MATHS

BOOKS - CENGAGE PUBLICATION

DIFFERENT PRODUCTS OF VECTORS AND THEIR GEOMETRICAL APPLICATIONS

Exercises

1. If $\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-a)^2 \end{vmatrix} = 0$ and vectors $\vec{A}, \vec{B}$ and $\vec{C}$, where $\vec{A} = a^2\hat{i} = a\hat{j} + \hat{k}$ etc. are non-coplanar, then prove that vectors $\vec{X}, \vec{Y}$ and $\vec{Z}$ where $\vec{X} = x^2\hat{i} + x\hat{j} + \hat{k}$. etc.may be coplanar.

2. If $OABC$ is a tetrahedron where O is the origin and $A, B, andC$ are the other three vertices with position vectors, $\vec{a}, \vec{b}, and\vec{c}$ respectively, then prove that the centre of the sphere circumscribing the tetrahedron is given by position vector $\dfrac{a^2(\vec{b} \times \vec{c}) + b^2(\vec{c} \times \vec{a}) + c^2(\vec{a} \times \vec{b})}{2[\vec{a}\vec{b}\vec{c}]}$.

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3. Let k be the length of any edge of a regular tetrahedron (a tetrahedron whose edges are equal in length is called a regular tetrahedron). Show that the angel between any edge and a face not containing the edge is $\cos^{-1}(1/\sqrt{3})$.

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4. In $\triangle ABC$, a point P is taken on AB such that $AP/BP = 1/3$ and point Q is taken on BC such that $CQ/BQ = 3/1$. If R is the point of intersection of the lines $AQandCP$, using vector method, find the area of ABC if the area of BRC is 1 unit

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5. Let O be an interior point of ΔABC such that $\overline{OA} + 2\overline{OB} + 3\overline{OC} = 0$. Then the ratio of a ΔABC to area of ΔAOC is

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6. The lengths of two opposite edges of a tetrahedron of $aandb$; the shortest distane between these edgesis d, and the angel between them if $\dot{\theta}$ Prove using vector4s that the volume of the tetrahedron is $\frac{abdisn\theta}{6}$.

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7. Find the volume of a parallelopiped having three coterminus vectors of equal magnitude $|a|$ and equal inclination θ with each other.

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8. $\vec{p}, \vec{q}, and \vec{r}$ are three mutually perpendicular vectors of the same magnitude. If vector $\vec{x}$ satisfies the equation $\vec{p} \times \left(\left(\vec{x} - \vec{q}\right) \times \vec{p}\right) + \vec{q} \times \left(\left(\vec{x} - \vec{r}\right) \times \vec{q}\right) + \vec{r} \times \left(\left(\vec{x} - \vec{p}\right) \times \vec{r}\right) = 0, then \vec{x}$ is given by $\frac{1}{2}\left(\vec{p} + \vec{q} - 2\vec{r}\right)$ b. $\frac{1}{2}\left(\vec{p} + \vec{q} + \vec{r}\right)$ c. $\frac{1}{3}\left(\vec{p} + \vec{q} + \vec{r}\right)$ d. $\frac{1}{3}\left(2\vec{p} + \vec{q} - \vec{r}\right)$

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9. Given the vectors $\vec{A}, \vec{B}, and \vec{C}$ form a triangle such that $\vec{A} = \vec{B} + \dot{\vec{C}}$ find $a, b, c, and d$ such that the area of the triangle is 56 where $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$ $\vec{B} = d\hat{i} + 3\hat{j} + 4\hat{k}$ $\vec{C} = 3\hat{i} + \hat{j} - 2\hat{k}$

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10. A line l is passing through the point $\vec{b}$ and is parallel to vector $\vec{c}$. Determine the distance of point A$\left(\vec{a}\right)$ from the line l in from

$$\left|\vec{b} - \vec{a} + \frac{\left(\vec{a} - \vec{b}\right)\vec{c}}{\left|\vec{c}\right|^2}\vec{c}\right| \text{ or } \frac{\left|\left(\vec{b} - \vec{a}\right) \times \vec{c}\right|}{\left|\vec{c}\right|}$$

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11. If $\vec{e}_1, \vec{e}_2, \vec{e}_3 and \vec{E}_1, \vec{E}_2, \vec{E}_3$ are two sets of vectors such that $\vec{e}_i . \vec{E}_j = 1,$ if $i = j and \vec{e}_i . \vec{E}_j = 0 and$ if $i \neq j,$ then prove that $\left[\vec{e}_1\vec{e}_2\vec{e}_3\right]\left[\vec{E}_1\vec{E}_2\vec{E}_3\right] = 1.$

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12. In a quadrilateral ABCD, it is given that $AB \mid \mid CD$ and the diagonals AC and BD are perpendicular to each other. Show that $AD. BC \geq AB. CD.$

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13. $OABC$ is regular tetrahedron in which D is the circumcentre of OAB and E is the midpoint of edge AC. Prove that DE is equal to half the edge of tetrahedron.

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14. If $A(\vec{a}), B(\vec{b}) and C(\vec{c})$ are three non-collinear points and origin does not lie in the plane of the points $A, B and C,$ then point $P(\vec{p})$ in the plane of the ABC such that vector $\vec{O}P$ is $\perp$ to planeof ABC , show that

$$\vec{O}P = \frac{\left[\vec{a}\vec{b}\vec{c}\right]\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)}{4^2}, where \text{ is the area of the } ABC.$$

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15. If $\vec{a}, \vec{b}, \vec{c}$ are three given non-coplanar vectors and any arbitrary vector $\vec{r}$ in space, where

$$\Delta_1 = \begin{vmatrix} \vec{r}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{r}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{r}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}, \Delta_2 = \left| \left(\vec{a}.\vec{a}, \vec{r}.\vec{a}, \vec{c}.\vec{a}\right), \left(\vec{a}.\vec{b}, \vec{r}.\vec{b}, \vec{c}.\vec{b}\right), \left(\vec{a}.\vec{c}, \vec{r}.\vec{c}\right.\right.$$

$$\Delta_3 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{r}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{r}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{r}.\vec{c} \end{vmatrix}' \Delta = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}, \text{ then prove that } \vec{r} = \frac{\Delta_1}{\Delta}\vec{a} + \frac{\Delta_2}{\Delta}$$

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Exercises Mcq

1. Two vectors in space are equal only if they have equal component in a. a given direction b. two given directions c. three given directions d. in any arbitrary direction

A. a given direction

B. two given directions

C. three given direction

D. in any arbitrary direaction

Answer: c

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2. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the three vectors having magnitudes, 1,5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \vec{c}$. Then $\tan\theta$ is equal to

A. 0

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{3}{4}$

Answer: d

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3. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors of equal magnitude such that the angle between each pair is $\frac{\pi}{3}$. If $\left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{6}$, then $\left|\vec{a}\right| =$

A. 2

B. -1

C. 1

D. $\sqrt{6}/3$

Answer: c

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4. Let $\vec{p} and \vec{q}$ be any two orthogonal vectors of equal magnitude 4 each. Let $\vec{a}, \vec{b}, and \vec{c}$ be any three vectors of lengths $7\sqrt{15} and 2\sqrt{33}$, mutually perpendicular to each other. Then find the distance of the vector

$$\left(\vec{a}\dot{\vec{p}}\right)\vec{p} + \left(\vec{a}\dot{\vec{q}}\right)\vec{q} + \left(\vec{a}\dot{\vec{p}} \times \vec{q}\right)\left(\vec{p} \times \vec{q}\right) + \left(\vec{b}\dot{\vec{p}}\right)\vec{p}\left(\vec{b}\dot{\vec{q}}\right)\vec{q} + \left(\vec{b}\dot{\vec{p}} \times \vec{q}\right)\left(\vec{p} \times \vec{q}\right) + ($$

from the origin.

A. $\vec{a} + \vec{b} + \vec{c}$

B. $\dfrac{\vec{a}}{|\vec{a}|} + \dfrac{\vec{b}}{|\vec{b}|} + \dfrac{\vec{c}}{|\vec{c}|}$

C. $\dfrac{\vec{a}}{|\vec{a}|^2} + \dfrac{\vec{b}}{|\vec{b}|^2} + \dfrac{\vec{c}}{|\vec{c}|^2}$

D. $|\vec{a}|\vec{a} - |\vec{b}|\vec{b} + |\vec{c}|\vec{c}$

Answer: b

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5. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$, then the point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is a. $(3, -1, 1)$ b. $(3, 1, -1)$ c. $(-3, 1, 1)$ d. $(-3, -1, -1)$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $3\hat{i} - \hat{j} + \hat{k}$

C. $3\hat{i} + \hat{j} - \hat{k}$

D. $\hat{i}$ - $\hat{j}$ - $\hat{k}$

Answer: c

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6. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\vec{a}.\vec{b} < 0$ and $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ then the angle between the vectors $\vec{a}$ and $\vec{b}$ is (a) π (b) $\frac{7\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

A. π

B. $7\pi/4$

C. $\pi/4$

D. $3\pi/4$

Answer: d

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7. If $\hat{a}, \hat{b}, and \hat{c}$ are three unit vectors, such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector and θ_1, θ_2 and θ_3 are angles between the vectors $\hat{a}, \hat{b}; \hat{b}, \hat{c} and \hat{c}, \hat{a}$ respectively, then among θ_1, θ_2 and θ_3. a. all are acute angles b. all are right angles c. at least one is obtuse angle d. none of these

A. all are acute angles

B. all are right angles

C. at least one is obtuse angle

D. none of these

Answer: c

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8. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}. \vec{b} = 0 = \vec{a}. \vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$.

A. $1/2$

B. 1

C. 2

D. none of these

Answer: b

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9. about to only mathematics

A. a plane containing the origian O and parallel to two non-collinear vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$

B. the surface of a sphere described on PQ as its diameter

C. a line passing through points P and Q

D. a set of lines parallel to line PQ

Answer: c

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10. Two adjacent sides of a parallelogram $ABCD$ are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Then the value of $|AC \times BD|$ is a. $20\sqrt{5}$ b. $22\sqrt{5}$ c. $24\sqrt{5}$ d. $26\sqrt{5}$

A. $20\sqrt{5}$

B. $22\sqrt{5}$

C. $24\sqrt{5}$

D. $26\sqrt{5}$

Answer: b

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11. If $\hat{a}, \hat{b}, and\hat{c}$ are three unit vectors inclined to each other at angle θ, then the maximum value of θ is $\frac{\pi}{3}$ b. $\frac{\pi}{4}$ c. $\frac{2\pi}{3}$ d. $\frac{5\pi}{6}$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{5}$

Answer: c

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12. Let the pairs $a, b,$ and c, d each determine a plane. Then the planes are parallel if a.$(\vec{a} \times \vec{c}) \times (\vec{b} \times \vec{d}) = \vec{0}$ b. $(\vec{a} \times \vec{c}).(\vec{b} \times \vec{d}) = \vec{0}$ c. $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$ d. $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = \vec{0}$

A. $(\vec{a} \times \vec{c}) \times (\vec{b} \times \vec{d}) = \vec{0}$

B. $(\vec{a} \times \vec{c}).(\vec{b} \times \vec{d}) = \vec{0}$

C. $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$

D. $(\vec{a} \times \vec{c}).(\vec{c} \times \vec{d}) = \vec{0}$

Answer: c

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13. If $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar, then

A. $\vec{r} \perp \left(\vec{c} \times \vec{a}\right)$

B. $\vec{r} \perp \left(\vec{a} \times \vec{b}\right)$

C. $\vec{r} \perp \left(\vec{b} \times \vec{c}\right)$

D. $\vec{r} = \vec{0}$

Answer: d

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14. If $\vec{a}$ satisfies $\vec{a} \times \left(\hat{i} + 2\hat{j} + \hat{k}\right) = \hat{i} - \hat{k}$ then $\vec{a}$ is equal to

A. $\lambda\hat{i} + (2\lambda - 1)\hat{j} + \lambda\hat{k}, \lambda \in R$

B. $\lambda\hat{i} + (1 - 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

C. $\lambda\hat{i} + (2\lambda + 1)\hat{j} + \lambda\hat{k}, \lambda \in R$

D. $\lambda\hat{i} + (1 + 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

Answer: c

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15. Vectors $3\vec{a} - 5\vec{b}$ and $2\vec{a} + \vec{b}$ are mutually perpendicular. If $\vec{a} + 4\vec{b}$ and $\vec{b} - \vec{a}$ are also mutually perpendicular, then the cosine of the angle between a and b is a. $\frac{19}{5\sqrt{43}}$ b. $\frac{19}{3\sqrt{43}}$ c. $\frac{19}{2\sqrt{45}}$ d. $\frac{19}{6\sqrt{43}}$

A. $\frac{19}{5\sqrt{43}}$

B. $\frac{19}{3\sqrt{43}}$

C. $\frac{19}{\sqrt{45}}$

D. $\frac{19}{6\sqrt{43}}$

Answer: a

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16. The unit vector orthogonal to vector $-\hat{i}+\hat{j}+2\hat{k}$ and making equal angles with the x and y-axis a.$\pm\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$ b. $\pm\frac{1}{3}\left(\hat{i}+\hat{j}-\hat{k}\right)$ c. $\pm\frac{1}{3}\left(2\hat{i}-2\hat{j}-\hat{k}\right)$ d. none of these

A. $\pm\frac{1}{3}\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

B. $\frac{19}{5\sqrt{43}}$

C. $\pm\frac{1}{3}\left(\hat{i}+\hat{j}-\hat{k}\right)$

D. none of these

Answer: a

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17. The value of x for which the angle between $\vec{a}=2x^2\hat{i}+4x\hat{j}+\hat{k}and\vec{b}=7\hat{i}-2\hat{j}+\hat{k}$ is obtuse and the angle between b and the z-axis acute and less than $\pi/6$ is given by

A. $a<x<1/2$

B. $1/2 < x < 15$

C. $x < 1/2$ or $x < 0$

D. none of these

Answer: b

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18. If vectors $\vec{a} and \vec{b}$ are two adjacent sides of a parallelogram, then the vector respresenting the altitude of the parallelogram which is the perpendicular to a is a.$\vec{b} + \frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ b. $\frac{\vec{a}\dot{\vec{b}}}{|\vec{b}|^2}$ c. $\vec{b}$ - $\frac{\vec{b}\dot{\vec{a}}}{|\vec{a}|^2}$ d. $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{b}|^2}$

A. $\vec{b} + \frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$

B. $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}$

C. $\vec{b}$ - $\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$

D. $\frac{\vec{a} \times \left(\vec{b} \times \vec{a}\right)}{\left|\vec{b}\right|^2}$

Answer: a

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19. A parallelogram is constructed on $2\vec{a} + \vec{b} and \vec{a} - 4\vec{b}, where \left|\vec{a}\right| = 6 and \left|\vec{b}\right| = 8, and \vec{a} and \vec{b}$ are anti-parallel. Then the length of the longer diagonal is 40 b. 64 c. 32 d. 48

A. 40

B. 64

C. 32

D. 48

Answer: c

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20. Let $\vec{a}.\vec{b} = 0$ where $\vec{a}$ and $\vec{b}$ are unit vectors and the vector $\vec{c}$ is inclined an anlge θ to both $\vec{a}$ and $\vec{b}$. $If \vec{c} = m\vec{a} + n\vec{b} + p\left(\vec{a} \times \vec{b}\right), (m, n, p \in R)$ then

A. $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$

B. $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

C. $0 \leq \theta \leq \frac{\pi}{4}$

D. $0 \leq \theta \leq \frac{3\pi}{4}$

Answer: a

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21. If $a\ and\ c$ are unit vectors and $|b| = 4$. The angel between $aandc$ is $\cos^{-1}(1/4) anda \times b = 2a \times c\ then, b - 2c = \lambda \dot{a}$ The value of λ is

A. 3,-4

B. 1/4,3/4

C. $-3, 4$

D. $-1/4, \frac{3}{4}$

Answer: a

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22. Let the position vectors of the points $PandQ$ be $4\hat{i} + \hat{j} + \lambda\hat{k} and 2\hat{i} - \hat{j} + \lambda\hat{k}$, respectively. Vector $\hat{i} - \hat{j} + 6\hat{k}$ is perpendicular to the plane containing the origin and the points $PandQ$. Then λ equals a $-1/2$ b. $1/2$ c. 1 d. none of these

A. $-1/2$

B. $1/2$

C. 1

D. none of these

Answer: a

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23. A vector of magnitude $\sqrt{2}$ coplanar with the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\vec{c} = \hat{i} + \hat{j} + \hat{k}$, is a.$-\hat{j} + \hat{k}$ b. $\hat{i} - \hat{k}$ c. $\hat{i} - \hat{j}$ d. $\hat{i} - \hat{j}$

