MATHS

BOOKS - CENGAGE PUBLICATION

BINOMIAL THEOREM

Others

1. Prove that $\left(.^{2n} C_0\right)^2 - \left(.^{2n} C_1\right)^2 + \left(.^{2n} C_2\right)^2 - .. + \left(.^{2n} C_{2n}\right)^2$ = $(-1)^n .^{2n} C_n$.

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2. The largest term in the expansion of $(3+2x)^{50}$, where x=1/5, is

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3. $\frac{1}{n!}+\frac{1}{2!(n-2)!}+\frac{1}{4!(n-4)!}+\ldots\ldots$ is equal to

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4. Find the sum of the last 30 coefficients in the expansion of $(1+x)^{59}$, when expanded in ascending powers of $x\cdot$

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5. If $x=1/3,$ find the greatest tem in the expansion of $(1+4x)^{8}\cdot$

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6. If the sum of coefficients in the expansion of $(x-2y+3z)^{n}$ is 128, then find the greatest coefficient in the expansion of $(1+x)^{n}\cdot$

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7. Find the sum of the coefficients in the expansion of $\left(1 + 2x + 3x^2 + nx^n\right)^2$.

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8. The number of terms in the expansion of $(1 + x)^{101}\left(1 + x^2 - x\right)^{100}$ in powers of x is

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9. Find the sum of coefficients in $\left(1 + x - 3x^2\right)^{4163}$.

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10. Find the middle term in the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$.

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11. In the expansion of $(1+x)^{50}$, find the sum of coefficients of odd powers of x.

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12. If $(1+x-2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots\ldots + a_{12}x^{12}$ then

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13. If the middle term in the binomial expansion of $\left(\frac{1}{x} + x\sin x\right)^{10}$ is equal to $\frac{63}{8}$, find the value of x.

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14. Find the sum $C_0 + 3C_1 + 3^2C_2 + \ldots + 3^nC_n$.

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15. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r$, then prove that $C_1 + 2C_2 + 3C_3 + \ldots. + nC_n = n2^{n-1}.$.

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16. If $T_0, T_1, T_2, , T_n$ represent the terms in the expansion of $(x+a)^n$, then find the value of $(T_0 - T_2 + T_4 -)^2 + (T_1 - T_3 + T_5 -)^2 n \in N$.

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17. If $(1+x+x^2)^n = a_0 + a_1 x + a_2 x^2 + \ + a_{2n} x^{2n}$, find the value of $a_0 + a_3 + a_6 + \ + , n \in N$.

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18. Find the sum $C_0 - C_2 + C_4 - C_6 + \ldots\ldots\ldots$,where $C_r = ^n C_r$.

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19. Prove that $.^n C_0 +^n C_3 +^n C_6 + \quad = \frac{1}{3}\left(2^n + 2\cos\left(\frac{n\pi}{3}\right)\right)$.

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20. Given that the 4^{th} term in the expansion of $\left(2 + \frac{3}{8}x\right)^{10}$ has the maximum numerical value. Find the range of the value of x for which this will be true.

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21. Find the greatest coefficient in the expansion of $(1 + 2x/3)^{15}\cdot$.

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22. Find the greatest term in the expansion of $\sqrt{3}\left(1 + \frac{1}{\sqrt{3}}\right)^{20}$.

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23. Find numerically the greatest term/terms in the expansion of $(3-5x)^{15}$ when $x=1/5$.

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24. Let n be an odd natural number greater than 1. Then , find the number of zeros at the end of the sum 99^n+1.

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25. Find the remainder when 27^{40} is divided by 12.

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26. In the expansion of $(1+x)^n$, 7th and 8th terms are equal. Find the value of $(7/x+6)^2$.

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27. Find the sum $\sum_{j=0}^{n}\left(\hat{}\,(4n+1)C_j +^{4n+1} C_{2n-j}\right)$.

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28. Show that no three consecutive binomial coefficients can be in G.P.

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29. Find the sum $\sum_{r=1}^{n} r^n \frac{\hat{}\ nC_r}{\hat{}\ nC_{r-1}}$.

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30. Show that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

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31. If the 3rd, 4th. 5th and sixth term in the expansion of $(x+\alpha)^n$ are a,b,c,d respectively, then prove that $\left(\frac{b^2-ac}{c^2-bd}\right)=\frac{5a}{3c}$.

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32. Find the remainder when 7^{98} is divided by 5.

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33. Show that $2^{4n+4}-15n-16, where$ n $\in N$ is divisible by 225.

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34. If $\left(2+\sqrt{3}\right)^n=I+f,$ where I and n are positive integers and

$0<f<1,$

show that I is an odd integer and $(1-f)(1+f)=1$

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35. Find the degree of the polynomial $\frac{1}{\sqrt{4x+1}}\left\{\left(\frac{1+\sqrt{4x+1}}{2}\right)^7-\left(\frac{1+\sqrt{4x+1}}{2}\right)^7\right\}$

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36. If 9^7+7^9 is divisible b 2^n, then find the greatest value of $n, where n \in N$.

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37. Prove that $\sqrt{10}\left[\left(\sqrt{10}+1\right)^{100}-\left(\sqrt{10}-1\right)^{100}\right]$ is an even integer .

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38. Find the remainder when $x=5^{5^{5^{5 \dots}}}$ (24 times 5) is divided by 24.

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39. Find the remainder when $1690^{2608} + 2608^{1690}$ is divided by 7.

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40. Find the value of $\left\{3^{2003}/28\right\}, where\{.\}$ denotes the fractional part.

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41. Find the remainder when 5^{99} is divided by 13.

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42. Find remainder when 7^{103} is divided by 25.

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43. Using binomial theorem prove that $6^n - 5n$ always leaves remainder I when divided by 25.

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44. If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is α and the coefficients of middle terms in the expansion of $(1+x)^{2n+1}$ are β and γ then relate $\alpha, \beta \text{ and } \gamma$.

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45. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1:7:42, then find the value of n.

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46. In the coefficients of rth, $(r+1)th, and(r+2)th$ terms in the binomial expansion of $(1+y)^m$ are in A.P., then prove that $m^2-m(4r+1)+4r^2-2=0.$

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47. Prove that $(C_0+C_1)(C_1+C_2)(C_2+C_3)(C_3+C_4)..........(C_{n-1}+C_n)$ = $\frac{C_0C_1C_2.....C_{n-1}(n+1)^n}{n!}$

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48. If the coefficients of four consecutive terms in the expansion of $(1+x)^n$ are a_1, a_2, a_3 and a_4 respectively. then prove that `a_1/(a_1+a_2)+a_3/(a_3+a_4)=2a_2/(a_2+a_3).

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49. Find the sum of $\sum_{r=1}^{n} r \cdot \frac{nC_r}{nC_{r-1}}$

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50. Find the positive integer just greater than $(1 + 0.0001)^{10000}$.

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51. Find the last digit of 17^{256}.

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52. If 10^m divides the number $101^{100} - 1$ then, find the greatest value of m.

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53. Using the principle of mathematical induction, prove that $\left(2^{3n}-1\right)$ is divisible by 7 for all $n \in N$

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54. If x is very large as compare to y, then prove that $\sqrt{\frac{x}{x+y}}\sqrt{\frac{x}{x-y}}=1+\frac{y^2}{2x^2}$.

