \quad$ find $\angle A$.

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3. In any $\triangle ABC$, prove that, $\dfrac{\sin B}{\sin C} = \dfrac{c - a \cos B}{b - a \cos C}$.

4. In any $\triangle ABC$, prove that $a(b \cos C - c \cos B) = b^2 - c^2$.

5. In any $\triangle ABC$, prove that, $\dfrac{b^2 - c^2}{a^2} \sin 2A + \dfrac{c^2 - a^2}{b^2} \sin 2B + \dfrac{a^2 - b^2}{c^2} \sin 2C = 0$.

6. Find the area of a $\triangle ABC$ in which $\angle A = 30^0, b = 6cm, c = 2cm$.

7. In any $\triangle ABC$, prove that $\Delta = \dfrac{b^2 + c^2 - a^2}{4 \cot A}$.

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8. In any $\triangle ABC$, prove that $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A = 4\Delta$.

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More Solved Examples

1. In any $\triangle ABC$, prove that $a \cos \left(\dfrac{B - C}{2} \right) = (b + c)\sin \left(\dfrac{A}{2} \right)$.

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2. In any $\triangle ABC$, prove that

(i) $\tan \dfrac{B - C}{2} = \dfrac{b - c}{b + c} \cot \dfrac{A}{2}$.

(ii) If $\angle B = 90°$, prove that $\tan\left(\dfrac{A}{2}\right) = \sqrt{\left(\dfrac{b-c}{b+c}\right)}$.

(iii) If $\angle C = 90°$, prove that $\dfrac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)} = \dfrac{a-b}{a+b}$.

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3. The sides of a triangle are three consecutive numbers and its largest angle is twice the smallest one. Find the sides of the triangle.

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4. In any $\triangle ABC$ prove that,

$a^3 \sin(B-C) + b^3 \sin(C-A) + c^3 \sin(A-B) = 0.$

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5. If $a\cos A = b\cos B$, then either the triangle is isosceles or right angled.

6. If A, B, C are the angles of a triangle ABC and if $\cos A = \dfrac{\sin B}{2\sin C}$, show that the triangle is isosceles.

7. If a^2, b^2, c^2 are in A.P., prove that $\cot A$, $\cot B$, $\cot C$ are in $A\dot{P}$.

8. In any $\triangle ABC$, prove that $\Delta = \dfrac{a^2 - b^2}{2} \dfrac{\sin A \sin B}{\sin(A - B)}$.

Exercise 7

1. In $\triangle ABC$,

if a = 2, b = 3, c = 4, prove that $\cos A = \dfrac{7}{8}$.

2. In $\triangle ABC$,

if the sides are $7, 4\sqrt{3}$ and $\sqrt{13}$ cm, prove that the smallest angle is $30°$.

3. In $\triangle ABC$,

if a = 9, b = 8, c = 4, prove that $6\cos C = 4 + 3\cos B$.

4. In $\triangle ABC$,

The sines of the angles of a triangle are in the ratio of 4 : 5 : 6, prove that

the cosines of the angles are 12 : 9 : 2.

5. In $\triangle ABC$,

If the two angles of a triangle are $30°$ and $45°$ and the included side is $\left(\sqrt{3} + 1\right)$ cm, find the area of the triangle.

6. In $\triangle ABC$,

If in a $\triangle ABC$, a = 6, b = 3 and $\cos(A - B) = \dfrac{4}{5}$, find its area.

7. In $\triangle ABC$,

In a triangle ABC, $\angle C = 60°$ and $\angle A = 75°$. If D is a point on AC such

that the area of the $\triangle BAD$ is $\sqrt{3}$ times the area of the $\triangle BCD$, find the $\angle ABD$.