A. $-\hat{j} + \hat{k}$

B. $\hat{i}$ and $\hat{k}$

C. $\hat{i} - \hat{k}$

D. hati- hatj`

Answer: a

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24. Let P be a point interior to the acute triangle ABC. If $PA + PB + PC$ is a null vector, then w.r.t traingel ABC, point P is its a. centroid b. orthocentre c. incentre d. circumcentre

A. centroid

B. orthocentre

C. incentre

D. circumcentre

Answer: a

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25. G is the centroid of triangle ABC and A_1 and B_1 are the midpoints of sides AB and AC , respectively. If Δ_1 is the area of quadrilateral GA_1AB_1 and Δ is the area of triangle ABC , then $\frac{\Delta}{\Delta_1}$ is equal to

a. $\frac{3}{2}$

b. 3

c. $\frac{1}{3}$

d. none of these

A. $\frac{3}{2}$

B. 3

C. $\frac{1}{3}$

D. none of these

Answer: b

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26. Points $\vec{a}, \vec{b}\vec{c}$ and $\vec{d}$ are coplanar and $(\sin\alpha)\vec{a} + (2\sin 2\beta)\vec{b} + (3\sin 3\gamma)\vec{c} - \vec{d} = \vec{0}$. Then the least value of $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma$ is

A. $1/14$

B. 14

C. 6

D. $1/\sqrt{6}$

Answer: a

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27. If $\vec{a} and \vec{b}$ are any two vectors of magnitudes 1 and 2, respectively, and $\left(1-3\vec{a}.\vec{b}\right)^2+\left|2\vec{a}+\vec{b}+3\left(\vec{a}\times\vec{b}\right)\right|^2=47,$ then the angel between $\vec{a} and \vec{b}$ is $\pi/3$ b. $\pi-\cos^{-1}(1/4)$ c. $\frac{2\pi}{3}$ d. $\cos^{-1}(1/4)$

A. $\pi/3$

B. $\pi-\cos^{-1}(1/4)$

C. $\frac{2\pi}{3}$

D. $\cos^{-1}(1/4)$

Answer: c

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28. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 2 and 3, respectively, such that $\left|2\left(\vec{a}\times\vec{b}\right)\right|+\left|3\left(\vec{a}.\vec{b}\right)\right|=k,$ then the maximum value of k is a. $\sqrt{13}$ b. $2\sqrt{13}$ c. $6\sqrt{13}$ d. $10\sqrt{13}$

A. $\sqrt{13}$

B. $2\sqrt{13}$

C. $6\sqrt{13}$

D. $10\sqrt{13}$

Answer: c

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29. $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vecrtors such that $\left|\vec{a} + \vec{b} + 3\vec{c}\right| = 4$ Angle between $\vec{a}$ and $\vec{b} is \theta_1$, between $\vec{b}$ and $\vec{c} is \theta_2$ and between $\vec{a}$ and $\vec{b}$ varies $[\pi/6, 2\pi/3]$. Then the maximum value of $\cos\theta_1 + 3\cos\theta_2$ is

A. 3

B. 4

C. $2\sqrt{2}$

D. 6

Answer: b

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30. If the vector product of a constant vector $\vec{O}A$ with a variable vector $\vec{O}B$ in a fixed plane OAB be a constant vector, then the locus of B is a straight line perpendicular to $\vec{O}A$ b. a circle with centre O and radius equal to $\left|\vec{O}A\right|$ c. a straight line parallel to $\vec{O}A$ d. none of these

A. a straight line perpendicular to $\overrightarrow{OA}$

B. a circle with centre O and radius equal to $\left|\overrightarrow{OA}\right|$

C. a striaght line parallel to $\overrightarrow{OA}$

D. none of these

Answer: c

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31. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2$ and $|\vec{w}| = 3$ if the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v}$ and $\vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals

A. 2

B. $\sqrt{7}$

C. $\sqrt{14}$

D. 14

Answer: c

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32. If the two adjacent sides of two rectangles are represented by vectors $\vec{p} = 5\vec{a} - 3\vec{b}; \vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}; \vec{s} = -\vec{a} + \vec{b},$ respectively, then the angel between the vector $\vec{x} = \frac{1}{3}(\vec{p} + \vec{r} + \vec{s})$ and $\vec{y} = \frac{1}{5}(\vec{r} + \vec{s})$

is a.$-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ b. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ c. $\pi - \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ d. cannot be evaluate

A. $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

B. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

C. $\pi\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

D. cannot of these

Answer: b

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33. if $\vec{\alpha} \mid \mid \left(\vec{\beta} \times \vec{\gamma}\right)$, then $\left(\vec{\alpha} \times \beta\right).\left(\vec{\alpha} \times \vec{\gamma}\right)$ equals to a.$\left|\vec{\alpha}\right|^2\left(\vec{\beta}.\vec{\gamma}\right)$ b. $\left|\vec{\beta}\right|^2\left(\vec{\gamma}.\vec{\alpha}\right)$ c. $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}.\vec{\beta}\right)$ d. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

A. $\left|\vec{\alpha}\right|^2\left(\vec{\beta}.\vec{\gamma}\right)$

B. $\left|\vec{\beta}\right|^2\left(\vec{\gamma}.\vec{\alpha}\right)$

C. $\left|\vec{\gamma}\right|^2\left(\vec{\alpha}.\vec{\beta}\right)$

D. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

Answer: a

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34. The position vectors of points A,B and C are $\hat{i}+\hat{j}, \hat{i}+5\hat{j}-\hat{k}$ and $2\hat{i}+3\hat{j}+5\hat{k}$, respectively the greatest angle of triangle ABC is

A. 120°

B. 90°

C. $\cos^{-1}(3/4)$

D. none of these

Answer: b

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35. Given three vectors $\vec{a}, \vec{b}, and \vec{c}$ two of which are non-collinear. Further if $\left(\vec{a} + \vec{b}\right)$ is collinear with $\vec{c}, \left(\vec{b} + \vec{c}\right)$ is collinear with $\vec{a}, \left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = \sqrt{2}$ Find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ a. 3 b. -3 c. 0 d. cannot be evaluated

A. 3

B. -3

C. 0

D. cannot of these

Answer: b

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36. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left(\vec{a} + \vec{b}\right).\left[\left(2\vec{a} + 3\vec{b}\right) \times \left(3\vec{a} - 2\vec{b}\right)\right] = 0$, then angle between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. π

D. indeterminate

Answer: d

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37. If in a right-angled triangle ABC, the hypotenuse $AB = p, then \vec{A}B \dot{A}C + \vec{B}C \dot{\vec{B}}A + \vec{C}A \dot{\vec{C}}B$ is equal to $2p^2$ b. $\frac{p^2}{2}$ c. p^2 d. none of these

A. $2p^2$

B. $\frac{p^2}{2}$

C. p^2

D. none of these

Answer: c

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38. Resolved part of vector $\vec{a}$ and along vector $\vec{b}$ is $\vec{a}1$ and that prependicular to $\vec{b}$ is $\vec{a}2$ then $\vec{a}1 \times \vec{a}2$ is equl to

A. $\dfrac{\left(\vec{a} \times \vec{b}\right).\vec{b}}{\left|\vec{b}\right|^2}$

B. $\dfrac{\left(\vec{a}.\vec{b}\right)\vec{a}}{\left|\vec{a}\right|^2}$

C. $\dfrac{\left(\vec{a}.\vec{b}\right)\left(\vec{b} \times \vec{a}\right)}{\left|\vec{b}\right|^2}$

D. $\dfrac{\left(\vec{a}.\vec{b}\right)\left(\vec{b} \times \vec{a}\right)}{\left|\vec{b} \times \vec{a}\right|}$

Answer: c

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39. Let $a = 2i - j + k$, $b = i + 2j - k$ and $c = i + j - 2k$ be three vectors. A vector r in the plane of b and c whose projection on a is of magnitude $\sqrt{\frac{2}{3}}$ is

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $-2\hat{i} - \hat{j} + 5\hat{k}$

C. $2\hat{i} + 3\hat{j} + 3\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: b

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40. If P is any arbitrary point on the circumcircle of the equilateral triangle of side length l units, then $\left|\vec{P}A\right|^2 + \left|\vec{P}B\right|^2 + \left|\vec{P}C\right|^2$ is always equal to $2l^2$ b. $2\sqrt{3}l^2$ c. l^2 d. $3l^2$

A. $2l^2$

B. $2\sqrt{3}l^2$

C. l^2

D. $3l^2$

Answer: a

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41. If $\vec{r} and \vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $\left|\vec{r}+b\vec{s}\right|$ is minimum, then the value of $\left|b\vec{s}\right|^2+\left|\vec{r}+b\vec{s}\right|^2$ is equal to $2\left|\vec{r}\right|^2$ b. $\left|\vec{r}\right|^2/2$ c. $3\left|\vec{r}\right|^2$ d. $|r|^2$

A. $2\left|\vec{r}\right|^2$

B. $\left|\vec{r}\right|^2/2$

C. $3\left|\vec{r}\right|^2$

D. $\left|\vec{r}\right|^2$

Answer: d

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42. $\vec{a}$ and $\vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that if equally inclined to $\vec{a}, \vec{b}$ and $\vec{a} \times \vec{b}$ is equal to

A. $\frac{1}{\sqrt{2}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

B. $\frac{1}{2}\left(\vec{a} \times \vec{b} + \vec{a} + \vec{b}\right)$

C. $\frac{1}{\sqrt{3}}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

D. $\frac{1}{3}\left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right)$

Answer: a

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43. Given that $\vec{a}, \vec{b}, \vec{p}, \vec{q}$ are four vectors such that $\vec{a} + \vec{b} = \mu\vec{p}, \vec{b}\dot{\vec{q}} = 0 and \left(\vec{b}\right)^2 = 1, where \mu$ is a scalar. Then $\left|\left(\vec{a}\dot{\vec{q}}\right)\vec{p} - \left(\vec{p}\dot{\vec{q}}\right)\vec{a}\right|$

is equal to $2\left|\vec{p}\vec{q}\right|$ b. $(1/2)\left|\vec{p}\vec{q}\right|$ c. $\left|\vec{p}\times\vec{q}\right|$ d. $\left|\vec{p}\vec{q}\right|$

A. $2\left|\vec{p}\vec{q}\right|$

B. $(1/2)\left|\vec{p}.\vec{q}\right|$

C. $\left|\vec{p}\times\vec{q}\right|$

D. $\left|\vec{p}.\vec{q}\right|$

Answer: d

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44. The position vectors of the vertices $A, BandC$ of a triangle are three unit vectors $\vec{a}, \vec{b}, and\vec{c}$, respectively. A vector $\vec{d}$ is such that $\vec{d}\cdot\vec{a}=\vec{d}\cdot\vec{b}=\vec{d}\cdot\vec{c} and \vec{d}=\lambda\left(\vec{b}+\vec{c}\right)$. Then triangle ABC is a. acute angled b. obtuse angled c. right angled d. none of these

A. acute angled

B. obtuse angled

C. right angled

D. none of these

Answer: a

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45. If a is real constant A, B and C are variable angles and $\sqrt{a^2 - 4\tan A} + a\tan B + \sqrt{a^2 + 4\tan C} = 6a,$ then the least value of $\tan^2 A + \tan^2 B + \tan^2 C$ is a. 6 b. 10 c. 12 d. 3

A. 6

B. 10

C. 12

D. 3

Answer: d

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46. The vertex A triangle ABC is on the line $\vec{r}=\hat{i}+\hat{j}+\lambda\hat{k}$ and the vertices $BandC$ have respective position vectors $\hat{i}and\hat{j}$ Let Delta be the area of the triangle and $\text{Delta}\left[3/2, \sqrt{33}/2\right]$. Then the range of values of λ corresponding to A is $[-8, 4] \cup [4, 8]$ b. $[-4, 4]$ c. $[-2, 2]$ d. $[-4, -2] \cup [2, 4]$

A. [-8, -4]cup[4,8]`

B. $[-4, 4]$

C. [-2,2]

D. $[-4, -2] \cup [2, 4]$

Answer: c

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47. A non-zero vector $\vec{a}$ is such that its projections along vectors $\frac{\hat{i}+\hat{j}}{\sqrt{2}}, \frac{-\hat{i}+\hat{j}}{\sqrt{2}}$ and $\hat{k}$ are equal, then unit vector along $\vec{a}$ is a. $\frac{\sqrt{2}\hat{j}-\hat{k}}{\sqrt{3}}$ b.

$\frac{\hat{j} - \sqrt{2}\hat{k}}{\sqrt{3}}$ c. $\frac{\sqrt{2}}{\sqrt{3}}\hat{j} + \frac{\hat{k}}{\sqrt{3}}$ d. $\frac{\hat{j} - \hat{k}}{\sqrt{2}}$

A. $\frac{\sqrt{2}\hat{j} - \hat{k}}{\sqrt{3}}$

B. $\frac{\hat{j} - \sqrt{2}\hat{k}}{\sqrt{3}}$

C. $\frac{\sqrt{2}}{\sqrt{3}}\hat{j} + \frac{\hat{k}}{\sqrt{3}}$

D. $\frac{\hat{j} - \hat{k}}{\sqrt{2}}$

Answer: a

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48. Position vector $\hat{k}$ is rotated about the origin by angle 135^0 in such a way that the plane made by it bisects the angel between $\hat{i} and \hat{j}$. Then its new position is $\pm\frac{\hat{i}}{\sqrt{2}} \pm \frac{\hat{j}}{\sqrt{2}}$ b. $\pm\frac{\hat{i}}{2} \pm \frac{\hat{j}}{2} - \frac{\hat{k}}{\sqrt{2}}$ c. $\frac{\hat{i}}{\sqrt{2}} - \frac{\hat{k}}{\sqrt{2}}$ d. none of these

A. $\pm\frac{\hat{i}}{\sqrt{2}} \pm \frac{\hat{j}}{\sqrt{2}}$

B. $\pm\frac{\hat{i}}{2}\pm\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$

C. $\frac{\hat{i}}{\sqrt{2}}-\frac{\hat{k}}{\sqrt{2}}$

D. none of these

Answer: d

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49. In a quadrilateral $ABCD, \vec{A}C$ is the bisector of $\vec{A}Band\vec{A}D$, angle between $\vec{A}Band\vec{A}D$ is $2\pi/3$, $15\left|\vec{A}C\right| = 3\left|\vec{A}B\right| = 5\left|\vec{A}D\right|$. Then the angle between $\vec{B}Aand\vec{C}D$ is $(a)\cos^{-1}\left(\frac{\sqrt{14}}{7\sqrt{2}}\right)$ b. $\cos^{-1}\left(\frac{\sqrt{21}}{7\sqrt{3}}\right)$ c. $\cos^{-1}\left(\frac{2}{\sqrt{7}}\right)$ d. $\cos^{-1}\left(\frac{2\sqrt{7}}{14}\right)$

A. $\cos^{-1}\frac{\sqrt{14}}{7\sqrt{2}}$

B. $\cos^{-1}\frac{\sqrt{21}}{7\sqrt{3}}$

C. $\cos^{-1}\frac{2}{\sqrt{7}}$

D. $\cos^{-1}\frac{2\sqrt{7}}{14}$

Answer: c

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50. In fig. $AB, DEandGF$ are parallel to each other and $AD, BGandEF$ are parallel to each other. If $CD:CE = CG:CB = 2:1,$ then the value of area $(AEG):$ area (ABD) is equal to $7/2$ b. 3 c. 4 d. $9/2$

A. $7/2$

B. 3

C. 4

D. $9/2$

Answer: b

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51. Vector $\vec{a}$ in the plane of $\vec{b} = 2\hat{i} + \hat{j} and \vec{c} = \hat{i} - \hat{j} + \hat{k}$ is such that it is equally inclined to $\vec{b} and \vec{d}$ where $\vec{d} = \hat{j} + 2\hat{k}$ The value of$\vec{a}$ is $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$ b. $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ c. $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$ d. $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$

A. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

B. $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

C. $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$

D. $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$

Answer: b

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52. Let $ABCD$ be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be 3, 4 and $5 sq. units$, respectively. Then the area of triangle BCD is

a.$5\sqrt{2}$

b. 5

c. $\frac{\sqrt{5}}{2}$

d. $\frac{5}{2}$

A. $5\sqrt{2}$

B. 5

C. $\frac{\sqrt{5}}{2}$

D. $\frac{5}{2}$

Answer: a

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53. Let $\overrightarrow{f}(t) = [t]\hat{i} + (t - [t])\hat{j} + [t + 1]\hat{k}, where[.]$ denotes the greatest integer function. Then the vectors $\overrightarrow{f}\left(\frac{5}{4}\right)and f(t), 0 < t < 1$ are(a) parallel to each other(b) perpendicular(c) inclined at $\cos^{-1}2\left(\sqrt{7\left(1-t^2\right)}\right)$ (d)inclined at