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55. Find the coefficient of x^n in the expansion of $\left(1-9x+20x^2\right)^{-1}$.

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56. Prove that the coefficient of x^r in the expansion of $(1-2x)^{-\frac{1}{2}}$ is $\frac{2r!}{(2^r)(r!)^2}$

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57. Find the sum: $1-\frac{1}{8}+\frac{1}{8}\times\frac{3}{16}-\frac{1\times3\times5}{8\times16\times24}+\dots$

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58. Show that $\sqrt{3}$ =1+$\frac{1}{3}$+($\frac{1}{3}$).($\frac{3}{6}$)+($\frac{1}{3}$).($\frac{3}{6}$).($\frac{5}{9}$)+.......

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59. Assuming x to be so small that x^2 and higher power of x can be neglected, prove that $\frac{\left(1+\frac{3x}{4}\right)^{-4}(16-3x)^{\frac{1}{2}}}{(8+x)^{\frac{2}{3}}}=1-\left(\frac{305}{96}\right)x$

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60. Find the sum `sumsum_(0lt=i

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61. Find the condition for which the formula $(a+b)^m = a^m + ma^{m-1}b + \frac{m(m-1)}{1 \times 2}a^{m-2}b^2 + \dots$ holds.

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62. Find the value of x, for which $\frac{1}{\sqrt{5+4x}}$ can be expanded as infinite series.

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63. Find the fourth term in the expansion of $(1-2x)^{3/2}$.

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64. Prove that $\hat{\ } nC_0{}^{2n}C_n -^n C_1^{2n-2}C_n +^n C_2^{2n-4}C_n = 2^n$

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65. Prove that $\hat{} \, nC_0 .^n C_0 -^{n+1} C_1 .^n C_1 +^{n+2} C_2 .^n C_2 \equiv (-1)^n.$

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66. The sum of the coefficients of all the integral powers of x in the expansion of $\left(1+2\sqrt{x}\right)^{40}$ is

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67. If the sum of the coefficient in the expansion of $\left(\alpha^2 x^2 - 2\alpha x + 1\right)^{51}$ vanishes, then find the value of α

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68. Prove that $\sum_{\alpha+\beta+\gamma=10} \frac{10!}{\alpha!\beta!\gamma!} = 3^{10}.$

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69. If $\left(1+x-2x^2\right)^{20}=a_0+a_1x+a_2x^2+a_3x^3+\ldots+a_{40}x^{40}$, then find the value of $a_1+a_3+a_5+\ldots+a_{39}$.

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70. Find the sum of the series $.^{15}C_0+^{15}C_1+^{15}C_2+\ldots\ldots\ldots\ldots..+^{15}C_7$

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71. Find the sum $\sum_{k=0}^{10}.^{20}C_k$.

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72. Find the sum of all the coefficients in the binomial expansion of $\left(x^2+x-3\right)^{319}$.

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73. If the sum of coefficient of first half terms in the expansion of $(x+y)^n$ is 256, then find the greatest coefficient in the expansion.

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74. Find the value of $\sum_{p=1}^{n}\left(\sum_{m=p}^{n}.^{n}C_{m}.^{m}C_{p}\right)$. And hence, find the value of $\lim_{n\to\infty}\frac{1}{3^n}\left(\sum_{m=p}^{n}.^{n}C_{m}.^{m}C_{p}\right)$.

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75. Show that the middle term in the expansion of $(x+1)^{2n}$ is $\frac{1.3.5.\ldots.(2n-1)}{n!}2^n.x^n$.

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76. If the middle term in the expansion of $\left(x^2 + 1/x\right)^n$ is $924\,x^6$, then find the value of n.

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77. The first three terms in the expansion of $(1 + ax)^n (n \neq 0)$ are $1, 6x$ and $16x^2$. Then find the value of a and n.

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78. If x^4 occurs in the rth term in the expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$, then find the value of r.

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79. Find the coefficient of x^{-10} in the expansion of $\left(\frac{a}{x} + bx\right)^{12}$.

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80. Find the constant term in the expansion of $\left(x - \frac{1}{x}\right)^6$.

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81. If the coefficients of $(r-5)th$ and $(2r-1)th$ terms in the expansion of $(1+x)^{34}$ are equal, find r.

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82. In $\left(2^{\frac{1}{3}} + \frac{1}{3^{\frac{1}{3}}}\right)^n$ if the ratio of 7th term from the beginning to the 7th term from the end is 1/6, then find the value of n.

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83. If the coefficient of 4th term in the expansion of $(a+b)^n$ is 56, then n is

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84. If p and q are positive, then prove that the coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ will be equal.

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85. Find the number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$.

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86. If x^p occurs in the expansion of $\left(x^2+\frac{1}{x}\right)^{2n}$, prove that its coefficient is $\frac{(2n)!}{\left[\frac{1}{3}(4n-p)\right]!\left[\frac{1}{3}(2n+p)\right]!}$.

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87. The co-efficient of $a^3b^4c^5$ in the expansion of $(bc + ca + ab)^6$ is

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88. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

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89. If the number of terms in the expansion of $(x + y + z)^n$ are 36, then find the value of n.

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90. The coefficient of a^3b^4c in the expansion of $(1 + a - b + c)^9$ is equal to

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91. The coefficient of x^4 in the expansion of $\left(1+x+x^2+x^3\right)^{11}$ is

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92. Find the number of terms which are free from radical signs in the expansion of $\left(y^{1/5}+x^{1/10}\right)^{55}$.

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93. Find the coefficient of x^5 in the expression of $\left(1+x^2\right)^5(1+x)^4$.

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94. Find the coefficient of x^{13} in the expansion of $(1-x)^5\times\left(1+x+x^2+x^3\right)^4$.

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95. Find the sum $.^{10}C_1 + ^{10}C_3 + ^{10}C_5 + ^{10}C_7 + ^{10}C_9$

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96. Find the sum of $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots,$

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97. If n is an even positive integer, then find the value of x if the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also.

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98. If $|x| < 1,$ then find the coefficient of x^n in the expansion of $(1 + 2x + 3x^2 + 4x^3 +)^{1/2}.$

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99. If $(r+1)th$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then find the value of r.

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100. If $|x|<1$, then find the coefficient of x^n in the expansion of $(1+x+x^2+......)^2$.

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101. If $|x|>1$, then expand $(1+x)^{-2}$.

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102. Find the cube root of 27

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103. Find the coefficient of x^2 in $\left(\frac{a}{a+x}\right)^{1/2} + \left(\frac{a}{a-x}\right)^{1/2}$

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104. Prove that

$:^{10}C_1(x-1)^2 -^{10}C_2(x-2)^2 +^{10}C_3(x-3)^2 \ldots\ldots -^{10}C_{10}(x-10)^2 = x^2$

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105. If the third term in the expansion of $(1+x)^m is -\frac{1}{8}x^2$, then find the value of $m\cdot$

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106. Prove that $\sum_{r=0}^{n} r(n-r)\left(.^{n}C_r\right)^2 = n^2\left(.^{2n-2}C_n\right)\cdot$

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107. Prove that

$$1 -^n C_1 \frac{1+x}{1+nx} +^n C_2 \frac{1+2x}{(1+nx)^2} -^n C_3 \frac{1+3x}{(1+nx)^3} + \ldots.(n+1) terms =$$

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108. Find the coefficient of x^{20} in $\left(x^2+2+\frac{1}{x^2}\right)^{-5}(1+x^2)^{40}$.