8. In any $\triangle ABC$, prove that

$$\frac{\sin A}{\sin(A + B)} = \frac{a}{c}$$

9. In any $\triangle ABC$, prove that

$$\frac{a - b}{a + b} = \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)}$$

10. In any $\triangle ABC$, prove that

$$ac \cos B - bc \cos A = a^2 - b^2$$

11. In any $\triangle ABC$, prove that

$$\frac{\sin(A - B)}{\sin(A + B)} = \frac{a^2 - b^2}{c^2}$$

12. In any $\triangle ABC$, prove that

$$a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$$

13. In any $\triangle ABC$, prove that

$$a\cos(A + B + C) - b\cos(B + A) - c\cos(A + C) = 0$$

14. In any $\triangle ABC$, prove that

$$a(\cos C - \cos B) = 2(b - c)\cos^2 \frac{1}{2}A$$

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15. In any $\triangle ABC$, prove that

$$a \sin \frac{1}{2}(B - C) = (b - c)\cos \frac{1}{2}A$$

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16. In any $\triangle ABC$, prove that

$$a \sin\left(\frac{A}{2} + B\right) = (b + c)\sin \frac{A}{2}$$

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17. In any $\triangle ABC$, prove that

$$c^2 = (a - b)^2 \cos^2 \frac{1}{2}C + (a + b)^2 \sin^2 \frac{1}{2}C$$

18. In any $\triangle ABC$, prove that

$$a \sin(B - C) + b \sin(C - A) + c \sin(A - B) = 0$$

19. In any $\triangle ABC$, prove that

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$$

20. In any $\triangle ABC$, prove that

$$\frac{1 + \cos(A - B)\cos C}{1 + \cos(A - C)\cos B} = \frac{a^2 + b^2}{a^2 + c^2}$$

21. In any $\triangle ABC$, prove that

$$(b^2 - c^2)\cot A + (c^2 - a^2)\cot B + (a^2 - b^2)\cot C = 0$$

22. In any $\triangle ABC$, prove that

$$a^3 \sin(B - C)\cos ec^2 A + b^3 \sin(C - A)\cos ec^2 B + c^3 \sin(A - B)\cos ec^2 C =$$

23. In any $\triangle ABC$, prove that

$$a^3 \cos(B - C) + b^3 \cos(C - A) + c^3 \cos(A - B) = 3abc$$

24. In a $\triangle ABC$, if $\dfrac{2\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{2\cos C}{c} = \dfrac{a}{bc} + \dfrac{b}{ca}$, prove that $\angle A = 90°$.

25. In a $\triangle ABC$, AD is the altitude from A. Given $b > c, \angle C = 23°$ and $AD = \dfrac{abc}{(b^2 - c^2)}$, find $\angle B$.

26. In $\triangle ABC$, $\dfrac{a^2 + b^2}{a^2 - b^2} = \dfrac{\sin(A + B)}{\sin(A - B)}$, prove that the triangle is isosceles or right triangle.

27. If $\dfrac{\sin A}{\sin C} = \dfrac{\sin(A - B)}{\sin(B - C)}$, prove that a^2, b^2, c^2 are in A.P.

28. If $\sin 2A + \sin 2B = \sin 2C$, prove that $A = 90°$ or $B = 90°$.

29. If in a triangle $ABC, \angle C = 60^0$, then prove that

$$\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c} .$$

30. In a $\triangle ABC$ the angles A, B, C are in A.P. show that

$$2\cos\frac{A-C}{2} = \frac{a+c}{\sqrt{(a^2 - ac + c^2)}}$$

31. If the angles A, B, C of $\triangle ABC$ are in A.P. and $b : c = \sqrt{3} : \sqrt{2}$, show that $A = 75°$.

32. If $A = 45°$ and $B = 75°$, show that $a + c\sqrt{2} = 2b$.

33. The angles A, B, C of a triangle are in the ratio $3:5:4$, prove that $a + c\sqrt{2} = 2b$.

34. In a $\triangle ABC$, if $\cos A = \dfrac{17}{22}$, $\cos C = \dfrac{1}{14}$, prove that the ratio of the sides is $7:9:11$.

35. The angle of a triangle are in the ratio $1:2:7$, prove that the ratio of the greatest side to the least side is $(\sqrt{5}+1):(\sqrt{5}-1)$.

36. If the sides of $\triangle ABC$ in the ratio 4 : 5 : 6, prove that one angle is twice that of the other.

37. Two sides and included angles of a triangle are respectively $3 + \sqrt{3}, 3 - \sqrt{3}$ and $60°$. Show that the remaining elements of the triangle are $105°$, $15°$, $3\sqrt{2}$.

38. In a $\triangle ABC$, if $B = 3C$, prove that

(i) $\cos C = \sqrt{\left(\dfrac{b+c}{4c}\right)}$ (ii) $\sin \dfrac{A}{2} = \dfrac{b-c}{2c}$.