$\cos^{-1}\left(\frac{8+t}{\sqrt{1+t^2}}\right)$;

A. parallel to each other

B. perpendicular to each other

C. inclined at $\frac{\cos^{-1}2}{\sqrt{7}\left(1-t^2\right)}$

D. inclined at $\frac{\cos^{-1}(8+t)}{9\sqrt{1+t^2}}$

Answer: d

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54. If $\vec{a}$ is parallel to $\vec{b}\times\vec{c}$, then $\left(\vec{a}\times\vec{b}\right).\left(\vec{a}\times\vec{c}\right)$ is equal to a. $\left|\vec{a}\right|^2\left(\vec{b}.\vec{c}\right)$ b. $\left|\vec{b}\right|^2\left(\vec{a}.\vec{c}\right)$ c. $\left|\vec{c}\right|^2\left(\vec{a}.\vec{b}\right)$ d. none of these

A. $\left|\vec{a}\right|^2\left(\vec{b}.\vec{c}\right)$

B. $\left|\vec{b}\right|^2\left(\vec{a}.\vec{c}\right)$

C. $\left|\vec{c}\right|^2\left(\vec{a}.\vec{b}\right)$

D. none of these

Answer: a

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55. The three vectors $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume: ____________

A. $1/3$

B. 4

C. $\left(3\sqrt{3}\right)/4$

D. $4\sqrt{3}$

Answer: d

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56. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is non-zero vector and $\left|\left(\vec{d} \cdot \vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d} \cdot \vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d} \cdot \vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0,$ then

a. $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$

b. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = |d|$

c. $\vec{a}, \vec{b}, and \vec{c}$ are coplanar

d. none of these

A. $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$

B. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = \left|\vec{d}\right|$

C. $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

D. none of these

Answer: c

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57. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 0$, then $\left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)\right)$ is equal to the given diagonal is $\vec{c} = 4\hat{k} = 8\hat{k}$ then , the volume of a parallelpiped is

A. $48\hat{b}$

B. $-48\hat{b}$

C. $48\hat{a}$

D. $-48\hat{a}$

Answer: a

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58. If the two diagonals of one its faces are $6\hat{i} + 6\hat{k} and 4\hat{j} + 2\hat{k}$ and of the edges not containing the given diagonals is $c = 4\hat{j} - 8\hat{k}$, then the volume of a parallelepiped is a. 60 b. 80 c. 100 d. 120

A. 60

B. 80

C. 100

D. 120

Answer: d

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59. The volume of a tetrahedron formed by the coterminous edges $\vec{a}, \vec{b}, and \vec{c}$ is 3. Then the volume of the parallelepiped formed by the coterminous edges $\vec{a} + \vec{b}, \vec{b} + \vec{c} and \vec{c} + \vec{a}$ is 6 b. 18 c. 36 d. 9

A. 6

B. 18

C. 36

D. 9

Answer: c

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60. If $\vec{a}, \vec{b},$ and $\vec{c}$ are three mutually orthogonal unit vectors, then the triple product $\left[\vec{a} + \vec{b} + \vec{c}\,\vec{a} + \vec{b}\,\vec{b} + \vec{c}\right]$ equals: (a.) 0 (b.) 1 or -1 (c.) 6 (d.) 3

A. 0

B. 1 or -1

C. 1

D. 3

Answer: b

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61. vector $\vec{c}$ are perpendicular to vectors $\vec{a} = (2, -3, 1)$ and $\vec{b} = (1, -2, 3)$ and satifies the condition $\vec{c}.\left(\hat{i} + 2\hat{j} - 7\hat{k}\right) = 10$ then vector $\vec{c}$ is equal to $(a)(7, 5, 1)$ $(b)(-7, -5, -1)$ $(c)(1, 1, -1)$ (d) none of these

A. 7,5,1

B. (-7, -5, -1)

C. 1,1,-1

D. none of these

Answer: a

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62. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j}, \vec{a} \perp \vec{b}, \vec{a}.\vec{c} = 4$ then find the value of $[\vec{a} \quad \vec{b} \quad \vec{c}]$.

A. $\left[\vec{a}\vec{b}\vec{c}\right]^2 = |\vec{a}|$

B. $\left[\vec{a}\vec{b}\vec{c}\right] = |\vec{a}|$

C. $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

D. $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

Answer: d

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63. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

A. 0

B. 1

C. $\frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

D. $\frac{3}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

Answer: c

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64. Let $\vec{r}, \vec{a}, \vec{b} and \vec{c}$ be four nonzero vectors such that

$\vec{r}\vec{a} = 0, \left|\vec{r} \times \vec{b}\right| = \left|\vec{r}\right|\left|\vec{b}\right| and \left|\vec{r} \times \vec{c}\right| = \left|\vec{r}\right|\left|\vec{c}\right|$ Then $[abc]$ is equal to $|a||b||c|$

b. - $|a||b||c|$ c. 0 d. none of these

A. |a||b||c|

B. - $|a||b||c|$

C. 0

D. none of these

Answer: c

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65. If $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 1, \vec{c} = \lambda\left(\vec{a} \times \vec{b}\right)$, angle between $\vec{c}$ and $\vec{b}$ is $2\pi/3$, $\left|\vec{a}\right| = \sqrt{2}, \left|\vec{b}\right| = \sqrt{3}$ and $\left|\vec{c}\right| = \frac{1}{\sqrt{3}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. (a) $\frac{\pi}{6}$

B. (b) $\frac{\pi}{4}$

C. (c) $\frac{\pi}{3}$

D. (d) $\frac{\pi}{2}$

Answer: b

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66. If $4\vec{a} + 5\vec{b} + 9\vec{c} = 0,$ then $\left(\vec{a} \times \vec{b}\right) \times \left[\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\right]$ is equal to a. vector perpendicular to the plane ofa, b, c b. a scalar quantity c. $\vec{0}$ d. none of these

A. a vector perpendicular to the plane of $\vec{a}, \vec{b}$ and $\vec{c}$

B. a scalar quantity

C. $\vec{0}$

D. none of these

Answer: c

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67. value of $\left[\vec{a} \times \vec{b} \vec{a} \times \vec{c} \vec{d}\right]$ is always equal to

A. $\left(\vec{a}.\vec{d}\right)\left[\vec{a}\vec{b}\vec{c}\right]$

B. `(veca.vecc)[veca vecb vecd]

C. $\left(\vec{a}.\vec{b}\right)\left[\vec{a}\vec{b}\vec{d}\right]$

D. none of these

Answer: a

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68. Let $\vec{a} and \vec{b}$ be mutually perpendicular unit vectors. Then for any arbitrary $\vec{r}$, $\vec{r} = \left(\vec{r}\dot{\hat{a}}\right)\hat{a} + \left(\vec{r}\dot{\hat{b}}\right)\hat{b} + \left(\vec{r}\dot{\hat{a}} \times \hat{b}\right)\left(\hat{a} \times \hat{b}\right)$

$\vec{r} = \left(\vec{r}\dot{\hat{a}}\right) - \left(\vec{r}\dot{\hat{b}}\right)\hat{b} - \left(\vec{r}\hat{a}\dot{\times}\hat{b}\right)\left(\hat{a}\times\hat{b}\right)$

$\vec{r} = \left(\vec{r}\dot{\hat{a}}\right)\hat{a} - \left(\vec{r}\dot{\hat{b}}\right)\hat{b} + \left(\vec{r}\hat{a}\dot{\times}\hat{b}\right)\left(\hat{a}\times\hat{b}\right)$ none of these

A. $\vec{r} = (\vec{r}.\hat{a})\hat{a} + (\vec{r}.\hat{b})\hat{b} + (\vec{r}.(\vec{a}\times\hat{b}))(\hat{a}\times\hat{b})$

B. $\vec{r} = (\vec{r}.\hat{a}) - (\vec{r}.\hat{b})\hat{b} - (\vec{r}.(\vec{a}\times\hat{b}))(\hat{a}\times\hat{b})$

C. $\vec{r} = (\vec{r}.\hat{a})\hat{a} - (\vec{r}.\hat{b})\hat{b} - (\vec{r}.(\vec{a}\times\hat{b}))(\hat{a}\times\hat{b})$

D. none of these

Answer: a

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69. Let $\vec{a}$ and $\vec{b}$ be unit vectors that are perpendicular to each other, then $\left[\vec{a} + \left(\vec{a}\times\vec{b}\right) + \left(\vec{a}\times\vec{b}\right)\right]$ is equal to

A. 1

B. 0

C. -1

D. none of these

Answer: a

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70. $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 4$ and $\vec{a}.$ Vecb =2. If vecc = (2vecaxx vecb) - 3vecb$thenf \in d\angle between$vecb and vecc`.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{6}$

Answer: d

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71. If $\vec{b}$ and $\vec{c}$ are unit vectors, then for any arbitary vector $\vec{a}$, $\left(\left(\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)\right)\times\left(\vec{b}\times\vec{c}\right)\right).\left(\vec{b}-\vec{c}\right)$ is always equal to

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72. If $\vec{a}.\vec{b}=\beta$ and $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}$ is

A. $\dfrac{\left(\beta\vec{a}-\vec{a}\times\vec{c}\right)}{|\vec{a}|^2}$

B. $\dfrac{\left(\beta\vec{a}+\vec{a}\times\vec{c}\right)}{|\vec{a}|^2}$

C. $\dfrac{\left(\beta\vec{c}+\vec{a}\times\vec{c}\right)}{|\vec{a}|^2}$

D. $\dfrac{\left(\beta\vec{c}+\vec{a}\times\vec{c}\right)}{|\vec{a}|^2}$

Answer: a

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73. If $a\left(\vec{\alpha}\times\vec{\beta}\right)+b\left(\vec{\beta}\times\vec{\gamma}\right)+c\left(\vec{\gamma}\times\vec{\alpha}\right)=0$ and at least one of $a, bandc$ is nonzero, then vectors $\vec{\alpha}, \vec{\beta} and \vec{\gamma}$ are a. parallel b. coplanar c. mutually perpendicular d. none of these

A. parallel

B. coplanar

C. mutually perpendicular

D. none of these

Answer: b

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74. if $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)=\vec{b}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-zero vectors, then

A. $\vec{a}, \vec{b}$ and $\vec{v}$ can be coplanar

B. $\vec{a}, \vec{b}$ and $\vec{c}$ must be coplanar

C. $\vec{a}, \vec{b}$ and $\vec{c}$ cannot be coplanar

D. none of these

Answer: c

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75. If $\vec{r}.\vec{a}=\vec{r}.\vec{b}=\vec{r}.\vec{c}=\frac{1}{2}$ for some non zero vector $\vec{r}$ and $\vec{a}, \vec{b}, \vec{c}$ are non coplanar, then the area of the triangle whose vertices are $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ is

A. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$

B. $\left|\vec{r}\right|$

C. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}\right|$

D. none of these

Answer: c

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76. A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point $P(1, 0)$ can be $6\hat{i} + 8\hat{j}$ b. $-8\hat{i} + 3\hat{j}$ c. $6\hat{i} - 8\hat{j}$ d. $8\hat{i} + 6\hat{j}$

A. $6\hat{i} + 8\hat{j}$

B. $-8\hat{i} + 3\hat{j}$

C. $6\hat{i} - 8\hat{j}$

D. $8\hat{i} + 6\hat{j}$

Answer: a

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77. If $\vec{a} and \vec{b}$ are two unit vectors incline at angle $\pi/3$, then $\left\{\vec{a} \times \left(\vec{b} + \vec{a} \times \vec{b}\right)\right\} \cdot \vec{b}$ is equal to $\frac{-3}{4}$ b. $\frac{1}{4}$ c. $\frac{3}{4}$ d. $\frac{1}{2}$

A. $\frac{-3}{4}$

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. $\frac{1}{2}$

Answer: a

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78. If $\vec{a}$ and $\vec{b}$ are othogonal unit vectors, then for a vector $\vec{r}$ non - coplanar with $\vec{a}$ and $\vec{b}$ vector $\vec{r} \times \vec{a}$ is equal to

A. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{b}$ - $\left(\vec{r}.\vec{b}\right)\left(\vec{b} \times \vec{a}\right)$

B. $\left[\vec{r}\vec{a}\vec{b}\right]\left(\vec{a} + \vec{b}\right)$

C. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{a} + \left(\vec{r}.\vec{a}\right)\vec{a} \times \vec{b}$

D. none of these

Answer: a

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79. If $\vec{a} + \vec{b}, \vec{c}$ are any three non- coplanar vectors then the equation

$$\left[\vec{b} \times \vec{c}\vec{c} \times \vec{a}\vec{a} \times \vec{b}\right]x^2 + \left[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}\right]x + 1 + \left[\vec{b} - \vec{c}\vec{c} - \vec{c} - \vec{a}\vec{a} - \vec{b}\right] = 0$$

has roots

A. real and distinct

B. real

C. equal

D. imaginary

Answer: c

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80. Sholve the simultasneous vector equations for $\vec{x}$ and $\vec{y}: \vec{x} + \vec{c} \times \vec{y} = \vec{a}$ and $\vec{y} + \vec{c} \times \vec{x} = \vec{b}, \vec{c} \neq 0$

A. $\vec{x} = \dfrac{\vec{b} \times \vec{c} + \vec{a} + (\vec{c}.\vec{a})\vec{c}}{1 + \vec{c}.\vec{c}}$

B. $\vec{x} = \dfrac{\vec{c} \times \vec{b} + \vec{b} + (\vec{c}.\vec{a})\vec{c}}{1 + \vec{c}.\vec{c}}$

C. $\vec{y} = \dfrac{\vec{a} \times \vec{c} + \vec{b} + (\vec{c}.\vec{b})\vec{c}}{1 + \vec{c}.\vec{c}}$

D. none of these

Answer: b

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81. The condition for equations $\vec{r} \times \vec{a} = \vec{b} and \vec{r} \times \vec{c} = \vec{d}$ to be consistent is

. . . .

a. $\vec{b}\vec{c} = \vec{a}\vec{d}$ b. $\vec{a}\vec{b} = \vec{c}\vec{d}$ c. $\vec{b}\vec{c} + \vec{a}\vec{d} = 0$ d. $\vec{a}\vec{b} + \vec{c}\vec{d} = 0$

A. $\vec{b}.\vec{c} = \vec{a}.\vec{d}$

B. $\vec{a}.\vec{b} = \vec{c}.\vec{d}$

C. $\vec{b}.\vec{c} + \vec{a}.\vec{d} = 0$

D. $\vec{a}.\vec{b} + \vec{c}.\vec{d} = 0$

Answer: c

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82. If $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ then $\left[\vec{a}\vec{b}\vec{i}\right]\hat{i} + \left[\vec{a}\vec{b}\vec{j}\right]\hat{j} + \left[\vec{a}\vec{b}\hat{k}\right]k$ is equal to

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83. If

$\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + \hat{j} + 2\hat{k}$ and $(1 + \alpha)\hat{i} + \beta(1 + \alpha)\hat{j} + \gamma(1 + \alpha)($

A. $-2, \ -4, \ -\frac{2}{3}$

B. $2, \ -4, \frac{2}{3}$

C. $-2, 4, \frac{2}{3}$

D. $2, 4, \ -\frac{2}{3}$

Answer: a

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84. Let $\left(\vec{a}(x) = (\sin x)\hat{i} + (\cos x)\hat{j}\right.$ and $\vec{b}(x) = (\cos 2x)\hat{i} + (\sin 2x)\hat{j}$ be two variable vectors $(x \in R)$. Then $\vec{a}(x)$ and $\vec{b}(x)$ are

A. collinear for unique value of x

B. perpendicular for infinte values of x.

C. zero vectors for unique value of x

D. none of these

Answer: b

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85. For any vectors $\vec{a}$ and $\vec{b}$, $\left(\vec{a} \times \hat{i}\right) + \left(\vec{b} \times \hat{i}\right) + \left(\vec{a} \times \hat{j}\right) . \left(\vec{b} \times \hat{j}\right) + \left(\vec{a} \times \hat{k}\right) . \left(\vec{b} \times \hat{k}\right)$ is always equal to

A. $\vec{a} . \vec{b}$

B. $2\vec{a}. Vecb$

C. zero

D. none of these

Answer: b

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86. $\vec{b} and \vec{c}$ are unit vectors. Then for any arbitrary vector $\vec{a}, \left(\left(\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)\right)\times\left(\vec{b}\times\vec{c}\right)\right)\vec{b}-\vec{c}$ is always equal to $|\vec{a}|$ b. $\frac{1}{2}|\vec{a}|$ c. $\frac{1}{3}|\vec{a}|$ d. none of these

A. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

B. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

C. $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

D. none of these

Answer: b

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87. Let $\vec{a}, \vec{b} and \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q} and \vec{r}$ the vectors defined by the relation $\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]} and \vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$. Then the value of the expression $\left(\vec{a} + \vec{b}\right)\dot{\vec{p}} + \left(\vec{b} + \vec{c}\right)\dot{\vec{q}} + \left(\vec{c} + \vec{a}\right)\dot{\vec{r}}$ is 0 b. 1 c. 2 d. 3

A. 3

B. 2

C. 1

D. 0

Answer: a

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88. $A(\vec{a}), B(\vec{b}) and C(\vec{c})$ are the vertices of triangle ABC and $R(\vec{r})$ is any point in the plane of triangle ABC, then $r\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is always equal to a. zero b. $[\vec{a}\vec{b}\vec{c}]$ c. - $[\vec{a}\vec{b}\vec{c}]$ d. none of these

A. zero

B. $[\vec{a}\vec{b}\vec{c}]$

C. - $[\vec{a}\vec{b}\vec{c}]$

D. none of these

Answer: b

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89. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar vectors and $\vec{a} \times \vec{c}$ is perpendicular to $\vec{a} \times (\vec{b} \times \vec{c})$, then the value of $[\vec{a} \times (\vec{b} \times \vec{c})] \times \vec{c}$ is equal to

A. $[\vec{a}\vec{b}\vec{c}]\vec{c}$

B. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{b}$

C. $\vec{0}$

D. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{a}$

Answer: c

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90. If V be the volume of a tetrahedron and V' be the volume of another tetrahedran formed by the centroids of faces of the previous tetrahedron and $V = KV', thenK$ is equal to a. 9 b. 12 c. 27 d. 81

A. 9

B. 12

C. 27

D. 81

Answer: c

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91. $\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\left(\vec{c} \times \vec{a}\right) \times \left(\vec{a} \times \vec{b}\right)\right]$ is equal to (where $\vec{a}, \vec{b} and \vec{c}$ are nonzero non-coplanar vector) a. $\left[\vec{a}\vec{b}\vec{c}\right]^2$ b. $\left[\vec{a}\vec{b}\vec{c}\right]^3$ c. $\left[\vec{a}\vec{b}\vec{c}\right]^4$ d. $\left[\vec{a}\vec{b}\vec{c}\right]$

A. $\left[\vec{a}\vec{b}\vec{c}\right]^2$

B. $\left[\vec{a}\vec{b}\vec{c}\right]^3$

C. $\left[\vec{a}\vec{b}\vec{c}\right]^4$

D. $\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: c

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92. If $\vec{r} = x_1\left(\vec{a} \times \vec{b}\right) + x_2\left(\vec{b} \times \vec{a}\right) + x_3\left(\vec{c} \times \vec{d}\right)$ and $4\left[\vec{a}\vec{b}\vec{c}\right] = 1$ then $x_1 + x_2 + x_3$ is equal to

A. $\frac{1}{2}\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

B. $\frac{1}{4}\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

C. $2\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

D. $4\vec{r}.\left(\vec{a}+\vec{b}+\vec{c}\right)$

Answer: d

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93. If the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other then a vector $\vec{v}$ in terms of $\vec{a}$ and $\vec{b}$ satisfying the equations $\vec{v}.\vec{a}=0, \vec{v}.\vec{b}=1$ and $\left[\vec{v}\ \ \vec{a}\ \ \vec{b}\right]=1$ is

A. $\frac{\vec{b}}{\left|\vec{b}\right|^2}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|^2}$

B. $\frac{\vec{b}}{\left|\vec{b}\right|}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|^2}$

C. $\frac{\vec{b}}{\left|\vec{b}\right|}+\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}$

D. none of these

Answer: a

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94. If $\vec{a}' = \hat{i} + \hat{j}, \vec{b}' = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c}' = 2\hat{i} - \hat{j} - \hat{k}$ then the altitude of the parallelepiped formed by the vectors, $\vec{a}, \vec{b}$ and $\vec{c}$ having base formed by $\vec{b}$ and $\vec{c}$ is (where $\vec{a}'$ is recipocal vector $\vec{a}$) $(a)1$ $(b)3\sqrt{2}/2$ $(c)1/\sqrt{6}$ $(d)1/\sqrt{2}$

A. 1

B. $3\sqrt{2}/2$

C. $1/\sqrt{6}$

D. $1/\sqrt{2}$

Answer: d

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95. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}, \vec{c} = \hat{k} + \hat{i}$ then in the reciprocal system of vectors $\vec{a}, \vec{b}, \vec{c}$ reciprocal $\vec{a}$ of vector $\vec{a}$ is

A. $\dfrac{\hat{i} + \hat{j} + \hat{k}}{2}$

B. $\dfrac{\hat{i} - \hat{j} + \hat{k}}{2}$

C. $\dfrac{-\hat{i} - \hat{j} + \hat{k}}{2}$

D. $\dfrac{\hat{i} + \hat{j} - \hat{k}}{2}$

Answer: d

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96. If unit vectors $\vec{a}$ and $\vec{b}$ are inclined at angle 2θ such that $\left|\vec{a} - \vec{b}\right| < 1 and 0 \leq \theta \leq \pi, then$ θ lies in interval a.$[0, \pi/6)$ b. $(5\pi/6, \pi]$ c. $[\pi/6, \pi/2]$ d. $[\pi/2, 5\pi/6]$

A. $[0, \pi/6)$

B. $(5\pi/6, \pi]$

C. $[\pi/6, \pi/2]$

D. $(\pi/2, 5\pi/6]$

Answer: a,b

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97. $\vec{a}, \vec{b} and \vec{c}$ are non-collinear if $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a}.\vec{b}\right)\vec{b} = (4 - 2x - \sin y)\vec{b} + \left(x^2 - 1\right)\vec{c}$ and $\left(\vec{c}.\vec{c}\right)\vec{a} = \vec{c}$ Then a. $x = 1$ b. $x = -1$ c. $y = (4n + 1)\pi/2, n \in I$ d. $y = (2n + 1)\pi/2, n \in I$

A. x =1

B. x = -1

C. $y = (4n + 1)\frac{\pi}{2}, n \in I$

D. $y(2n + 1)\frac{\pi}{2}, n \in I$

Answer: a,c

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98. Let $\vec{a}\vec{b} = 0, where \vec{a} and \vec{b}$ are unit vectors and the unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a} and \vec{b}$. If $\vec{c} = m\vec{a} + n\vec{b} + p\left(\vec{a} \times \vec{b}\right), (m, n, p \in R)$, then $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ b. $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$ c. $0 \leq \theta \leq \frac{\pi}{4}$ d. $0 \leq \theta \leq \frac{3\pi}{4}$

A. $\alpha = \beta$

B. $\gamma^2 = 1 - 2\alpha^2$

C. $\gamma^2 = -\cos 2\theta$

D. $\beta^2 = \frac{1 + \cos 2\theta}{2}$

Answer: a,b,c,d

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99. If vectors $\vec{a} and \vec{b}$ are two adjacent sides of a parallelogram, then the vector respresenting the altitude of the parallelogram which is the perpendicular to a is a.$\vec{b} + \frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ b. $\frac{\vec{a}\dot{\vec{b}}}{|\vec{b}|^2}$ c. $\vec{b}$ - $\frac{\vec{b}\dot{\vec{a}}}{|\vec{a}|^2}$ d. $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{b}|^2}$

A. $\frac{(\vec{a}.\vec{b})}{|\vec{a}|^2}\vec{a} - \vec{b}$

B. $\frac{1}{|\vec{a}|^2}\left\{|\vec{a}|^2\vec{b} - (\vec{a}.\vec{b})\vec{a}\right\}$

C. $\frac{\vec{a} \times (\vec{a} \times \vec{b})}{|\vec{a}|^2}$

D. $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{b}|^2}$

Answer: a,b,c

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100. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is perpendicular to $\left(\vec{a} \times \vec{b}\right) \times \vec{c}$, we may have a. $\left(\vec{a}.\vec{c}\right)\left|\vec{b}\right|^2 = \left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{c}\right)\left(\vec{c}.\vec{a}\right)$ b. $\vec{a}\vec{b} = 0$ c. $\vec{a}\vec{c} = 0$ d. $\vec{b}\vec{c} = 0$

A. $\left(\vec{a}.\vec{b}\right)\left|\vec{b}\right|^2 = \left(\vec{a}.\vec{b}\right)\left(\vec{b}.\vec{c}\right)$

B. $\vec{a}.\vec{b} = 0$

C. $\vec{a}.\vec{c} = 0$

D. $\vec{b}.\vec{c} = 0$

Answer: a,c

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101. If $\vec{p} = \dfrac{\vec{b} \times \vec{c}}{\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]}, \vec{q} = \dfrac{\vec{c} \times \vec{a}}{\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]}, \vec{r} = \dfrac{\vec{a} \times \vec{b}}{\left[\vec{a} \quad \vec{b} \quad \vec{b}\right]}$ where $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then the value of the expression $\left(\vec{a} + \vec{b} + \vec{c}\right).\left(\vec{p} + \vec{q} + \vec{r}\right)$ is

A. $x\left[\vec{a}\vec{b}\vec{c}\right] + \dfrac{\left[\vec{p}\vec{q}\vec{r}\right]}{x}$ has least value 2

B. $x^2\left[\vec{a}\vec{b}\vec{c}\right]^2 + \frac{\left[\vec{p}\vec{q}\vec{r}\right]}{x^2}$ has least value $\left(3/2^{2/3}\right)$

C. $\left[\vec{p}\vec{q}\vec{r}\right] > 0$

D. none of these

Answer: a,c

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102. $a_1, a_2, a_3 \in R - \{0\}$ and $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ " for all " x in R then (a) vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other (b)vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ are parallel to each each other (c)if vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then on of the ordered trippplet $\left(a_1, a_2, a_3\right) = (1, -1, -2)$ (d)if $2a_1 + 3a_2 + 6a_3 = 26$, then $\left|\vec{a}\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

A. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other

B. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ are parallel to each each other

C. if vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then on of the ordered trippplet $(a_1, a_2, a_3) = (1, -1, -2)$

D. if $2a_1 + 3a_2 + 6a_3 + 6a_3 = 26, \text{ then } \left|\vec{a}\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

Answer: a,b,c,d

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103. If $\vec{a} and \vec{b}$ are two vectors and angle between them is θ, then

$\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}\dot{}\vec{b}\right)^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$ $\left|\vec{a} \times \vec{b}\right| = \left(\vec{a}\dot{}\vec{b}\right), \quad \text{if} \quad \theta = \pi/4$

$\vec{a} \times \vec{b} = \left(\vec{a}\dot{}\vec{b}\right)\hat{n}$, ($where \hat{n}$ is unit vector,) if $\theta = \pi/4$ $\left(\vec{a} \times \vec{b}\right)\dot{}\vec{a} + \vec{b} = 0$

A. $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$

B. $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2, \quad \text{if} \quad \theta = \pi/4$

C. $\vec{a} \times \vec{b} = \left(\vec{a}. Vecb\right)\hat{n}$ (where $\hat{n}$ is a normal unit vector) $\text{if} \quad \theta f = \pi/4$

D. $\left(\vec{a} \times \vec{b}\right). \left(\vec{a} + \vec{b}\right) = 0$

Answer: a,b,c,d

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104. Let $\vec{a}$ and $\vec{b}$ be two non- zero perpendicular vectors. A vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$ can be

A. $\vec{b}$ - $\dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

B. $2\vec{b}$ - $\dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

C. $\left|\vec{a}\right|\vec{b}$ - $\dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

D. $\left|\vec{b}\right|\vec{b}$ - $\dfrac{\vec{a} \times \vec{b}}{\left|\vec{b}\right|^2}$

Answer: a,b,cd,

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105. If vector $\vec{b} = \left(tan\alpha, \ -1, 2\sqrt{\sin\alpha/2}\right) and \vec{c} = \left(tan\alpha, tan\alpha, \ -\dfrac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and vector $\vec{a} = (1, 3, \sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is

A. $\alpha = (4n+1)\pi + \tan^{-1}2$

B. $\alpha = (4n+1)\pi - \tan^{-1}2$

C. $\alpha = (4n+2)\pi + \tan^{-1}2$

D. $\alpha = (4n+2)\pi - \tan^{-1}2$

Answer: b,d

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106. Let $\vec{r}$ be a unit vector satisfying $\vec{r} \times \vec{a} = \vec{b}$, where $\left|\vec{a}\right| = \sqrt{3}$ and $\left|\vec{b}\right| = \sqrt{2}$, then $(a)\vec{r} = \dfrac{2}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$ (b)

$\vec{r} = \frac{1}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$ (c) $\vec{r} = \frac{2}{3}\left(\vec{a} - \vec{a} \times \vec{b}\right)$ (d) $\vec{r} = \frac{1}{3}\left(-\vec{a} + \vec{a} \times \vec{b}\right)$

A. $\vec{r} = \frac{2}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

B. $\vec{r} = \frac{1}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

C. $\vec{r} = \frac{2}{3}\left(\vec{a} - \vec{a} \times \vec{b}\right)$

D. $\vec{r} = \frac{1}{3}\left(-\vec{a} + \vec{a} \times \vec{b}\right)$

Answer: b,d

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107. If $\vec{a}$ and $\vec{b}$ are unequal unit vectors such that $\left(\vec{a} - \vec{b}\right) \times \left[\left(\vec{b} + \vec{a}\right) \times \left(2\vec{a} + \vec{b}\right)\right] = \vec{a} + \vec{b}$ then angle θ between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. $\pi/4$

D. π

Answer: b,d

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108. If $\vec{a}$ and $\vec{b}$ are two unit vectors perpenicualar to each other and $\vec{c}=\lambda_1\vec{a}+\lambda_2\vec{b}+\lambda_3\left(\vec{a}\times\vec{b}\right)$, then which of the following is (are) true ?

A. $\lambda_1=\vec{a}.\vec{c}$

B. $\lambda_2=\left|\vec{b}\times\vec{c}\right|$

C. $\lambda_3=\ |\ \left(\vec{a}\times\vec{b}\right|\times\vec{c}\right|$

D. $\lambda_1\vec{a}+\lambda_2\vec{b}+\lambda_3\left(\vec{a}\times\vec{b}\right)$

Answer: a,d

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109. If vectors $\vec{a}$ and $\vec{b}$ are non collinear then $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$ is (A) a unit vector (B) in the plane of $\vec{a}$ and $\vec{b}$ (C) equally inclined to $\vec{a}$ and $\vec{b}$ (D) perpendicular to $\vec{a} \times \vec{b}$

A. a unit vector

B. in the plane of $\vec{a}$ and $\vec{b}$

C. equally inclined to $\vec{a}$ and $\vec{b}$

D. perpendicular to $\vec{a} \times \vec{b}$

Answer: b,c,d

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110. If $\vec{a}$ and $\vec{b}$ are non - zero vectors such that $|\vec{a} + \vec{b}| = |\vec{a} - 2\vec{b}|$ then

A. $2\vec{a}.\vec{b} = |\vec{b}|^2$

B. $\vec{a}.\vec{b} = |\vec{b}|^2$

C. least value of $\vec{a}.\vec{b}+\dfrac{1}{\left|\vec{b}\right|^2+2}$ is $\sqrt{2}$

D. least value of $\vec{a}.\vec{b}+\dfrac{1}{\left|\vec{b}\right|^2+2}$ is $\sqrt{2}-1$

Answer: a,d

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111. Let $\vec{a}\vec{b}$ and $\vec{c}$ be non- zero vectors aned $\vec{V}_1=\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ and $\vec{V}_2=\left(\vec{a}\times\vec{b}\right)\times\vec{c}$.vectors $\vec{V}_1$ and $\vec{V}_2$ are equal . Then

A. $\vec{a}$ and $\vec{b}$ ar orthogonal

B. $\vec{a}$ and $\vec{c}$ are collinear

C. $\vec{b}$ and $\vec{c}$ ar orthogonal

D. $\vec{b}=\lambda\left(\vec{a}\times\vec{c}\right)$ when λ is a scalar

Answer: b,d

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112. Vectors $\vec{A}$ and $\vec{B}$ satisfying the vector equation $\vec{A}+\vec{B}=\vec{a}, \vec{A}\times\vec{B}=\vec{b}$ and $\vec{A}.\vec{a}=1$. where veca and $\vec{b}$ are given vectosrs, are