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109. The number of terms in the expansion of $(a+b+c)^n$ where $n \in N$ is

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110. Find the coefficient of x^{50} in the expansion of $(1+x)^{101} \times (1-x+x^2)^{100}$.

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111. Find the coefficient of x^4 in the expansion of $(2 - x + 3x^2)^6$.

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112. Find the coefficient of $x^k \in 1 + (1 + x) + (1 + x)^2 + + (1 + x)^n (0 \leq k \leq n)$.

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113. The term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3}{2}(x^2) - \frac{1}{3x}\right)^9$

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114. If $aandb$ are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, wherever n is a positive integer.

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115. Find a, b and n in the expansion of $(a+b)^n$ if the first three term s of the expansion are 729, 7290 and 30375, respectively.

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116. Find the coefficient of x^{25} in expansion of expression $\sum_{r=0}^{50} \hat{} (50)C_r(2x-3)^r(2-x)^{50-r}$.

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117. If the sum of the coefficients of the first, second, and third terms of the expansion of $\left(x^2+\frac{1}{x}\right)^m$ is 46, then find the coefficient of the term that does not contain x.

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118. If p+q=1, then show that $\sum_{r=0}^{n} r^2 \hat{} nC_r p^r q^{n-r} = npq + n^2p^2$

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119. If every pair from among the equations $x^2 + ax + bc = 0. x^2 + bx + ca = 0, and x62 + cx + ab = 0$ has a common root, then the sum of the three common roots is $-1/2(a+b+c)$ the sum of the three common roots is $2(a+b+c)$ the product of the three common roots is abc the product of the three common roots is $a^2b^2c^2$

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120. Prove that

$\hat{} mC_1^n C_m -^m C_2^{2n} C_m +^m C_3^{3n} C_m - \dots . = (-1)^{m-1} n^m.$

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121. Prove that $\hat{} \; nC_0{}^{2n}C_n -^n C_1^{2n-2}C_n +^n C_2^{2n-4}C_n = 2^n$

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122. Find the sum $\sum_{r=0} .^{n+r} C_r$.

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123. Find the value of $\left(\sum\sum\right)_{0 \leq i < j \leq n}(i+j)\left({}^nC_i + {}^nC_j\right)$.

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124. Find the sum `sumsum_(0lt=i

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125. Find the value of $\sum\sum_{0 \leq i < j \leq n} (.^n C_i + .^n C_j)$.

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126. Find the sum $\left(\sum\sum\right)_{0 \leq i < j \leq n} {}^nC_i . {}^nC_j.$

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127. Prove that $\sum_{r=0}^{s} \sum_{s=1}^{n} \hat{} \; nC_s^s C_r = 3^n - 1.$

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128. Find the sum $\sum \sum_{0 \leq i < j \leq n} {}^nC_i$

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129. Find the coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$.

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130. Find the term in $\left(3\sqrt{\left(\frac{a}{\sqrt{b}}\right)}+\left(\sqrt{\frac{b}{3\sqrt{a}}}\right)\right)^{21}$ which has the same power of a and b.

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131. Using the binomial theorem, evaluate $(102)^5$.

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132. Find the 6th term in expansion of $\left(2x^2-\frac{1}{3x^2}\right)^{10}$.

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133. Find a, if 17th and 18th terms in the expansions of $(2+a)^{50}$ are equal.

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134. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^{n}$ is $\sqrt{6}:1$.

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135. Simplify: $x^5+10x^4a+40x^3a^2+80x^2a^3+80xa^4+32a^5$.

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136. Find the value of

$$\frac{18^3+7^3+3.18.7.25}{3^6+6.243.2+15.81.4+20.27.8+15.9.16+6.3.32+64}$$

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137. Find the approximation of $(0.99)^5$ using the first three terms of its expansion.

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138. If for $n \in N, \sum_{k=0}^{2n} (-1)^k \left(.^{2n} C_k\right)^2 = A$, then find the value of $\sum_{k=0}^{2n} (-1)^k (k = 2n) \left(.^{2n} C_k\right)^2$.

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139. There are two bags each of which contains n balls. A man has to select an equal number of balls from both the bags. Prove that the number of ways in which a man can choose at least one ball from each bag $is^{2n} C_n - 1$.

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140. Find the sum $\sum_{i=0}^{r} .^{n_1} C_{r-i} .^{n_2} C_i$.

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141. Prove that $\sum_{r=0}^{2n}\left(r.^{2n}C_r\right)^2 = n^{4n}C_{2n}$.

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142. If k and n are positive integers and $S_k = 1^k + 2^k + 3^k + \ldots\ldots + n^k$,

then prove that $\sum_{r=1}^{m} .^{m+1}C_r s_r = (n+1)^{m+1} - (n+1)$

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143. Prove that $\sum_{r=1}^{n}(-1)^{r-1}\left(1+\frac{1}{2}+\frac{1}{3}+ \quad + \frac{1}{r}\right)(.^nC_r) = \frac{1}{n}$.

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144. Prove that

$$\frac{C_1}{1} - \frac{C_2}{2} + \frac{C_3}{3} - \frac{C_4}{4} + \ldots. + \frac{(-1)^{n-1}}{n}C_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$$

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145. Prove that $\sum_{r=0}^{n} .^{n} C_r \sin rx. \cos(n-r)x = 2^{n-1} \times \sin nx.$

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146. Find the last two digits of the number $(23)^{14}$.

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147. Find the last two digits of the number 27^{27}.

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148. Find the number of nonzero terms in the expansion of $\left(1+3\sqrt{2}x\right)^9+\left(1-3\sqrt{2}x\right)^9$.

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149. Find the value of $\left(\sqrt{2}+1\right)^6-\left(\sqrt{2}-1\right)^6$.

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150. Using the binomial theorem (without using the formula for $.^n C_r$), prove that

$$.^n C_4+.^m C_2-.^m C_1.^n C_2=.^m C_4-.^{m+n} C_1.^m C_3+.^{m+n} C_2.^m C_2-.^{m-}$$

.

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151. Find the value of $.^{4n} C_0+^{4n} C_4+^{4n} C_8+\ldots+^{4n}C_{4n}$.

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152. Find the degree of the polynomial

$$\frac{1}{\sqrt{4x+1}}\left\{\left(\frac{1+\sqrt{4x+1}}{2}\right)^7-\left(\frac{1+\sqrt{4x+1}}{2}\right)^7\right\}$$

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153. Find the coefficient of x^n in the polynomial $(x + .^n C_0)(x + 3.^n C_1) \times (x + 5.^n C_2)\ldots.[x + (2n+1).^n C_n]$.

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154. If $(1+x)^{15} = C_0 + C_1 x + C_2 x^2 + \quad + C_{15} x^{15}$, then find the sum of $C_1 + 2C_3 + 3C_4 + \quad + 14C_{15}$.

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155. Prove that $\frac{.^n C_0}{1} + \frac{.^n C_2}{3} + \frac{.^n C_4}{5} + \frac{.^n C_6}{7} + \ldots = \frac{2^n}{n+1}$.

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156. Find the sum `sumsum_(0lt=i

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157. Show that the integer next above $(\sqrt{3}+1)^{2m}$ contains 2^{m+1}, as a factor.

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158. Prove that

$$\frac{1^2}{3}{}^nC_1+\frac{1^2+2^2}{5}{}^nC_2+\frac{1^1+2^2+3^2}{7}{}^nC_3+...+\frac{1^2+2^2+\ +n^2}{2n+1}{}^nC_n$$

$$=\left(\frac{n(n+3)}{6}\right)\cdot 2^{n-2}.$$

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159. Prove that

$$\frac{1}{n+1}=\frac{.^nC_1}{2}-\frac{2(.^nC_2)}{3}+\frac{3(.^nC_3)}{4}-....+(-1)^{n+1}\frac{n(.^nC_n)}{n+1}.$$

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160. Find the sum

$$2.\,.^{10}C_0 + \frac{2^2}{2}.^{10}C_1 + \frac{2^3}{3}.^{10}C_2 + \frac{2^4}{4}.^{10}C_3 + ... + \frac{2^{11}}{11}.^{10}C_{10}.$$

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161. If in the expansion of $(2x+5)^{10}$, the numerically greatest term in equal to the middle term, then find the values of x

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162. Find the value of

$$\frac{1}{81^n} - \frac{10}{81^n}.^{2n}C_1 + \frac{10^2}{81^n}.^{2n}C_2 - \frac{10^3}{81^n}.^{2n}C_3 + ... + \frac{10^{2n}}{81^n}.$$

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163. Find the value of $5C_3 + 4C_2$

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164. Find the sum $.^1 C_0 +^2 C_1 +^3 C_2 + ... +^{n+1} C_n, where C_r =^n C_r.$

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165. If $\left(1 + x + x^2 + \ldots\ldots + x^n\right)^n = a_0 + a_1 x + a_2 x^2 + \ldots . a_{np} x^{np},$ then find the value of $a_1 + 2a_2 + 3a_3 + \ldots\ldots + npa_{np}.$

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166. If $n > 2,$ then prove that $C_1(a - 1) - C_2 \times (a - 2) + \ldots .. + (- 1)^{n-1} C_n(a - n) = a,$ where $C_r = .^n C_r.$

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167. Find the sum $C_0 - C_2 + C_4 - C_6 + \ldots\ldots\ldots$,where $C_r =^n C_r$.

A. $n(n+1)2^n - 1$

B. $n(n+3)2^n - 2$

C. $2n.^{2n} C_n$

D. none of these

Answer: null

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168. If $x+y=1$, prove that $\sum_{r=0}^{n} .^n C_r x^r y^{n-r} = 1.$

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169. Find the sum $3C_1 + 5C_2$

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170. Prove that $\frac{.^n C_1}{2} + \frac{.^n C_3}{4} + \frac{.^n C_5}{6} + \ldots = \frac{2^n - 1}{n+1}$.

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171. If $(1+x)^n = \sum_{r=0}^{n} \hat{} \, nC_r$, show that $C_0 + \frac{C_1}{2} + \ + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$.

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172. If $\sum_{r=0}^{2n} a_r (x-2)^r = \sum_{r=0}^{2n} b_r, (x-3)^r$ and $a_k = 1$ for all $k \geq n$, then b_n is equal to

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173. $3^{2n+2} - 8n - 9$ is divisible by

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174. Statement 1: The number of distinct terms in $\left(1+x+x^2+x^3+x^4\right)^{1000} is 4001.$

Statement 2: The number of distinct terms in expansion $(a_1+a_2+\ +a_m)^n is^{n+m-1}C_{m-1}$

Only conclusion I follows Only

conclusion II follows

Either I or II follows

Neither I nor II follows

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175. The product of 3rd and 8th term of a GP is 243. If its 4th term is 3. find its 7th term.

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176. The value of

$^{30}C_0.^{30}C_{10}+^{30}C_1.^{30}C_{11}+^{30}C_2.^{30}C_{12}+\ldots\ldots+^{30}C_{20}.^{30}C_{30}$

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177. If $f(x)=x^n, f(1)+\frac{f^1(1)}{1}+\frac{f^2(1)}{2!}+\ldots\ldots.\frac{f^n(1)}{n!}, where f^r(x)$ denotes the rth order derivative of $f(x)$ with respect to $x,$ is a. n b. 2^n c. 2^{n-1} d. none of these

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178. The fractional part of =$\frac{2^{4n}}{15}$ is

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179. The value of $.^{15}C_0^2-.^{15}C_1^2+.^{15}C_2^2-\ldots.-.^{15}C_{15}^2$ is

a. 15

b. -15

c. 0

d. 51

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180. If the sum of the coefficients in the expansion of $\left(1-3x+10x^2\right)^n isa$ and if the sum of the coefficients in the expansion of $\left(1+x^2\right)^n isb,$ then a. $a=3b$ b. $a=b^3$ c. $b=a^3$ d. none of these

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181. If $\left(1+x-2x^2\right)^6=1+a_1x+a_2x^2+\ldots\ldots+a_{12}x^{12}$ then

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182. Maximum sum of coefficient in the expansion of $\left(1-x\sin\theta+x^2\right)^n$ is

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183. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is

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184. The number of distinct terms in the expansion of $\left(x+\frac{1}{x}+x^2+\frac{1}{x^2}\right)^{15}$ is/are (with respect to different power of x is a) 255 b. 61 c. 127 d. none of these

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185. The sum of the coefficients of even power of x in the expansion of $\left(1+x+x^2+x^3\right)^5 is$ 256 b. 128 c. 512 d. 64

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186. Second term of a GP is 6 and its 5th term is 9th time of its 3rd term. Find the GP. Consider each of the GP is positive.

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187. The 4th term and the 9th term of a GP is 54 and 13122 respectively. Find its general term.

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188. If a,b,c are in GP Then prove that, $\log a, \log b, \log c$ are in AP.

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189. If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{b}x\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$ then a and b satisfy the relation

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190. If the binomial coefficient of the $(2r+4)^{th}$ term and $(r-2)^{th}$ term in the expansion of $(1+x)^{18}$ are equal find the value of r.

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191. If the coefficients of the rth, $(r+1)th, (r+2)th$ terms is the expansion of $(1+x)^{14}$ are in A.P, then the largest value of r is.

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192. If the three consecutive coefficients in the expansion of $(1+x)^{n}$ are 28, 56, and 70, then the value of n is.

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193. The expression $\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}\right)^6$ is polynomial of degree

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194. Least positive integer just greater than $(1+0.00002)^{50000}$ is ____.

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195. If $U_n=\left(\sqrt{3}+1\right)^{2n}+\left(\sqrt{3}-1\right)^{2n}$, then prove that $U_{n+1}=8U_n-4U_{n-1}$

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196. Prove that the coefficient of x^n in the expansion of $\frac{1}{(1-x)(1-2x)(1-3x)}$ is $\frac{1}{2}\left(3^{n+2}-2^{n+3}+1\right)$

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197. The value of $(30, 0)(30, 10) - (30, 1)(30, 11) + (30, 2)(30, 12) - \ldots\ldots. + (30, 20)(30, 3$

, where $(n, r) = nC_r$ is

a. $(30, 10)$

b. $(30, 15)$

c. $(60, 30)$

d. $(31, 10)$

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198. If a, b, c are in AP, a, x, b are in GP , where as b, y and c also in GP. Then prove that x^2, b^2, y^2 are in AP.