A. $\vec{A}=\dfrac{\left(\vec{a}\times\vec{b}\right)-\vec{a}}{a^2}$

B. $\vec{B}=\dfrac{\left(\vec{b}\times\vec{a}\right)+\vec{a}\left(a^2-1\right)}{a^2}$

C. $\vec{A}=\dfrac{\left(\vec{a}\times\vec{b}\right)+\vec{a}}{a^2}$

D. $\vec{B}=\dfrac{\left(\vec{b}\times\vec{a}\right)-\vec{a}\left(a^2-1\right)}{a^2}$

Answer: b,c,

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113. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} and \vec{c} = 3\hat{j} - 2\hat{k}$ Let $\vec{x}, \vec{y}, and \vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a},$ respectively. Then a.$\vec{x}. \vec{d} = -1$ b. $\vec{y}. \vec{d} = 1$ c. $\vec{z}. \vec{d} = 0$ d. $\vec{r}. \vec{d} = 0, where \vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

A. $\vec{x}. \vec{d} = -1$

B. $\vec{y}. \vec{d} = 1$

C. vecz.vecd=0`

D. vecr.vecd=0, " where " vecr=lambda vecx + mu vecy +deltavecz`

Answer: c.d

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114. Vectors perpendicular to $\hat{i} - \hat{j} - \hat{k}$ and in the plane of $\hat{i} + \hat{j} + \hat{k} and - \hat{i} + \hat{j} + \vec{k}$ are $\hat{i} + \hat{k}$ b. $2\hat{i} + \hat{j} + \hat{k}$ c. $3\hat{i} + 2\hat{j} + \hat{k}$ d. $-4\hat{i} - 2\hat{j} - 2\hat{k}$

A. $\hat{i} + \hat{k}$

B. $2\hat{i} + \hat{j} + \hat{k}$

C. $3\hat{i} + 2\hat{j} + \hat{k}$

D. $-4\hat{i} - 2\hat{j} - 2\hat{k}$

Answer: b,d

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115. If side $\vec{A}B$ of an equilateral trangle ABC lying in the x-y plane $3\hat{i}$, then side $\vec{C}B$ can be a. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$ b. $\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$ c. $-\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$ d. $\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

A. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$

B. $-\frac{3}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$

C. $-\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

D. $\frac{3}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$

Answer: b,d

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116. Let $\hat{a}$ be a unit vector and $\hat{b}$ a non zero vector non parallel to $\vec{a}$. Find the angles of the triangle tow sides of which are represented by the vectors. $\sqrt{3}\left(\hat{} \times \vec{b}\right)$ and $\vec{b} - \left(\hat{a}.\vec{b}\right)\hat{a}$

A. $\tan^{-1}\left(\sqrt{3}\right)$

B. $\tan^{-1}\left(1/\sqrt{3}\right)$

C. $\cot^{-1}(0)$

D. tant^(-1)(1)`

Answer: a,b,c

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117. $\vec{a}, \vec{b}, and \vec{c}$ are unimodular and coplanar. A unit vector $\vec{d}$ is perpendicular to then. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$, and the angel between $\vec{a} and \vec{b}$ is $30^0, then \vec{c}$ is $\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)/3$ b.$\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right)/3$ c. $\left(2\hat{i} + 2\hat{j} - \hat{k}\right)/3$ d. $\left(-2\hat{i} - 2\hat{j} + \hat{k}\right)/3$

A. $\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)/3$

B. $\left(- \hat{i} + 2\hat{j} - 2\hat{k}\right)/3$

C. $\left(- \hat{i} + 2\hat{j} - \hat{k}\right)/3$

D. $\left(- 2\hat{i} - 2\hat{j} + \hat{k}\right)/3$

Answer: a,b

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118. If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$ then $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} =$

A. $2\left(\vec{a} \times \vec{b}\right)$

B. $6\left(\vec{b} \times \vec{c}\right)$

C. $3\left(\vec{c} \times \vec{a}\right)$

D. $\vec{0}$

Answer: c,d

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119. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - \left(\vec{a}.\vec{b}\right)\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $\left|\vec{v}\right|$ is

A. $\left|\vec{u}\right|$

B. $\left|\vec{u}\right| + \left|\vec{u}.\vec{b}\right|$

C. $\left|\vec{u}\right| + \left|\vec{u}.\vec{a}\right|$

D. none of these

Answer: b,d

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120. if $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a},$ where $\vec{c} \neq \vec{0}$ then (a) $\left|\vec{a}\right| = \left|\vec{c}\right|$ (b) $\left|\vec{a}\right| = \left|\vec{b}\right|$ (c) $\left|\vec{b}\right| = 1$ (d) $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = 1$

A. $\left|\vec{a}\right| = \left|\vec{c}\right|$

B. $\left|\vec{a}\right| = \left|\vec{b}\right|$

C. $\left|\vec{b}\right| = 1$

D. $\left|\vec{a}\right| = \vec{b}| = |\vec{c}\ | \ = 1$

Answer: a,c

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121. $\vec{b} and \vec{c}$ are unit vectors. Then for any arbitrary vector $\vec{a}, \left(\left(\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)\right)\times\left(\vec{b}\times\vec{c}\right)\right)\vec{b}-\vec{c}$ is always equal to $\left|\vec{a}\right|$ b. $\frac{1}{2}\left|\vec{a}\right|$ c. $\frac{1}{3}\left|\vec{a}\right|$ d. none of these

A. $\dfrac{\vec{d}.\left(\vec{a}+\vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]} = 2$

B. $\dfrac{\vec{d}.\left(\vec{a}+\vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]} = -2$

C. minimum value of $x^2 + y^2 is \pi^2/4$

D. minimum value of $x^2 + y^2 is 5\pi^2/4$

Answer: b,d

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122. If $\vec{a}, \vec{b}, and \leftrightarrow c$ are three unit vecrtors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{1}{1}\vec{b}, then \left(\vec{b} and \vec{c}\right.$ being non-parallel) angle between $\vec{a} and \vec{b}$ is$\pi/3$ b.a n g l eb e t w e e n$\vec{a} and \vec{c}$i s$\pi/3$ c. a. angle between $\vec{a} and \vec{b}$ is$\pi/2$ d. a. angle between $\vec{a} and \vec{c}$ is$\pi/2$

A. angle between $\vec{a}$ and $\vec{b} is \pi/3$

B. angle between $\vec{a}$ and $\vec{c} is \pi/3$

C. angle between $\vec{a}$ and $\vec{b} is \pi/2$

D. angle between $\vec{a}$ and $\vec{c} is \pi/2$

Answer: b,c

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123. If in triangle ABC, $\overrightarrow{AB} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|}$ and $\overrightarrow{AC} = \frac{2\vec{u}}{|\vec{u}|}$, where $|\vec{u}| \neq |\vec{v}|$, then $(a) 1 + \cos 2A + \cos 2B + \cos 2C = 0$ (b)$\sin A = \cos C$ (c)projection of AC on BC is equal to BC (d) projection of AB on BC is equal to AB

A. $1 + \cos 2A + \cos 2B + \cos 2C = 0$

B. $\sin A = \cos C$

C. projection of AC on BC is equal to BC

D. projection of AB on BC is equal to AB

Answer: a,b,c

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124. $\left[\vec{a} \times \vec{b} \quad \vec{c} \times \vec{d} \quad \vec{e} \times \vec{f}\right]$ is equal to

A. $\left[\vec{a}\vec{b}\vec{d}\right]\left[\vec{c}\vec{e}\vec{f}\right] - \left[\vec{a}\vec{b}\vec{c}\right]\left[\vec{d}\vec{e}\vec{f}\right]$

B. $\left[\vec{a}\vec{b}\vec{e}\right]\left[\vec{f}\vec{c}\vec{d}\right] - \left[\vec{a}\vec{b}\vec{f}\right]\left[\vec{e}\vec{c}\vec{d}\right]$

C. $\left[\vec{c}\,\vec{d}\,\vec{a}\right]\left[\vec{b}\,\vec{e}\,\vec{f}\right]$ - $\left[\vec{a}\,\vec{d}\,\vec{b}\right]\left[\vec{a}\,\vec{e}\,\vec{f}\right]$

D. $\left[\vec{a}\,\vec{c}\,\vec{e}\right]\left[\vec{b}\,\vec{d}\,\vec{f}\right]$

Answer: a,b,c

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125. The scalars l and m such that $l\vec{a} + m\vec{b} = \vec{c}$,where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are given vectors, are equal to

A. $l = \dfrac{\left(\vec{c} \times \vec{b}\right).\left(\vec{a} \times \vec{b}\right)}{\left(\vec{a} \times \vec{b}\right)^2}$

B. $l = \dfrac{\left(\vec{c} \times \vec{a}\right).\left(\vec{b} \times \vec{a}\right)}{\left(\vec{b} \times \vec{a}\right)}$

C. $m = \dfrac{\left(\vec{c} \times \vec{a}\right).\left(\vec{b} \times \vec{a}\right)}{\left(\vec{b} \times \vec{a}\right)^2}$

D. $m = \dfrac{\left(\vec{c} \times \vec{a}\right).\left(\vec{b} \times \vec{a}\right)}{\left(\vec{b} \times \vec{a}\right)}$

Answer: a,c

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126. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right). \left(\vec{a} \times \vec{d}\right) = 0$ then which of the following may be true ?

A. $\vec{a}, \vec{b}$ and $\vec{d}$ are nenessarily coplanar

B. $\vec{a}$ lies iin the plane of $\vec{c}$ and $\vec{d}$

C. $\vec{v}b$ lies in the plane of $\vec{a}$ and $\vec{d}$

D. $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{d}$

Answer: b,c,d

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127. $A, B, CandD$ are four points such that $\vec{AB} = m\left(2\hat{i} - 6\hat{j} + 2\hat{k}\right), \vec{BC} = \left(\hat{i} - 2\hat{j}\right)and\vec{CD} = n\left(- 6\hat{i} + 15\hat{j} - 3\hat{k}\right)^{\cdot}$ If CD

intersects AB at some point E, then a. $m \geq 1/2$ b.$n \geq 1/3$ c. $m = n$ d. $m < n$

A. $m \geq 1/2$

B. $n \geq 1/3$

C. m= n

D. $m < n$

Answer: a,b

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128. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar and l,m,n are distinct real numbers, then $[(l\vec{a} + m\vec{b} + n\vec{c}\Big)\Big(l\vec{b} + m\vec{c} + n\vec{a}\Big)\Big(l\vec{c} + m\vec{a} + n\vec{b})]$ = 0, implies (A) $lm + mn + nl = 0$ (B) $l + m + n = 0$ (C) $l^2 + m^2 + n^2 = 0$

A. $l + m + n = 0$

B. roots of the equation $lx^2 + mx + n = 0$ are equal

C. $l^2 + m^2 + n^2 = 0$

D. $l^3 + m^2 + n^3 = 3lmn$

Answer: a,b,d

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129. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \vec{\beta} = b\hat{i} + c\hat{j} + a\hat{k}$ and $\vec{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplnar vectors with $a \neq b$, and $\vec{v} = \hat{i} + \hat{j} + \hat{k}$. Then $\vec{v}$ is perpendicular to

A. $\vec{\alpha}$

B. $\vec{\beta}$

C. $\vec{\gamma}$

D. none of these

Answer: a,b,c

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130. If vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{B} = \hat{i} + \hat{j} + 5\hat{k}$ and $\vec{C}$ form a left-handed system, then $\vec{C}$ is a.$11\hat{i} - 6\hat{j} - \hat{k}$ b.$-11\hat{i} + 6\hat{j} + \hat{k}$ c. $11\hat{i} - 6\hat{j} + \hat{k}$ d. $-11\hat{i} + 6\hat{j} - \hat{k}$

A. $11\hat{i} - 6\hat{j} - \hat{k}$

B. $-11\hat{i} - 6\hat{j} - \hat{k}$

C. $-11\hat{i} - 6\hat{j} + \hat{k}$

D. $-11\hat{i} + 6\hat{j} - \hat{k}$

Answer: b,d

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131. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k}$, then $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is

(a)parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$ (b)orthogonal to $\hat{i} + \hat{j} + \hat{k}$ (c)orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$ (d)orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

A. parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$

D. orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

Answer: a,b,c,d

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132. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$ then

A. $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{0}$

B. $\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$

C. $\vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

D. $\vec{c} \times \vec{a} \times \vec{b} = \vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

Answer: a,c,d

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133. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, ; \vec{c}, \vec{a},$ respectively. Then

A. $\vec{z}.\vec{d} = 0$

B. $\vec{x}.\vec{d} = 1$

C. $\vec{y}.\vec{d} = 32$

D. $\vec{r}.\vec{d} = 0, \text{ where } \vec{r} = \lambda\vec{x} + \mu\vec{y} + \gamma\vec{z}$

Answer: a,d

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134. A parallelogram is constructed on the vectors $\vec{a} = 3\vec{\alpha} - \vec{\beta}, \vec{b} = \vec{\alpha} + 3\vec{\beta}. If \left|\vec{\alpha}\right| = \left|\vec{\beta}\right| = 2$ and angle between $\vec{\alpha}$ and $\vec{\beta} is \frac{\pi}{3}$ then the length of a diagonal of the parallelogram is

A. $4\sqrt{5}$

B. $4\sqrt{3}$

C. $4\sqrt{7}$

D. none of these

Answer: b,c

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Reasoning Type

1. (a)Statement 1: Vector $\vec{c} = -5\hat{i} + 7\hat{j} + 2\hat{k}$ is along the bisector of angle between $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = 8\hat{i} + \hat{j} - 4\hat{k}$.

Statement 2 : $\vec{c}$ is equally inclined to $\vec{a}$ and $\vec{b}$.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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2. Statement 1: A component of vector $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in the direction perpendicular totehdirectin of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k} is \hat{i} - \hat{j}$ Statement 2: A component of vector in the direction of $\vec{a} = \hat{i} + \hat{j} + \hat{k} is 2\hat{i} + 2\hat{j} + 2\hat{k}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: c

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3. Statement 1: Distance of point D(1,0,-1) from the plane of points A(1,-2,0) , B (3, 1,2) and C(-1,1,-1) is $\frac{8}{\sqrt{229}}$

Statement 2: volume of tetrahedron formed by the points A,B, C and D is $\frac{\sqrt{229}}{2}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: d

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4. Let $\vec{r}$ be a non - zero vector satisfying $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ for given non- zero vectors $\vec{a}\vec{b}$ and $\vec{c}$

Statement 1: $\left[\vec{a} - \vec{b}\vec{b} - \vec{c}\vec{c} - \vec{a}\right] = 0$

Statement 2: $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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5. Statement 1: If $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b}\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are three mutually perpendicular unit vectors then $a_1\hat{i} + b_1\hat{j} + c_1\hat{k}, a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ and $a_3\hat{i} + b_3\hat{j} + c_3\hat{k}$ may be mutually perpendicular unit vectors.

Statement 2 : value of determinant and its transpose are the same.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: a

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6. Statement 1: $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{B} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$ then $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = 243$ Statement 2: $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = \left|\vec{A}\right|^2 \left|\left[\vec{A}\vec{B}\vec{C}\right]\right|$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: d

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7. Statement 1: $\vec{a}, \vec{b}, and \vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are non-coplanar. If

$\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] = 1, then \vec{d} = \vec{a} + \vec{b} + \vec{c}.$ Statement 2: $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right]$; then $\vec{d}$ equally inclined to $\vec{a}$,$\vec{b}$ and $\vec{c}$.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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8. Consider three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$

Statement 1: $\vec{a} \times \vec{b} = \left(\left(\hat{i} \times \vec{a}\right).\vec{b}\right)\hat{i} + \left(\left(\hat{j} \times \vec{a}\right).\vec{b}\right)\hat{j} + \left(\hat{k} \times \vec{a}\right).\vec{b}\right)\hat{k}$

Statement 2: $\vec{c} = \left(\hat{i}.\vec{c}\right)\hat{i} + \left(\hat{j}.\vec{c}\right)\hat{j} + \left(\hat{k}.\vec{c}\right)\hat{k}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: a

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Comprehension Type

1. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} = 3/2, \vec{a}.\vec{v} = 7/4$ and

Vector $\vec{u}$ is

A. $\vec{a} - \frac{2}{3}\vec{b} + \vec{c}$

B. $\vec{a} + \frac{4}{3}\vec{b} + \frac{8}{3}\vec{c}$

C. $2\vec{a} - \vec{b} + \frac{1}{3}\vec{c}$

D. $\frac{4}{3}\vec{a} - \vec{b} + \frac{2}{3}\vec{c}$

Answer: b

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2. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} = 3/2, \vec{a}.\vec{v} = 7/4$ and |

Vector $\vec{u}$ is

A. (a) $\vec{a} - \frac{2}{3}\vec{b} + \vec{c}$

B. (b) $\vec{a} + \frac{4}{3}\vec{b} + \frac{8}{3}\vec{c}$

C. (c) $2\vec{a} - \vec{b} + \frac{1}{3}\vec{c}$

D. (d) $\frac{4}{3}\vec{a} - \vec{b} + \frac{2}{3}\vec{c}$

Answer: c

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3. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u}+\vec{v}+\vec{w}=\vec{a}, \vec{u}\times\left(\vec{v}\times\vec{w}\right)=\vec{b}, \left(\vec{u}\times\vec{v}\right)\times\vec{w}=\vec{c}, \vec{a}.\vec{u}=3/2, \vec{a}.\vec{v}=7/4$ and

Vector $\vec{u}$ is

A. (a) $\vec{a}-\frac{2}{3}\vec{b}+\vec{c}$

B. (b) $\vec{a}+\frac{4}{3}\vec{b}+\frac{8}{3}\vec{c}$

C. (c) $2\vec{a}-\vec{b}+\frac{1}{3}\vec{c}$

D. (a) $\frac{4}{3}\vec{a}-\vec{b}+\frac{2}{3}\vec{c}$

Answer: d

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4. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x}\times\left(\vec{y}\times\vec{z}\right)=\vec{a}, \vec{y}\times\left(\vec{z}\times\vec{x}\right)=\vec{b}$ and $\vec{x}\times\vec{y}=\vec{c}$. Find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.