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199. Prove that

$$\frac{1}{m!}.^nC_0 + \frac{n}{(m+1)!}.^nC_1 + \frac{n(n-1)}{(m+2)!}.^nC_2 + \ldots\ldots + \frac{n(n-1)\ldots..2\times 1}{(m+n)!}$$

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200. If $n = 12m(m \in N),$ prove that

$$.^nC_0 - \frac{.^nC_2}{\left(2+\sqrt{3}\right)^2} + \frac{.^nC_4}{\left(2+\sqrt{3}\right)^4} - \frac{.^nC_6}{\left(2+\sqrt{3}\right)^6} + \ldots\ldots. = (-1)^m\left(\frac{2\sqrt{2}}{1+\sqrt{3}}\right)^n.$$

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201. Prove that in the expansion of $(1+x)^n(1+y)^n(1+z)^n$, the sum of the coefficient of the terms of degree r is $.^{3n}C_r$.

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202. Prove that

$.^{100}C_0^{100}C_2 +^{100}C_2^{100}C_4 +^{100}C_4^{100}C_6 + \quad +^{100}C_{98}^{100}C_{100} = \frac{1}{2}\left[.^{200}C_{98} -^{100}\right.$

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203. Prove that $\sum_{r=1}^{m-1} \frac{2r^2 - r(m-2) + 1}{(m-r)^m C_r} = m - \frac{1}{m}.$

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204. Find the coefficients of x^{50} in the expression

$(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots + 1001x^{1000}$.

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205. If a, b, c are in GP and a, x, b, y are in AP Then prove that, $\frac{a}{x} + \frac{c}{y} = 2$

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206. If $.^{n+1}C_{r+1} : ^nC_r : ^{n-1}C_{r-1} = 11 : 6 : 3,$ then $nr =$ 20 b. 30 c. 40 d. 50

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207. If the last tem in the binomial expansion of $\left(2^{\frac{1}{3}} - \frac{1}{\sqrt{2}}\right)^n is \left(\frac{1}{3^{\frac{5}{3}}}\right)^{\log_3 8}$, then 5th term from the beginning is 210 b. 420 c. 105 d. none of these

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208. Find the last two digits of the number $(23)^{14}$.

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209. The value of x for which the sixth term in the expansion of

$$\left[2^{\log_2 \sqrt{9^{x-1}+7}} + \frac{1}{2^{\frac{1}{5}\log_2(3^{x-1}+1)}}\right]^7$$ is 84 is

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210. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}} + x^2 \log_{10} x\right)^8$ is 5600, then x equals

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211. The total number of terms which are dependent on the value of x in the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^n$ is equal to $2n+1$ b. $2n$ c. n d. $n+1$

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212. In the expansion of $\left(3^{-x/4} + 3^{5x/4}\right)^n$ the sum of binomial coefficient is 64 and term with the greatest binomial coefficient exceeds the third by $(n-1)$, the value of x must be 0 b. 1 c. 2 d. 3

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213. If n is an integer between 0 and 21, then the minimum value of $n!(21-n)!$ is attained for $n=$ a. 1 b. 10 c. 12 d. 20

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214. If R is remainder when $6^{83}+8^{83}$ is divided by 49, then find the value of $\frac{R}{5}$.

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215. Let a and b be the coefficient of x^3 in $\left(1+x+2x^2+3x^3\right)^4$ and $\left(1+x+2x^2+3x^3+4x^4\right)^4$, respectively. Then the value of $4a/b$ is _____ .

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216. Let $1+\sum_{r=1}^{10}\left(3^r.\,.^{10}C_r+r.\,.^{10}C_r\right)=2^{10}\left(\alpha.\,4^5+\beta\right)$ where $\alpha,\beta\in N$ and $f(x)=x^2-k^2+1$. If α,β lies between the roots of $f(x)=$, the smalles positive integral value of k is ____.

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217. Let $a=3^{1/224}+1$ and for all $n\geq 3$,

let

$f(n)={}^nC_0a^{n-1}-{}^nC_1a^{n-2}+{}^nC_2a^{n-3}+\ldots+(-1)^{n-1}\cdot{}^nC_{n-1}\cdot a^0.$

If the value of f(2016)+f(2017)=3^k, the value of K is

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218. If the constant term in the binomial expansion of $\left(x^2-\frac{1}{x}\right)^n$, $n\in N$ is 15, then find the value of n.

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219. The largest value of x for which the fourth tem in the expansion

$$\left(5^{\left(\frac{2}{5}\right)(\log)_5\sqrt{4^x+44}}+\frac{1}{5^{\log_5}\left(2^{(x-1)}+7\right)^{\frac{1}{3}}}\right)^8$$ is 336 is.

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220. The number of values in set of values of r for which

$^{23}C_r+2^{23}C_{r+1}+{}^{23}C_{r+2}\geq{}^{25}C_{15}$ is

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221. If the second term of the expansion $\left[a^{\frac{1}{13}}+\left(\frac{a}{\sqrt{a^{-1}}}\right]is$14a^(5/2)

and $thevalueof$(^nC_3)/("^nC_2) $=\lambda$ then λ is

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222. Given $(1-2x+5x^2-10x^3)(1+x)^n = 1 + a_1x + a_2x^2 + ...$

and that $a_1^2 = 2a_2$, then the value of n is

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223. If $A+B=90^o$and tanA= $\frac{4}{3}$,find cosecB.

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224. If X−k divides $x^3 - 6x^2 + 11x -$ 6=0, then k can't be equal to, (a) 1. (b) 2. (c) 3. (d) 4

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225. Prove that $\sum_{r=1}^{k} (-3)^{r-1} (3n)^{C} \_ (2r-1) = 0$, where $k = 3n/2$ and n is an even integer.

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226. The coefficient of the middle term in the binomial expansion in powers of x of$(1 + \alpha x)^4$ and of$(1 - \alpha x)^6$ is the same, if α equals

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227. The coefficient of three consecutive terms in the expansion of $(1 + x)^n$ are a, b, c respectively prove that $\dfrac{2ac + b(a + c)}{b^2 - ac} = n.$

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228. The sum of G. P $3, 6, 12, \ldots 1536.$

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229. Prove that $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$ for all $n = 1, 2, \ldots$

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230. The coefficient of $1/x$ in the expansion of $(1+x)^n(1+1/x)^n$ is (a). $\frac{n!}{(n-1)!(n+1)!}$ (b). $\frac{(2n)!}{(n-1)!(n+1)!}$ (c). $\frac{(2n)!}{(2n-1)!(2n+1)!}$ (d). none of these

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231. Find the coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+...+(1+x)^{30}$.

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232. If x^m occurs in the expansion of $\left(x+\left(\frac{1}{x^2}\right)\right)^{2n}$ then the coefficient of x^m is

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233. If the coefficients of $5^{th}, 6^{th}$ and 7^{th} terms in the expansion of $(1+x)^n$ are in A.P. then $n =$

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234. If $\left(1+2x+x^2\right)^n = \sum_{r=0}^{2n} a_r x^r$, then $a_r =$

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235. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n, n \in N$, if the sum of the coefficients of $x^5 and x^{10}$ is zero , then n is a. 25 b. 20 c. 15 d. none of these

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236. If the coefficients of rth and $(r+1)th$ terms in the expansion of $(3+7x)^{29}$ are equal, then r is equals to a. 15 b. 21 c. 14 d. none of these

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237. In the expansion of $(1+3x+2x^2)^6$, find the coefficient of x^{11}.