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5. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$. Find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.

A. $\frac{1}{2}\left[\left(\vec{a} + \vec{c}\right) \times \vec{b} - \vec{b} - \vec{a}\right]$

B. $\frac{1}{2}\left[\left(\vec{a} - \vec{c}\right) \times \vec{b} + \vec{b} + \vec{a}\right]$

C. $\frac{1}{2}\left[\left(\vec{a} - \vec{b}\right) \times \vec{c} + \vec{b} + \vec{a}\right]$

D. $\frac{1}{2}\left[\left(\vec{a} - \vec{c}\right) \times \vec{a} + \vec{b} - \vec{a}\right]$

Answer: c

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6. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{\times} x\left(\vec{y} \times \left(\vec{z} \times \vec{x}\right) = \vec{b}nd \vec{\times} x\vec{y} = \vec{c}, f \in d\vec{x}, \vec{y}, \vec{z}\right.$ in terms of

$\vec{a}, \vec{b}$ and $\vec{c}$.

A. $\frac{1}{2}\left[\left(\vec{a} - \vec{c}\right) \times \vec{c} - \vec{b} + \vec{a}\right]$

B. $\frac{1}{2}\left[\left(\vec{a} - \vec{b}\right) \times \vec{c} + \vec{b} - \vec{a}\right]$

C. $\frac{1}{2}\left[\vec{c} \times \left(\vec{a} - \vec{b}\right) + \vec{b} + \vec{a}\right]$

D. none of these

Answer: b

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7. If $\vec{x} \times \vec{y} = \vec{a}, \vec{y} \times \vec{z} = \vec{b}, \vec{x}.\vec{b} = \gamma, \vec{x}.\vec{y} = 1$ and $\vec{y}.\vec{z} = 1$ then find x,y,z in terms of `vec a , vec band y .

A. $\frac{1}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

B. $\frac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \vec{b} - \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

C. $\frac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \vec{b} + \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

D. none of these

Answer: b

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8. Vectors $\vec{x}, \vec{y}, \vec{z}$ each of magnitude $\sqrt{2}$ make angles of 60^0 with each other. If $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$. Find $\vec{x}, \vec{y}, \vec{z}$ in terms of $\vec{a}, \vec{b}, \vec{c}$.

A. $\dfrac{\vec{a} \times \vec{b}}{\gamma}$

B. $\vec{a} + \dfrac{\vec{a} \times \vec{b}}{\gamma}$

C. $\vec{a} + \vec{b} + \dfrac{\vec{a} \times \vec{b}}{\gamma}$

D. none of these

Answer: a

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9. If $\vec{x} \cdot x\vec{y} = \vec{a}, \vec{y} \times \vec{z} = \vec{b}, \vec{x}.\vec{b} = \gamma, \vec{x}.\vec{y} = 1$ and $\vec{y}.\vec{z} = 1$ then find x,y,z in terms of `veca,vecb and gamma.

A. $\frac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} \times \left(\vec{a} \times \vec{b}\right)\right]$

B. $\frac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} - \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

C. $\frac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} + \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

D. none of these

Answer: c

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10. Given two orthogonal vectors $\vec{A}$ and $\vec{B}$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then $\left(\vec{P} \times \vec{B}\right) \times \vec{B}$ is equal to

A. $\vec{P}$

B. $-\vec{P}$

C. $2\vec{B}$

D. $\vec{A}$

Answer: b

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11. Given two orthogonal vectors $\vec{A}$ and $\vec{B}$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then $\vec{P}$ is equal to

A. $\frac{\vec{A}}{2} + \frac{\vec{A} \times \vec{B}}{2}$

B. $\frac{\vec{A}}{2} + \frac{\vec{B} \times \vec{A}}{2}$

C. $\frac{\vec{A} \times \vec{B}}{2} - \frac{\vec{A}}{2}$

D. $\vec{A} \times \vec{B}$

Answer: b

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12. Given two orthogonal vectors $\vec{A}$ and $VecB$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then which of the following statements is false ?

A. vectors $\vec{P}, \vec{A}$ and $\vec{P} \times \vec{B}$ ar linearly dependent.

B. vectors $\vec{P}, \vec{B}$ and $\vec{P} \times \vec{B}$ ar linearly independent

C. $\vec{P}$ is orthogonal to $\vec{B}$ and has length $\frac{1}{\sqrt{2}}$.

D. none of these

Answer: d

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13. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then $\vec{a}_2$ is equal to

A. $\frac{943}{49}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

B. $\frac{943}{49^2}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

C. $\frac{943}{49}\left(-2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

D. $\frac{943}{49^2}\left(-2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

Answer: b

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14. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then $\vec{a}_1 . \vec{b}$ is equal to

A. -41

B. $-41/7$

C. 41

D. 287

Answer: a

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15. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then $\vec{a}_2$ is equal to

A. $\vec{a}$ and $vcea_2$ are collinear

B. $\vec{a}_1$ and $\vec{c}$ are collinear

C. $\vec{a}m\vec{a}_1$ and $\vec{b}$ are coplanar

D. $\vec{a}, \vec{a}_1$ and a_2 are coplanar

Answer: c

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16. Consider a triangular pyramid ABCD the position vectors of whose angular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 2, 3)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCD. The length of the vec AG is

A. $\sqrt{17}$

B. $\sqrt{51}/3$

C. $3/\sqrt{6}$

D. $\sqrt{59}/4$

Answer: b

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17. Consider a triangular pyramid ABCD the position vectors of whone agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCT. The length of the perpendicular from the vertex D on the opposite face

A. 24

B. $8\sqrt{6}$

C. $4\sqrt{6}$

D. none of these

Answer: c

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18. Consider a triangular pyramid ABCD the position vectors of whose agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCD. The length of the vector $\overline{AG}$ is

A. $14/\sqrt{6}$

B. $2/\sqrt{6}$

C. $3/\sqrt{6}$

D. none of these

Answer: a

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19. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D (x,y,z) The distance between the parallel lines AB and CD is

A. $\sqrt{6}$

B. $3\sqrt{6/5}$

C. $2\sqrt{2}$

D. 3

Answer: c

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20. Vertices of a parallelogram taken in order are A(2,-1,4)B(1,0,-1)C(1,2,3) and D.

Distance of the point P (8, 2,-12) from the plane of the parallelogram is

A. $\frac{4\sqrt{6}}{9}$

B. $\frac{32\sqrt{6}}{9}$

C. $\frac{16\sqrt{6}}{9}$

D. none

Answer: b

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21. Vertices of a parallelogram taken in order are A(2,-1,4)B(1,0,-1)C(1,2,3) and D.

Distance of the point P (8, 2,-12) from the plane of the parallelogram is

A. 14, 4,2

B. 2,4,14

C. 4,2,14

D. 2,14,4

Answer: d

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22. Let $\vec{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\vec{r}.\left(10\hat{j} - 8\hat{i} - \vec{r}\right) = 40$ and $p_1 = \max\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}, p_2 = \min\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}$. A tangent line is drawn to the curve $y = \frac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B

A. 9

B. $2\sqrt{2} - 1$

C. $6\sqrt{6} + 3$

D. $9 - 4\sqrt{2}$

Answer: d

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23. Let $\vec{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\vec{r}.\left(10\hat{j} - 8\hat{i} - \vec{r}\right) = 40$ and $p_1 = \max\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}, p_2 = \min\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}$. Then $p_1 + p_2$ is equal to

A. 2

B. 10

C. 18

D. 5

Answer: c

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24. Let $\vec{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\vec{r}.\left(10\hat{j} - 8\hat{i} - \vec{r}\right) = 40$ and $p_1 = \max\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}, p_2 = \min\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}$. Then $p_1 + p_2$ is equal to

A. 1

B. 2

C. 3

D. 4

Answer: c

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25. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and

A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC} = \vec{b}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$ vector $\overrightarrow{AB}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} - \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

D. none of these

Answer: a

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26. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and

A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$ vector $\overrightarrow{AD}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} - \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

D. none of these

Answer: b

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27. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and

A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC} = \vec{b}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a}$ is $\frac{|\vec{a}|}{3}$ vector $\overrightarrow{AB}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} - \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

D. none of these

Answer: c

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Martrix Match Type

1.

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2.

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3.

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4. Given two vectors $\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{j} - \hat{k}$

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5. Given two vectors $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$

find $\left|\vec{a} \times \vec{b}\right|$

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6.

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7. Volume of parallelpiped formed by vectors $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ is 36 sq. units.

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8.

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9.

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10.

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Integer Type

1. If $\vec{a} and \vec{b}$ are any two unit vectors, then find the greatest positive integer in the range of $\frac{3\left|\vec{a} + \vec{b}\right|}{2} + 2\left|\vec{a} - \vec{b}\right|$.

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2. Let $\vec{u}$ be a vector on rectangular coodinate system with sloping angle 60° suppose that $\left|\vec{u} - \hat{i}\right|$ is geomtric mean of $\left|\vec{u}\right|$ and $\left|\vec{u} - 2\hat{i}\right|$, where $\hat{i}$ is the unit vector along the x-axis . Then find the value of $\dfrac{\sqrt{2} - 1}{\left|\vec{u}\right|}$

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3. Find the absolute value of parameter t for which the area of the triangle whose vertices the $A(-1, 1, 2); B(1, 2, 3) and C(5, 1, 1)$ is minimum.

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4. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ and

$$\left[3\vec{a} + \vec{b} \;\; 3\vec{b} + \vec{c} \;\; 3\vec{c} + \vec{a}\right] = \lambda \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \text{ then find the value of } \frac{\lambda}{4}$$

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5. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = \alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}, and \vec{c} = 2\hat{i} - \alpha\hat{j} + \hat{k}$. Find thevalue of 6α, such that $\left\{\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\right\} \times \left(\vec{c} \times \vec{a}\right) = 0.$

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6. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying $\left[(a - 2)\alpha^2 + (b - 3)\alpha + c\right]\vec{x} + \left[(a - 2)\beta^2 + (b - 3)\beta + c\right]\vec{y} + \left[(a - 2)\gamma^2 + (b - 3)\gamma + c$

are three distinct real numbers, then find the value of $\left(a^2 + b^2 + c^2 - 4\right)$.

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7. Let $\vec{u}$ and $\vec{v}$ be unit vectors such that $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \vec{v}$. Find the value of $\left[\vec{u}\ \vec{v}\ \vec{w}\right]$.

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8. Find the value of λ if the volume of a tetrahedron whose vertices are with position vectors $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 7\hat{k}$, $5\hat{i} - \hat{j} + \lambda\hat{k}$ and $7\hat{i} - 4\hat{j} + 7\hat{k}$ is 11 cubic unit.

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9. Given that $\vec{u} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{v} = 2\hat{i} + \hat{k} + 4\hat{k}, \vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$ and $\left(\vec{u}.\vec{R} - 15\right)\hat{i} + \left(\vec{c}.\vec{R} - 30\right)\hat{j} +$

. Then find the greatest integer less than or equal to $\left|\vec{R}\right|$.

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10. Let a three dimensional vector $\vec{V}$ satisfy the condition, $2\vec{V} + \vec{V} \times \left(\hat{i} + 2\hat{j}\right) = 2\hat{i} + \hat{k}$ If $3\left|\vec{V}\right| = \sqrt{m}$ Then find the value of m

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11. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = 0 = \vec{a}.\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$.

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12. Let $\vec{O}A = \vec{a}, \hat{O}B = 10\vec{a} + 2\vec{b} and \vec{O}C = \vec{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB$, and let q denote the area of parallelogram with $OA and OC$ as adjacent sides. If $p = kq$, then find$k$.

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13. Find the work done by the force $F = 3\hat{i} - \hat{j} - 2\hat{k}$ acrting on a particle such that the particle is displaced from point $A(-3, -4, 1) \rightarrow B(-1, -1, -2)$.

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14. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$ then find the value of $\left(2\vec{a} + \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$

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15. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = i + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$ then find the value of $\vec{r}.\vec{b}$.

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16. If $\vec{a}, \vec{b} and \vec{c}$ are unit vectors satisfying $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9,$ then $\left|2\vec{a} + 5\vec{b} + 5\vec{c}\right|$ is.

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17. Let $\vec{a}, \vec{b}$, and $\vec{c}$ be three non coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$ where p,q,r are scalars then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

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Subjective Type

1. From a point O inside a triangle ABC, perpendiculars $OD, OEandOf$ are drawn to rthe sides $BC, CAandAB$, respecrtively. Prove that the perpendiculars from $A, B, andC$ to the sides $EF, FDandDE$ are concurrent.

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3. If c is a given non-zero scalar, and $\vec{A}$ and $\vec{B}$ are given non-zero vector such that $\vec{A} \perp \vec{B}$, then find vector $\vec{X}$ which satisfies the equation $\vec{A}.\vec{X} = c$ and $\vec{A} \times \vec{X} = \vec{B}$.

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4. $A, B, CandD$ are any four points in the space, then prove that $\left|\vec{A}B \times \vec{C}D + \vec{B}C \times \vec{A}D + \vec{C}A \times \vec{B}D\right| = 4$ (area of ABC).

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5. If the vectors $\vec{a}, \vec{b},$ and $\vec{c}$ are coplanar show that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = 0$$

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6. Let $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}$ $\vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ Determine a vector $\vec{R}$ satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}.\vec{A} = 0$.

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7. Determine the value of c so that for all real x, vectors $cx\hat{i} - 6\hat{j} - 3\hat{k}$ and $x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other.

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8. If vectors, $\vec{b}, \vec{c}$ and $\vec{d}$ are not coplanar, the prove that vector $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) + \left(\vec{a} \times \vec{c}\right) \times \left(\vec{d} \times \vec{b}\right) + \left(\vec{a} \times \vec{d}\right) \times \left(\vec{b} \times \vec{c}\right)$ is parallel to $\vec{a}$.

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9. The position vectors of the vertices A, B and C of a tetrahedron ABCD are $\hat{i} + \hat{j} + \hat{k}$, $\hat{k}$, $\hat{i}$ and $\hat{3}i$,respectively. The altitude from vertex D to the opposite face ABC meets the median line through Aof triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is2√2/3, find the position vectors of the point E for all its possible positions

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10. Let $\vec{a}, \vec{b}, and \vec{c}$ be non-coplanar unit vectors, equally inclined to one another at an angleθ then $\left[\vec{a}\vec{b}\vec{c}\right]$ in terms of θ is equal to :

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11. If $\vec{A}, \vec{B}$ and $\vec{C}$ are vectors such that $\left|\vec{B}\right| = \left|\vec{C}\right|$. Prove that $\left[\left(\vec{A} + \vec{B}\right) \times \left(\vec{A} + \vec{C}\right)\right] \times \left(\vec{B} + \vec{C}\right). \left(\vec{B} + \vec{C}\right) = 0$

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12. For any two vectors $\vec{u}$ and $\vec{v}$ prove that $\left(1 + |\vec{u}|^2\right)\left(1 + |\vec{v}|^2\right) = (1 - \vec{u}.\vec{v})^2 + \left|\vec{u} + \vec{v} + (\vec{u} \times \vec{v})\right|^2$

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13. Let $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w} + \vec{w} \times \vec{u} = \vec{v}$, then prove that $\left|(\vec{u} \times \vec{v}).\vec{w}\right| \leq \frac{1}{2}$ and that the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

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14. Find 3-dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying $\vec{v}_1 \cdot \vec{v}_1 = 4, \vec{v}_1 \cdot \vec{v}_2 = -2, \vec{v}_1 \cdot \vec{v}_3 = 6,$ $\vec{v}_2 \cdot \vec{v}_2 = 2, \vec{v}_2 \cdot \vec{v}_3 = -5, \vec{v}_3 \cdot \vec{v}_3 = 29$

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15. Let V be the volume of the parallelopiped formed by the vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If a_r, b_r and c_r, where r = 1, 2, 3, are non-negative real numbers and $\sum_{r=1}^{3}\left(a_r + b_r + c_r\right) = 3L$ show that $V \leq L^3$

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16. $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-coplanar unit vecrtors and α, β and γ are the angles between$\vec{u}$ and $\vec{v}$, $\vec{v}$and $\vec{w}$, $and\vec{w}$ and $\vec{u}$, respectively, and $\vec{x}, \vec{y}$ and $\vec{z}$ are unit vectors along the bisectors of the angles $\alpha, \beta and \gamma$, respectively.