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238. If $.^{n-1}C_r = (k^2-3)^n C_{r+1}$, then k belongs to

(a) $(-\infty, -2]$

(b) $[2, \infty)$

(c) $[-\sqrt{3}, \sqrt{3}]$

(d) $[\sqrt{3}, 2]$

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239. Prove that $\frac{3!}{2(n+3)} = \sum_{r=0}^{n} (-1)^r \left(\frac{nC_r}{(r+3)C_3}\right)$

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240. Find the $5th$ term of the GP. $\frac{5}{2}$, 1, ...

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241. The expression $\left(x+\frac{\left(x^3-1\right)^{\frac{1}{2}}}{2}\right)^5+\left(x-\frac{\left(x^3-1\right)^{\frac{1}{2}}}{2}\right)^5$ is a polynomial of degree

a. 5 b. 6 c. 7 d. 8

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242. Find $\left(\frac{dy}{dx}\right)$ of $\sin(\cos x)$ is

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243. In the binomial expansion of $(a-b)^n, n \geq 5,$ the sum of the 5th and 6th term is zero. Then a/b equals $(n-5)/6$ b. $(n-4)/5$ c.

$n/(n-4)$ d. $6/(n-5)$

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244. Coefficient of x^{11} in the expansion of $\left(1+x^2\right)^4\left(1+x^3\right)^7\left(1+x^4\right)^{12}$ is

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245. Give the integers $r>1, n>2$ and c0-efficients of $(3r)^t h$ and $(r+2)^{th}$ term in the binomial expansion of $(1+x)^{2n}$ are equal then

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246. Find the coefficient of x^4 in the expansion of $\left(x/2-3/x^2\right)^{10}$.

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247. If C_r stands for nC_r, then the sum of the series $\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - .... + (-1)^n(n+1)C_n^2\right]$,where n is an even positive integer, is

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248. If the sum $1 + 2 + 2^2 + ... + 2^{n-1}$is 255, then find the number of terms.

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249. The coefficient of X^{24}in the expansion of $\left(1 + X^2\right)^{12}\left(1 + X^{12}\right)\left(1 + X^{24}\right)$

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250. Find the sum of the GP $1 + 3 + 9 + 27 + ... 12 terms$

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251. The coefficient of x^{53} in the expansion of $\sum_{m=0}^{100} \hat{} \, 100C_m(x-3)^{100-m}2^m$ is equal to

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252. The coefficient of the term independent of x in the expansion of $\left[\frac{(x+1)}{x^{2/3}-x^{1/3}+1}-\frac{(x-1)}{x-x^{1/2}}\right]^{10}$ is

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253. In the expansion of $\left(1+x+x^3+x^4\right)^{10}$, the coefficient of x^4 is

a..$^{40}C_4$ b. .$^{10}C_4$ c. 210 d. 310

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254. If coefficient of $a^2b^3c^4$ in $(a+b+c)^m$ (where $m \in N$) is L $(L \neq 0)$.

Then in same expansion coefficient of $a^4b^4c^1$ will be

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255. The last two digits of the number 3^{400} are:

(A) 81 (B) 43 (C) 29 (D) 01

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256. The expression $\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}\right)^6$ is polynomial of degree

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257. A GP has common ratio 3, last term 486, if the sum of its terms is 728, find its first term.

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258. If $\left(1 + 2x + 3x^2\right)^{10} = a_0 + a_1x + a_2x^2 + \dots . + a_{20}x^{20}, then a_1$ equals a.10 b. 20 c. 210 d. none of these

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259. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1024}$.

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260. For which of the following values of x, $5th$ term is the numerically greatest term in the expansion of $(1 + x / 3)^{10}$

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261. For natural numbers m, n, if $(1-y)^{m}(1+y)^{n}=1+a_{1}y+a_{2}y^{2}+\ldots$, and $a_{1}=a_{2}=10$, t

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262. If the middle term in the expansion of $\left(\frac{x}{2}+2\right)^{8}$ is 1120, then find the sum of possible real values of x.

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263. If $(1+x)^{n}=C_{0}+C_{1}x+C_{2}x^{2}+\ldots+C_{n}x^{n}$,

then $C_{0}-(C_{0}+C_{1})+(C_{0}+C_{1}+C_{2})-(C_{0}+C_{1}+C_{2}+C_{3})+\ldots+(-1)^{n-1}(C_{0}+C_{1}+C_{n-1})$, where n a) is even integer b) is a positive value c) a negative value d) divisible by2^{n-1}

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264. In the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n, n \in N,$

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265. The value of $.^n C_1 + .^{n+1} C_2 + .^{n+2} C_3 + \ldots\ldots + .^{n+m-1} C_m$ is equal to

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266. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, $n \in N$,then $C_0 - C_1 + C_2 - \ldots . + (-1)^{n-1} C_{m-1}$, is equal to $(m < n)$

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267. The 10th term of $\left(3 - \sqrt{\frac{17}{4} + 3\sqrt{2}}\right)^{20}$ is (a) a irrational number (b) a rational number (c) a positive integer (d) a negative integer

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268. Find the geometric mean between $2a$ and $8a^3$

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269. Let $(1+x^2)^2(1+x)^n = \sum_{k=0}^{n+4} a_k x^k$.. If a_1, a_2 and a_3 aer in arithmetic progression, then the possible value/values of n is/are

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270. The middle term in the expansion of $\left(\frac{x}{2}+2\right)^8$ is 1120, then $x \in R$ is equal to a. -2 b. 3 c. -3 d. 2

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271. The sum of three numbers of GP is $\frac{39}{10}$ and their product is 1. Find the numbers.

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272. The first two term of a G.P are 125, and 25 respectively. Find the 5th and 6th term of the GP.

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273. If $\sum_{r=0}^{n}\left(\frac{r+2}{r+1}\right).^{n}C_{r}=\frac{2^{8}-1}{6}$. then nis

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274. Let $f(x)=a_0+a_1x+a_2x^2+\ldots.+a_nx^n+\ldots.$ abd $\frac{f(x)}{1-x}=b_0+b_1x+b_2x^2+\ldots.+b_nx^n+\ldots.$, then

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275. If $\left(1+x^2\right)^n = \sum_{r=0}^{n} a_r x^r = \left(1+x+x^2+x^3\right)^{100}$. If $a = \sum_{r=0}^{300} a_r$, then $\sum_{r=0}^{300} r a_r$ is

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276. The value of $\sum_{r=1}^{n+1} \left(\sum_{k=1}^{n}\right.$ ^kC_(r-1))$(wherer$, k, n in N`) is equal to

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277. If $\frac{x^2+x+1}{1-x} = a_0 + a_1 x + a_2 x^2 + \ldots$., then $\sum_{r=1}^{50} a_r$ equal to

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278. Find $\frac{dy}{dt}$, if y = $\frac{1-\cos t}{1+\cos t}$ is

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279. The coefficient of x^9 in the expansion of $(1+x)(1+x^2)(1+x^3)\dots(1+x^{100})$ is

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280. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n= ______

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281. If $(1-x)^{-n}=a_0+a_1x+a_2x^2+\;+a_rx^r+,\,then a_0+a_1+a_2+\;+a_r$ is equal to

a. $\dfrac{n(n+1)(n+2)(n+r)}{r!}$

b. $\dfrac{(n+1)(n+2)(n+r)}{r!}$

c. $\dfrac{n(n+1)(n+2)(n+r-1)}{r!}$

d. none of these

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282. The value of $\sum_{r=0}^{20} r(20-r)$(^(20)C_r)^2` is equal to ?