Prove that $\left[\vec{x} \times \vec{y}\vec{y} \times \vec{z}\vec{z} \times \vec{x}\right] = \frac{1}{16}\left[\vec{u}\vec{v}\vec{w}\right]^2 \sec^2\left(\frac{\alpha}{2}\right)\sec^2\left(\frac{\beta}{2}\right)\sec^2\left(\frac{\gamma}{2}\right)$.

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17. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ ar distinct vectors such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$. Prove that $\left(\vec{a} - \vec{d}\right).\left(\vec{c} - \vec{b}\right) \neq 0, i.e., \vec{a}.\vec{b} + \vec{d}.\vec{c} \neq \vec{d}.\vec{b} + \vec{a}.\vec{c}$.

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18. P_1ndP_2 are planes passing through origin L_1andL_2 are two lines on P_1andP_2, respectively, such that their intersection is the origin. Show that there exist points $A, BandC,$ whose permutation $A^{'}, B^{'}andC^{'}$, respectively, can be chosen such that (1)A is on $L_1, BonP_1$ but not on L_1andC not on P_1; (2)A' is on $L_2, B' onP_2$ but not on L_2andC' not on $P_2^{.}$

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Fill In The Blanks

1. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be vectors of legth , 3,4and 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}, \vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to$\vec{A}+\vec{B}$ then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is ________.

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2. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1)$ and $(0, 2, 1)$.

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3. The area of the triangle whose vertices are $A(1, -1, 2), B(2, 1 -1) C(3, -1, 2)$ is ....

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4. If $\vec{A}, \vec{B}, \vec{C}$ are non-coplanar vectors then $\dfrac{\vec{A}.\vec{B}\times\vec{C}}{\vec{C}\times\vec{A}.\vec{B}} + \dfrac{\vec{B}.\vec{A}\times\vec{C}}{\vec{C}.\vec{A}\times\vec{B}} =$

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5. If $\vec{A} = (1, 1, 1)$ and $\vec{C} = (0, 1, -1)$ are given vectors then find a vector $\vec{B}$ satisfying equations $\vec{A}\times\vec{B} = \vec{C}$ and $\vec{A}.\vec{B} = 3$

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6. Let $\vec{b} = 4\hat{i} + 3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. Find all vetors in te same plane having projection 1 and 2 along $\vec{b}$ and $\vec{c}$ respectively.

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7. The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are _______ and _________, respectively.

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8. A unit vector coplanar with $\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$ and perpendicular to $\vec{i} + \vec{j} + \vec{k}$ is ______

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9. A non vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\vec{i}, \vec{i} + \vec{j}$ and thepane determined by the vectors $\vec{i} - \vec{j}, \vec{i} + \vec{k}$ then angle between $\vec{a}$ and $\vec{i} - 2\vec{j} + 2\vec{k}$ is = (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

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10. If $\vec{b}$ and $\vec{c}$ are any two mutually perpendicular unit vectors and $\vec{a}$ is any vector, then $\left(\vec{a}.\vec{b}\right)\vec{b} + \left(\vec{a}.\vec{c}\right)\vec{c} + \frac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left|\vec{b}\times\vec{c}\right|^2}\left(\vec{b}\times\vec{c}\right) =$ (A) 0 (B) $\vec{a}(C)$ veca /2(D)2veca`

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11. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1,1 and 2 resectively. If $\vec{a}\times\left(\vec{a}\times\vec{c}\right) + \vec{b} = \vec{0}$ then the acute angel between $\vec{a}$ and $\vec{c}$ is

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12. A, B C and D are four points in a plane with position vectors, $\vec{a}, \vec{b}\vec{c}$ and $\vec{d}$ respectively, such that $\left(\vec{a}-\vec{d}\right).\left(\vec{b}-\vec{c}\right) = \left(\vec{b}-\vec{d}\right).\left(\vec{c}-\vec{a}\right) = 0$ then point D is the ______ of triangle ABC.

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13. If $\vec{A} = \lambda(\vec{u} \times \vec{v}) + \mu(\vec{v} \times \vec{w}) + v(\vec{w} \times \vec{u})$ and $[\vec{u}\vec{v}\vec{w}] = \frac{1}{5} then \lambda + \mu + v =$

(A) 5 (B) 10 (C) 15 (D) none of these

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14. If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle , then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is

____

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True And False

1. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be unit vectors such that $\vec{A}.\vec{B} = \vec{A}.\vec{C} = 0$ and the angle between $\vec{B}$ and $\vec{C}$ be$\pi/3$. Then $\vec{A} = \pm 2(\vec{B} \times \vec{C})$.

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2. If $\vec{x}.\vec{a} = 0\vec{x}.\vec{b} = 0$ and $\vec{x}.\vec{c} = 0$ for some non zero vector $\vec{x}$ then show that $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

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3. for any three vectors, $\vec{a}, \vec{b}$ and $\vec{c}$, $\left(\vec{a} - \vec{b}\right).\left(\vec{b} - \vec{c}\right) \times \left(\vec{c} - \vec{a}\right)$ =

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Exercise 2 1

1. Find $\left|\vec{a}\right|$ and $\left|\vec{b}\right|$, if $\left(\vec{a} + \vec{b}\right).\left(\vec{a} - \vec{b}\right) = 8$ and $\left|\vec{a}\right| = 8\left|\vec{b}\right|$

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2. Show that $\left|\vec{a}\right|\vec{b} + \left|\vec{b}\right|\vec{a}$ is a perpendicular to $\left|\vec{a}\right|\vec{b} - \left|\vec{b}\right|\vec{a}$, for any two non-zero vectors $\vec{a} and \vec{b}$.

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3. If the vectors A, B, C of a triangle ABC are $(1, 2, 3), (- 1, 0, 0), (0, 1, 2)$, respectively then find $\angle ABC$.

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4. If $|a| = 3, |b| = 4 and$ the angle between $a and b$ is 120^0 , then find the value of $|4a + 3b|$.

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5. If vectors $\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other, then find the locus of th point (x,y).

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6. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be pairwise mutually perpendicular vectors, such that $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$, the find the length of $\vec{a} + \vec{b} + \vec{c}$.

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7. If $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$,$|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$ find the angle between the vectors $\vec{a}$ and $\vec{b}$.

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8. If the angel between unit vectors $\vec{a} and \vec{b} 60^0$, then find the value of $|\vec{a} - \vec{b}|$.

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9. Let $\vec{u} = \hat{i} + \hat{j}, \vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$, $|\vec{w}.\hat{n}|$ is equal to (A) 0 (B) 1 (C) 2 (D) 3

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10. A,B,C and d are any four points prove that $\vec{AB}.\vec{CD} + \vec{BC}.\vec{AD} + \vec{CA}.\vec{BD} = 0$

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11. $P(1, 0, -1), Q(2, 0, -3), R(-1, 2, 0) and S(3, -2, -1)$, then find the projection length of $\vec{P}Qon\vec{R}S$.

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12. If the vectors $3\vec{p} + \vec{q}; 5p - 3\vec{q} and 2\vec{p} + \vec{q}; 3\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors, then find the angle between vectors $\vec{p} and \vec{q}$.

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13. Let $\vec{A}$ and $\vec{B}$ be two non-parallel unit vectors in a plane. If $\left(\alpha\vec{A} + \vec{B}\right)$ bisects the internal angle between $\vec{A}$ and $\vec{B}$, then find the value of α.

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14. Let $\vec{a}, \vec{b} and \vec{c}$ be unit vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{x}, \vec{a}.\vec{x} = 1, \vec{b}.\vec{x} = \frac{3}{2}, \left|\vec{x}\right| = 2$. Then find the angle between $\vec{c}$ and $\vec{x}$

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15. If $\vec{a}$ and $\vec{b}$ are unit vectors, then find the greatest value of $\left|\vec{a} + \vec{b}\right| + \left|\vec{a} - \vec{b}\right|$.

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16. Constant forces $P_1 = \hat{i} + \hat{j} + \hat{k}, P_2 = -\hat{i} + 2\hat{j} - \hat{k} and P_3 = -\hat{j} - \hat{k}$ act on a particle at a point $\dot{A}$ Determine the work done when particle is displaced from position $A\left(4\hat{i} - 3\hat{j} - 2\hat{k}\right)$ to $B\left(6\hat{i} + \hat{j} - 3\hat{k}\right)^{\cdot}$

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17. If $|\vec{a}| = 4, \left|\vec{a} - \vec{b}\right| = 6 \text{ and } \left|\vec{a} + \vec{b}\right| = 8$ then find $\left|\vec{b}\right|$

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18. If A, B, C, D are four distinct point in space such that $\overrightarrow{AB}$ is not perpendicular to $\overrightarrow{CD}$ and satisfies $\overrightarrow{AB}.\overrightarrow{CD} = k\left(\left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BC}\right|^2 - \left|\overrightarrow{AC}\right|^2 - \left|\overrightarrow{BD}\right|^2\right)$, then find the value of $\dot{k}$

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Exercise 2 2

1. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k}$ and $\vec{a} \times \vec{b} = \vec{0}$, then find (m, n)

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2. If $|\vec{a}| = 3, |\vec{b}| = 6$ and $|\vec{a} \times \vec{b}| = 9$ then find the value between $|\vec{a}|$ and $|\vec{b}|$.

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3. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq 0, where \vec{a}, \vec{b}, and \vec{c}$ are coplanar vectors, then for some scalar k prove that $\vec{a} + \vec{c} = k\vec{b}$

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4. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}, \vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{c} = \vec{i} + \vec{j} + \vec{k}$, then find the value of $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$

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5. If the vectors $\vec{c}, \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right-handed system, then find $\vec{c}$.

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6. Given that $\vec{a}.\vec{b} = \vec{a}.\vec{c}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a}$ is not a zero vector. Show that $\vec{b} = \vec{c}$.

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7. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\vec{a} \times \vec{b}$ and give a geometrical interpretation of it.

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8. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\left|\vec{z}\right| = \frac{2}{\sqrt{7}}$ such that $\vec{z} + \vec{z} \times \vec{x} = \vec{y}$ then find the angle θ between $\vec{x}$ and $\vec{z}$

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9. Prove that $\left(\vec{a}.\hat{i}\right)\left(\vec{a} \times \hat{i}\right) + \left(\vec{a}.\hat{j}\right)\left(\vec{a} \times \hat{j}\right) + \left(\vec{a}.\hat{k}\right)\left(\vec{a} \times \hat{k}\right) = \vec{0}$

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10. Let a,b,c be three non-zero vectors such that $a + b + c = 0$, then $\lambda b \times a + b \times c + c \times a = 0, where \lambda$ is

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11. A particle has an angular speed of 3 rad/s and the axis of rotation passes through the points $(1, 1, 2) and (1, 2, -2)$. Find the velocity of the particle at point $P(3, 6, 4)$.

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12. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = 0 = \vec{a}.\vec{c}$. It the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$ then find $\vec{a}$.

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13. if $\left(\vec{a} \times \vec{b}\right)^2 + \left(\vec{a}.\vec{b}\right)^2 = 36$ and $\left|\vec{a}\right| = 3$ the find the value of $\left|\vec{b}\right|$

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14. Given $\left|\vec{a}\right| = \left|\vec{b}\right| = 1$ and $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$ if $\vec{c}$ is a vector such that $\vec{c} - \vec{a} - 2\vec{b} = 3\left(\vec{a} \times \vec{b}\right)$ then find the value of $\vec{c}.\vec{b}$.

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15. Find the moment of $\vec{F}$ about point (2, -1, 3), where force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is acting on point (1, -1, 2).

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Exercise 2 3

1. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four non-coplanar unit vectors such that $\vec{d}$ makes equal angles with all the three vectors $\vec{a}, \vec{b}, \vec{c}$ then prove that $\left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right]$

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2. If $\vec{l}, \vec{m}, \vec{n}$ are three non coplanar vectors prove that $\left[\vec{\ }\right.$ vecm vecn]

(vecaxxvecb) =|(vec1.veca, vec1.vecb, vec1),(vecm.veca, vecm.vecb, vecm),

(vecn.veca, vecn.vecb, vecn)|`

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3. If the volume of a parallelepiped whose adjacent edges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ is 15, then find the value of α if $(\alpha > 0)^{\cdot}$

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4. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find the vector $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$.

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5. If $\vec{x}. Veca = 0, \vec{x}. Vecb = 0$ and $\vec{x}. \vec{c} = 0$ for some non-zero vector $\vec{x}$. Then prove that $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

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6. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find the vector $\vec{c}$ such that $\vec{a}. \vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$.

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7. If $\vec{a}, \vec{b},$ and $\vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b},$ then prove that $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$.

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8. If $\vec{a} = \vec{P} + \vec{q}, \vec{P} \times \vec{b} = \vec{0}$ and $\vec{q}. \vec{b} = 0$ then prove that $\dfrac{\vec{b} \times \left(\vec{a} \times \vec{b}\right)}{\vec{b}. \vec{b}} = \vec{q}$

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9. Prove that $\left(\vec{a}.\left(\vec{b}\times\hat{i}\right)\right)\hat{i}+\left(\vec{a}.\left(\vec{b}\times\hat{j}\right)\right)\hat{j}+\left(\vec{a}.\left(\vec{b}\times\hat{k}\right)\right)\hat{k}=\vec{a}\times\vec{b}$

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10. For any four vectors, $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ prove that $\vec{d}.\left(\vec{a}\times\left(\vec{b}\times\left(\vec{c}\times\vec{d}\right)\right)\right)=\left(\vec{b}.\vec{d}\right)\left[\vec{a}\ \vec{c}\ \vec{d}\right]$.

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11. If $\vec{a}$ and $\vec{b}$ be two non-collinear unit vector such that $\vec{a}\times\left(\vec{a}\times\vec{b}\right)=\frac{1}{2}\vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

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12. show that $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ if and only if $\vec{a}$ and $\vec{c}$ are collinear or $\left(\vec{a} \times \vec{c}\right) \times \vec{b} = \vec{0}$

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13. If $\vec{a}, \vec{b},$ and $\vec{c}$ be non-zero vectors such that no two are collinear or $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If θ is the acute angle between vectors $\vec{b}$ and $\vec{c}$, then find the value of $\sin\theta$.

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14. If $\vec{p}, \vec{q}, \vec{r}$ denote vector $\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}$, respectively, show that $\vec{a}$ is parallel to $\vec{q} \times \vec{r}$, $\vec{b}$ is parallel $\vec{r} \times \vec{p}$, $\vec{c}$ is parallel to $\vec{p} \times \vec{q}$.

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15. Let $\vec{a}, \vec{b}, and \vec{c}$ be non-coplanar vectors and let the equation $\vec{a}^{\prime}, \vec{b}^{\prime}, \vec{c}'$ are reciprocal system of vector $\vec{a}, \vec{b}, \vec{c},$ then prove that $\vec{a} \times \vec{a}^{\prime} + \vec{b} \times \vec{b}^{\prime} + \vec{c} \times \vec{c}'$ is a null vector.

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16. Given unit vectors $\hat{m}\hat{n}$ and $\hat{p}$ such that angle between $\hat{m}$ and $\hat{n} is \alpha$ and angle between $\hat{p}$ and $\hat{m} X \hat{n} is \alpha$if [n p m] = 1/4 find alpha

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17. $\vec{a}, \vec{b}, and \vec{c}$ are three unit vectors and every two are inclined to each other at an angel $\cos^{-1}(3/5)^{\cdot}$ If $\vec{a} \times \vec{b} = p\vec{a} + q\vec{b} + r\vec{c}, where p, q, r$ are scalars, then find the value of $q^{\cdot}$

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18. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ gve three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b} is \frac{\pi}{6}$, then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} p = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$$

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Single Correct Answer Type

1. The scalar $\vec{A}\left(\left(\vec{B} + \vec{C}\right) \times \left(\vec{A} + \vec{B} + \vec{C}\right)\right)$ equals

a.0 b. $\left[\vec{A}\vec{B}\vec{C}\right] + \left[\vec{B}\vec{C}\vec{A}\right]$ c. $\left[\vec{A}\vec{B}\vec{C}\right]$ d. none of these

A. 0

B. $\left[\vec{A}\vec{B}\vec{C}\right] + \left[\vec{B}\vec{C}\vec{A}\right]$

C. $\left[\vec{A}\vec{B}\vec{C}\right]$

D. none of these

Answer: a

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2. For non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $\left|\left(\vec{a} \times \vec{b}\right). \vec{c} = \left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$ holds if and only if

A. $\vec{a}. \vec{b} = 0, \vec{b}. \vec{c} = 0$

B. $\vec{b}. \vec{c} = 0, \vec{c}, \vec{a} = 0$

C. $\vec{c}. \vec{a} = 0, \vec{a}, \vec{b} = 0$

D. $\vec{a}. \vec{b} = \vec{b}. \vec{c} = \vec{c}. \vec{a} = 0$

Answer: d

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3. The volume of he parallelepiped whose sides are given by $\vec{O}A = 2i - 2j, \vec{O}B = i + j - kand\vec{O}C = 3i - k$ is a. $\frac{4}{13}$ b. 4 c. $\frac{2}{7}$ d. 2

A. $4/13$

B. 4

C. $2/7$

D. 2

Answer: d

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4. Let $\vec{a}, \vec{b}and\vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}and\vec{r}$ the vectors defined by the relation $\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}and\vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$. Then the value of the expression $\left(\vec{a} + \vec{b}\right) \cdot \vec{p} + \left(\vec{b} + \vec{c}\right) \cdot \vec{q} + \left(\vec{c} + \vec{a}\right) \cdot \vec{r}$ is 0 b. 1 c. 2 d. 3

A. 0

B. 1

C. 2

D. 3

Answer: d

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5. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k}, \vec{c} = \hat{k} - \hat{i}. If \hat{d}$ is a unit vector such that $\vec{a}. \hat{d} = 0 = \left[\vec{b}\vec{c}\vec{d}\right]$ then $\hat{d}$ equals

A. $\pm \dfrac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$

B. $\pm \dfrac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

C. $\pm \dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

D. $\pm \hat{k}$

Answer: a

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6. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \dfrac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is a. $3\pi/4$ b. $\pi/4$ c. $\pi/2$ d. π

A. $3\pi/4$

B. $\pi/4$

C. $\pi/2$

D. π

Answer: a

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7. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$ if $\left|\vec{u}\right| = 1, \left|\vec{v}\right| = 2$ and $\left|\vec{w}\right| = 3$ then $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$ is

A. 7

B. -25

C. 0

D. -7

Answer: b

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8. If $\vec{a}, \vec{b} and \vec{c}$ are three non coplanar vectors, then $(\vec{a}+\vec{b}+\vec{c})\left[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})\right]$ is :

A. 0

B. $\left[\vec{a}\vec{b}\vec{c}\right]$

C. $2\left[\vec{a}\vec{b}\vec{c}\right]$

D. $-\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: d

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9. Let $\vec{p}, \vec{q}, \vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation

$$\vec{p} \times \{\vec{x} - \vec{q}) \times \vec{p}\} + \vec{q} \times \{\vec{x} - \vec{r}) \times \vec{q}\} + \vec{r} \times \{\vec{x} - \vec{p}) \times \vec{r}\} = \vec{0},$$

then $\vec{x}$ is given by

A. (a) $\frac{1}{2}(\vec{p} + \vec{q} - 2\vec{r})$

B. (b) $\frac{1}{2}(\vec{p} + \vec{q} + \vec{r})$

C. (c) $\frac{1}{3}(\vec{p} + \vec{q} + \vec{r})$

D. (d) $\frac{1}{3}(2\vec{p} + \vec{q} - \vec{r})$

Answer: b

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10. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\vec{b} = \hat{i} + \hat{j}$ if c is a vector such that $\vec{a}.\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{i} s 30^{\circ}$, then $|(\vec{a} \times \vec{b})| \times \vec{c}|$ is equal to

A. $2/3$

B. $3/2$

C. 2

D. 3

Answer: b

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11. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ is a. $\frac{1}{\sqrt{2}}\left(-\hat{j} + \hat{k}\right)$ b. $\frac{1}{\sqrt{3}}\left(-\hat{i} - \hat{j} - \hat{k}\right)$ c. $\frac{1}{\sqrt{5}}\left(-\hat{k} - 2\hat{j}\right)$ d. $\frac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} - \hat{k}\right)$

A. $\frac{1}{\sqrt{2}}(-j+k)$

B. $\frac{1}{\sqrt{3}}(i-j-k)$

C. $\frac{1}{\sqrt{5}}(i-2j)$

D. $\frac{1}{\sqrt{3}}(i-j-k)$

Answer: a

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12. If the vectors $\vec{a}, \vec{b}, and \vec{c}$ form the sides $BC, CA and AB,$ respectively, of triangle $ABC, then$

A. $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$

B. $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

C. $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}$

D. $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

Answer: b

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13. Let vectors $\vec{a}, \vec{b}, \vec{c}, and \vec{d}$ be such that $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = 0.$ Let $P_1 and P_2$ be planes determined by the pair of vectors $\vec{a}, \vec{b}, and \vec{c}, \vec{d},$ respectively. Then the angle between $P_1 and P_2$ is 0 b. $\pi/4$ c. $\pi/3$ d. $\pi/2$

A. 0

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

Answer: a

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14. If $\vec{a}, \vec{b} and \vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a} - \vec{b} 2\vec{b} - \vec{c} 2\vec{c} - \vec{a}\right]$ is 0 b. 1 c. $-\sqrt{3}$ d. $\sqrt{3}$

A. 0

B. 1

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: a

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15. If $\hat{a}, \hat{b}, and \hat{c}$ are unit vectors, then $\left|\hat{a} - \hat{b}\right|^2 + \left|\hat{b} - \hat{c}\right|^2 + \left|\hat{c} - \hat{a}\right|^2$ does not exceed

A. 4

B. 9

C. 8

D. 6

Answer: b

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16. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is

A. 45°

B. 60°

C. $\cos^{-1}(1/3)$

D. $\cos^{-1}(2/7)$

Answer: b

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17. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k} and \vec{W} = \hat{i} + 3\dot{\hat{k}}$ If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is a.-1 b. $\sqrt{10} + \sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

A. -1

B. $\sqrt{10}+\sqrt{6}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: c

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18. Find the value of a so that the volume of the parallelepiped formed by vectors $\hat{i}+a\hat{j}+k, \hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum.

A. -3

B. 3

C. $1/\sqrt{3}$

D. $\sqrt{3}$

Answer: c

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19. If $\vec{a} = \left(\hat{i} + \hat{j} + \hat{k}\right), \vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k},$ then $\vec{b}$ is

A. $\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{i} - \hat{k}$

C. $\hat{i}$

D. $2\hat{i}$

Answer: c

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20. The unit vector which is orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k} and \hat{i} - \hat{j} + \hat{k}$ is $\dfrac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$ b. $\dfrac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$ c. $\dfrac{3\hat{j} - \hat{k}}{\sqrt{10}}$

d. $\dfrac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

A. $\dfrac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$

B. $\dfrac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$

C. $\dfrac{3\hat{j} - \hat{k}}{\sqrt{10}}$

D. $\dfrac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

Answer: c

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21. if $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero, non- coplanar vectors and

$$\vec{b}_1 = \vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}, \vec{b}_2 = \vec{b} + \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}, \vec{c}_1 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1, \vec{c}_2 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}}{|\vec{b}}$$

, then the set of orthogonal vectors is

A. $(\vec{a}, \vec{b}_1, \vec{c}_3)$

B. $(\vec{c}a, \vec{b}_1, \vec{c}_2)$

C. $(\vec{a}, \vec{b}_1, \vec{c}_1)$

D. $(\vec{a}, \vec{b}_2, \vec{c}_2)$

Answer: c

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22. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k} and \vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a} and \vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is given by a. $\hat{i} - 3\hat{j} + 3\hat{k}$ b. $-3\hat{i} - 3\hat{j} + 3\hat{k}$ c. $3\hat{i} - \hat{j} + 3\hat{k}$ d. $\hat{i} + 3\hat{j} - 3\hat{k}$

A. $4\hat{i} - \hat{j} + 4\hat{k}$

B. $3\hat{i} + \hat{j} - 3\hat{k}$

C. $2\hat{i} + \hat{j} - 2\hat{k}$

D. $4\hat{i} + \hat{j} - 4\hat{k}$

Answer: a

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23. Let two non-collinear unit vector $\hat{a}$ a n d $\hat{b}$ form an acute angle. A point P moves so that at any time t, the position vector $OP(whereO$ is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t When P$ is farthest from origin O, let M be the length of $OP and \hat{u}$ be the unit vector along OP Then (a) $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|} and M = \left(1 + \hat{a}\dot{\hat{b}}\right)^{1/2}$ (b) $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|} and M = \left(1 + \hat{a}\dot{\wedge}\right)^{1/2}$ (c) $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|} and M = \left(1 + 2\hat{a}\dot{\hat{b}}\right)^{1/2}$ (d) $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|} and M = \left(1 + 2\hat{a}\dot{\hat{b}}\right)^{1/2}$

A. , $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = \left(1 + \hat{a}.\hat{b}\right)^{1/2}$

B. , $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = \left(1 + \hat{a}.\hat{b}\right)^{1/2}$

C. $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = \left(1 + 2\hat{a}.\hat{b}\right)^{1/2}$

D. , $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = \left(1 + 2\hat{a}.\hat{b}\right)^{1/2}$

Answer: a

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24. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a} \times \vec{b}\right).\vec{c} \times \vec{d} = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$ then a) $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar b) $\vec{b}, \vec{c}, \vec{d}$ are non -coplanar c)$\vec{b}, \vec{d}$ are non parallel d) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

A. $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar

B. $\vec{b}, \vec{c}$ and $\vec{d}$ are non-coplanar

C. $\vec{b}$ and $\vec{d}$ are non- parallel

D. $\vec{a}$ and $\vec{d}$ are parallel and $\vec{b}$ and $\vec{c}$ are parallel

Answer: c

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25. Two adjacent sides of a parallelogram $ABCD$ are given by $\vec{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k} and \vec{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}^{.}$ The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes $AD^{'.}$

If AD' makes a right angle with the side AB, then the cosine of the angel α is given by $\frac{8}{9}$ b. $\frac{\sqrt{17}}{9}$ c. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: b

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26. Let P, Q, R and S be the points on the plane with position vectors $-2i - j, 4i, 3i + 3j and - 3i + 2j,$ respectively. The quadrilateral $PQRS$ must be (a) Parallelogram, which is neither a rhombus nor a rectangle (b) Square (c) Rectangle but not a square (d) Rhombus, but not a square

A. Parallelogram, which is neither a rhombus nor a rectangle

B. square

C. rectangle, but not a square

D. rhombus, but not a square.

Answer: a

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27. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vectors $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is given by

A. $\hat{i} - 3\hat{j} + 3\hat{k}$

B. $-3\hat{i} - 3\hat{j} + \hat{k}$

C. $3\hat{i} - \hat{j} + 3\hat{k}$

D. $\hat{i} + 3\hat{j} - 3\hat{k}$

Answer: c

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28. Let $\vec{P}R = 3\hat{i} + \hat{j} - 2\hat{k} and \vec{S}Q = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS, and \vec{P}T = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determine by the vectors $\vec{P}T$, $\vec{P}Q$ and $\vec{P}S$ is 5 b. 20 c. 10 d. 30

A. 5

B. 20

C. 10

D. 30

Answer: c

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Multiple Correct Answers Type

1. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non- zero vectors such that $\vec{c}$ is a unit vectors perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$ then

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

A. (a) 0

B. (b) 1

C. (c) $\frac{1}{4}\left(a_1^2 + a_2^2 + a_2^2\right)\left(b_1^2 + b_2^2 + b_2^2\right)$

D. (d) $\frac{3}{4}\left(a_1^2 + a_2^2 + a_2^2\right)\left(b_1^2 + b_2^2 + b_2^2\right)\left(c_1^2 + c_2^2 + c_2^2\right)$

Answer: c

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2. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0) and \vec{b} = (0, 1, 1)$ is a. one b. two c. three d. infinite

A. one

B. two

C. three

D. infinite

Answer: b

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3. Let $a = 2i - j + k$, $b = i + 2j - k$ and $c = i + j - 2k$ be three vectors. A vector r in the plane of b and c whose projection on a is of magnitude $\sqrt{\frac{2}{3}}$ is

A. $2\hat{i} + 3\hat{j}$ - $3\hat{k}$

B. $2\hat{i} + 3\hat{j} + 3\hat{k}$

C. $-2\hat{i} - \hat{j} + 5\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: a,c

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4. For three vectors $\vec{u}, \vec{v} and \vec{w}$ which of the following expressions is not equal to any of the remaining three ? $\vec{u} \dot{\vec{v}} \times \vec{w}$ b. $\left(\vec{v} \times \vec{w}\right) \dot{\vec{u}}$ c. $\vec{v} \dot{\vec{u}} \times \vec{w}$ d. $\left(\vec{u} \times \vec{v}\right) \dot{\vec{w}}$

A. (a) $\vec{u}. \left(\vec{v} \times \vec{w}\right)$

B. (b) $\left(\vec{v} \times \vec{w}\right). \vec{u}$

C. (c) $\vec{v}. \left(\vec{u} \times \vec{w}\right)$

D. (d) $\left(\vec{u} \times \vec{v}\right). \vec{w}$

Answer: c

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5. Which of the following expressions are meaningful? a. $\vec{u}.\left(\vec{v}\times\vec{w}\right)$ b. $\vec{u}.\vec{v}.\vec{w}$ c. $\left(\vec{u}\vec{v}\right).\vec{w}$ d. $\vec{u}\times\left(\vec{v}.\vec{w}\right)$

A. $\vec{u}.\left(\vec{v}\times\vec{w}\right)$

B. $\left(\vec{u}.\vec{v}\right).\vec{w}$

C. $\left(\vec{u}.\vec{v}\right)\vec{w}$

D. $\vec{u}\times\left(\vec{v}.Vecw\right)$

Answer: a,c

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6. If $\vec{a}$ and $\vec{b}$ are two non collinear vectors and $\vec{u}=\vec{a}-\left(\vec{a}.\vec{b}\right)\vec{b}$ and $\vec{v}=\vec{a}\times\vec{b}$ then $\left|\vec{v}\right|$ is

A. $\left|\vec{u}\right|$

B. $\left|\vec{u}\right|+\left|\vec{u}.Veca\right|$

C. $\left|\vec{u}\right| + \left|\vec{u}.\vec{b}\right|$

D. $\left|\vec{u}\right| + \vec{u}.\left(\vec{a} + \vec{b}\right)$

Answer: a,c

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7. $\vec{P} = \left(2\hat{i} - 2\hat{j} + \hat{k}\right)$, then find $\left|\vec{P}\right|$

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8. Let $\vec{A}$ be a vector parallel to the line of intersection of planes $P_1 and P_2$. Plane P_1 is parallel to vectors $2\hat{j} + 3\hat{k} and 4\hat{j} - 3k and P_2$ is parallel to $\hat{j} - \hat{k} and 3\hat{i} + 3\hat{j}$. Then the angle betweenvector $\vec{A}$ and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is $\pi/2$ b. $\pi/4$ c. $\pi/6$ d. $3\pi/4$

A. $\pi/2$

B. $\pi/4$

C. $\pi/6$

D. $3\pi/4$

Answer: b,d

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9. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to vector $\hat{i}+\hat{j}+\hat{k}$, is/are a. $\hat{j}-\hat{k}$ b. $-\hat{i}+\hat{j}$ c. $\hat{i}-\hat{j}$ d. $-\hat{j}+\hat{k}$

A. $\hat{j}-\hat{k}$

B. $-\hat{i}+\hat{j}$

C. $\hat{i}-\hat{j}$

D. $-\hat{j}+\hat{k}$

Answer: a,d

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10. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vector each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. if $vcea$ is a non - zero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

A. (a) $\vec{b} = (\vec{b}.\vec{z})(\vec{z} - \vec{x})$

B. (b) $\vec{a} = (\vec{a}.\vec{y})(\vec{y} - \vec{z})$

C. (c) $\vec{a}.\vec{b} = -(\vec{a}.\vec{y})(\vec{b}.\vec{z})$

D. (d) $\vec{a} = (\vec{a}.\vec{y})(\vec{z} - \vec{y})$

Answer: a,b,c

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11. Let ΔPQR be a triangle Let $\vec{a} = \overline{QR}, \vec{b} = \overline{RP}$ and $\vec{c} = \overline{PQ}$ if $|\vec{a}| = 12, |\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is (are) true ?

A. (a) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

B. (b) $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 30$

C. (c) $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$

D. (d) $\vec{a}.\vec{b} = -72$

Answer: a,c,d

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