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283. The coefficient of x^{10} in the expansion of $\left(1+x^2-x^3\right)^8$ is

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284. If the term independent of x in the $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then k equals $2, \ -2$ b. $3, \ -3$ c. $4, \ -4$ d. $1, \ -1$

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285. The coefficient of $x^2 y^3$ in the expansion of $(1-x+y)^{20}$ is (a) $\frac{20!}{213!}$ b. $-\frac{20!}{213!}$ c. $\frac{20!}{5!2!3!}$ d. none of these

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286. The coefficient of x^5 in the expansion of $\left(x^2 - x - 2\right)^5$ is

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287. The coefficient of $a^8 b^4 c^9 d^9$ in $(abc + abd + acd + bcd)^{10}$ is

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288. If the coefficient of x^7in $\left[ax^2 + \left(\frac{1}{b}x\right)\right]^{11}$ equals the coefficient of x^{-7}in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$ then a and b satisfy the relation

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289. If $(1+x)^5 = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$, then the value of $(a_0 - a_2 + a_4)^2 + (a_1 - a_3 + a_5)^2$ is equal to 243 b. 32 c. 1 d.

2^{10}

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290. The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is

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291. The coefficient of x^{28} in the expansion of $\left(1+x^2-x^6\right)^{30}$ is

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292. The coefficient of x^n in $(1+x)^{101}\left(1-x+x^2\right)^{100}$ is nonzero, then n cannot be of the form

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293. prove that $\sum_{r=0}^{n}(-1)^r \hat{} nC_r . [\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+$ ......up to m terms]= $\frac{2^{mn}-1}{2^{mn}(2^n-1)}$

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294. In the expansion of $\left(7^{1/3}+11^{1/9}\right)^{6561}$,

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295. If for z as real or complex, $\left(1+z^2+z^4\right)^8=C_0+C_1z^2+C_2z^4+\ldots+C_{16}z^{32} then$ prove that $C_0-C_1+C_2-C_3+\ldots\ldots+C_{16}=1$ and $C_0+C_3+C_6+C_{12}+C_{15}=3^7$

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296. The sum of the coefficient in the expansion of $\left(1+ax-2x^2\right)^n$ is

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297. if the 4^{th} term in the expansion of $(ax+1)^n$ is $5/2$, then

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298. Number of values of r satisfying the equation $,^{69}C_{3r-1}-^{69}C_{r^2}=^{69}C_{r^2-1}-^{69}C_{3r}$ is

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299. If $(4+\sqrt{15})^n=I+f$, where n is an odd natural number, I is an integer and ,then

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300. In the expansion of $(x+a)^n$ if the sum of odd terms is P and the sum of even terms is Q, then

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301. If the coefficients of the rth, $(r+1)th, (r-2)th$ terms is the expansion of $(1+x)^{14}$ are in A.P, then the largest value of r is.

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302. The value/value of x in the expression $\left(x+x^{\log_{10}x}\right)^5$ if the third term in the expansion is $10, 00, 000$is/are

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303. Let $R=\left(5\sqrt{5}+11\right)^{2n+1}$ and f=R-[R] where [] is the greatest integer function. Prove that Rf=4^{2n+1}

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304. If $|x| < 1$, then the coefficient of x^n in expansion of $(1 + x + x^2 + x^3 + \dots..)^2$ is

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305. The coefficient of $x^5 \in (1 + 2x + 3x^2 +)^{-3/2} is(|x| < 1)$

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306. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{\frac{3}{2}} - \left(1 + \frac{1}{2x}\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

A. $3x + \dfrac{3}{8}x^2$

B. $1 - \dfrac{3}{8}x^2$

C. $\dfrac{x}{2} - \dfrac{3}{x^2}$

D. $-\dfrac{3}{8}x^2$

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307. If x is positive, the first negative term in the expansion of $(1+x)^{27/5} is (|x|<1)$

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308. Value of $\sum_{k=1}^{\infty} \sum_{r=0}^{k} \frac{1}{3^k}(kC_r)$ is $\frac{2}{3}$ b. $\frac{4}{3}$ c. 2 d. 1

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309. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is a$a_0+a_1x+a_2x^2+a_3x^3+, then a_n is$ a.$\frac{b^n-a^n}{b-a}$ b. $\frac{a^n-b^n}{b-a}$ c. $\frac{b^{n+1}-a^{n+1}}{b-a}$ d. $\frac{a^{n+1}-b^{n+1}}{b-a}$

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310. If $f(x) = 1 - x + x^2 - x^3 + .. - x^{15} + x^{16} - x^{17}$, then the coefficient of $x^2 \in f(x-1)$ is a. 826 b. 816

c. 822 d. none of these

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311. The sum of rational in $\left(\sqrt{2} + \sqrt[3]{3} + \sqrt[6]{5}\right)^{10}$ is equal to

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312. The value of $\sum_{r=0}^{10} (r).^{20} C_r$ is equal to

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313. If $P = \left(8 + 3\sqrt{7}\right)^n$ and $f = p - [p]$, where $[\cdot]$ denotes the greatest integers function, then the value of $p(1-f)$ is equal to

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314. Find the GP whose first term is 64 and next term is 32.

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315. The fifth term of GP is 81 and second term is 24. Find the GP

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316. The value of x for which the sixth term in the expansion of

$$\left[2^{\log_2 \sqrt{9^{x-1}+7}} + \frac{1}{2^{\frac{1}{5}\log_2\left(3^{x-1}+1\right)}}\right]^7$$ is 84 is

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317. Find the 7th term of the GP, $\sqrt{3}+1, 1, \frac{\sqrt{3}-1}{2}$,......

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318. The number $51^{49} + 51^{48} + 51^{47} + \ldots\ldots. + 51 + 1$ is divisible by a. 10 b. 20 c. 25 d. 50

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319. If $\sum_{r=0}^{n} \frac{r}{{}^{n}C_{r}} = \sum_{r=0}^{n} \frac{n^2 - 3n + 3}{2.\ {}^{n}C_{r}}$, then find n

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320. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$, then show that the sum of the products of the coefficients taken two at a time, represented by $\sum \sum_{0 \le i < j \le n} {}^{n}c_i\ {}^{n}c_j$ is equal to $2^{2n-1} - \frac{(2n)!}{2(n!)^2}$

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321. If $\sum_{r=0}^{n} \{a_r (x - \alpha + 2)^r - b_r (\alpha - x - 1)^r\} = 0$, then

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322. Let $a = \left(2^{1/401} - 1\right)$ and for each $n \geq 2, let b_n = ^n C_1 + ^n C_2 \dot{a} + ^n C_3 a^2 + \ldots\ldots + ^n C_n \cdot a^{n-1}$. Find the value of $\left(b_{2006} - b_{2005}\right)$.

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323. Prove that $\sum_{r=0}^{n} .^n C_r(-1)^r \left[i^r + i^{2r} + i^{3r} + i^{4r}\right]$
$= 2^n + 2^{n+1} \cos(n\pi/4)$, where $i = \sqrt{-1}$

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324. The coefficients of x^n in $\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots\ldots + \frac{x^n}{n!}\right)^2$ is

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325. If the first and third term is 2 and 8 respectively. Find its second term.

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326. Find the next two terms of the series $2, \ -6, 18, \ -54, \ldots .$

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327. If n is a positive integer, prove that $1-2n+\frac{2n(2n-1)}{2!}-\frac{2n(2n-1)(2n-2)}{3!}+\ldots\ldots+(-1)^{n-1}\frac{2n(2n}{(}$

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328. Find the third term from the end term of the GP. $\frac{2}{27}, \frac{2}{9}, \frac{2}{8}, \ldots\ldots 162.$

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329. Given,

$s_n = 1 + q + q^2 + \ldots. + q^n, S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + \ldots + \left(\frac{q+1}{2}\right)^n, q \neq 1$ prove that ${}^{n+1}C_1 + {}^{n+1}C_2 s_1 + {}^{n+1}C_3 s_2 + \ldots\ldots + {}^{n+1}C_{n+1} s_n = 2^n S_n.$

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330. The sum of $1 + n\left(1 - \frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1 - \frac{1}{x}\right)^2 + \ldots. \infty$

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331. $\sum_{k=1}^{\infty} k\left(1 - \frac{1}{n}\right)^{k-1}$ =?

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332. The coefficient of x in the expansion of $\left\{\sqrt{1+x^2} - x\right\}^{-1}$ in ascending powers of x, when $|x| < 1, is$ a. 1 b. $\frac{1}{2}$ c. $-\frac{1}{2}$ d. $-\frac{1}{8}$

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333. $1+\frac{1}{3}x+\frac{1\times 4}{3\times 6}x^2+\frac{1\times 4\times 7}{3\times 6\times 9}x^3+$ ------ is equal to

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334. The value of $\sum_{r=1}^{15}\frac{r2^r}{(r+2)!}$ is (a). $\frac{(17)!-2^{16}}{(17)!}$ (b). $\frac{(18)!-2^{17}}{(18)!}$ (c). $\frac{(16)!-2^{15}}{(16)!}$ (d). $\frac{(15)!-2^{14}}{(15)!}$

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335. $(n+2).^n C_0 2^{n+1}.^n C_1 2^n+n.^n C_2 2^{n-1}-\ldots$ is equal to

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336. The value of $\sum_{r=0}^{20}(-1)^r.\frac{^{50}C_r}{r+2}$ is equal to

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337. In the expansion of $[(1+x)(1-x)]^2$, the coefficient of x^n will be

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338. If a, b, c are in GP and a, x, b, y, c ar in AP. then prove that, $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$

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339. Statement 1: ${}^mC_r+{}^mC_{r-1}({}^nC_1)+{}^mC_{r-2}({}^nC_2)+\ldots.+{}^nC_r=0$, if $m+n<r$

Statement 2: ${}^nC_r=0$, if $n<r$

(a) Statement 1 and Statement 2, both are correct. Statement 2 is the correct explanation for Statement 1.

(b) Statement 1 and Statement 2, both are correct. Statement 2 is not the correct explanation for Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 2 is true but Statement 1 is false.

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340. $1+\left(\frac{1}{4}\right)+\left(\frac{1\cdot 3}{4\cdot 8}\right)+\left(\frac{1\cdot 4\cdot 7}{4\cdot 8\cdot 12}\right)+\ldots .=$

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341. If $|x|<1, then 1+n\left(\frac{2x}{1+x}\right)+\frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2+\ldots\ldots$ is equal to

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342. If a, b, c are in AP also in GP Then show that, $a=b=c$

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343. The sum of the GP $\frac{x+y}{x-y}, 1, \frac{x-y}{x+y}, \ldots .$

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344. Statement 1: In the expansion of $(1+x)^{41}\left(1-x+x^2\right)^{40}$, the coefficient of x^{85} is zero.

Statement 2: In the expansion of $(1+x)^{41} and \left(1-x+x^2\right)^{40}, x^{85}$ term does not occur.

(a) Statement 1 and Statement 2, both are correct. Statement 2 is the correct explanation for Statement 1

(b) Statement 1 and Statement 2, both are correct. Statement 2 is not the correct explanation for Statement 1

(c) Statement 1 is correct but Statement 2 is not correct.

(d) Both Statement 1 and Statement 2 are not correct.

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345. The coefficient of x^n in $\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots . .+\frac{x^n}{n!}\right)^3$ is

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346. Evaluate the following

$.^{3}C_{2}$

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347. Evaluate

$.^{5}C_{2}$

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348. The value of $\sum_{r=0}^{10} r^{10}C_r 3^r(-2)^{10-r}$ is (a) 20 (b) 10 (c) 300 (d) 30

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349. Find n if $nP_1 = 2$

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350. Evaluate

$.^5 P_2$

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351. The value of $\frac{.^n C_0}{n} + \frac{.^n C_1}{n+1} + \frac{.^n C_2}{n+2} + ... + \frac{.^n C_n}{2n}$

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352. The value of

$.^{20} C_0 + .^{20} C_1 + .^{20} C_2 + .^{20} C_3 + .^{20} C_4 + .^{20} C_{12} + .^{20} C_{13} + .^{20} C_{14} + .^{20}$

is

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353. If $(3 + x^{2008} + x^{2009})^{2010} = a_0 + a_1 x^2 + ... + a_n x^n,\ a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3$ ..... is

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354. Find the seventh term of the G.P: $1,\ \sqrt{3},\ 3,\ 3\sqrt{3},\ \ldots..$

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355. Find the 10th term of the G.P. : $12,\ 4,\ 1\frac{1}{3}$,....

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356. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots.. + C_n x^n$, then $C_0 C_2 + C_1 C_3 + C_2 C_4 + \ldots\ldots + C_{n-2} C_n =$

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357. The value of $(\lim)_{n \to \infty} \sum_{r=1}^{n} \left(\sum_{t=0}^{r-1} \frac{1}{5^n} \cdot {}^n C_r \cdot {}^r C_t \cdot 3^t \right)$ is equal to

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358. If $n > 2$, then prove that $C_1(a-1) - C_2 \times (a-2) + \ldots .. + (-1)^{n-1} C_n(a-n) = a$, where $C_r = .^n C_r$.

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359. Find the nth term of the series: 1, 2, 4, 8, ...

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360. The remainder, if $1 + 2 + 2^2 + 2^3 + \ldots . + 2^{1999}$ in divided by 5 is

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361. Largest real value for x such that $\sum_{k=0}^{4}\left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right)=\frac{32}{3}$

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362. If in the expansion of $(a-2b)^n$, the sum of 5^{th} and 6^{th} terms is 0, then the values of $\frac{a}{b}$ is

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363. The number of real negative terms in the binomial expansion of $(1+ix)^{4n-2}, n \in N, x > 0$ is

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